CAP6657: Assignment #3, Due Date:Oct 19th 2016

Make sure that you writing is legible, or else, type your answers using your favorite text formatter.

1. In template matching, we define the mean-squared error (MSE) at any position (m, n) in a picture as:

$$\begin{aligned} MSE(m,n) &= \sum_i \sum_j (f(i,j) - g(i,j))^2 = \sum_i \sum_j [f^2(i,j) + \\ &\quad g^2(i-m, j-n) - 2f(i,j)g(i-m, j-n)] \end{aligned}$$

   Where f is a template and g is the image function, (i, j) such that $(i-m, j-n)$ are in the domain of definition of the template. If the template is of a smaller spatial extent than the image (they usually are), the image energy in the template window $\sum\sum g^2$ in general varies with (m, n). Suggest a way (give the expression) in which this effect can be normalized.

2. Show that the curve $f(x)$ that minimizes the integral

$$\int_1^2 \frac{\sqrt{1+f'^2}}{x} dx, \quad with \quad f(1) = 0, \quad and \quad f(2) = 1.$$

   is a circle. What is its radius and what are the coordinates of its center?

3. Given n data points $\{x_i, y_i\}_{i=1,..,n}$ such that $f(x_i) = y_i$, how many unknowns are there if a quadratic spline is to be fitted to this data? Deduce the conditions required to solve the unknowns. Do you have enough conditions? If not, what conditions can you impose?

4. Given a set of sample measurements of a one dimensional curve in the image plane, $f(x)$, what is the purpose of minimizing the following functional:

$$E(S) = \int \{\lambda(S'(x))^2 + (f(x) - S(x))^2 \sum_k \delta(x - x_k)\} dx$$

   Describe the significance of each term on the right hand side of the above equation. Assume that λ is a constant regularization parameter. Then, write down the Euler-Lagrange equation for this minimization problem.

5. We know that the length element of a curve $y(x)$ is given by $ds = \sqrt{1 + (y')^2} dx$. Show that the area element of the image surface $I(x, y)$ is given by $dA = \sqrt{1 + I_x^2 + I_y^2} dxdy$. Next, find the Euler-Lagrange equations for the functional $\int dA$.