**CAP5416: Assignment #1, Due Date: 09/12/2016**

Make sure that your writing is legible, or else, please type your answers using your favorite text formatter. Your solutions are to be submitted via CANVAS on the due date and no later. No late assignments (please read the policies on the class website).

1. The following is an example of an affine map

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

   Show that this affine map is a composition of a shear followed by a rotation (by an angle), then by a magnification and then a translation. You are required to explicitly show the numerical values in the transformation matrices. Develop MATLAB code to apply each of these transformations, and then their composition to, (i) a square and show the resulting figure (draw the input and the output), (ii) a picture of your face taken using your cellphone camera. Show the output image after the application of each of the transformations and the composition. PLease include the MATLAB code in your submission. *Caution: You will have to use some kind of an interpolation technique, e.g., bilinear interpolation, to get a meaningful image when applying transformations to an image, else, your output image will contain holes. Do not submit results without interpolation.*

2. Show that a line at infinity $\mathbf{l}_\infty$ remains a line at infinity under the projectivity $H$ if and only if $H$ is an affinity. Note that you have to prove both sides of the implication.

3. Rotation in 2D about the origin can be achieved via multiplication of any 2D point $(x, y)^t$ by the matrix $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$. An equivalent way to achieve this is via complex number representation of 2D points and simply using the operation of multiplication over complex numbers. For this problem, you will prove that planar rotation can be achieved via multiplication over the complex representation of the 2D points. In this representation, every 2D point $(x, y)^t$ is represented by $(x + iy)$. Next, show that a rotation by $\theta_1$ followed by a rotation by $\theta_2$ can be accomplished simply via multiplication of the corresponding complex representation of these rotations. Note that a rotation by $\theta$ is represented in complex numbers by $(cos\theta + isin\theta)$. Thus, derive the formulas for $cos(\theta_1 + \theta_2)$ and $sin(\theta_1 + \theta_2)$ , which you should be familiar with from high school trigonometry.

4. Given a projective transformation that takes points $\mathbf{x} \rightarrow \mathbf{x}'$ via $\mathbf{x}' = H\mathbf{x}$, prove that $Cross(\mathbf{x_1}', \mathbf{x_2}', \mathbf{x_3}', \mathbf{x_4}') = Cross(\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}, \mathbf{x_4})$ i.e., the cross ratio is a projective invariant.

5. Prove that an image line $\mathbf{l}$ defines a plane through the camera center with normal direction $\mathbf{n} = K^t\mathbf{l}$ measured in the camera's Euclidean coordinate frame.

6. In DLT, if $\|A\mathbf{h}\|$ is minimized subject to the constraint $h_0 = H_{33} = 1$, then the result is invariant to scaling but not translation of coordinates.