# Machine Learning on Graphs - Homework 2

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## 1 Question 1

### 1.1 A

The magnitude of Katz Index indicates the number of walks with respect to the neighborhood of the current node $u, v$. If the Katz number is big, it is known that there are a large number of walks with short length between $u, v$.

The vectorized form is:

$$S_{Katz} = \sum_{l}^{\infty} \beta^l A^l = (I - \beta A)^{-1} - I$$

where $A$ is the adjacency matrix and $\beta$ is the discount factor.

### 1.2 B

We want the walks with longer length to have less importance in this index. Thus, we use the discount factor $\beta$ as a number between 0 and 1 which reduces the importance of walks with larger lengths.

### 1.3 C

The absolute value of All the eigen values of the matrix should be smaller than 1.

$$\begin{aligned} &Av = \lambda v \rightarrow A(Av) = \lambda(Av) \rightarrow A^2 v = \lambda(\lambda v) \rightarrow A^2 v = \lambda^2 v \\ &\implies A^i v = \lambda^i v \text{ and } v \neq [0]_{n\times 1} \\ &\implies \lim_{i\to\infty} A^i = [0]_{n\times n} \implies \lim_{i\to\infty} \lambda^i = 0 \implies |\lambda| < 1 \end{aligned}$$

### 1.4 D

$$\begin{aligned} &S = \sum_{i=0}^{n} A^i \rightarrow SA = \sum_{i=0}^{n} A^{i+1} \implies SA - S = \sum_{i=0}^{n} A^{i+1} - \sum_{i=0}^{n} A^i \\ &\implies S(A - I) = I - A^{n+1} \implies S = (I - A^{n+1})(A - I)^{-1} \\ &\xrightarrow{\text{Based On C}} S = (I - O)(A - I)^{-1} = (A - I)^{-1} \end{aligned}$$

The required conditions are that $(A - I)^{-1}$ needs to be invertible matrix and from part C we have that the absolute value of All the eigen values of the matrix should be smaller than 1.

### 1.5 E

The discount factor parameter $\beta$ needs to be between 0 and 1 for the summation in part D to be convergent.

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## 2 Question 2

### 2.1 A

A graphlet is a induced connected non-isomorphic rooted subgraph which shows the local structure around the root node in the main graph.

### 2.2 B

There are 3 graphlets with 3 nodes and 11 graphlets with 4 nodes. Graphlets with 3 nodes are:

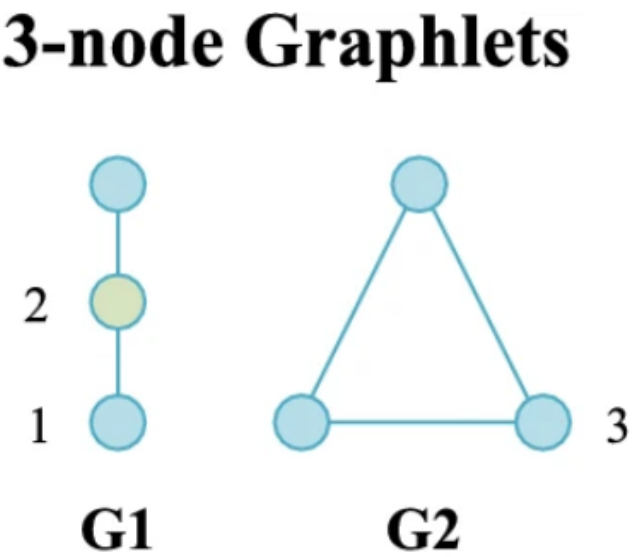


Figure 1: Graphlets with 3 nodes

### 2.3 C

Yes, They are different. Graphlets that are used for graphs are not rooted and can be disconnected graphs.

### 2.4 D

First, we indicate the graphlets that we want to use as graph or node features. Then, for each graph or node, we count the number of each graphlet for that graph or node. Thus, for each graph or node, we have a vector of graphlet count.

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# 3 Question 3

## 3.1 A

Consider $M_{n\times n}$ as a Stochastic matrix

$$\det(M^T - \lambda I) = \det((M - \lambda I)^T) = \det(M - \lambda I)$$

$$M^T v = \lambda v \implies M^T \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} M_{1,j} \\ \sum_{k=1}^{n} M_{2,j} \\ \vdots \\ \sum_{k=1}^{n} M_{n,j} \end{bmatrix} = 1 \times \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$\implies \lambda = 1$$

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The Rest of the Answers is in `./HW2-MohammadBahrami-9724133.ipynb`