11/29/16

ASSIGNMENT 4

Ans-2 Given $R(p,q) = \frac{1 + p_s p + q_s q}{(\sqrt{1+p^2+q^2})(\sqrt{1+p_s^2+q_s^2})}$ — ①

where the light source lies in the direction $(-p_s, -q_s, 1)^T$

Now, we can express $R(p,q)$ as a dot product of two unit vectors $\hat{a}$ & $\hat{b}$

Hence, if $R(p,q) = \hat{a} \cdot \hat{b}$ — ①

then we have

$$\hat{a} = \frac{[p_s, q_s, 1]^T}{\sqrt{1+p_s^2+q_s^2}} \quad \& \quad \hat{b} = \frac{[p, q, 1]^T}{\sqrt{1+p^2+q^2}}$$

Now, from eqn ① we have

$R(p,q) = |\hat{a}||\hat{b}| \cos\theta$ where θ is the angle between the two unit vectors.

Now, we know that since magnitude of unit vectors is 1 we have the reflectance map $R(p,q)$ depending only on the cosine of angle b/w the two vectors, which is maximum only when $\theta = 0°$. This makes the maxima unique and global since the solution is unique $(\theta = 0°)$.

This means that both vectors $\hat{a}$ & $\hat{b}$ are aligned with each other (coincident) which is possible only

when : $p = p_s$ & $q = q_s$
This means $p = p_s$ & $q = q_s$ maximizes the given expression $R(p,q)$.

b) For what values of p & q, is $R(p,q) = 0$.
This will happen when both unit vectors $\hat{a}$ & $\hat{b}$ are perpendicular to each other i.e $\theta = 90°$. So all those values for which the unit vectors $\hat{a}$ & $\hat{b}$ are perpendicular.
A given instance would be
when $p = \frac{-1}{2p_s}$ & $q = \frac{-1}{2q_s}$
When above values for p & q are substituted in eq^n ① we get $R(p,q) = 0$

$$\because R(p,q) = \frac{\cancel{1} + \frac{-1}{\cancel{2p_s}} \times \cancel{p_s} + \frac{-1}{\cancel{2q_s}} \times \cancel{q_s}}{(\sqrt{1+p^2+q^2})(\sqrt{1+p_s^2+q_s^2})} = \underline{\underline{0}}$$

Hence,

$$\hat{a} = \frac{[p_s, q_s, 1]^T}{\sqrt{1+p_s^2+q_s^2}} \quad \& \quad \hat{b} = \frac{[-1/2p_s, -1/2q_s, 1]^T}{\left(\sqrt{1+\frac{1}{4p_s^2}+\frac{1}{4q_s^2}}\right)}$$

Also, the corresponding contour in the gradient space (u,v) is a <u>straight line</u>.
(cited by fig 10-13 by BKP Horn)

Ans-3 Given surface shape near the origin is $z = z_0 + \frac{1}{2}(x \;\; y)\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

$$\Rightarrow z = z_0 + \frac{1}{2}\begin{pmatrix} ax + by \\ bx + cy \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow z = z_0 + \frac{1}{2}\left[ax^2 + 2bxy + cy^2\right] \quad \text{— (1)}$$

& Reflectance Map $R(p, q) = \frac{1}{2}(p^2 + q^2)$ — (8)

To Show $E(x, y) = \frac{1}{2}(x \;\; y)\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ — (2)

$$\Rightarrow E(x, y) = \frac{1}{2}\begin{pmatrix} ax + by \\ bx + cy \end{pmatrix}^T \begin{pmatrix} ax + by \\ bx + cy \end{pmatrix}$$

$$E(x, y) = \frac{1}{2}\left[(ax + by)^2 + (bx + cy)^2\right] \quad \text{— (5)}$$

$$= \frac{1}{2}\left[a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + c^2y^2 + 2bcxy\right]$$

$$E(x, y) = \frac{1}{2}\left[(a^2 + b^2)x^2 + (b^2 + c^2)y^2 + 2bxy(a + c)\right]$$ — (3)

Now we know that p is partial derivative of z wrt x & q is partial derivative of z wrt y

i.e $p = \frac{\delta z}{\delta x}$ & $q = \frac{\delta z}{\delta y}$

$\Rightarrow p = \frac{\delta z}{\delta x} = \frac{\delta}{\delta x}\left(z_0 + \frac{1}{2}[ax^2 + cy^2 + 2bxy]\right)$ from (1)

$p = \frac{1}{2}[2ax + 2by] = ax + by$ — (4)

Now, $q = \frac{\delta z}{\delta y} = \frac{\delta}{\delta y}\left(z_0 + \frac{1}{2}[ax^2 + cy^2 + 2bxy]\right)$

$q = \frac{1}{2}[2cy + 2bx] = bx + cy$ — (6)

Now we know $E(x,y) = R(p,q)$ (7) which is the image irradiance eq^n cited by BKP Horn page 244

From (7) & (8) we have,

$E(x,y) = \frac{1}{2}(p^2 + q^2)$

Now substitute values of p & q from (4) & (6) we have, $E(x,y) = \frac{1}{2}[(ax+by)^2 + (bx+cy)^2]$ (9)

$\Rightarrow E(x,y) = \frac{1}{2}\begin{pmatrix} ax+by \\ bx+cy \end{pmatrix}^T \begin{pmatrix} ax+by \\ bx+cy \end{pmatrix}$

$\Rightarrow E(x,y) = \frac{1}{2}(x \ y)\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

$\Rightarrow E(x,y) = \frac{1}{2}(x \ y)\begin{pmatrix} a & b \\ b & c \end{pmatrix}^2\begin{pmatrix} x \\ y \end{pmatrix}$

proved

Scanned by CamScanner

The number of solutions locally for the surface shape are :- $\boxed{4}$ cited from BKP Horn Page 256 since even though it generates three 2^{nd} order polynomials in 3 unknowns which can have upto 8 soln's. The three eqn's relevant to this case are of a special form.
The soln's are related with each other by eqn's:

$$E_{xx} = a^2 + b^2 \quad E_{xy} = (a+c)b \quad \& \quad E_{yy} = b^2 + c^2$$

This can be found out by differentiating eqn 9 wrt x we get & wrt y we get, as follows:

$$E_x = (a^2+b^2)x + (a+c)by$$

$$\& \quad E_y = (a+c)bx + (b^2+c^2)y$$

Ans-4 b) We wish to minimize the sum of the brightness error and the deviatn from integrability,

$$\iint_I \left((E(x,y) - R(p,q))^2 + \lambda (p_y - q_x)^2 \right) dx\,dy \quad -(1)$$

by suitable choice of $p(x,y)$ & $q(x,y)$

The appropriate Euler equations are stated as follows:-

For function $p(x,y)$

$$\frac{\delta F}{\delta p} - \frac{\delta}{\delta x}(F_{p_x}) - \frac{\delta}{\delta y}(F_{p_y}) = 0 \quad \text{— (A)}$$

For function $q(x,y)$

$$\frac{\delta F}{\delta q} - \frac{\delta}{\delta x}(F_{q_x}) - \frac{\delta}{\delta y}(F_{q_y}) = 0 \quad \text{—— (B)}$$

Here, $F = \left((E(x,y) - R(p,q))^2 + \lambda (p_y - q_x)^2 \right)$ — (2) from (1)

Hence substituting value of (2) in E-L eq^n (A) we get

$$(-)2(E(x,y) - R(p,q)) \times R_p \,(-)\, 2\frac{\delta}{\delta y}(\lambda(p_y - q_x)) = 0$$

$$\Rightarrow [E(x,y) - R(p,q)] R_p + \lambda(p_{yy} - q_{xy}) = 0$$

$$\Rightarrow \underline{[E(x,y) - R(p,q)] R_p} = (-)\lambda(p_{yy} - q_{xy}) = \lambda(q_{xy} - p_{yy})$$

Hence substituting value of (2) in E-L eq^n (B) we get

$$(-)2[E(x,y) - R(p,q)] \times R_q + \lambda \frac{\delta}{\delta x}[2(p_y - q_x)] = 0$$

$$\Rightarrow \underline{[E(x,y) - R(p,q)] R_q} = (-)\lambda(q_{xx} - p_{xy}) = \lambda(p_{xy} - q_{xx})$$

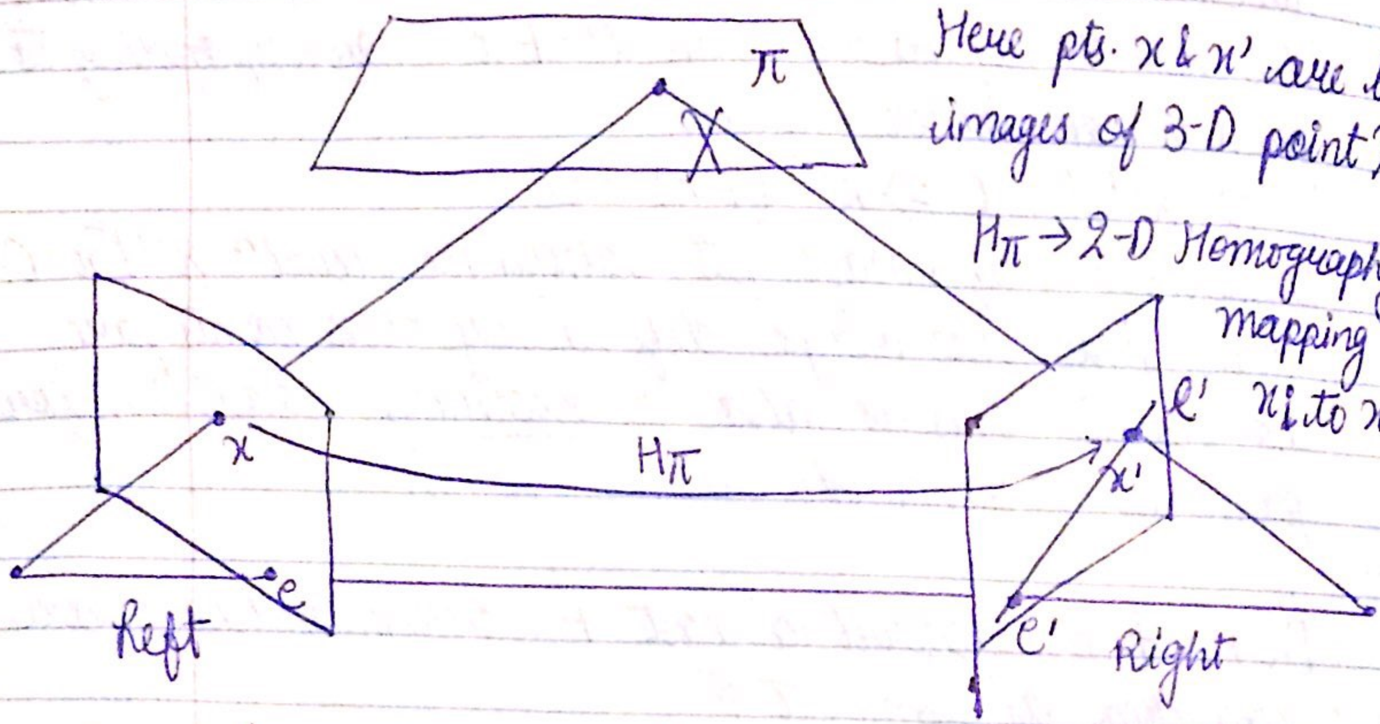
visit: http://courses.csail.mit.edu/6.801/2009f/errata.pdf

Error in the question!!

a) Since the iterative shape from shading in problem 11-11 is subjected to an integrability constraint $\boxed{p_y = q_x}$ in addition to the minimi-zation of the brightness error. This makes the parameter λ as a lagrangian function $\lambda(x,y)$.

Whereas λ in the problem 11-12 is simply a constant which is introduced while minimizing $e_s + \lambda e_i$ where $e_i \rightarrow$ brightness error ; $e_s \Rightarrow$ penalizes rapid changes in f & g which is equivalent to p & q in this problem.
Here in problem 11-12 λ is a parameter that weights the errors in the image irradiance eq^n re-lative to departure from smoothness.
This parameter is large if brightness measurements are accurate.
It is small when the brightness measurements are noisy.

Ans-1) If the coordinate points taken into consideration in the left & right camera are x & x' respectively corresponding to each other in a stereo camera system and we are given a fundamental matrix F. Then the equation of the epipolar line in the right camera for any point x in the left camera in terms of the fundamental matrix F is given by:-



Here pts. x & x' are both images of 3-D point X

$H_\pi \rightarrow$ 2-D Homography mapping each x_i to x_i'

A point x in the left image is transferred via the plane π to a matching point x' in the right image. The epipolar line through x' is obtained by joining x' to the epipole e'.
Hence we write $x' = H_\pi x$

& $l' = [e']_\times H_\pi x = Fx$ taken by right camera.
$l' \rightarrow$ epipolar line for the right image, where we have the fundamental matrix $F = [e']_\times H_\pi$

Now, the fundamental matrix satisfies the condition is that for any pair of corresponding points x & x' in the left & right images is:-

$$x'^T F x = 0$$

because if only the points x & x' correspond can x' lie on the epipolar line $l' = Fx$ corresponding to point x. In other words,

$$\Rightarrow x'^T l' = 0 \Rightarrow x'^T (Fx) = 0$$

Conversely, if image pts satisfy the relat^n $x'^T F x = 0$ then only the rays defined by these points are co-planar. This is also a <u>necessary</u> condition for points to correspond.

DOF = 7 In general atleast 7 correspondences are required to compute F.
It has 8 independent ratios (there are 9 elements and the common scaling is not significant) However F also satisfies the constraint $\det F = 0$ which removes one degree of freedom.

Cited from Wikipedia

Ans-5 The Lucas-Kanade method assumes that the displacement of the image ~~consists~~ contents between two nearby instants is approximately constant within a neighbourhood of the point p under consideration. Hence, the optical flow equation can be assumed to hold for all pixels within a window centered at p. Namely the local image flow (velocity) vector (V_x, V_y) must satisfy

$$I_x(q_1)V_x + I_y(q_1)V_y = -I_t(q_1)$$
$$I_x(q_2)V_x + I_y(q_2)V_y = -I_t(q_2)$$
$$\vdots$$
$$I_x(q_n)V_x + I_y(q_n)V_y = -I_t(q_n)$$

where

$q_1, q_2, \ldots q_n \rightarrow$ pixels inside the window

$I_x(q_i) \rightarrow$ Partial derivative of image I eval. at pt. q_i wrt x

$I_y(q_i) \rightarrow$ —"— evaluated at pt. q_i wrt y

$I_t(q_i) \rightarrow$ —"— t and at the current time.

Hence, in Matrix form $Av = b$

where,

$$A = \begin{bmatrix} I_x(q_1) & I_y(q_1) \\ I_x(q_2) & I_y(q_2) \\ \vdots & \\ I_x(q_n) & I_y(q_n) \end{bmatrix}, \quad v = \begin{bmatrix} V_x \\ V_y \end{bmatrix} \; \& \; b = \begin{bmatrix} -I_t(q_1) \\ -I_t(q_2) \\ \vdots \\ -I_t(q_n) \end{bmatrix}$$

Here the method obtains a compromise solution by the least squares principle. Namely it solves the 2x2 system

$$A^T A v = A^T b \quad \text{— (1)}$$

$$\Rightarrow v = (A^T A)^{-1} A^T b \quad \text{— (2)}$$

But the above method gives uniform weightage to all n pixels q_i in the window. (as done in class) Hence it is better to give more weight to pixels closer to central pixel p. For that one uses weighted versions of (1) & (2) i.e

$$A^T W A v = A^T W b \quad \text{— (3)}$$

$$\Rightarrow v = (A^T W A)^{-1} A^T W b \quad \text{— (4)}$$

where W is an $n \times n$ diagonal matrix containg weights $W_{ii} = w_i$ to be assigned to the eq^n of pixel q_i. It computes from (4)

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \sum_i w_i I_x(q_i)^2 & \sum_i w_i I_x(q_i) I_y(q_i) \\ \sum_i w_i I_x(q_i) I_y(q_i) & \sum_i w_i I_y(q_i)^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum_i w_i I_x(q_i) I_t(q_i) \\ -\sum_i w_i I_y(q_i) I_t(q_i) \end{bmatrix}$$

Here the weight w_i is usually set to a Gaussian function of ~~distat~~ distance b/w q_i & p