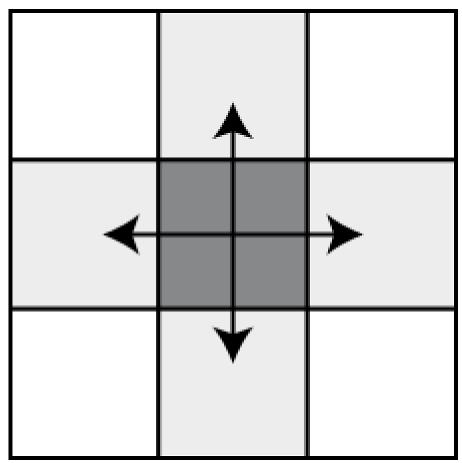
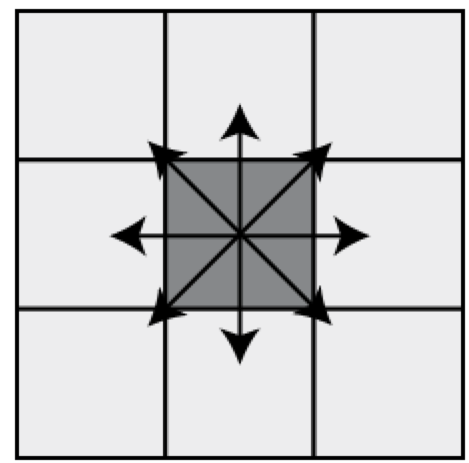
CST382-3 Digital Image Processing

Pixel Relationship

- Neighborhood
- Connectivity
- Adjacency

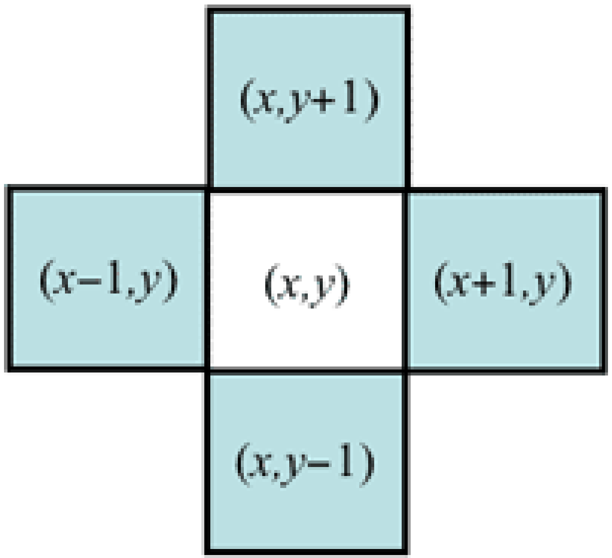


4-Connectivity

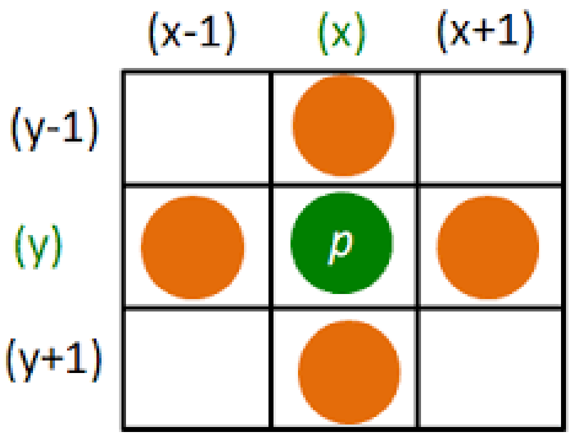


8-Connectivity

Neighborhood $N_4(p)$

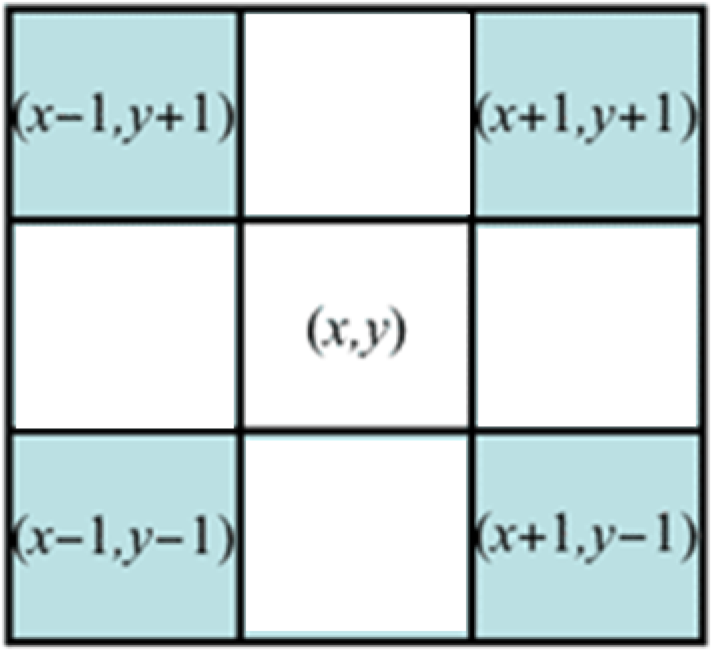


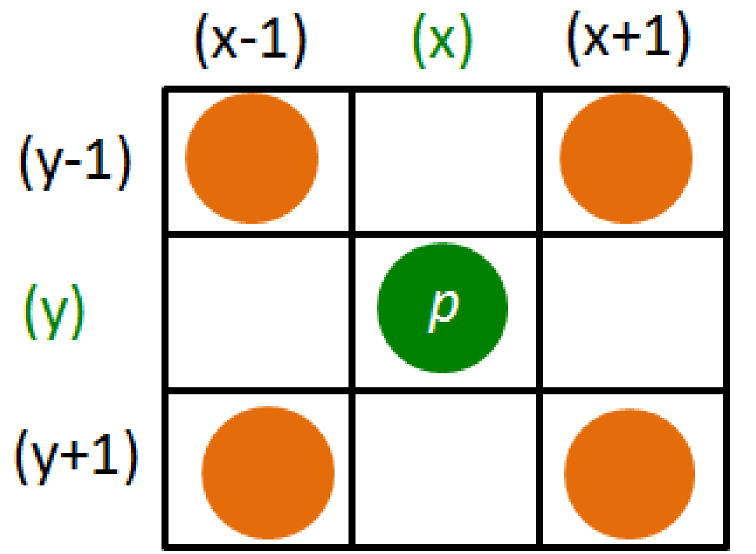
4-neighbourhood



4-neighbors N4(p)

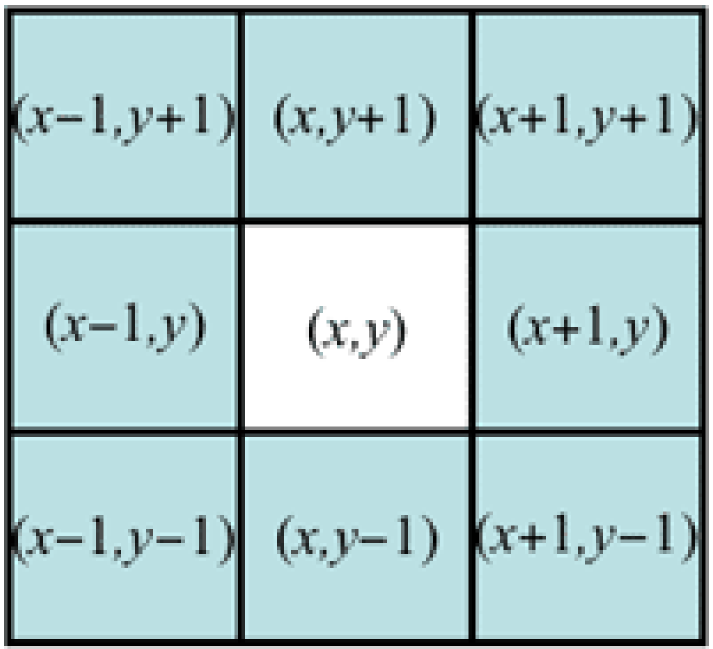
Neighborhood $N_D(p)$



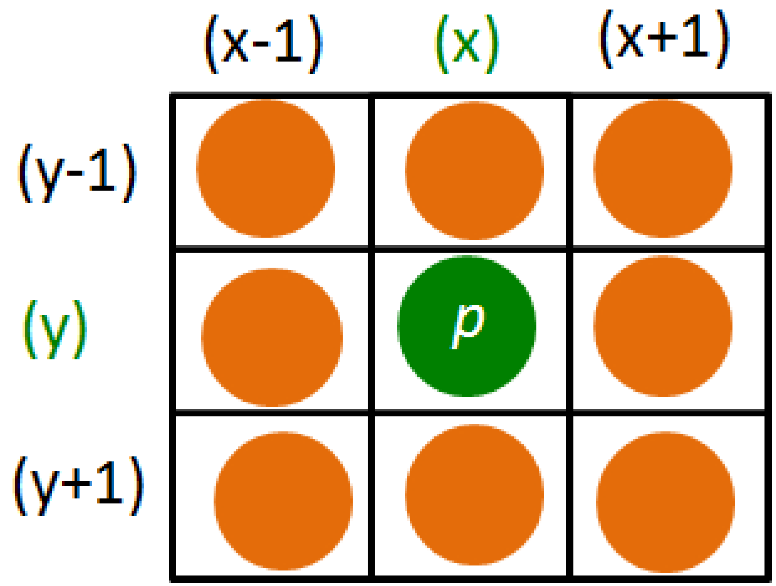


Diagonal neighbors $N_d(p)$

Neighborhood $N_8(p)$



8-neighbourhood



8-neighbors $N_8(p)$

Object and Connection

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	0	1	1	1	1	0	0	0	1	1	1	1	0	0	0
0	0	1	1	1	1	0	0	0	1	1	1	0	0	1	1	0
0	1	1	1	0	0	1	1	0	0	0	1	1	1	0	0	0
0	0	1	1	0	0	0	0	0	1	1	0	0	0	1	1	0
0	0	0	0	0	0	1	1	1	1	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Adjacency and Connectivity

- Let V: a set of intensity values used to define adjacency and connectivity.
- In a binary image, $V = \{1\}$, if we are referring to adjacency of pixels with value 1.
- In a gray-scale image, the idea is the same, but V typically contains more elements, for example, $V = \{180, 181, 182, \ldots, 200\}$
- If the possible intensity values 0 – 255, V set can be any subset of these 256 values.

Type of Adjacency

1. **4-adjacency:** Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.
2. **8-adjacency:** Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.
3. **m-adjacency =(mixed)** Two pixels p and q with the values from V are m-adjacent if
   - q is in N4(p), or
   - q is in $N_D(p)$ **and** the set $N4(p) \cap N4(q)$ has no pixels whose values are from V.

Question

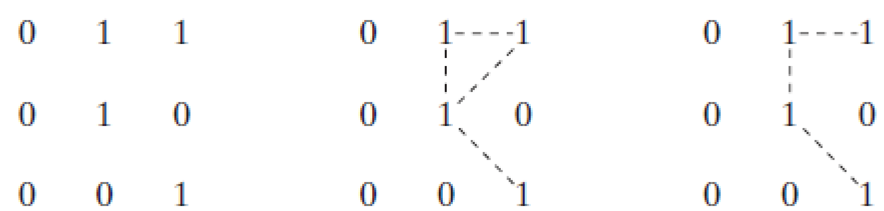
Consider the two image subsets, and shown in the following figure. For determine whether these two subsets are (a) 4-adjacent, (b) 8-adjacent, or (c) *m*-adjacent.

	S_1				S_2				
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	0	0
0	0	1	1	1	0	0	0	0	0
0	0	1	1	1	0	0	1	1	1

Digital Path

- A digital path (or curve) from pixel *p* with coordinate (x,y) to pixel *q* with coordinate (s,t) is a sequence of distinct pixels with
  - coordinates (x_0,y_0), (x_1,y_1), …, (x_n, y_n) where $(x_0,y_0) = (x,y)$ and $(x_n, y_n) = (s,t)$ and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$
- n is the length of the path
- If $(x_0,y_0) = (x_n, y_n)$, the path is closed.
- We can specify 4-, 8- or m-paths depending on the type of adjacency specified.

Digital Path



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Connectivity

- Let S represent a subset of pixels in an image.
- Two pixels p and q are said to be ***connected*** in S if there exists a path between them consisting entirely of pixels in S.
- For any pixel p in S, the set of pixels that are connected to it in S is called a **connected component** of S.
  - S = {**p,q,r**,*s*}
- Only one connected component, then set S is called a ***connected set***.

Connectivity

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	0	1	1	1	1	0	0	0	1	1	1	1	0	0	0
0	0	1	1	1	1	0	0	0	1	1	1	0	0	1	1	0
0	1	1	1	0	0	1	1	0	0	0	1	1	1	0	0	0
0	0	1	1	0	0	0	0	0	1	1	0	0	0	1	1	0
0	0	0	0	0	0	1	1	1	1	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Region and Boundary

Region

- Let R be a subset of pixels in an image, we call R a **region** of the image if R is a connected set.
- Two regions, and are said to be ***adjacent*** if their union forms a connected set.
- Regions that are not adjacent are said to be ***disjoint***.
- We consider **4- and 8-adjacency** when referring to regions.

$$\left.\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\right\} R_i$$
$$\left.\begin{matrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}\right\} R_j$$

Region and Boundary

- **Boundary**

  The *boundary* (also called the *border* or *contour*) of a region *R* is

  – the set of points that are adjacent to points in the complement of *R*.

OR

  – the set of pixels in the region that have at least one background neighbor.

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

1	**1**	**1**	**1**
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

0	**0**	**0**	**0**
1	1	1	0
1	1	1	0
1	1	1	0
0	0	0	0

- If *R* happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns in the image.

- This extra definition is required because an image has no neighbors **beyond its borders**

- Normally, when we refer to a region, we are referring to subset of an image, and any pixels in the boundary of the region that happen to coincide with the border of the image are included implicitly as part of the region boundary.

Distance Measures

- For pixels p, q and z, with coordinates (x,y), (s,t) and (v,w), respectively, D is a distance function if:

  (a) $D(p,q) \geq 0$ ($D(p,q) = 0$ iff $p = q$),

  (b) $D(p,q) = D(q,p)$, and

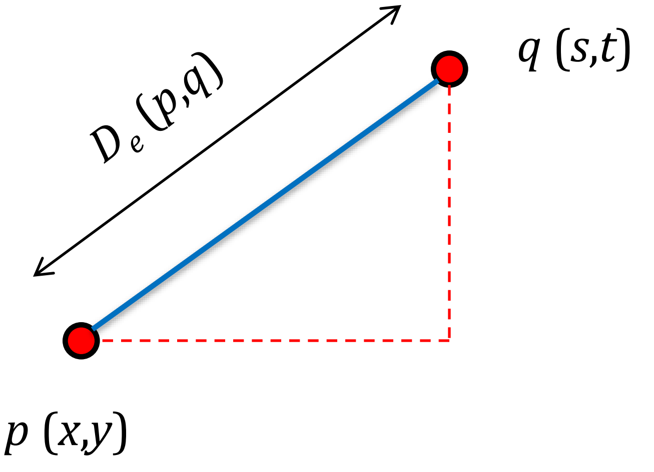
  (c) $D(p,z) \leq D(p,q) + D(q,z)$.

Distance Measures

- The *Euclidean Distance* between *p* and *q* is defined as:

$$D_e(p,q) = [(x - s)^2 + (y - t)^2]^{1/2}$$

Pixels having a distance less than or equal to some value r from (x,y) are the points contained in a disk of radius r centered at (x,y)

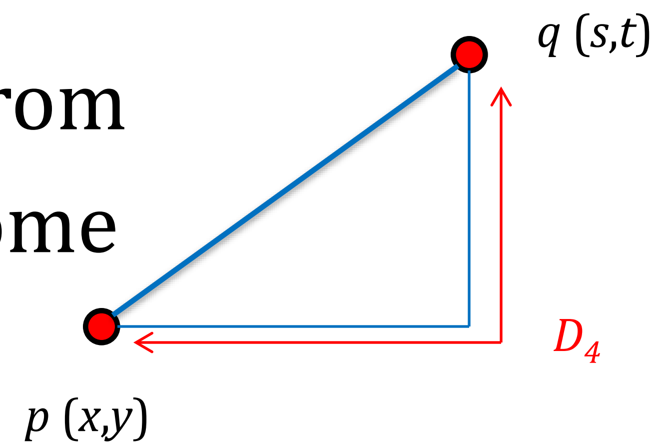


Distance Measures

- The D_4 *distance* (also called *city-block distance*) between p and q is defined as:

  $D_4\,(p,q) = |\,x - s\,| + |\,y - t\,|$

Pixels having a D_4 distance from (x,y), less than or equal to some value r form a Diamond centered at (x,y)



Distance Measures

Example:

The pixels with distance $D_4 \le 2$ from (x,y) form the following contours of constant distance.

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

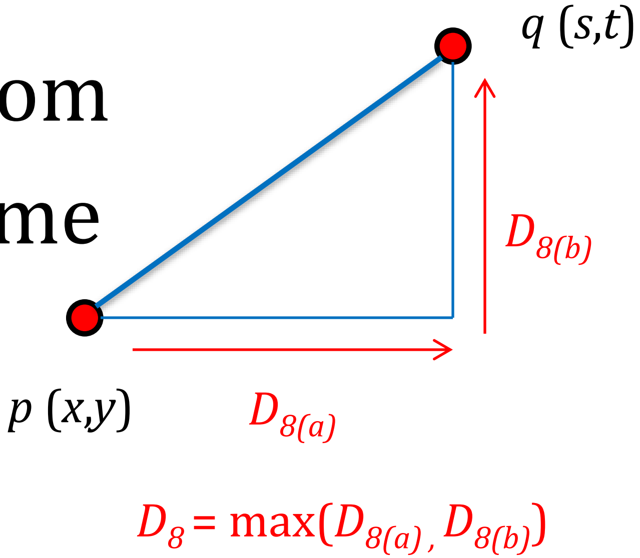
The pixels with $D_4 = 1$ are the 4-neighbors of (x,y)

Distance Measures

- The D_8 *distance* (also called *chessboard distance*) between *p* and *q* is defined as:

  $D_8(p,q) = \max(|x - s|, |y - t|)$

Pixels having a D_8 distance from (x,y), less than or equal to some value r form a square Centered at (x,y)



Distance Measures

Example:

D_8 distance ≤ 2 from (x,y) form the following contours of constant distance.

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Distance Measures

- **Dm distance:**

  is defined as the shortest m-path between the points.

  In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.

Distance Measures

- Example:

  Consider the following arrangement of pixels and assume that p, p_2, and p_4 have value 1 and that p_1 and p_3 can have a value of 0 or 1

  Suppose that we consider the adjacency of pixels values 1 (i.e. $V = \{1\}$)

	p_3	p_4
p_1	p_2	
p		

Distance Measures

- Cont. Example:

  Now, to compute the D_m between points p and p_4

  Here we have 4 cases:

  Case1: If p_1 =0 and p_3 = 0

  The length of the shortest m-path (the D_m *distance) is 2 (p, p_2, p_4)*

	0	1
0	1	
1		

Distance Measures

- Cont. Example:

  Case2: If p_1 =1 and p_3 = 0

  now, *p_1 and p will no longer be adjacent (see m-adjacency definition)*

  then, the length of the shortest path will be 3 (p, p_1, p_2, p_4)

	0	1
1	1	
1		

Distance Measures

- Cont. Example:

  Case3: If p_1 =0 and p_3 = 1

  The same applies here, and the shortest –m-path will be 3 (p, p_2, p_3, p_4)

	1	1
0	1	
1		

Distance Measures

- Cont. Example:

Case4: If p_1 =1 and p_3 = 1

The length of the shortest m-path will be 4

(p, p_1, p_2, p_3, p_4)

	1	1
1	1	
1		