

CAP5416: Assignment #2, Due Date: Oct. 3, 2016

Make sure that your writing is legible, or else, type your answers using your favorite text formatter.

1. Show that if $f(x, y)$ is separable, its Fourier transform is also separable.
2. Using the scaling and shifting theorems, write down the 2D Fourier transform of (i) $rect(x/3, y/2)$, (ii) $rect(x - 4, y - 5)$, (iii) $sinc(x - 5, 2y - 7)$, (iv) $3rect(x - 8, 3y)$, and (v) $exp\{j16\pi x\}sinc(x, y/3)$.
3. If the modulation transfer function of an optical telescope is $H(u, v) = P(u, v) * P(u, v)$, where $P(u, v)$ is given by

$$P(u, v) = \begin{cases} 1 & \text{for } |u| \leq 1/2 \text{ and } |v| \leq 1/2. \\ 0 & \text{Otherwise.} \end{cases}$$

Find $H(u, v)$ and the corresponding point spread function $h(x, y)$.

4. Compute the following convolutions and write the solution in matrix form.

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix} * \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

5. Prove that the squared gradient is a rotationally symmetric operator. You can show this by proving that the squared gradient of an image function and a rotated image function are identical.
6. Problem 8.3 (a) and (b) from the text book (BKP Horn).