Algorithms: MTH 524
Department of Mathematics and Statistics
Indian Institute of Technology - Kanpur March 25, 2015

Problem Set 3

Exercises are for extra practice and should not be turned in.

Exercise 1 *30.1-2 from CLRS :* Another way to evaluate a polynomial $A(x)$ of degree-bound n at a given point x_0 is to divide $A(x)$ by the polynomial $(x - x_0)$, obtaining a quotient polynomial $q(x)$ of degree-bound $n-1$ and a remainder r, such that

$$A(x) = q(x)(x - x_0) + r$$

Clearly, $A(x_0) = r$. Show how to compute the remainder r and the coefficients of $q(x)$ in time $\Theta(n)$ from x_0 and the coefficients of A.

Exercise 2 *30.2-4 from CLRS :* Write pseudocode to compute DFT^{-1} in $\Theta(n \lg n)$ time.

Exercise 3 *30.2-5 from CLRS :* Describe the generalization of the FFT procedure to the case in which n is a power of 3. Give a recurrence for the running time, and solve the recurrence.

Exercise 4 *33.1-1 from CLRS :* Prove that if $p_1 \times p_2$ is positive, then vector p_1 is clockwise from vector p_2 with respect to the origin $(0, 0)$ and that if this cross product is negative, then p_1 is counterclockwise from p_2.

Exercise 5 *33.1-4 from CLRS :* Show how to determine in $O(n^2 \lg n)$ time whether any three points in a set of n points are collinear.

Exercise 6 *33.1-8 from CLRS :* Show how to compute the area of an n-vertex simple, but not necessarily convex, polygon in $\Theta(n)$ time.

Exercise 7 *33.2-1 from CLRS :* Show that a set of n line segments may contain $\Theta(n^2)$ intersections.

Exercise 8 *33.2-4 from CLRS :* Give an $O(n \lg n)$-time algorithm to determine whether an n-vertex polygon is simple.

Exercise 9 *33.2-5 from CLRS :* Give an $O(n \lg n)$-time algorithm to determine whether two simple polygons with a total of n vertices intersect.

Exercise 10 *33.2-6 from CLRS :* A disk consists of a circle plus its interior and is represented by its center point and radius. Two disks intersect if they have any point in common. Give an $O(n \lg n)$- time algorithm to determine whether any two disks in a set of n intersect.

Exercise 11 *33.3-1 from CLRS :* Prove that in the procedure GRAHAM-SCAN, points p_1 and p_m must be vertices of CH(Q).

Exercise 12 *33.3-3 from CLRS :* Given a set of points Q, prove that the pair of points farthest from each other must be vertices of CH(Q).

The following problem is due **Friday, April 17** at **11:59 PM**.
Steps you should follow :

1. You should do all your work for this problem set in a directory called GroupID-PS3 where ID stands for your group number. For example, if you are homework group 05, then the directory name will be Group05-PS3; if you are homework group 13, then the directory name will be Group13-PS3.

2. Upon completion of the work, once you are ready for submission, go the directory that contains the directory GroupID-PS3 and compress it using following command at the xterm/terminal command prompt

   ```
   $ tar cvfz GroupID-PS3.tgz GroupID-PS3
   ```

   which will create a file called GroupID-PS3.tgz .

3. Attach this file in an email with subject GroupID-PS3 and send it to akasha@iitk.ac.in by 11.59 pm on April 17. Late submission will not get any credit.

Recall the problem A in *Problem Set 1.* As a continuation of that problem, let $R = \{r^1, \ldots, r^K\}$ be a given set of points in C, $P = \{p^1, \ldots, p^N\}$ be a given set of points on the boundary of $C_{(0,0,0)}$ and $Q = \{q^1, \ldots, q^M\}$ be a given set of points on $S_{(0,0,0)}$. Given a set of K real numbers, say $\Gamma = \{\gamma^1, \ldots, \gamma^K\}$, define, for $q \in \mathbb{R}^3$,

$$\mathcal{F}_\ell^\Gamma(q) = \sum_{\{j | r^j \in R_\ell\}} \frac{\sin|q - r^j|}{|q - r^j|}\gamma^j.$$

Now, for each ℓ, let the vector $\Psi_\ell = [\psi_\ell^1, \ldots, \psi_\ell^N]^T$ be the *least-squares solution* of the following system of linear equations:

$$\sum_{j=1}^{N} \frac{\sin|(q^i + \ell/L) - (p^j + \ell/L)|}{|(q^i + \ell/L) - (p^j + \ell/L)|}\psi_\ell^j = F_\ell^\Gamma(q^i + \frac{\ell}{L}), \quad i = 1, \ldots, M.$$

In other words, Ψ_ℓ solves

$$\mathcal{A}^T\mathcal{A}\Psi_\ell = \mathcal{A}^T f_\ell^\Gamma,$$

where

$$\mathcal{A}_{ij} = \frac{\sin|q^i - p^j|}{|q^i - p^j|}$$

and

$$(f_\ell^\Gamma)_i = \mathcal{F}_\ell^\Gamma(q^i + \frac{\ell}{L}).$$

1. Write an algorithm to **SOLVE-LS(**A**,**b**)** that solves the linear system $Ax = b$ and returns the solution x.
2. Given L and the sets R, P, Q and Γ, write **SOLVE(**L**,** R**,** P**,** Q**,** Γ**)** to find Ψ_ℓ for all ℓ, that uses your **SOLVE-LS**, **PARTITION-ALL** and **BUILD-MATRIX** routines.
3. Implement your **SOLVE** with function prototype

```
bool solve
(int L, char* inputFileR, char* inputFileP, char* inputFileQ,
 char* inputFileGamma, char* outputFile);
```

   Input and output file formats remain same as given in *Problem Set 1*.