Research Statement

Indranil Ghosh

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1 Introduction

My research interests are primarily in modelling various complex phenomena in terms of dynamical systems. These phenomena range from piecewise-smooth systems to models of neurons among other applications. My interests include developing theoretical and numerical tools for the applications with a view towards further understanding the models. Certainly, a career goal is to make substantial contributions not only in an academic setting but also in an industrial context. It is hoped that my contribution will not only lead to a deeper understanding of applications but also that I am able to use this expertise to solve concrete problems arising in industry.

In this research statement I first summarize my current research abilities and academic achievements. This is followed by a research plan that is closely linked to my current research. A strength of this application, however, is that I have a relatively broad background and secondary research interests, which lends some flexibility to any research plan and facilitates collaborations.

2 Primary Research Interest

Piecewise linear maps provide a useful tool to explore complex aspects of dynamical systems, such as chaos. My research mainly focuses on the *border-collision normal form* given by

$$f_\xi(x,y) = \begin{cases} \big(\tau_L x + y + 1, -\delta_L x\big), & x \le 0, \\ \big(\tau_R x + y + 1, -\delta_R x\big), & x \ge 0. \end{cases} \tag{1}$$

This form models the dynamics created in *border-collision bifurcations*, where a fixed point of a piecewise-smooth map collides with a switching manifold [5, 23, 24]. In this context, piecewise linear maps have been used to explain bifurcations in diverse applications such as power converters [26] and mechanical systems with stick/slip friction [25].

The dynamics of the border-collision normal form can be very rich even in two dimensions, (cf. [2, 14, 22]). Banerjee, Yorke, and Grebogi in their pioneering paper [3] identified an open parameter region, also known as the region of *robust chaos* throughout which the two-dimensional border-collision normal form has a chaotic attractor [15]. Robust chaos is a desirable phenomenon in chaos-based cryptography [17], because periodic windows in the key space can be taken over by an attacker to break the encryption [1]. In addition, robust chaos can be useful to devices that have advantageous functional characteristics when operated in a chaotic regime, for example, power converters [4], optical resonators [19], and energy harvestors [18].

3 Secondary Research Interests

The study of neurons as dynamical systems has been a current topic of interest among physicists, applied mathematicians, and neuroscientists. Being a physicist by training and a Ph.D. in applied mathematics, I have dabbled in this topic in the past few years with my colleagues across different universities. My main focus area of this research is looking into the plethora of rich dynamics that arise in an ensemble of neurons arranged in specific topologies. I specifically study various coupling strategies (pairwise and higher-order interactions for example) among the neurons (both map-based models and continuous systems) and how these drive synchronization among these neurons.

4 Research outputs

My research outputs are in the form of publications and can be partitioned into those relevant to my aforementioned primary research and those stemming from my secondary interests. The main results are summarised briefly by publication.

4.1 Primary Research Output

In [11], we studied the robust chaos parameter region and applied renormalisation to partition this region by the number of connected components of a Chaotic Milnor attractor, concisely revealing previously undescribed bifurcation structure. We strengthened the existence of robust chaos throughout this region satisfying Devaney's definition of Chaos in [12]. Then in [7] we constructed a trapping region in phase space and an invariant expanding cone in tangent space, showing that the normal form exhibits a chaotic attractor throughout an open region of parameter space, generalising the construction to include the non-invertible and orientation-reversing normal form. We also applied renormalisation to the non-invertible and orientation-reversing cases in [8], exploring numerically and showing how renormalisation appears to remain effective in this more general setting. Finally in [13] we identified the existence of robust chaos to the $n-$dimensional piecewise-linear map, with two pieces, each having exactly one unstable direction.

4.2 Secondary Research Output

In [6], we investigated the effects of external periodic current and electromagnetic flux on the dynamical properties of the improved *denatured* Morris-Lecar neuron model, which is a continuous system. Simultaneously my colleagues and I also looked into the effect of electromagnetic flux on a map-based neuron model called the Chialvo neuron [20], which opens up further research questions. Next, our team went on to look into the effect of noise on the coupling strength in a ring-star network of memristive Chialvo neurons [9]. We further looked into how a small heterogeneous chain of bidirectionally coupled neurons behave and synchronize [10]. We were also able to report a collection of intriguing dynamical behaviour and route to chaos in the smallest ring-star network of Chialvo neurons, however, this time connected via higher-order couplings [21].

5 Research plan

1. **Primary Research Plan 1a:** The first goal of my research plan is to verify analytically the conjectures (existence of robust chaos in noninvertible and orientation-reversing piecewise-

linear maps) in [8] and to understand in detail the dynamics of the renormalisation operator by extending the calculations in [11]. I also want to explore renormalisation schemes based on other symbolic substitution rules with a view towards explaining parameter regimes where the border-collision normal form has attractors with other numbers of components, e.g. three. The number of connected components the attractors in [11] consist of is an integer power of 2. As a first step, we have focused on the case where the map has one direction of instability for each piece of the map. However, maps with multiple directions of instability should be just as relevant, giving the possibility of so-called *wild chaos* [16], and it remains to treat these scenarios.

2. **Primary Research Plan 1b:** The dynamics of piecewise-smooth maps are extraordinarily rich because they have an extreme form of nonlinearity. I hope to prove the uniqueness of the attractor for the normal form. This is an open problem which is an important and exciting direction to look into. The uniqueness of the attractor can also be tested for the more generalised orientation-reversing and non-invertible cases.

3. **Application:** I want to apply $n-$dimensional construction as the key space for an encryption scheme [17]. This is a novel application area that needs rigour and calls for cross-disciplinary collaboration. Another interesting avenue of application is in neural networks where the robustness of chaos is an existing phenomenon. In addition, the theory of piecewise-smooth systems can be extended to study artificial neural networks with multiple switches as dynamical systems.

4. **Secondary Research plan:** I plan to implement various control strategies to better understand various neuron network models and how they perform under various adaptive behaviour. Heterogeneous neuron networks with various coupling functions (electrical, chemical, magnetic, and mean-filed) are able to exhibit useful and interesting dynamics and can be explained via bifurcation studies. Currently, researchers are going beyond studying pairwise interactions and looking into higher-order couplings between neurons which give rise to an array of intriguing phenomena. I am mostly interested in investigating "small" networks of neurons coupled under various strategies and how they synchronize. This might give rise to behaviours like "chimera" and "solitary nodes" and I envision applying various analytical and numerical techniques to detect them.

5. **Collaboration:** Certainly part of my research plan is to establish a wide network of collaborators from both experimental and theoretical disciplines. I want to bring to the table projects that I have been already involved with and also contribute to the research goals of the lab I am working in. That being said, I will be more than happy to participate in various expository activities on international platforms. On top of that, I am willing to supervise student projects and guide them toward fruition.

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