Q3:-

Quadratic spline :- ax^2+bx+c

For each piece, there are 3 unknowns, so total $3(n-1)$ unknowns.

For each piece, continuity of first derivative imposes $(n-2)$ linear constraints and interpolation imposes $2(n-1)$ constraints, thus totalling $3n-4$ constraint. The 1 more linear constraint, we can take as $f'|_{x_1} = 0$ where f is the spline.

Q1:- Replace f by $\bar{f}$ where $\bar{f}(i,j) = \dfrac{f(i,j) - f_\mu}{f_\sigma}$

and replace g by $\bar{g}$ where $\bar{g}(i,j) = \dfrac{g(i,j) - g_\mu}{g_\sigma}$

where f_μ, g_μ are mean and f_σ, g_σ are std. dev.

Q2:- The E-L eq^n is

$$\frac{d}{dx}\frac{f'}{x\sqrt{1+f'^2}} = 0 \Rightarrow \frac{f'}{x\sqrt{1+(f')^2}} = c, \ c \in \mathbb{R}$$

Integrating, we get

$$f(x) = -\sqrt{5-x^2} + 2$$

So, center is $(0, 2)$, radius $\sqrt{5}$.

Q4:-

The first term is the regularization term and the second term is data fidelity term. Purpose is to find $S(x)$ which is smooth and best approximate $f(x)$.

The E-L eq^n is:-

$$-2(f(x)-S(x))\sum_{k}\delta(x-x_k) - \frac{d}{dx}(2\lambda S'(x)) = 0$$

$$\Rightarrow \lambda S''(x) + (-S(x)+f(x))\sum_{k}\delta(x-x_k) = 0$$

Q5:-

$$dA = \left| (dx, 0, \frac{\partial I}{\partial x}dx)^t \times (0, dy, \frac{\partial I}{\partial y}dy)^t \right|$$

$$= \cancel{\sqrt{1+\left(\frac{dI}{dx}\right)^2+\left(\frac{dI}{dy}\right)^2}\,dx\,dy} = \sqrt{1+\left(\frac{\partial I}{\partial x}\right)^2+\left(\frac{\partial I}{\partial y}\right)^2}\,dx\,dy$$

$$= \sqrt{1+I_x^2+I_y^2}\;dx\,dy$$

the EL eq^n is

$$F_I - \frac{d}{dx}F_{I_x} - \frac{d}{dy}F_{I_y} = 0$$

$$F_I = 0,\quad F_{I_x} = \frac{I_x}{\sqrt{1+I_x^2+I_y^2}},\quad F_{I_y} = \frac{I_y}{\sqrt{1+I_x^2+I_y^2}}$$

solving, we get

$$\frac{I_{xx} + I_{xx}I_y^2 + I_{yy} + I_{yy}I_x^2 - 2I_yI_xI_{xy}}{(1+I_x^2+I_y^2)^{3/2}} = 0$$