Programming Assignment Report-2

Answer-1: We have three different random curves with three different time step values

The red curve denotes the initial curve generated by the random closed curve generator and the blue curves denote the curves generated after propagation of curvature flow equation implemented by Euler-Lagrange minimization with the respective parameters. We can increase the number of iterations to shrink the curve successively inside the contour.

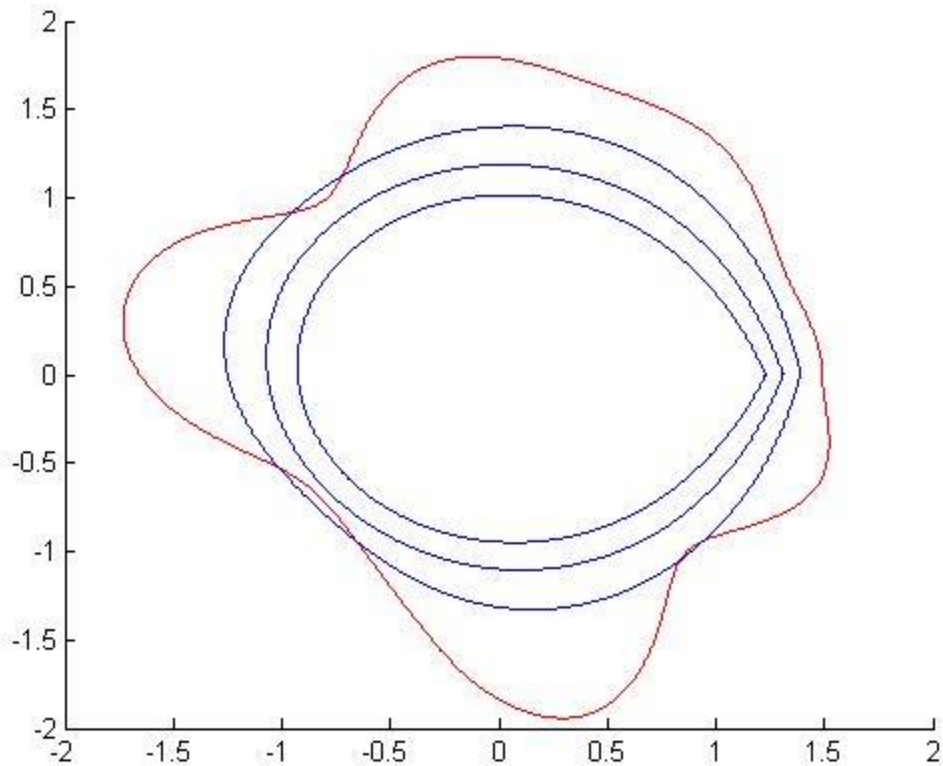


Figure-2 Time Step=0.5 and number of iterations=90000 each stage after 30000 iterations

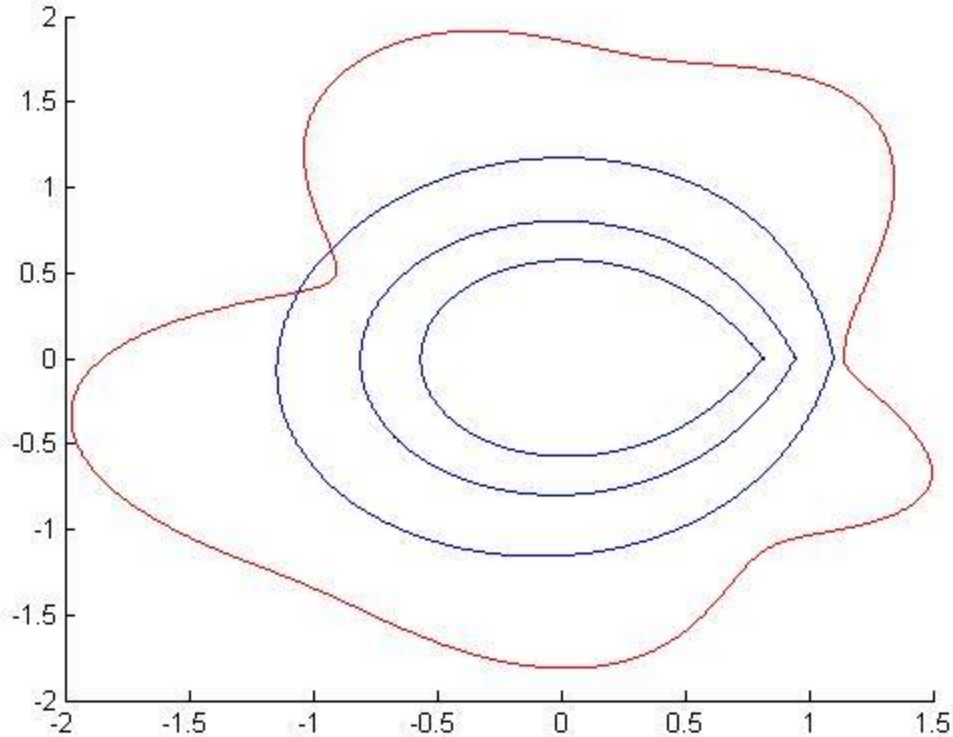


Figure-2 Time Step=1.2 number of iterations=90000 each stage after 30000 iterations

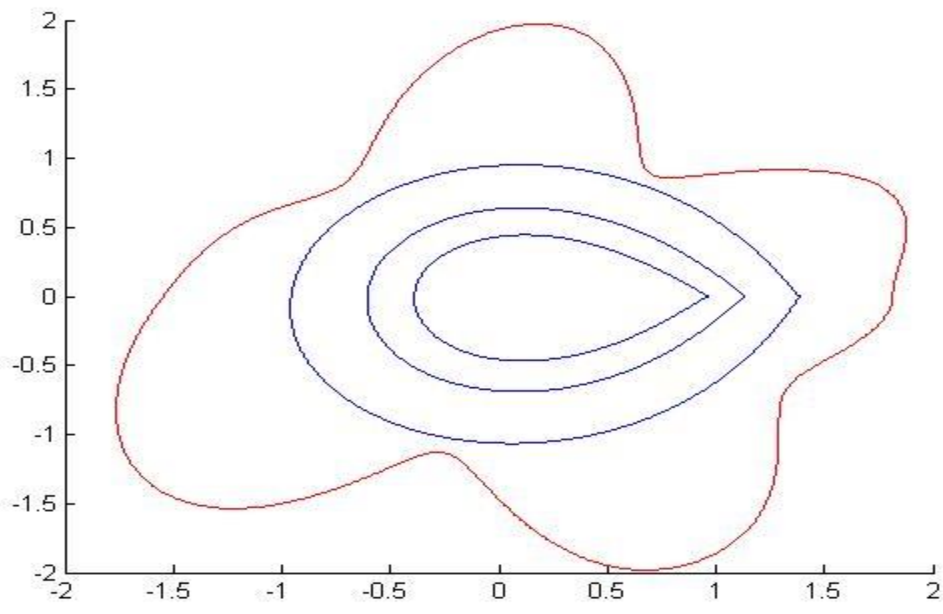


Figure-3 Time Step=1.6 and number of iterations=90000 each stage after 30000 iterations

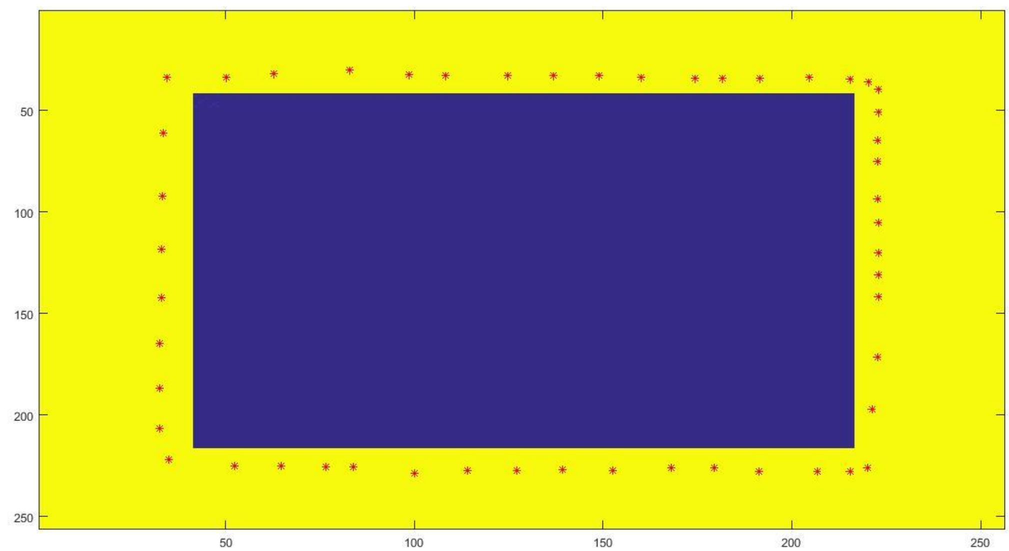
- It can be clearly seen that when you increase the time step value for the curve for each iteration the initialized curve seems to be shrinking at a faster rate.
- Curvature Equation=derivative of (C(s;t) wrt t)=KN; where N->Normal Vector
- As the number of iterations keep on increasing the contour formation shrinks at a slower rate since the gradient value gradually decreases.

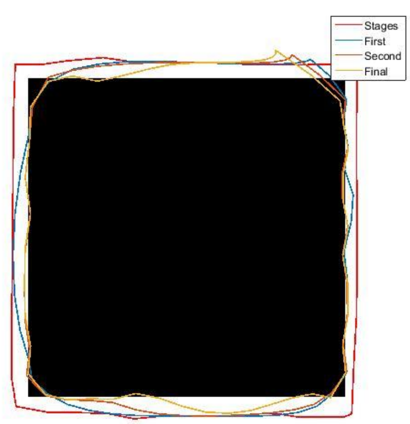
Answer-2:

Energy equation of snake=(alpha*v(ss)-Beta*v(ssss)) +Eext(Image)-> (Gradient Vector Field V)

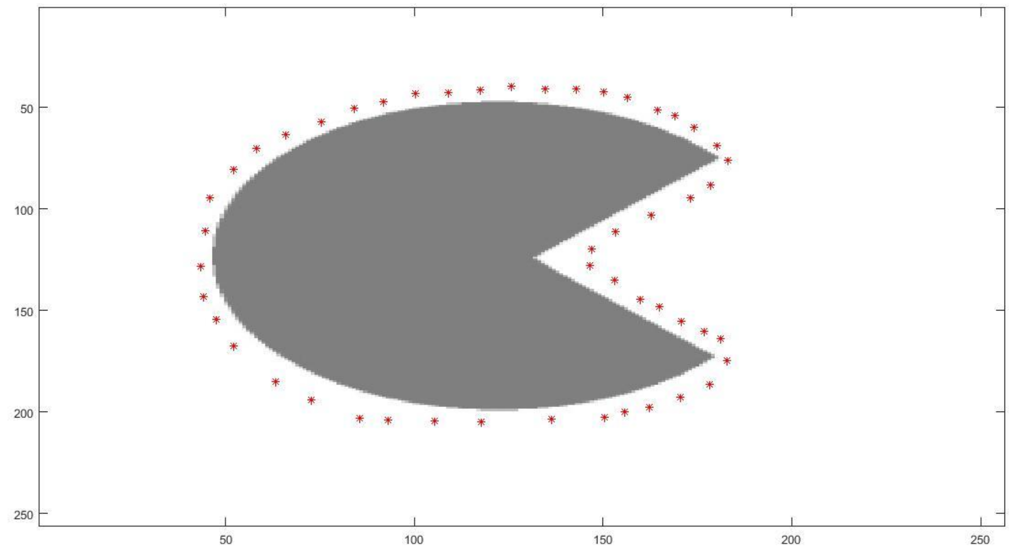
. In this question, I have implemented the traditional snake instead of the GVF snake.

<u>Test Cases:</u> Square Image





Pacman Image



Note: -All the parameters that have been chosen for the image are specific to it and are adjusted per the nature of image.

Active contours are elastically deformable curves whose behaviour is based on elasticity theory. They can deform under externally applied forces & come to rest under force balance. Active contours are represented using parametric curves.

parametric curve : $v(s) = (x(s), y(s))$

$$\mathcal{E}_{snake} = \mathcal{E}_{ext} + \mathcal{E}_{int} + \mathcal{E}_{constraint}$$

$$\mathcal{E}_{int} = \mathcal{E}_{bending} + \mathcal{E}_{elastic} = \int_S \frac{1}{2} (\alpha |v_s|^2 + \beta |v_{ss}|^2 ds$$

$$\mathcal{E}_{ext} = \int_S \mathcal{E}_{image}(v(s))\, ds$$

$$\mathcal{E}_{image}(x, y) = -|\nabla I(x,y)|^2$$

$$\mathcal{E}_{image}(x,y) = -|\nabla(G_\sigma(x,y) * I(x,y))|^2$$

The problem is to find a $v(s)$ that minimizes the energy functional :-

$$\mathcal{E}_{snake} = \int_S \frac{1}{2}(\alpha |v_s|^2 + \beta |v_{ss}|^2) + \mathcal{E}_{image}(v(s))\, ds$$

Applying the E-L eqn (Variational Calculus)

we have :- $\alpha v_{ss} - \beta v_{ssss} - \nabla \mathcal{E}_{image} = 0$

Each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces.

$$F_{elastic} = \alpha V_{ss}$$

$$F_{ext} = -\nabla \mathcal{E}_{image}$$

Consider the snake also to be a function of time i.e $V_t(s,t)$

then: $\alpha V_{ss}(s,t) - \beta V_{ssss}(s,t) - \nabla \mathcal{E}_{image} = V_t(s,t)$

where: $V_t(s,t) = \dfrac{\delta V(s,t)}{\delta t}$

Snakes are very sensitive to false local minima which leads to wrong convergence. Note that they were never meant to be stand alone in segmentation tools.

Model for GVF snake

$$V(x,y) = (u(x,y), v(x,y))$$

Force eqn of GVF snake

$$\alpha V_{ss} - \beta V_{ssss} + V = 0$$

$V(x,y)$ is defined such that it minimizes the functional

$$\int \mu (u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |V - \nabla f|^2 dx dy$$

$|\nabla f|$ & $f(x,y) \rightarrow$ edge map of the image

GVF field can be obtained by solving the following eqn's:-

$$\mu \nabla^2 u - (u - f_x)(f_x^2 + f_y^2) = 0 \quad \text{--- } (1)$$

$$\mu \nabla^2 v - (v - f_y)(f_x^2 + f_y^2) = 0 \quad \text{--- } (2)$$

$\nabla^2 \rightarrow$ Laplacian Operator