EECE7398 – Homework 1

Problem 1.

Input (x)	Label (y)	SVM Hinge Loss	SoftMax Loss
[1.52, 2.63, 5.37, 4.94]	Cat	0	0
[8.87, 1.25, 4.49, 0.12]	Cat	71.53	44.33
[3.22, 4.63, 3.55, 5.41]	Dog	64.91	60.98
[1.38, 0.63, 2.90, 8.52]	Horse	158.73	103.04

$$x_1 = \begin{bmatrix} 1.52 \\ 2.63 \\ 5.37 \\ 4.94 \end{bmatrix}, \quad y = \text{cat}$$

$$z_1 = W \cdot x_1 = \begin{bmatrix} -0.57 & 1.24 & -3.37 & 6.43 \\ -5.53 & -1.13 & -8.05 & 3.21 \\ 4.23 & 0.98 & -2.53 & -7.67 \\ -2.31 & -1.84 & 6.93 & -8.66 \end{bmatrix} \cdot \begin{bmatrix} 1.52 \\ 2.63 \\ 5.37 \\ 4.94 \end{bmatrix} = \begin{bmatrix} 16.0621 \\ -38.7486 \\ -42.4689 \\ -13.9217 \end{bmatrix}$$

SoftMax Loss

$$L = -\log\left(\frac{e^{z_y}}{\sum_j e^{z_j}}\right)$$

$$L = -\log\left(\frac{e^{z_{\text{cat}}}}{\sum_{j\in\{\text{cat, dog, cow, horse}\}} e^{z_j}}\right)$$

$$= -\log\left(\frac{e^{16.0621}}{e^{16.0621} + e^{-38.7486} + e^{-42.4689} + e^{-13.9217}}\right)$$

$$= -\log(1)$$

$$= 0$$

SVM Hinge Loss

$$L = \sum_{j\neq y} \max(0,\, z_j - z_y + 1)$$

$$L = \sum_{j\neq \text{cat}} \max(0,\, z_j - z_{\text{cat}} + 1)$$

$$= \max(0,\, z_{\text{dog}} - z_{\text{cat}} + 1) + \max(0,\, z_{\text{cow}} - z_{\text{cat}} + 1) + \max(0,\, z_{\text{horse}} - z_{\text{cat}} + 1)$$

$$= \max(0,\, -38.7486 - 16.0621 + 1) + \max(0,\, -42.4689 - 16.0621 + 1) + \max(0,\, -13.9217 - 16.0621 + 1)$$

$$= 0$$

$$x_2 = \begin{bmatrix} 8.87 \\ 1.25 \\ 4.49 \\ 0.12 \end{bmatrix}, \quad y = \text{cat}$$

$$z_2 = W \cdot x_2 = \begin{bmatrix} -0.57 & 1.24 & -3.37 & 6.43 \\ -5.53 & -1.13 & -8.05 & 3.21 \\ 4.23 & 0.98 & -2.53 & -7.67 \\ -2.31 & -1.84 & 6.93 & -8.66 \end{bmatrix} \cdot \begin{bmatrix} 8.87 \\ 1.25 \\ 4.49 \\ 0.12 \end{bmatrix} = \begin{bmatrix} -17.8656 \\ -86.2379 \\ 26.466 \\ 7.2968 \end{bmatrix}$$

SoftMax Loss

$$\begin{aligned} L &= -\log\left(\frac{e^{z_y}}{\sum_j e^{z_j}}\right) \\ L &= -\log\left(\frac{e^{z_{\text{cat}}}}{\sum_{j\in\{\text{cat, dog, cow, horse}\}} e^{z_j}}\right) \\ &= -\log\left(\frac{e^{-17.8656}}{e^{-17.8656} + e^{-86.2379} + e^{26.466} + e^{7.2968}}\right) \\ &= -\log\left(5.59 \times 10^{-20}\right) \\ &\approx 44.33 \end{aligned}$$

SVM Hinge Loss

$$\begin{aligned} L &= \sum_{j\neq y} \max\left(0,\ z_j - z_y + 1\right) \\ L &= \sum_{j\neq \text{cat}} \max\left(0,\ z_j - z_{\text{cat}} + 1\right) \\ &= \max\left(0,\ z_{\text{dog}} - z_{\text{cat}} + 1\right) + \max\left(0,\ z_{\text{cow}} - z_{\text{cat}} + 1\right) + \max\left(0,\ z_{\text{horse}} - z_{\text{cat}} + 1\right) \\ &= \max\left(0,\ -86.2379 + 17.8656 + 1\right) + \max\left(0,\ 26.466 + 17.8656 + 1\right) + \max\left(0,\ 7.2968 + 17.8656 + 1\right) \\ &= 0 + 45.3516 + 26.1824 \\ &= 71.534 \end{aligned}$$

$$x_3 = \begin{bmatrix} 3.22 \\ 4.63 \\ 3.55 \\ 5.41 \end{bmatrix}, \quad y = \text{dog}$$

$$z_3 = W \cdot x_3 = \begin{bmatrix} -0.57 & 1.24 & -3.37 & 6.43 \\ -5.53 & -1.13 & -8.05 & 3.21 \\ 4.23 & 0.98 & -2.53 & -7.67 \\ -2.31 & -1.84 & 6.93 & -8.66 \end{bmatrix} \cdot \begin{bmatrix} 3.22 \\ 4.63 \\ 3.55 \\ 5.41 \end{bmatrix} = \begin{bmatrix} 26.7286 \\ -34.2499 \\ -32.3182 \\ -38.2065 \end{bmatrix}$$

SoftMax Loss

$$\begin{aligned} L &= -\log\left(\frac{e^{z_y}}{\sum_j e^{z_j}}\right) \\ L &= -\log\left(\frac{e^{z_{\text{dog}}}}{\sum_{j\in\{\text{cat, dog, cow, horse}\}} e^{z_j}}\right) \\ &= -\log\left(\frac{e^{-34.2499}}{e^{26.7286} + e^{-34.2499} + e^{-32.3182} + e^{-38.2065}}\right) \\ &= -\log\left(3.29 \times 10^{-27}\right) \\ &\approx 60.98 \end{aligned}$$

SVM Hinge Loss

$$
\begin{aligned}
L &= \sum_{j \neq y} \max\left(0,\ z_j - z_y + 1\right) \\
L &= \sum_{j \neq \text{dog}} \max\left(0,\ z_j - z_{\text{dog}} + 1\right) \\
&= \max\left(0,\ z_{\text{cat}} - z_{\text{dog}} + 1\right) + \max\left(0,\ z_{\text{cow}} - z_{\text{dog}} + 1\right) + \max\left(0,\ z_{\text{horse}} - z_{\text{dog}} + 1\right) \\
&= \max\left(0,\ 26.7286 + 34.2499 + 1\right) + \max\left(0,\ -32.3182 + 34.2499 + 1\right) + \max\left(0,\ -38.2065 + 34.2499 + 1\right) \\
&= 61.9785 + 2.9317 + 0 \\
&= 64.9102
\end{aligned}
$$

$$
x_4 = \begin{bmatrix} 1.38 \\ 0.63 \\ 2.90 \\ 8.52 \end{bmatrix}, \quad y = \text{horse}
$$

$$
z_4 = W \cdot x_4 = \begin{bmatrix} -0.57 & 1.24 & -3.37 & 6.43 \\ -5.53 & -1.13 & -8.05 & 3.21 \\ 4.23 & 0.98 & -2.53 & -7.67 \\ -2.31 & -1.84 & 6.93 & -8.66 \end{bmatrix} \cdot \begin{bmatrix} 1.38 \\ 0.63 \\ 2.90 \\ 8.52 \end{bmatrix} = \begin{bmatrix} 45.0052 \\ -4.3391 \\ -66.2306 \\ -58.0332 \end{bmatrix}
$$

SoftMax Loss

$$
\begin{aligned}
L &= -\log\left(\frac{e^{z_y}}{\sum_j e^{z_j}}\right) \\
L &= -\log\left(\frac{e^{z_{\text{horse}}}}{\sum_{j \in \{\text{cat, dog, cow, horse}\}} e^{z_j}}\right) \\
&= -\log\left(\frac{e^{-58.0332}}{e^{45.0052} + e^{-4.3391} + e^{-66.2306} + e^{-58.0332}}\right) \\
&= -\log\left(1.78 \times 10^{-45}\right) \\
&\approx 103.04
\end{aligned}
$$

SVM Hinge Loss

$$
\begin{aligned}
L &= \sum_{j \neq y} \max\left(0,\ z_j - z_y + 1\right) \\
L &= \sum_{j \neq \text{horse}} \max\left(0,\ z_j - z_{\text{horse}} + 1\right) \\
&= \max\left(0,\ z_{\text{cat}} - z_{\text{horse}} + 1\right) + \max\left(0,\ z_{\text{dog}} - z_{\text{horse}} + 1\right) + \max\left(0,\ z_{\text{cow}} - z_{\text{horse}} + 1\right) \\
&= \max\left(0,\ 45.0052 + 58.0332 + 1\right) + \max\left(0,\ -4.3391 + 58.0332 + 1\right) + \max\left(0,\ -66.2306 + 58.0332 + 1\right) \\
&= 104.0384 + 54.6941 + 0 \\
&= 158.7325
\end{aligned}
$$