1.3.5.11. Measures of Skewness and Kurtosis

Skewness and Kurtosis

A fundamental task in many statistical analyses is to characterize the *location* and *variability* of a data set. A further characterization of the data includes skewness and kurtosis.

Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point.

Kurtosis is a measure of whether the data are heavy-tailed or light-tailed relative to a normal distribution. That is, data sets with high kurtosis tend to have heavy tails, or outliers. Data sets with low kurtosis tend to have light tails, or lack of outliers. A uniform distribution would be the extreme case.

The histogram is an effective graphical technique for showing both the skewness and kurtosis of data set.

Definition of Skewness

For univariate data $Y_1, Y_2, ..., Y_N$, the formula for skewness is:

$$g_1 = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^3/N}{s^3}$$

where $\bar{Y}$ is the mean, s is the standard deviation, and N is the number of data points. Note that in computing the skewness, the s is computed with N in the denominator rather than N - 1.

The above formula for skewness is referred to as the Fisher-Pearson coefficient of skewness. Many software programs actually compute the adjusted Fisher-Pearson coefficient of skewness

$$G_1 = \frac{\sqrt{N(N-1)}}{N-2}\frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^3/N}{s^3}$$

This is an adjustment for sample size. The adjustment approaches 1 as N gets large. For reference, the adjustment factor is 1.49 for N = 5, 1.19 for N = 10, 1.08 for N = 20, 1.05 for N = 30, and 1.02 for N = 100.

The skewness for a normal distribution is zero, and any symmetric data should have a skewness near zero. Negative values for the skewness indicate data that are skewed left and positive values for the skewness indicate data that are skewed right. By skewed left, we mean that the left tail is long relative to the right tail. Similarly, skewed right means that the right tail is long relative to the left tail. If the data are multi-modal, then this may affect the sign of the skewness.

Some measurements have a lower bound and are skewed right. For example, in reliability studies, failure times cannot be negative.

It should be noted that there are alternative definitions of skewness in the literature. For example, the Galton skewness (also known as Bowley's skewness) is defined as

$$\text{Galton skewness} = \frac{Q_1 + Q_3 - 2Q_2}{Q_3 - Q_1}$$

where Q_1 is the lower quartile, Q_3 is the upper quartile, and Q_2 is the median.

The Pearson 2 skewness coefficient is defined as

$$S_{k_2} = 3\frac{(\bar{Y} - \tilde{Y})}{s}$$

where $\tilde{Y}$ is the sample median.

There are many other definitions for skewness that will not be discussed here.

Definition of Kurtosis

For univariate data $Y_1, Y_2, ..., Y_N$, the formula for kurtosis is:

$$\text{kurtosis} = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^4/N}{s^4}$$

where $\bar{Y}$ is the mean, s is the standard deviation, and N is the number of data points. Note that in computing the kurtosis, the standard deviation is computed using N in the denominator rather than N - 1.

Alternative Definition of Kurtosis

The kurtosis for a standard normal distribution is three. For this reason, some sources use the following definition of kurtosis (often referred to as "excess kurtosis"):

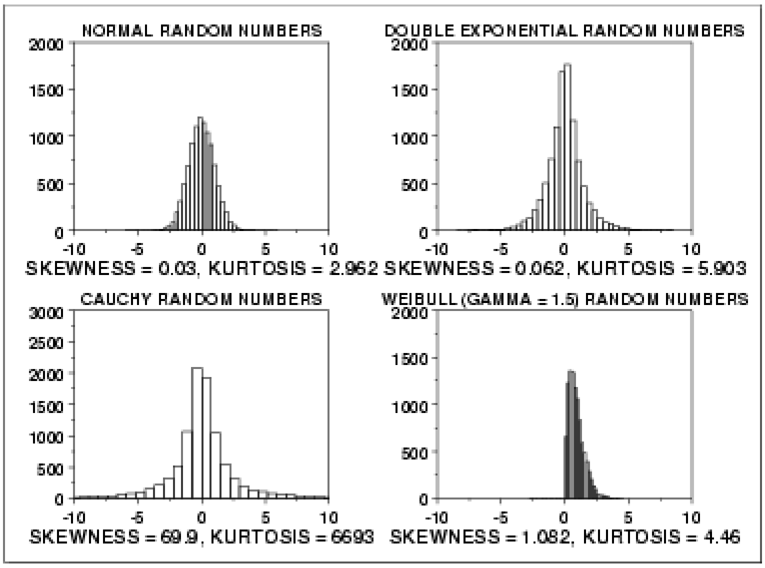
$$\text{kurtosis} = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^4/N}{s^4} - 3$$

This definition is used so that the standard normal distribution has a kurtosis of zero. In addition, with the second definition positive kurtosis indicates a "heavy-tailed" distribution and negative kurtosis indicates a "light tailed" distribution.

Which definition of kurtosis is used is a matter of convention (this handbook uses the original definition). When using software to compute the sample kurtosis, you need to be aware of which convention is being followed. Many sources use the term kurtosis when they are actually computing "excess kurtosis", so it may not always be clear.

Examples

The following example shows histograms for 10,000 random numbers generated from a normal, a double exponential, a Cauchy, and a Weibull distribution.



Normal Distribution

The first histogram is a sample from a normal distribution. The normal distribution is a symmetric distribution with well-behaved tails. This is indicated by the skewness of 0.03. The kurtosis of 2.96 is near the expected value of 3. The histogram verifies the symmetry.

Double Exponential Distribution

The second histogram is a sample from a double exponential distribution. The double exponential is a symmetric distribution. Compared to the normal, it has a stronger peak, more rapid decay, and heavier tails. That is, we would expect a skewness near zero and a kurtosis higher than 3. The skewness is 0.06 and the kurtosis is 5.9.

Cauchy Distribution

The third histogram is a sample from a Cauchy distribution.

For better visual comparison with the other data sets, we restricted the histogram of the Cauchy distribution to values between -10 and 10. The full data set for the Cauchy data in fact has a minimum of approximately -29,000 and a maximum of approximately 89,000.

The Cauchy distribution is a symmetric distribution with heavy tails and a single peak at the center of the distribution. Since it is symmetric, we would expect a skewness near zero. Due to the heavier tails, we might expect the kurtosis to be larger than for a normal distribution. In fact the skewness is 69.99 and the kurtosis is 6,693. These extremely high values can be explained by the heavy tails. Just as the mean and standard deviation can be distorted by extreme values in the tails, so too can the skewness and kurtosis measures.

Weibull Distribution

The fourth histogram is a sample from a Weibull distribution with shape parameter 1.5. The Weibull distribution is a skewed distribution with the amount of skewness depending on the value of the shape parameter. The degree of decay as we move away from the center also depends on the value of the shape parameter. For this data set, the skewness is 1.08

and the kurtosis is 4.46, which indicates moderate skewness and kurtosis.

Dealing with Skewness and Kurtosis

Many classical statistical tests and intervals depend on normality assumptions. Significant skewness and kurtosis clearly indicate that data are not normal. If a data set exhibits significant skewness or kurtosis (as indicated by a histogram or the numerical measures), what can we do about it?

One approach is to apply some type of transformation to try to make the data normal, or more nearly normal. The Box-Cox transformation is a useful technique for trying to normalize a data set. In particular, taking the log or square root of a data set is often useful for data that exhibit moderate right skewness.

Another approach is to use techniques based on distributions other than the normal. For example, in reliability studies, the exponential, Weibull, and lognormal distributions are typically used as a basis for modeling rather than using the normal distribution. The probability plot correlation coefficient plot and the probability plot are useful tools for determining a good distributional model for the data.

Software

The skewness and kurtosis coefficients are available in most general purpose statistical software programs.

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