EECE5643 – Homework 2

1.

a.

```
for 1000 jobs
   average interarrival time =  9.87
   average service time .... =  7.12
   average delay ........... =  18.59
   average wait ............ =  25.72
   maximum delay ........... = 118.76
   number of jobs in the serivce node at t=400 = 7
   proportion of jobs delayed = 0.72
   server utilization = 0.72
```

b.

Maximum delay experienced: 118.76

c.

Jobs in the service node at t=400: 7

Each job i spends exactly $(c_i - a_i)$ units of time in the system from arrival a_i to departure c_i as the job i is in the system whenever $a_i \leq t < c_i$. The number of jobs in the service node at time t t is $l(t) = \sum_{i=1}^{n} [a_i \leq t < c_i]$.

Theorem 1.2.1 states:

$$\int_0^{c_n} l(t)\,dt = \sum_{i=1}^{n} (c_i - a_i)$$

Summing across all jobs:

$$\int_0^{c_n} l(t)\,dt = \int_0^{c_n} \sum_{i=1}^{n} [a_i \leq t < c_i]\,dt = \sum_{i=1}^{n} \int_0^{c_n} [a_i \leq t < c_i]\,dt = \sum_{i=1}^{n} (c_i - a_i)$$

d.

Proportion of delayed jobs: 0.72

Server Utilization: 0.72

The server is busy 72% of the time, and consequently 72% of jobs are delayed because they arrive to find the server already busy and must be queued. The fraction of jobs that arrive with a delay matches the fraction of time the server is busy, i.e., the utilization, as utilization measures the long-term fraction of time the server is not idle.

2.

```
for 500 jobs
   average interarrival time =  4.08
   average service time .... =  3.03
   average delay ........... =  4.54
   average wait ............ =  7.57
   server utilization   = 0.74
   traffic intensity    = 0.74
```

a.

Average service time: 3.03
Server utilization: 0.74
Traffic intensity: 0.74

b.

For an incoming job i, if the arrival time a_i is less than the completion time c_{i-1} of the previous job, the job is delayed because it must wait for the previous job to finish executing.

$$\text{If } a_i < c_{i-1} \quad \text{(job } i \text{ arrives during the service of job } i-1\text{)}: \quad s_i = c_i - c_{i-1}$$

For an incoming job i, if the arrival time a_i is greater than or equal to the completion time c_{i-1} of the previous job, then job i starts executing immediately when it arrives.

$$\text{If } a_i \geq c_{i-1} \quad \text{(job } i \text{ arrives after job } i-1 \text{ has finished)}: \quad s_i = c_i - a_i$$

Combining these two cases:

$$s_i = c_i - \max\{a_i, c_{i-1}\}$$

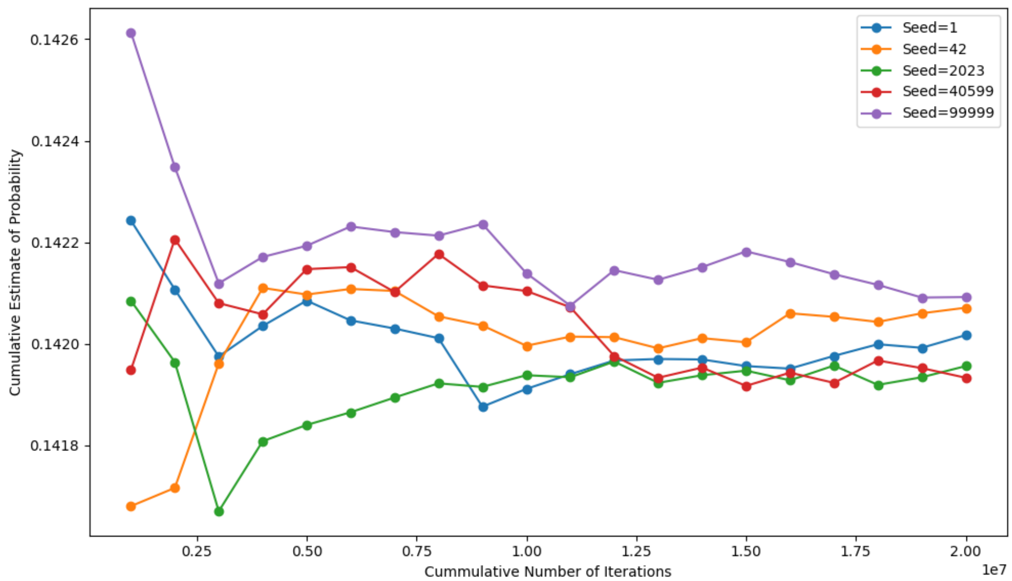
3.

a.

The event is the sum of the two dice or the two up faces equals 7. The event is $\{X + Y = 7\}$ where X is the outcome of the first die and Y is the outcome of the second die.

The random components are the individual die rolls that produce the random variables X and Y. Each die roll follows the specified unfair probability distribution.

Seed	$P(X + Y = 7)$
1	0.14224
42	0.14168
2023	0.14209
40599	0.14195
99999	0.14261



b.

Possible ways to get $\{X+Y=7\}$ with two dice:

$$(1,6),\ (6,1),\ (2,5),\ (5,2),\ (3,4),\ (4,3)$$

$$P(1)=\frac{1}{13},\quad P(6)=4\cdot\frac{1}{13}=\frac{4}{13},\quad P(2)=P(3)=P(4)=P(5)=2\cdot\frac{1}{13}=\frac{2}{13}$$

$$\begin{aligned}P(X+Y=7)&=P(1)\,P(6)+P(6)\,P(1)+P(2)\,P(5)+P(5)\,P(2)+P(3)\,P(4)+P(4)\,P(3)\\&=\left(\frac{1}{13}\right)\left(\frac{4}{13}\right)+\left(\frac{4}{13}\right)\left(\frac{1}{13}\right)+\left(\frac{2}{13}\right)\left(\frac{2}{13}\right)+\left(\frac{2}{13}\right)\left(\frac{2}{13}\right)+\left(\frac{2}{13}\right)\left(\frac{2}{13}\right)+\left(\frac{2}{13}\right)\left(\frac{2}{13}\right)\\&=2\times\frac{1\cdot 4}{13\cdot 13}+2\times\frac{2\cdot 2}{13\cdot 13}+2\times\frac{2\cdot 2}{13\cdot 13}\\&=\frac{8}{169}+\frac{8}{169}+\frac{8}{169}=\frac{24}{169}\approx 0.1420\end{aligned}$$

4.

a.

The event is the distance between two randomly chosen points on the circumference of a circle or the chord between those two points is greater than the radius. The event is $(\text{distance}(\theta_1,\theta_2)>\rho)$ where the distance or length of the chord is $2\rho\sin\left(\frac{|\theta_1-\theta_2|}{2}\right)$.

The random components are the positions of the two individual points on the circumference of a circle. Angle-based notation is θ_1 and θ_2. Cartesian-based notation is (X_1,Y_1) and (X_2,Y_2).

b.

The probability that the distance between two random points on the circumference of a circle is greater than the circle's radius does not depend on the circle's radius ρ.

Seed	$P(\text{distance}(\theta_1, \theta_2) > \rho)$
1	0.66671
42	0.66659
2023	0.66686
40599	0.66770
99999	0.66653

$$
\begin{aligned}
2\rho \sin\left(\frac{|\theta_1 - \theta_2|}{2}\right) &> \rho \\
\Rightarrow 2\sin\left(\frac{|\theta_1 - \theta_2|}{2}\right) &> 1 \\
\Rightarrow \sin\left(\frac{|\theta_1 - \theta_2|}{2}\right) &> \frac{1}{2}
\end{aligned}
$$

The expression only depends on the angles θ_1 and θ_2 and does not depend on ρ.