Black-Litterman Model

An Alternative to the Markowitz Asset Allocation Model

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What is the Black-Litterman Model?

- The Black-Litterman Model is used to determine optimal asset allocation in a portfolio
- Black-Litterman Model takes the Markowitz Model one step further
  - Incorporates an investor's own views in determining asset allocations

Two Key Assumptions

- Asset returns are normally distributed
  - Different distributions could be used, but using normal is the simplest
- Variance of the prior and the conditional distributions about the true mean are known
  - Actual true mean returns are not known

Basic Idea

1. Find implied returns
2. Formulate investor views
3. Determine what the expected returns are
4. Find the asset allocation for the optimal portfolio

Implied vs. Historical Returns

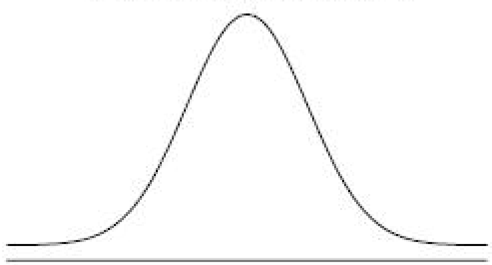
- Analogous to implied volatility
- CAPM is assumed to be the true price such that given market data, implied return can be calculated
- Implied return will not be the same as historical return

Implied Returns + Investor Views = Expected Returns

Risk Aversion Coefficient	Covariance Matrix	Market capitalization weights
• $\delta = (E(r) - r_f)/\sigma^2$	• Σ	• w_{mkt}

Views	Uncertainty of Views
• (P) • (Q)	• (Ω)

Prior Equilibrium Distribution

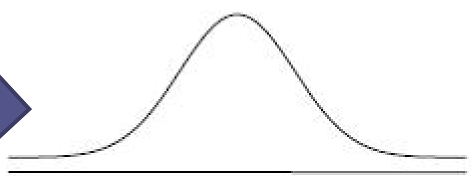


$$N \sim (\Pi, \tau\Sigma)$$

Implied return vector

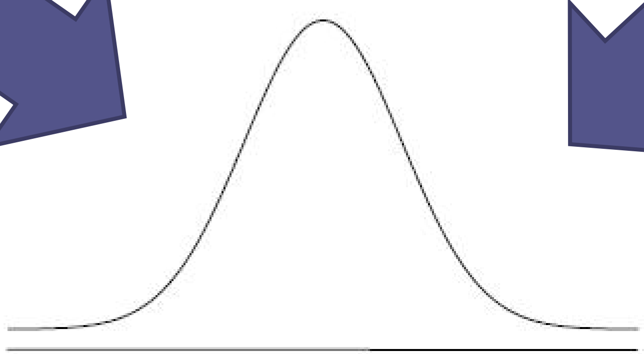
• $\Pi = \delta\, \Sigma\, w_{mkt}$

View Distribution



$$N \sim (Q, \Omega)$$

New Combined Return Distribution



$$N \sim \left(E[R], \left[(\tau\Sigma)^{-1} + \left(P'\Omega^{-1}P\right)\right]^{-1}\right)$$

Bayesian Theory

- Traditionally, personal views are used for the prior distribution
- Then observed data is used to generate a posterior distribution
- The Black-Litterman Model assumes implied returns as the prior distribution and personal views alter it

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Expected Returns

$$E(R) = [(\tau\,\Sigma)^{-1} + P^T\,\Omega P]^{-1}\,[(\tau\,\Sigma)^{-1}\,\Pi + P^T\,\Omega Q]$$

- Assuming there are N-assets in the portfolio, this formula computes E(R), the expected new return.
- τ = A scalar number indicating the uncertainty of the CAPM distribution (0.025-0.05)

Expected Returns: Inputs

$$\Pi = \delta \, \Sigma \, w_{\mathrm{mkt}}$$

- Π = The equilibrium risk premium over the risk free rate (Nx1 vector)
- $\delta = (E(r) - r_f)/\sigma^2$, risk aversion coefficient
- Σ = A covariance matrix of the assets (NxN matrix)

Expected Returns: Inputs

- P = A matrix with investors views; each row a specific view of the market and each entry of the row represents the portfolio weights of each assets (KxN matrix)
- Ω = A diagonal covariance matrix with entries of the uncertainty within each view (KxK matrix)
- Q = The expected returns of the portfolios from the views described in matrix P (Kx1 vector)

Breaking down the views

- Asset A has an absolute return of 5%
- Asset B will outperform Asset C by 1%
- Omega is the covariance matrix

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \qquad Q = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.5 \end{bmatrix}$$

From expected returns to weights

$E(R) = [(\tau\,\Sigma)^{-1} + P^T\,\Omega P]^{-1}\,[(\tau\,\Sigma)^{-1}\,\Pi + P^T\,\Omega Q]$

- Expected returns

$M = [(\tau\,\Sigma)^{-1} + P^T\,\Omega P]^{-1}$

- Uncertainty of returns

$\Sigma_p = \Sigma + M$

- New covariance matrix

$w = (\delta\,\Sigma_p\,)^{-1}\,\Pi$

- Weights

Example 1

- Using Black-Litterman model to determine asset allocation of 12 sectors
  - View: Energy Sector will outperform Manufacturing by 10% with a variance of .025^2
    - 67% of the time, Energy will outperform Manufacturing by 7.5 to 12.5%

Complications

- Assets by sectors
  - We did not observe major differences between BL asset allocation given a view and market equilibrium weights
  - Inconsistent model was difficult to analyze
    - There should have been an increase in weight of Energy and decrease in Manufacturing

Example 2
Model in Practice

- Example illustrated in Goldman Sachs paper
- Determine weights for countries
  - View: Germany will outperform the rest of Europe by 5%

Statistical Analysis

Country Metrics

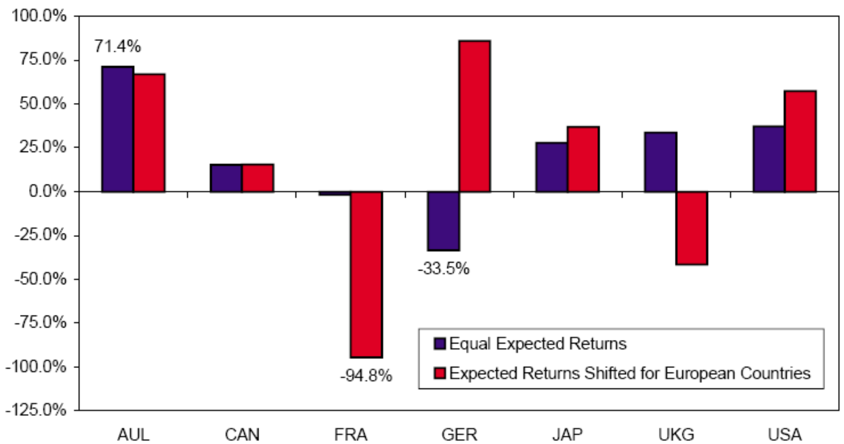
Country	Equity Index Volatility (%)	Equilibrium Portfolio Weight (%)	Equilibrium Expected Returns (%)
Australia	16.0	1.6	3.9
Canada	20.3	2.2	6.9
France	24.8	5.2	8.4
Germany	27.1	5.5	9.0
Japan	21.0	11.6	4.3
UK	20.0	12.4	6.8
USA	18.7	61.5	7.6

Covariance Matrix

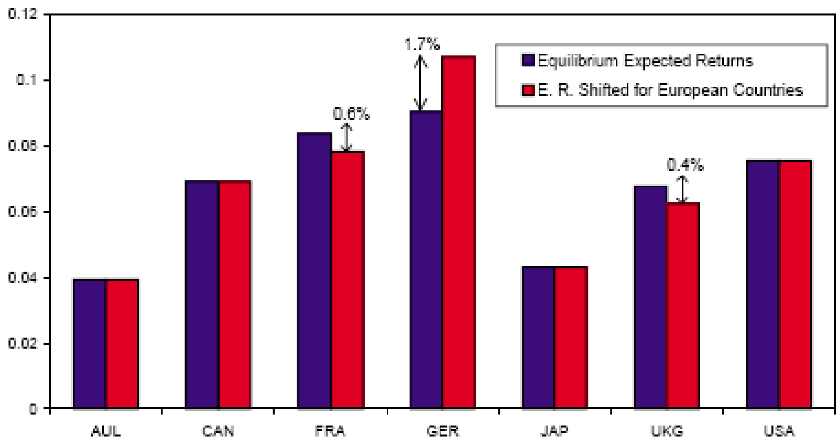
	AUS	CAN	FRA	GER	JAP	UK	USA
AUS	0.0256	0.01585	0.018967	0.02233	0.01475	0.016384	0.014691
CAN	0.01585	0.041209	0.033428	0.036034	0.027923	0.024685	0.024751
FRA	0.018967	0.033428	0.061504	0.057866	0.018488	0.038837	0.030979
GER	0.02233	0.036034	0.057866	0.073441	0.020146	0.042113	0.033092
JAP	0.01475	0.013215	0.018488	0.020146	0.0441	0.01701	0.012017
UK	0.016384	0.024685	0.038837	0.042113	0.01701	0.04	0.024385
USA	0.014691	0.029572	0.030979	0.033092	0.012017	0.024385	0.034969

Traditional Markowitz Model

Portfolio Asset Allocation



Expected Returns



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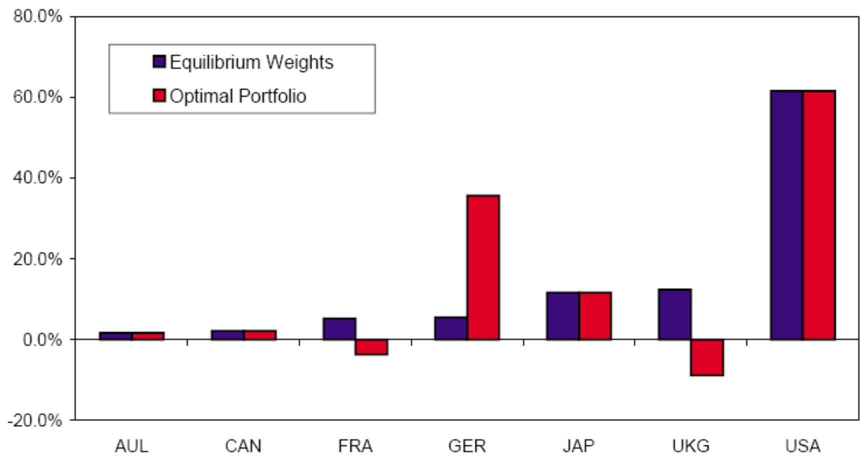
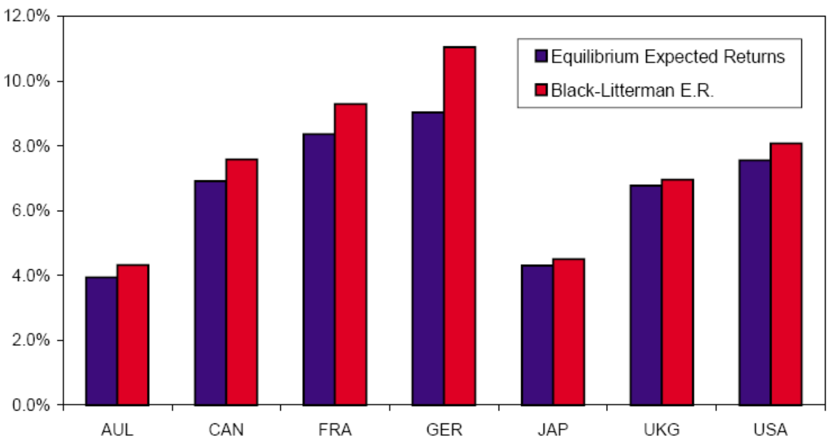
Portfolio Asset Allocation



Expected Returns



Advantages and Disadvantages

- Advantages
  - Investor's can insert their view
  - Control over the confidence level of views
  - More intuitive interpretation, less extreme shifts in portfolio weights

- Disadvantages
  - Black-Litterman model does not give the best possible portfolio, merely the best portfolio given the views stated
  - As with any model, sensitive to assumptions
    - Model assumes that views are independent of each other

Conclusion

- Further Developments

Author(s)	τ	View Uncertainty	Posterior Variance
He and Litterman	Close to 0	diag($\tau P \Sigma P$)	Updated
Idzorek	Close to 0	Specified as %	Use prior variance
Satchell and Scowcroft	Usually 1	N/A	Use prior variance

Table obtained from http://blacklitterman.org/methods.html

Bibliography

- Black, F. and Litterman, R. (1991). "Global Asset Allocation with Equities, Bonds, and Currencies." *Fixed Income Research*, Goldman, Sachs & Company, October.
- He, G. and Litterman, R. (1999). "The Intuition Behind Black-Litterman Model Portfolios." *Investment Management Research*, Goldman, Sachs & Company, December.
- Black, Fischer and Robert Litterman, "Asset Allocation: Combining Investor Views With Market Equilibrium." Goldman, Sachs & Co., Fixed Income Research, September 1990.
- Idzorek, Thomas M. "A Step-by-Step Guide to the Black-Litterman Model." Zehyr Associates, Inc. July, 2004.
- Satchell, S. and Scowcroft, A. (2000). "A Demystification of the Black-Litterman Model: Managing Quantitative and Traditional Construction." Journal of Asset Management, September, 138-150.