

P. FINELLI

NUMERICAL PROJECTS FOR NUCLEAR PHYSICS

UNIBO -2022

Legenda

- **Difficulty level 1**

The student is supposed to develop its own numerical project from scratch. Maximum evaluation obtainable: 28 (more than 28 only for particular reasons). The evaluation will be based on: 1) Script for the code (readability, performance, documentation) and 2) the presentation (25-35 minutes).

- **Difficulty level 2**

Challenging problems that could lead to a 30 cum laude evaluation. The student is supposed to develop its own numerical project from scratch. The evaluation will be based on: 1) Script for the code (readability, performance, documentation) and 2) the presentation (25-35 minutes).

- **Difficulty level 3 (available for student's collaborations)**

More challenging problems that could (but not necessarily) be done within a small group of students (max. 2). The student is supposed to develop its own numerical project from scratch. Maximum evaluation obtainable: 30 cum laude. The evaluation will be based on: 1) Script for the code (readability, performance, documentation) and 2) the presentation (25-35 minutes).

Lepage's Analysis

Repeat the analysis of P. Lepage concerning the construction of effective theories in non-relativistic quantum mechanics¹:

1. Choose a potential (Coulomb + short range, I would suggest square well)

$$V(r) = -\frac{\alpha}{r} + V_s(r)$$

2. Solve the Schrödinger equation, find the eigenvalues and the eigenfunctions

$$H|\psi_k\rangle = E_k|\psi_k\rangle$$

3. Introduce the contact interaction

$$H = \frac{p^2}{2m} - \frac{\alpha}{r} + c\delta(r)$$

and repeat the calculations

4. Use the EFT approach

$$H_{eff} = \frac{p^2}{2m} + V_{eff}(r)$$

and reproduce the Lepage's analysis

5. (Optional) Look at the operator's side

Developing the full analysis of Lepage, the problem could be considered for a two students **collaboration project**.

Numerics

- Derivatives
- Integration techniques
- Differential equations

¹ G. Peter Lepage. How to renormalize the schrodinger equation, 1997. URL <https://iol.unibo.it/course/view.php?id=39785>

Lippmann-Schwinger Equation

The student could follow two lecture notes² that will provide all the needed theoretical equipment to deal with the LS equation.

1. Use the simple potential suggested by M. Hjorth Jensen
2. Look for scattering solutions, you can find lecture and video here³
3. Look at bound-state solutions, you can find lecture and video here⁴
4. (Optional) Use realistic potentials. For this application, it could be useful to look into the work of R. Cenzato⁵

With the optional step, the problem could be considered for a two students **collaboration project**.

Numerics

- Derivatives
- Differential equations
- Integration techniques

² M. Hjorth-Jensen. Nuclear structure from nuclei to neutron stars. URL http://www.int.washington.edu/NNPSS/2000_talks/jensen.pdf; and Talent / int course on nuclear forces. URL http://www.int.washington.edu/PROGRAMS/talent13/exercises/TALENT_exercises_Monday_1.pdf

³ R. Landau. Integral equations for qm scattering, a. URL http://sites.science.oregonstate.edu/~landaur/Books/CPbook/eBook/Lectures/Modules/IntEqn_Scatt/IntEqn_Scatt.html

⁴ R. Landau. Integral equations for qm bound states, b. URL http://sites.science.oregonstate.edu/~landaur/Books/CPbook/eBook/Lectures/Modules/IntEqn_Bound/IntEqn_Bound.html

⁵ R. Cenzato. Analisi e soluzione numerica dell'equazione di lippmann-schwinger, 2018. URL <https://amslaurea.unibo.it/16366/1/CenzatoRebeccaTesi.pdf>

Similarity Renormalization Group with Phenomenological Forces

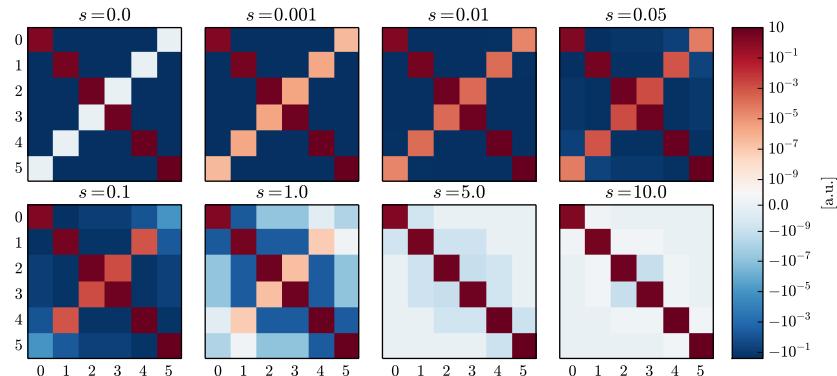
Generally speaking the student is supposed to follow the scheme:

1. Develop a SRG function/routine/class that solves the SRG flow equations

$$\frac{d}{d\alpha} \text{Tr}(H^2) = \frac{d}{d\alpha} \sum_{i,j} (H_{ii}^2 + |H_{ij}|^2) = 0$$

2. Apply your code to a simplified potential (following the lectures we discussed during the lecture)
3. (Optional) Apply your code to a realistic potential

A very useful resource could be found in the lecture *In-Medium Similarity Renormalization Group Approach to the Nuclear Many-Body Problem*⁶ where analytical and simplified interactions are proposed and discussed from the numerical point of view.



With the optional step, the project could be considered **difficult**.

Numerics

- Derivatives
- Differential equations
- Matrix manipulations

⁶ Heiko Hergert, Scott K. Bogner, Justin G. Lietz, Titus D. Morris, Samuel J. Novario, Nathan M. Parzuchowski, and Fei Yuan. *In-Medium Similarity Renormalization Group Approach to the Nuclear Many-Body Problem*. Springer International Publishing, 2017. URL https://doi.org/10.1007/978-3-319-53336-0_10

Deuteron Equations

The ground state of the deuteron is characterized by a non vanishing electric quadrupole moment. This indicates that the interaction between proton and neutron is not central.

From the analysis of the non-central term of the interaction, it emerges that the Schroedinger equation that describes the deuteron can be rewritten in terms of two differential equations (Rarita-Schwinger)⁷

$$\begin{aligned} \left[\frac{\hbar^2}{M} \frac{d^2}{dr^2} + E - V_c(r) \right] u_S(r) &= \sqrt{8} V_T(r) u_D(r) \\ \left[\frac{\hbar^2}{M} \left(\frac{d^2}{dr^2} - \frac{6}{r^2} \right) + E + 2V_T(r) - V_C(r) \right] u_D(r) &= \sqrt{8} V_T(r) u_S(r), \end{aligned}$$

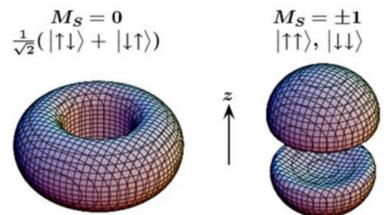
where $V_C(r)$ is a central force which is just dependent on the relative distance r and $V_T(r)$ is called the tensor interaction and the associated operator structure is often denoted as \hat{S}_{12} .

1. Solve the previous equations for a simplified potential and find eigenvalues and eigenfunctions (you can imagine a square well potential with the depth V_0 determined reproducing the deuteron binding energy and then feel free to play with the tensor potential)
2. Calculate the quadrupole moment and compare/discuss your results in comparison with the experimental data
3. (Optional) Repeat the previous steps with the 1π exchange potential (see R. Schavilla notes⁸)

With the optional step, the project could be considered **difficult**.

Numerics

- Derivatives
- Integration techniques
- Coupled Channel Differential equations



⁷ Ji Xiangdong. Phys 741 - lecture notes, 2012. URL <https://www.physics.umd.edu/courses/Phys741/xji/chapter6.pdf>; M. Hjorth-Jensen, M. P. Lombardo, and U. van Kolck. An advanced course in computational nuclear physics. URL <https://github.com/NuclearTalent/ManyBody2018/blob/master/doc/Literature/LNP936.pdf>; Repository for LNP936. URL <https://github.com/ManyBodyPhysics/LectureNotesPhysics>; and F. Convenga. Il deutone in teoria effettiva pionless, 2015. URL http://www.infn.it/thesis/thesis_dettaglio.php?tid=8962

⁸ R. Schavilla. Nustec class notes. URL <https://indico.fnal.gov/event/8047/material/0/0>

Triton Equations (SRG with Two- and Three-body Forces)

See Robert Roth's project⁹. In this project you have to solve the quantum many-body problem for the ground state of the triton (${}^3\text{H}$) using an SRG-transformed chiral Hamiltonian (given in a predetermined basis) in the No-Core Shell Model (No-Core simply means that all particles are active and there is no inert core).

1. Read (and check) the matrix elements from data file
2. Implement the SRG-evolution of the Hamiltonian using the kinetic energy T and the potential matrix elements
3. Solve the flow equation

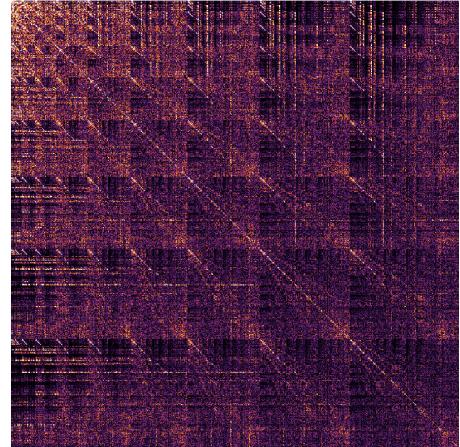
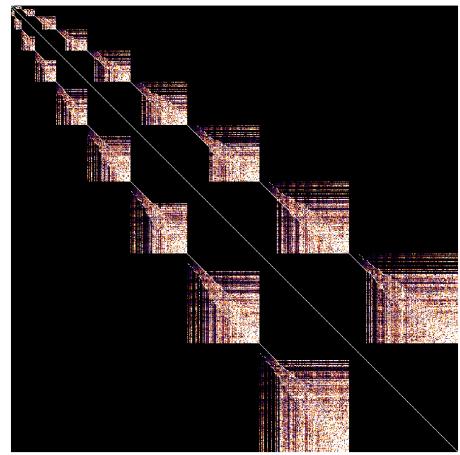
$$\begin{aligned}\frac{d}{ds}H(s) &= M_n^2 [[T, H(s)], H(s)] \\ &= M_n^2 (TH(s)H(s) - 2H(s)TH(s) + H(s)H(s)T)\end{aligned}$$

4. Solve this matrix evolution equation using the simplest possible algorithm for treating ordinary differential equations numerically, the so-called Euler algorithm
5. Finally, you can solve the exact Schrödinger equation for the triton via a matrix eigenvalue problem, which corresponds to a full-fledged No-Core Shell Model calculation.

Numerics

- Derivatives
- Integration techniques
- Differential equations
- Matrix manipulations

⁹ R. Robert. Ect* doctoral training program, 2017. URL http://chimera.roma1.infn.it/OMAR/ECTSTAR_DTP/roth/



Hartree-Fock Equations

The project aims at the development of an Hartree-Fock code for a simplified interaction. For a general overview of the theory I suggest the lectures of M. Hjorth Jensen¹⁰ (Chap. 10). A short description on how to implement a simple code is also given (see also the repository¹¹).

To develop the code, my suggestion is to follow the lectures presented at the TALENT 4 school (2016) freely available online¹².

Another interesting source of informations/numerical programs is available at the TALENT 3 school¹³, where detailed projects on Hartree-Fock are sketched out.

Depending on the potential used in the calculation, the project could be considered **difficult**.

Numerics

- Derivatives
- Integration techniques
- Differential equations
- Matrix manipulations

¹⁰ M. Hjorth-Jensen, M. P. Lombardo, and U. van Kolck. An advanced course in computational nuclear physics. URL <https://github.com/NuclearTalent/ManyBody2018/blob/master/doc/Literature/LNP936.pdf>

¹¹ Repository for LNP936. URL <https://github.com/ManyBodyPhysics/LectureNotesPhysics>

¹² J. Dobaczewski. URL https://www.fuw.edu.pl/~dobaczew/TALENT_DFT_2016/manuale_hf.pdf

¹³ Talent 3 school Projects. URL <https://wikihost.nscl.msu.edu/TalentDFT/doku.php?id=projects>; and Talent 3 school Codes. URL <https://wikihost.nscl.msu.edu/TalentDFT/doku.php?id=codes>

Random-Phase Approximation

Because the development of a numerical code for realistic calculations is beyond the scope of the course, it is anyway possible to perform calculations for the Lipkin-Meshkov-Glick (LMG) model.

It is an algebraic model introduced in 1965 which played an important role in nuclear theory. The LMG model describes a two level system where one level is situated just below the Fermi level, the other just above, separated by an energy difference of ϵ . The levels are N -fold degenerate, and the system is filled with N fermions. Each state is characterized by a quantum number σ that assumes the value +1 in the upper level and -1 in the lower one, and a quantum number p specifying the particular degenerate state within the shell. A two body interaction which does not change the value of p is considered. A complete analysis can be found here¹⁴.

N=2	$\bullet\bullet$	$ 1, 1\rangle$
	$\bullet\bullet$	$ 1, 0\rangle$
	$\bullet\bullet$	$ 1, -1\rangle$
N=3	$\bullet\bullet\bullet$	$ \frac{3}{2}, \frac{3}{2}\rangle$
	$\bullet\bullet\bullet$	$ \frac{3}{2}, \frac{1}{2}\rangle$
	$\bullet\bullet\bullet$	$ \frac{3}{2}, -\frac{1}{2}\rangle$
	$\bullet\bullet\bullet$	$ \frac{3}{2}, -\frac{3}{2}\rangle$
N=4	$\bullet\bullet\bullet\bullet$	$ 2, 2\rangle$
	$\bullet\bullet\bullet\bullet$	$ 2, 1\rangle$
	$\bullet\bullet\bullet\bullet$	$ 2, 0\rangle$
	$\bullet\bullet\bullet\bullet$	$ 2, -1\rangle$
	$\bullet\bullet\bullet\bullet$	$ 2, -2\rangle$

Numerics

- Derivatives
- Differential equations
- Matrix manipulations

¹⁴ Giampaolo Co' and Stefano De Leo. Hartree-fock and random phase approximation theories in a many-fermion solvable model. *Modern Physics Letters A*, 30(36):1550196, Nov 2015. ISSN 1793-6632. DOI: 10.1142/s0217732315501965. URL <http://dx.doi.org/10.1142/S0217732315501965>; Giampaolo Co' and Stefano De Leo. Analytical and numerical analysis of the complete lipkin-meshkov-glick hamiltonian. *International Journal of Modern Physics E*, 27(05):1850039, May 2018. ISSN 1793-6608. DOI: 10.1142/s0218301318500398. URL <http://dx.doi.org/10.1142/S0218301318500398>; and R. Romano. A generalization of the lipkin model as a testing ground for many-body theories, 2018. URL <http://www0.mi.infn.it/~jroca/doc/thesis/thesis-riccardo-romano-magistrale.pdf>

Neutron Stars (Tolman-Oppenheimer-Volkoff equations)

The full explanation of the project could be found in the lectures of M. Hjorth Jensen¹⁵ (Chap.8, pg. 271).

The gravitational force which acts on an element of volume at a distance r is given by

$$F = -\frac{GM}{r^2}\rho, \quad (1)$$

where G is the gravitational constant, $\rho(r)$ is the mass density and $M(r)$ is the total mass inside a radius r . The latter is given by

$$M(r) = 4\pi \int_0^r \rho(x)x^2 dx \quad (2)$$

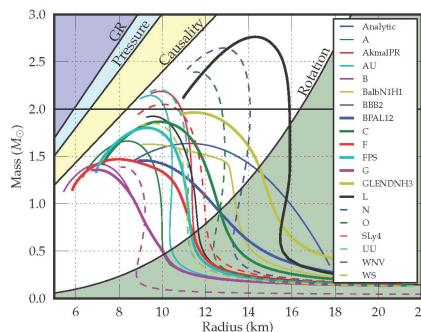
which gives rise to a differential equation for mass and density

$$\frac{dM}{dr} = 4\pi r^2 \rho(r). \quad (3)$$

When the star is in equilibrium we have

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r). \quad (4)$$

Relativity corrections can also be applied¹⁶. The last equations give us two coupled first-order differential equations which determine the structure of a neutron star when the EoS is known. The initial conditions are dictated by the mass being zero at the center of the star, i.e., when $r = 0$, we have $m(r = 0) = 0$. The other condition is that the pressure vanishes at the surface of the star. This means that at the point where we have $P = 0$ in the solution of the differential equations, we get the total radius R of the star and the total mass $m(r = R)$. The mass-energy density when $r = 0$ is called the central density ρ_s . Since both the final mass M and total radius R will depend on ρ_s , a variation of this quantity will allow us to study stars with different masses and radii. Additional material can be found in the lectures of Silbar and Reddy¹⁷ and Sagert¹⁸.



If realistic interactions are used, the project can be considered difficult.

Numerics

- Derivatives
- Differential equations
- Matrix manipulations

¹⁵ M. Hjorth-Jensen, M. P. Lombardo, and U. van Kolck. An advanced course in computational nuclear physics. URL <https://github.com/NuclearTalent/ManyBody2018/blob/master/doc/Literature/LNP936.pdf>

¹⁶

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2} \left[1 + \frac{P(r)}{\rho(r)c^2} \right] \left[1 + 4\pi r^3 \frac{P(r)}{M(r)c^2} \right] \left[1 - \frac{2GM(r)}{c^2 r} \right]^{-1}$$

¹⁷ Richard R. Silbar and Sanjay Reddy. Neutron stars for undergraduates. *American Journal of Physics*, 72(7):892–905, 2004. DOI: 10.1119/1.1703544. URL <https://arxiv.org/abs/nucl-th/0309041>

¹⁸ Irina Sagert, Matthias Hempel, Carsten Greiner, and Jurgen Schaffner-Bielich. Compact stars for undergraduates. *Eur. J. Phys.*, 27:577–610, 2006. DOI: 10.1088/0143-0807/27/3/012. URL <https://arxiv.org/abs/astro-ph/0506417>

Many-Body Perturbation Theory

Exercise 8.10 of Ref. ¹⁹ (Pgs. 386) concerning a simplified Hamiltonian consisting of an unperturbed Hamiltonian and a so-called pairing interaction term. To study this system, a mix of many-body perturbation theory (MBPT), Hartree-Fock (HF) theory and full configuration interaction (FCI) theory will be used.

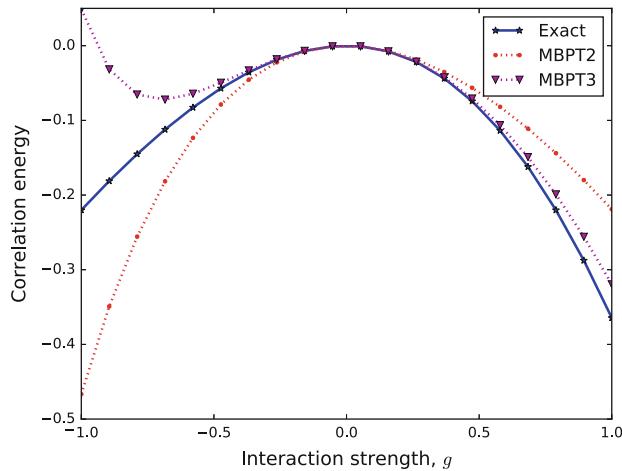


Fig. 8.4 Correlation energy for the pairing model with exact diagonalization, MBPT2 and perturbation theory to third order MBPT3 for a range of interaction values. A canonical Hartree-Fock basis has been employed in all MBPT calculations

A complete set of calculations could lead to consider the project as **difficult**.

Numerics

- Derivatives
- Matrix manipulations
- Integration techniques

¹⁹ M. Hjorth-Jensen, M. P. Lombardo, and U. van Kolck. An advanced course in computational nuclear physics. URL <https://github.com/NuclearTalent/ManyBody2018/blob/master/doc/Literature/LNP936.pdf>

Coupled-Cluster Theory

Section 8.7.4 of Ref. ²⁰ (Pgs. 359) concerning a CCD (Coupled Cluster with Doubles) code for infinite matter.

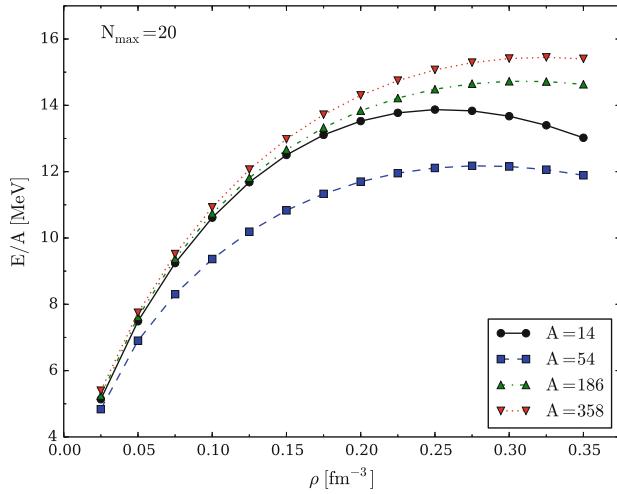


Fig. 8.7 Energy per particle of pure neutron matter computed in the CCD approximation with the Minnesota interaction model [57] for different numbers of particles with $N_{\max} = 20$

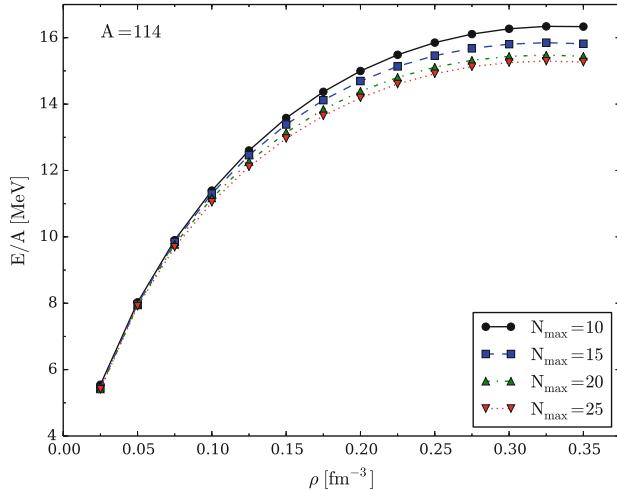


Fig. 8.8 Energy per particle of pure neutron matter computed in the CCD approximation with the Minnesota interaction model [57] for different model space sizes with $A = 114$

Numerics

- Derivatives
- Matrix manipulations
- Integration techniques

²⁰ M. Hjorth-Jensen, M. P. Lombardo, and U. van Kolck. An advanced course in computational nuclear physics. URL <https://github.com/NuclearTalent/ManyBody2018/blob/master/doc/Literature/LNP936.pdf>

Variational Monte Carlo

Application of the Variational Monte Carlo method to the 1D harmonic oscillator and then, after validation of the code, a study of the ground-state of the ${}^4\text{He}$ for a spin/isospin independent interaction. For a complete reference, see *Monte Carlo methods in Quantum Many-Body Theories* by R. Guardiola²¹.

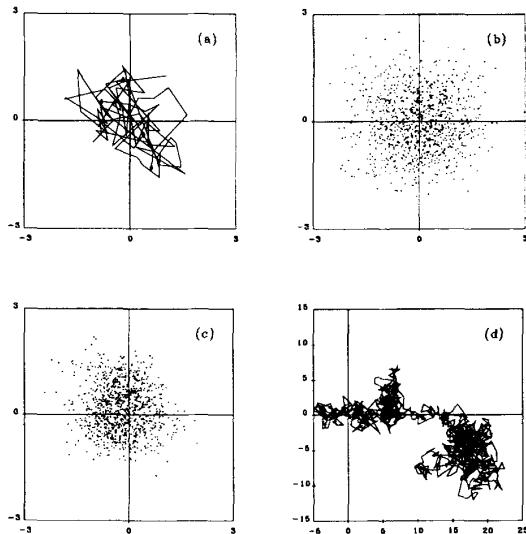


Fig. 4. Some results corresponding to the calculation of ${}^4\text{He}$. **a:** The first 20 moves of the particles, without showing the intermediate thermalization moves. The diamonds are the starting positions with respect to the cm. **b:** Plot of the position of one particle after 1000 moves, with thermalization, referred to the cm. **c:** Same as **b** but without thermalization. **d:** Same as **b** but referred to an external origin. All positions are in fm.

Numerics

- Derivatives
- Differential equations
- Matrix manipulations
- Integration techniques

²¹ Rafael Pardo Guardiola. Monte carlo methods in quantum many-body theories. 1998. URL <https://iol.unibo.it/course/view.php?id=39785>

Diffusion Monte Carlo

Application of the Diffusion Monte Carlo method to the 1D harmonic oscillator and then, after validation of the code, a study of the ground-state of the ${}^4\text{He}$ for a spin/isospin independent interaction. For a complete reference, see *Monte Carlo methods in Quantum Many-Body Theories* by R. Guardiola²².

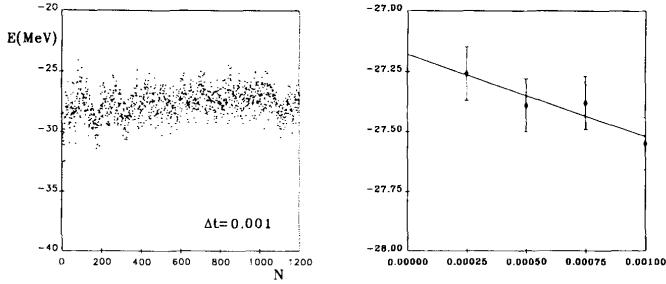


Fig. 7. The time evolution of the average energy (left) as a function of the number of time steps N and the evaluation of the average for different time steps (right). The continuous line represents a linear fit for $t = 0$ extrapolation.

Numerics

- Derivatives
- Differential equations
- Matrix manipulations
- Integration techniques

²² Rafael Pardo Guardiola. Monte carlo methods in quantum many-body theories. 1998. URL <https://iol.unibo.it/course/view.php?id=39785>

Path Integral Monte Carlo

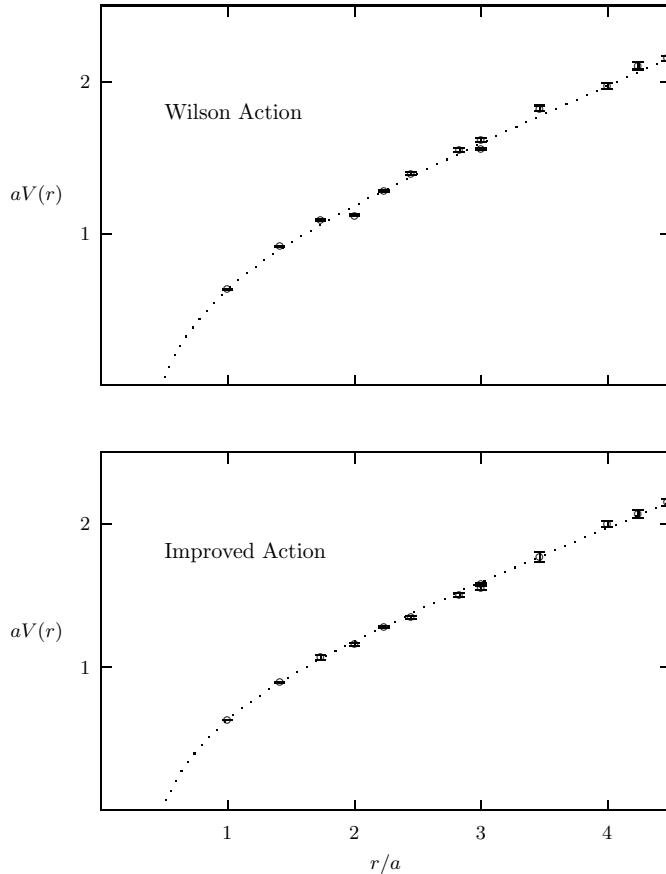
Follow the lectures *Lattice QCD for novices* of P. Lepage²³.

The lectures are for novices who are interested in learning how to do lattice QCD simulations. The work starts with simple one-dimensional quantum mechanics. Then the project jumps to quantum field theories with a simple application to gluon dynamics in QCD.

Numerics

- Derivatives
- Differential equations
- Integration techniques

²³ G. Peter Lepage. Lattice qcd for novices. 2005. URL <https://arxiv.org/abs/hep-lat/0506036>



Developing the full analysis of Lepage, the problem could be considered for a two students [collaboration project](#).

Monte Carlo Simulations for Instantons

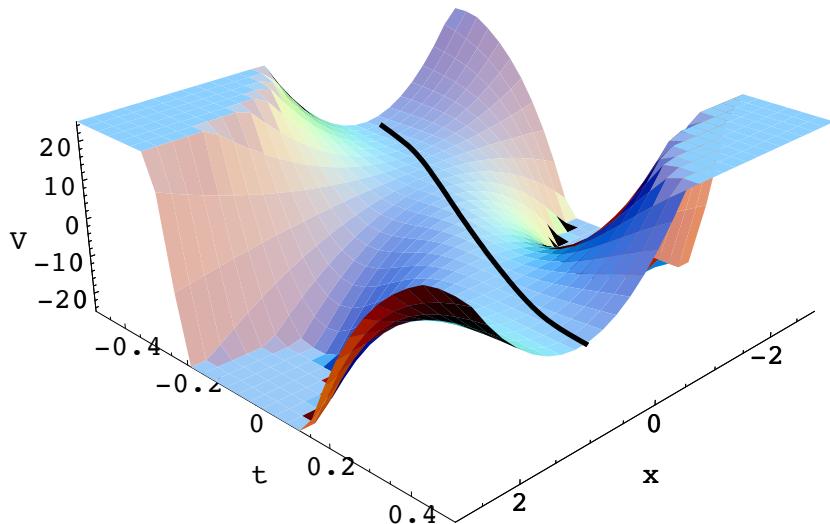
Follow the lectures *Instantons and Monte Carlo Methods in Quantum Mechanics* of T. Schäfer²⁴.

In these lectures the author describe the use of Monte Carlo simulations in understanding the role of tunneling events, instantons, in a quantum mechanical toy model. The student will study, in particular, a variety of methods that have been used in the QCD context, such as Monte Carlo simulations of the partition function, cooling and heating, the random and interacting instanton liquid model, and numerical simulations of non-Gaussian corrections to the semi-classical approximation.

Numerics

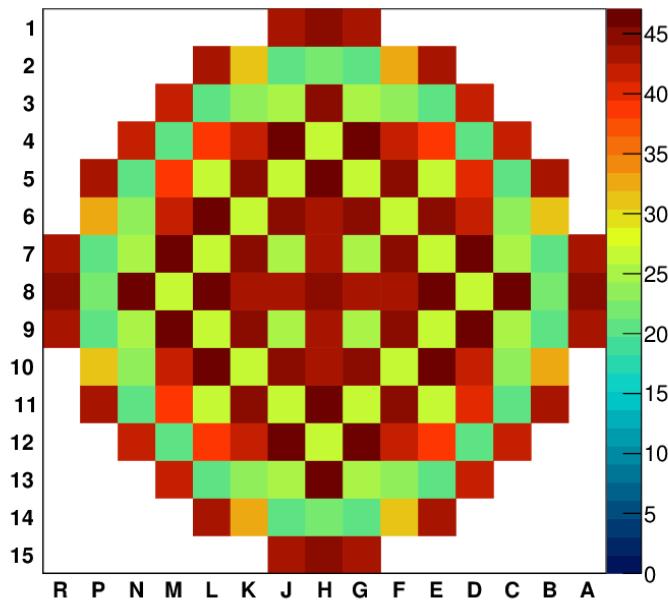
- Derivatives
- Differential equations
- Integration techniques

²⁴ Thomas Schäfer. Instantons and Monte Carlo methods in quantum mechanics. 11 2004. URL <https://arxiv.org/abs/hep-lat/0411010>



Neutron Diffusion Equation in a Reactor Core

Solution of the neutron diffusion equation in simple geometries among the many available, as for example explained in the books of S. Marguet²⁵. As a companion, for the numerical part an interesting document can be found at Ref.²⁶.



Numerics

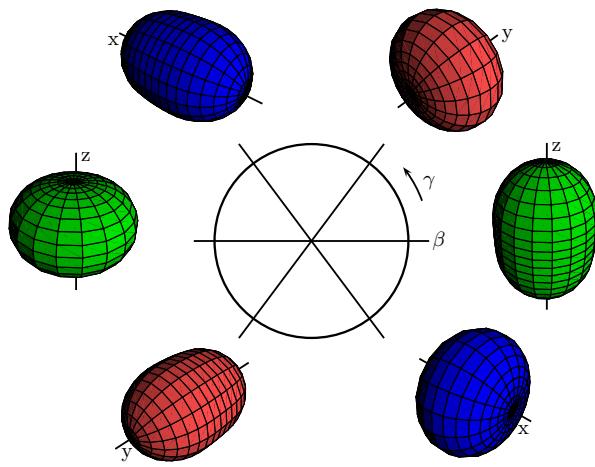
- Derivatives
- Differential equations
- Matrix manipulations

²⁵ Serge Marguet. *The Physics of Nuclear Reactors*. Springer International Publishing. URL <https://doi.org/10.1007/978-3-319-59560-3>

²⁶ Reactor Physics: Numerical Methods. URL <https://www.nuceng.ca/ep4d3/text/4-numeric-r1.pdf>

Solutions for the Bohr-Hamiltonian

The Bohr Hamiltonian, also called collective Hamiltonian, is one of the cornerstones of nuclear physics and a wealth of solutions of the associated eigenvalue equation have been proposed over more than half a century. Each particular solution is associated with a peculiar form for the $V(\beta, \gamma)$ potential. The large number and the different details of the mathematical derivation of these solutions, as well as their increased and renewed importance for nuclear structure and spectroscopy are collected in Ref. ²⁷.



Numerics

- Derivatives
- Differential equations
- Integration techniques

²⁷ Lorenzo Fortunato. Solutions of the Bohr Hamiltonian, a compendium. *Eur. Phys. J. A*, 26S1:1–30, 2005. DOI: 10.1140/epjad/i2005-07-115-8. URL <https://arxiv.org/abs/nucl-th/0411087>

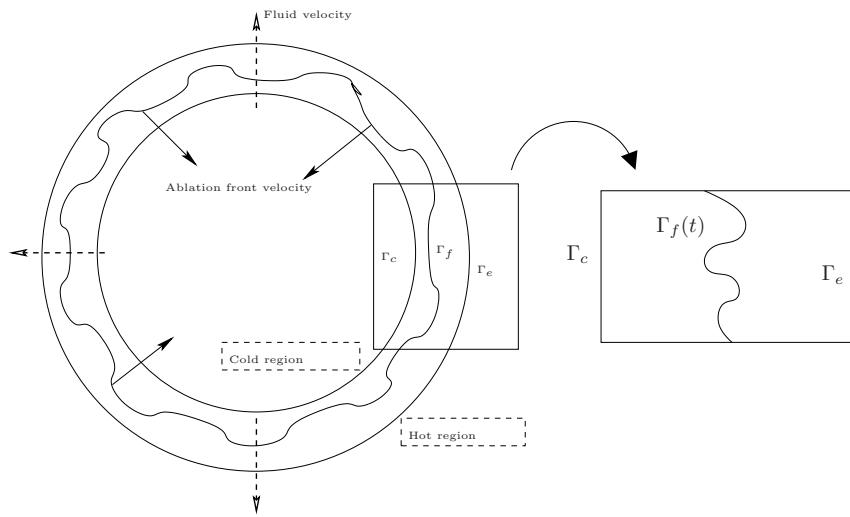
Simulations for the Inertial Confinement Fusion

Nuclear fusion reactions is the source of energy of stars, and is the major source of energy in the visible universe. In stellar objects, gravitational forces confine the fusion fuel, which reaches conditions for nuclear fusion reactions. In this lecture ²⁸, different aspects of the inertial confinement problem are discussed with numerical examples. The student is free to choose one of the topics.

Numerics

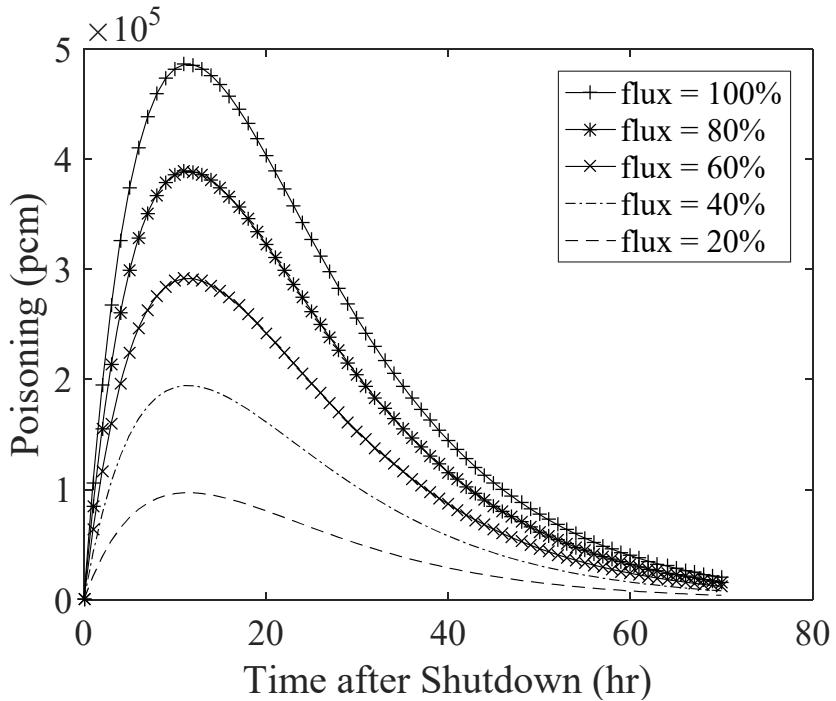
- Derivatives
- Differential equations
- Integration techniques
- Matrix manipulations

²⁸ X. Blanc and B. Després. Numerical methods for inertial confinement fusion, 2010. URL <http://smai.emath.fr/cemracs/cemracs10/PROJ/Blanc-Despres-lectures.pdf>



Bateman's Equations

After a nuclear reactor is shutdown, xenon-135, an isotope with a very high thermal neutron absorption cross-section, will build up and reduce the reactivity considerably for a while. This is known as poisoning. However, the concentration of xenon-135 would gradually decrease through decaying or absorbing neutron, making it necessary to suppress the reactivity in order to prevent the reactor to go critical or supercritical. Thus, it is important to predict the relationship between xenon poisoning and time after the reactor is shutdown to ensure the safety of the reactor. In order to make the prediction, the Bateman equations of xenon (Xe) and iodine (I), which is of the form of an Ordinary Differential Equation (ODE) system, need to be solved as illustrated in Ref. ²⁹.



Numerics

- Derivatives
- Differential equations
- Integration techniques
- Matrix manipulations

²⁹ Ding, Zechuan. Solving bateman equation for xenon transient analysis using numerical methods. *MATEC Web Conf.*, 186:01004, 2018. DOI: 10.1051/matecconf/201818601004. URL <https://doi.org/10.1051/matecconf/201818601004>

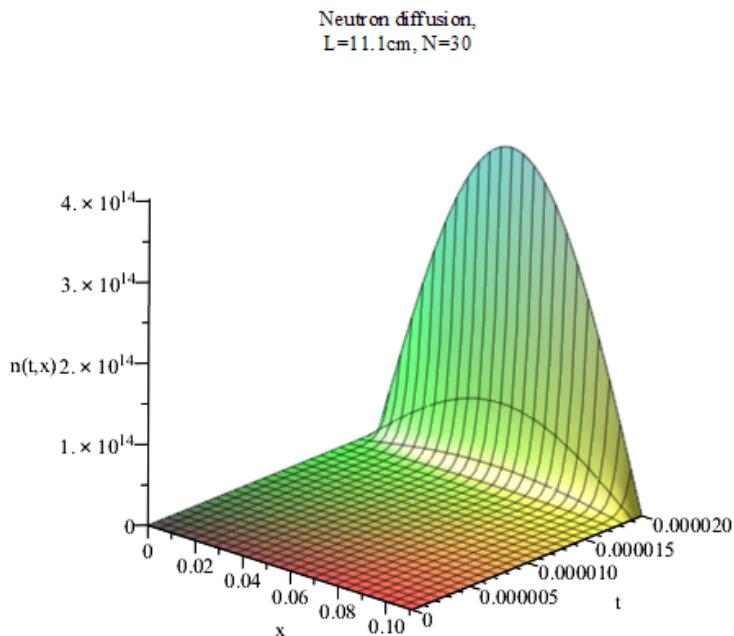
Critical Mass of Nuclear Weapons

In Ref. ³⁰ is presented the diffusion of neutrons in fissile material where collisions between free neutrons and nuclei result in the release of secondary neutrons. As the fissile material increases in size, and depending upon the mean free path for neutron travel, the radioactive material becomes critical when the total density of neutrons increases exponentially. The result is a runaway nuclear reaction that can lead to an intense explosion. The calculations proposed establishes the size at which criticality occurs. Calculations are mainly based upon simplified Dirichlet boundary conditions, whereby the neutron density is assumed to fall to zero at the edges of the core, i.e. no neutrons escape.

Numerics

- Derivatives
- Differential equations
- Integration techniques

³⁰ Graham Griffiths. Neutron diffusion. 02 2018. URL https://www.researchgate.net/publication/323035158_Neutron_diffusion



Bibliography

Talent / int course on nuclear forces. URL http://www.int.washington.edu/PROGRAMS/talent13/exercises/TALENT_exercises_Monday_1.pdf.

X. Blanc and B. Després. Numerical methods for inertial confinement fusion, 2010. URL <http://smai.emath.fr/cemracs/cemracs10/PROJ/Blanc-Despres-lectures.pdf>.

R. Cenato. Analisi e soluzione numerica dell'equazione di lippmann-schwinger, 2018. URL <https://amslaurea.unibo.it/16366/1/CenatoRebeccaTesi.pdf>.

Giampaolo Co' and Stefano De Leo. Hartree-fock and random phase approximation theories in a many-fermion solvable model. *Modern Physics Letters A*, 30(36):1550196, Nov 2015. ISSN 1793-6632. DOI: [10.1142/S0217732315501965](https://doi.org/10.1142/S0217732315501965). URL <http://dx.doi.org/10.1142/S0217732315501965>.

Giampaolo Co' and Stefano De Leo. Analytical and numerical analysis of the complete lipkin-meshkov-glick hamiltonian. *International Journal of Modern Physics E*, 27(05):1850039, May 2018. ISSN 1793-6608. DOI: [10.1142/S0218301318500398](https://doi.org/10.1142/S0218301318500398). URL <http://dx.doi.org/10.1142/S0218301318500398>.

F. Convenga. Il deutone in teoria effettiva pionless, 2015. URL http://www.infn.it/thesis/thesis_dettaglio.php?tid=8962.

Ding, Zechuan. Solving bateman equation for xenon transient analysis using numerical methods. *MATEC Web Conf.*, 186:01004, 2018. DOI: [10.1051/matecconf/201818601004](https://doi.org/10.1051/matecconf/201818601004). URL <https://doi.org/10.1051/matecconf/201818601004>.

J. Dobaczewski. URL https://www.fuw.edu.pl/~dobaczew/TALENT_DFT_2016/manuale_hf.pdf.

Repository for LNP936. URL <https://github.com/ManyBodyPhysics/LectureNotesPhysics>.

- Lorenzo Fortunato. Solutions of the Bohr Hamiltonian, a compendium. *Eur. Phys. J. A*, 26S1:1–30, 2005. doi: 10.1140/epjad/i2005-07-115-8. URL <https://arxiv.org/abs/nucl-th/0411087>.
- Graham Griffiths. Neutron diffusion. 02 2018. URL https://www.researchgate.net/publication/323035158_Neutron_diffusion.
- Rafael Pardo Guardiola. Monte carlo methods in quantum many-body theories. 1998. URL <https://iol.unibo.it/course/view.php?id=39785>.
- Heiko Hergert, Scott K. Bogner, Justin G. Lietz, Titus D. Morris, Samuel J. Novario, Nathan M. Parzuchowski, and Fei Yuan. *In-Medium Similarity Renormalization Group Approach to the Nuclear Many-Body Problem*. Springer International Publishing, 2017. URL https://doi.org/10.1007/978-3-319-53336-0_10.
- M. Hjorth-Jensen. Nuclear structure from nuclei to neutron stars. URL http://www.int.washington.edu/NNPSS/2000_talks/jensen.pdf.
- M. Hjorth-Jensen, M. P. Lombardo, and U. van Kolck. An advanced course in computational nuclear physics. URL <https://github.com/NuclearTalent/ManyBody2018/blob/master/doc/Literature/LNP936.pdf>.
- R. Landau. Integral equations for qm scattering, a. URL http://sites.science.oregonstate.edu/~landaur/Books/CPbook/eBook/Lectures/Modules/IntEqn_Scatt/IntEqn_Scatt.html.
- R. Landau. Integral equations for qm bound states, b. URL http://sites.science.oregonstate.edu/~landaur/Books/CPbook/eBook/Lectures/Modules/IntEqn_Bound/IntEqn_Bound.html.
- G. Peter Lepage. How to renormalize the schrodinger equation, 1997. URL <https://iol.unibo.it/course/view.php?id=39785>.
- G. Peter Lepage. Lattice qcd for novices. 2005. URL <https://arxiv.org/abs/hep-lat/0506036>.
- Serge Marguet. *The Physics of Nuclear Reactors*. Springer International Publishing. URL <https://doi.org/10.1007/978-3-319-59560-3>.
- Reactor Physics: Numerical Methods. URL <https://www.nuceng.ca/ep4d3/text/4-numeric-r1.pdf>.
- R. Robert. Ect* doctoral training program, 2017. URL http://chimera.roma1.infn.it/OMAR/ECTSTAR_DTP/roth/.

R. Romano. A generalization of the lipkin model as a testing ground for many-body theories, 2018. URL <http://www0.mi.infn.it/~jroca/doc/thesis/thesis-riccardo-romano-magistrale.pdf>.

Irina Sagert, Matthias Hempel, Carsten Greiner, and Jurgen Schaffner-Bielich. Compact stars for undergraduates. *Eur. J. Phys.*, 27:577–610, 2006. DOI: 10.1088/0143-0807/27/3/012. URL <https://arxiv.org/abs/astro-ph/0506417>.

Thomas Schäfer. Instantons and Monte Carlo methods in quantum mechanics. 11 2004. URL <https://arxiv.org/abs/hep-lat/0411010>.

R. Schavilla. Nustec class notes. URL <https://indico.fnal.gov/event/8047/material/0/0>.

Talent 3 school Codes. URL <https://wikihost.nscl.msu.edu/TalentDFT/doku.php?id=codes>.

Talent 3 school Projects. URL <https://wikihost.nscl.msu.edu/TalentDFT/doku.php?id=projects>.

Richard R. Silbar and Sanjay Reddy. Neutron stars for undergraduates. *American Journal of Physics*, 72(7):892–905, 2004. DOI: 10.1119/1.1703544. URL <https://arxiv.org/abs/nucl-th/0309041>.

Ji Xiangdong. Phys 741 - lecture notes, 2012. URL <https://www.physics.umd.edu/courses/Phys741/xji/chapter6.pdf>.