1.

a.

To determine the number of multiplications/divisions required for LU factorization of an $n \times n$ matrix A using Gaussian elimination:

Form the multipliers at pivot k: For each pivot $k = 1, 2, \ldots, n-1$, we eliminate entries below the pivot a_{kk}. For each row $i = k+1, \ldots, n$:

$$m_{ik} = \frac{a_{ik}}{a_{kk}}.$$

Hence, the number of divisions at step k is

$$\text{Divisions at step } k = (n-k).$$

Update the trailing submatrix at pivot k: For each $i = k+1, \ldots, n$ and each $j = k+1, \ldots, n$:

$$a_{ij} \leftarrow a_{ij} - m_{ik}\, a_{kj}.$$

This update requires one multiplication per entry of the $(n-k) \times (n-k)$ trailing submatrix, i.e.,

$$\text{Multiplications at step } k = (n-k)^2.$$

Sum over all pivots:

$$\text{Total divisions} = \sum_{k=1}^{n-1} (n-k) = \frac{n(n-1)}{2}.$$

$$\text{Total multiplications} = \sum_{k=1}^{n-1} (n-k)^2 = \frac{(n-1)n(2n-1)}{6}.$$

b.

Forward substitution: $Lc = b$ with $\ell_{ii} = 1$. The i-th equation is

$$c_i = b_i - \sum_{j=1}^{i-1} \ell_{ij}\, c_j, \qquad i = 1, 2, \ldots, n.$$

For row i, the sum has $(i-1)$ products, so

$$\text{Multiplications at row } i = (i-1), \qquad \text{Divisions at row } i = 0 \quad (\text{since } \ell_{ii} = 1).$$

Summing over all rows:

$$\text{Multiplications for } Lc = b = \sum_{i=1}^{n} (i-1) = \frac{n(n-1)}{2}, \qquad \text{Divisions for } Lc = b = 0.$$

Back substitution: $Ux = c$. The i-th equation (solved for $i = n, \dots, 1$) is

$$x_i = \frac{c_i - \sum_{j=i+1}^{n} u_{ij}\, x_j}{u_{ii}}.$$

For row i, the sum has $(n-i)$ products, plus one division by u_{ii}:

$$\text{Multiplications at row } i = (n-i), \qquad \text{Divisions at row } i = 1.$$

Summing over all rows:

$$\text{Multiplications for } Ux = c = \sum_{i=1}^{n}(n-i) = \frac{n(n-1)}{2}, \qquad \text{Divisions for } Ux = c = n.$$

Totals:

$$\text{Total multiplications} = \frac{n(n-1)}{2} + \frac{n(n-1)}{2} = n(n-1),$$

$$\text{Total divisions} = 0 + n = n.$$

Solving one system costs $n(n-1)$ multiplications and n divisions.

c.

Given k linear systems with the same coefficient matrix:

$$Ax = b_i, \qquad i = 1, 2, \dots, k.$$

Assuming we count only multiplications/divisions. Since A is fixed and only b_i changes, we can factor A into LU once and then reuse the factorization for all systems.

Cost of LU factorization (from part (a)):

$$\text{Multiplications} = \frac{(n-1)n(2n-1)}{6}, \qquad \text{Divisions} = \frac{n(n-1)}{2}.$$

Cost of solving one system (from part (b)):

$$\text{Multiplications} = n(n-1), \qquad \text{Divisions} = n.$$

Cost for k systems using LU once:

$$\text{Multiplications} = \frac{(n-1)n(2n-1)}{6} + k\, n(n-1),$$

$$\text{Divisions} = \frac{n(n-1)}{2} + k\, n.$$

Cost with Gaussian elimination done separately: For each system:

$$\text{Multiplications} = \frac{(n-1)n(2n-1)}{6} + n(n-1), \qquad \text{Divisions} = \frac{n(n-1)}{2} + n.$$

For k systems:

$$\text{Multiplications} = k\left(\frac{(n-1)n(2n-1)}{6} + n(n-1)\right),$$

$$\text{Divisions} = k\left(\frac{n(n-1)}{2} + n\right).$$

For $k = 1$, both LU factorization and Gaussian elimination require the same number of multiplications/divisions. For $k > 1$, LU factorization is more efficient, because the factorization cost is only incurred once:

$$\text{Savings in multiplications } = (k-1)\,\frac{(n-1)n(2n-1)}{6}, \qquad \text{Savings in divisions } = (k-1)\,\frac{n(n-1)}{2}.$$

2.

$$A = \begin{bmatrix} -1 & 2 & -2 \\ -3 & 8 & -3 \\ 2 & 4 & 5 \end{bmatrix}, \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Choosing pivot $d_1 = a_{11} = -1$

$$\hat{u}_{12} = \frac{a_{12}}{d_1} = \frac{2}{-1} = -2, \qquad \hat{u}_{13} = \frac{a_{13}}{d_1} = \frac{-2}{-1} = 2$$

$$m_{21} = \frac{a_{21}}{d_1} = \frac{-3}{-1} = 3, \qquad m_{31} = \frac{a_{31}}{d_1} = \frac{2}{-1} = -2$$

$$R_2 \leftarrow R_2 - m_{21}R_1 = R_2 - 3R_1 = \begin{bmatrix} -3 & 8 & -3 \end{bmatrix} - 3\begin{bmatrix} -1 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \end{bmatrix}$$

$$R_3 \leftarrow R_3 - m_{31}R_1 = R_3 - (-2)R_1 = \begin{bmatrix} 2 & 4 & 5 \end{bmatrix} + 2\begin{bmatrix} -1 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 8 & 1 \end{bmatrix}$$

Choosing pivot $d_2 = a_{22} = 2$

$$\hat{u}_{23} = \frac{a_{23}}{d_2} = \frac{3}{2}, \qquad m_{32} = \frac{a_{32}}{d_2} = \frac{8}{2} = 4$$

$$R_3 \leftarrow R_3 - m_{32}R_2 = \begin{bmatrix} 0 & 8 & 1 \end{bmatrix} - 4\begin{bmatrix} 0 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -11 \end{bmatrix}$$

$$U = \begin{bmatrix} -1 & 2 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & -11 \end{bmatrix}$$

Using multipliers $m_{21} = 3,\ m_{31} = -2,\ m_{32} = 4, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 4 & 1 \end{bmatrix}$

Extract the diagonal of U into D: $\quad D = \operatorname{diag}(-1,\ 2,\ -11) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -11 \end{bmatrix}$

Scale U to unit diagonal: $\quad \hat{U} = D^{-1}U = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{11} \end{bmatrix} \begin{bmatrix} -1 & 2 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & -11 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$

Using $\hat{u}_{12} = -2,\ \hat{u}_{13} = 2,\ \hat{u}_{23} = \frac{3}{2}, \quad \hat{U} = \begin{bmatrix} 1 & \hat{u}_{12} & \hat{u}_{13} \\ 0 & 1 & \hat{u}_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$

LDU decomposition of A: $\qquad A = LD\hat{U} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 4 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -11 \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$

3.

$$A \xrightarrow{r_3 \leftarrow r_3 - r_1} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \xrightarrow{P\,(r_2 \leftrightarrow r_3)} \begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix} \xrightarrow{r_3 \leftarrow r_3 + r_2} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Sequence of Row Operations:

1. $r_3 \ \leftarrow\ r_3 - r_1$
2. $r_2 \ \leftrightarrow\ r_3$ (row swap, permutation P)
3. $r_3 \ \leftarrow\ r_3 + r_2$

Let I_3 be the 3×3 identity matrix. The corresponding elementary matrices are:

$$E_1 = I_3 + (-1)e_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad E_3 = I_3 + e_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

By construction,

$$E_3 E_2 E_1 A = U$$

For Recovering A from U, the inverses follow directly from the row-operation rules:

- Swapping rows: $E^{-1} = E$.
- Scaling r_i by k: inverse is scaling by $1/k$.
- Subtracting kr_j from r_i: inverse is adding kr_j to r_i.

Thus,

$$E_1^{-1} = I_3 + e_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad E_2^{-1} = E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad E_3^{-1} = I_3 - e_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}.$$

By construction,

$$A = E_1^{-1} E_2^{-1} E_3^{-1} U$$

Recovering A from U:

1. Apply E_3^{-1} ($r_3 \leftarrow r_3 - r_2$):

$$U = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}.$$

2. Apply E_2^{-1} (swap r_2 and r_3):

$$\Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

3. Apply E_1^{-1} ($r_3 \leftarrow r_3 + r_1$):

$$r_3 = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \end{bmatrix}.$$

$$\Rightarrow A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

U is the upper-triangular matrix after all the row operations (including swaps) have been applied. By definition,

$$U = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

P is the product of the swap matrices used during elimination. Here there is only one swap, so P is that single permutation matrix:

$$P \; = \; E_2 \; = \; \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$PA = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\0 & -1 & 1\\2 & 1 & 2\end{bmatrix} = \begin{bmatrix}2 & 0 & 1\\2 & 1 & 2\\0 & -1 & 1\end{bmatrix}.$$

Gaussian elimination on PA:

Pivot $d_1 = a_{11} = 2$,

$$m_{21} = \frac{a_{21}}{d_1} = \frac{2}{2} = 1, \qquad m_{31} = \frac{a_{31}}{d_1} = \frac{0}{2} = 0$$

$$R_2 \leftarrow R_2 - m_{21}R_1 = \begin{bmatrix}2 & 1 & 2\end{bmatrix} - 1\begin{bmatrix}2 & 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 1 & 1\end{bmatrix},$$
$$R_3 \leftarrow R_3 - m_{31}R_1 = \begin{bmatrix}0 & -1 & 1\end{bmatrix} - 0\begin{bmatrix}2 & 0 & 1\end{bmatrix} = \begin{bmatrix}0 & -1 & 1\end{bmatrix}$$

Pivot $d_2 = a_{22} = 1$,

$$m_{32} = \frac{a_{32}}{d_2} = \frac{-1}{1} = -1$$

$$R_3 \leftarrow R_3 - m_{32}R_2 = \begin{bmatrix}0 & -1 & 1\end{bmatrix} - (-1)\begin{bmatrix}0 & 1 & 1\end{bmatrix} = \begin{bmatrix}0 & 0 & 2\end{bmatrix}$$

Thus $L = \begin{bmatrix}1 & 0 & 0\\m_{21} & 1 & 0\\m_{31} & m_{32} & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\0 & -1 & 1\end{bmatrix}$

$$L = \begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\0 & -1 & 1\end{bmatrix}, \quad U = \begin{bmatrix}2 & 0 & 1\\0 & 1 & 1\\0 & 0 & 2\end{bmatrix}, \quad P = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix} \quad A = \begin{bmatrix}2 & 0 & 1\\0 & -1 & 1\\2 & 1 & 2\end{bmatrix}$$

4.

To find $p \in \mathcal{P}_2(\mathbb{F}_3)$ such that $p(0) = 2,\ p(1) = 2,\ p(2) = 1.$

$$\mathbb{F}_3 = \{0, 1, 2\} \quad \text{(arithmetic mod 3)}.$$

Let $p(x) = a + bx + cx^2, \qquad a, b, c \in \mathbb{F}_3.$

Evaluating at the given points:

$$p(0) = a = 2,$$

$$p(1) = a + b + c \equiv 2 \pmod 3,$$
$$p(2) = a + 2b + 4c \equiv 1 \pmod 3.$$

Since $4 \equiv 1 \pmod 3$ and $4 \notin \mathbb{F}_3$,

$$p(2) \equiv a + 2b + c \equiv 1 \pmod 3.$$

The corresponding augmented system matrix (over $\mathbb{F}_3$) is:

$$\begin{bmatrix}1 & 0 & 0 & 2\\1 & 1 & 1 & 2\\1 & 2 & 1 & 1\end{bmatrix}.$$

A matrix is in RREF if it satisfies all of the following conditions:

1. Any rows consisting entirely of zeros appear at the bottom of the matrix.

2. In each nonzero row, the first nonzero entry (the pivot) is equal to 1.
3. Each pivot is the only nonzero entry in its column.
4. The pivot in each row appears to the right of the pivot in the row above it.

Choosing pivot $d_1 = a_{11} = 1$,

$$m_{21} = \tfrac{a_{21}}{d_1} = \tfrac{1}{1} = 1, \qquad m_{31} = \tfrac{a_{31}}{d_1} = \tfrac{1}{1} = 1.$$

$$\begin{aligned} R_2 &\leftarrow R_2 - m_{21}R_1 \\ &= \begin{bmatrix}1 & 1 & 1 \mid 2\end{bmatrix} - 1 \cdot \begin{bmatrix}1 & 0 & 0 \mid 2\end{bmatrix} \\ &= \begin{bmatrix}0 & 1 & 1 \mid 0\end{bmatrix} \end{aligned}$$

$$\begin{aligned} R_3 &\leftarrow R_3 - m_{31}R_1 \\ &= \begin{bmatrix}1 & 2 & 1 \mid 1\end{bmatrix} - 1 \cdot \begin{bmatrix}1 & 0 & 0 \mid 2\end{bmatrix} \\ &= \begin{bmatrix}0 & 2 & 1 \mid -1\end{bmatrix} \equiv \begin{bmatrix}0 & 2 & 1 \mid 2\end{bmatrix} \pmod 3. \end{aligned}$$

The transformed matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

Choosing pivot $d_2 = a_{22} = 1$,

$$m_{32} = \frac{a_{32}}{d_2} = \frac{2}{1} = 2$$

$$R_3 \leftarrow R_3 - m_{32}R_2 = \begin{bmatrix}0 & 2 & 1 \mid 2\end{bmatrix} - 2\begin{bmatrix}0 & 1 & 1 \mid 0\end{bmatrix} = \begin{bmatrix}0 & 0 & -1 \mid 2\end{bmatrix} \equiv \begin{bmatrix}0 & 0 & 2 \mid 2\end{bmatrix} \pmod 3.$$

The transformed matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

With pivot $d_3 = a_{33} = 2$, the matrix is not yet in RREF since the pivot is not 1.

Since $2^{-1} \equiv 2 \pmod 3$, scale R_3 by 2:

$$R_3 \leftarrow 2R_3 \quad \Rightarrow \quad \begin{bmatrix}0 & 0 & 2 \mid 2\end{bmatrix} \mapsto \begin{bmatrix}0 & 0 & 4 \mid 4\end{bmatrix} \equiv \begin{bmatrix}0 & 0 & 1 \mid 1\end{bmatrix} \pmod 3.$$

Thus the matrix becomes

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

Choosing pivot $d_3 = a_{33} = 1$, the matrix is not yet in RREF because column 3 has a nonzero entry above the pivot ($a_{23} = 1$).

Pivot: $d_3 = a_{33} = 1$,

$$m_{23} = \tfrac{a_{23}}{d_3} = \tfrac{1}{1} = 1.$$

$$R_2 \leftarrow R_2 - m_{23}R_3: \quad \begin{bmatrix}0 & 1 & 1 \mid 0\end{bmatrix} - \begin{bmatrix}0 & 0 & 1 \mid 1\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 \mid -1\end{bmatrix} \equiv \begin{bmatrix}0 & 1 & 0 \mid 2\end{bmatrix} \pmod 3.$$

Thus the matrix becomes

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

RREF obtained:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$a = 2, \quad b = 2, \quad c = 1 \quad \Rightarrow \quad p(x) \equiv 2 + 2x + x^2 \pmod 3.$$

Check:

$$p(0) = 2, \quad p(1) = 2 + 2 + 1 = 5 \equiv 2, \quad p(2) = 2 + 4 + 4 = 10 \equiv 1 \pmod 3.$$

$$p(x) = 2 + 2x + x^2 \pmod 3$$

5.

To find $p \in \mathcal{P}_1(\mathbb{R})$ such that $p(1) = 2$, $p'(1) = 3$.

a. Standard-basis solution $\{1, x\}$

$$p(x) = c_0 + c_1 x, \qquad p'(x) = c_1.$$

From $p(1) = 2$ and $p'(1) = 3$ we get the linear system

$$\begin{cases} c_0 + c_1 = 2, \\ c_1 = 3. \end{cases} \qquad \Longleftrightarrow \qquad \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$

With pivot $d_2 = a_{22} = 1$, the matrix is not yet in RREF because column 2 still has a nonzero entry above the pivot ($a_{12} = 1$). To fix this, we clear that entry:

$$\text{Pivot: } d_2 = a_{22} = 1, \quad m_{12} = \tfrac{a_{12}}{d_2} = \tfrac{1}{1} = 1.$$

$$R_1 \leftarrow R_1 - m_{12}R_2 \quad \Rightarrow \quad [1 \;\; 1 \mid 2] - [0 \;\; 1 \mid 3] = [1 \;\; 0 \mid -1].$$

Thus the matrix in RREF form is

$$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 3 \end{array}\right] \quad \Longrightarrow \quad c_0 = -1, \; c_1 = 3.$$

Hence

$$p(x) = -1 + 3x.$$

$$\text{Check:} \quad p(1) = -1 + 3 = 2, \qquad p'(x) = 3 \;\Rightarrow\; p'(1) = 3.$$

b. Constructing a basis $\{q_1, q_2\} \subset \mathcal{P}_1(\mathbb{R})$

To find polynomials such that

$$q_1(1) = 1, \quad q_1'(1) = 0, \qquad q_2(1) = 0, \quad q_2'(1) = 1.$$

Let $q(x) = a + bx$, so $q'(x) = b$.
For q_1: The conditions are $a_1 + b_1 = 1,\ b_1 = 0 \Rightarrow a_1 = 1$. Thus $q_1(x) = 1$.
For q_2: The conditions are $a_2 + b_2 = 0,\ b_2 = 1 \Rightarrow a_2 = -1$. Thus $q_2(x) = x - 1$.

Hence

$$q_1(x) = 1, \qquad q_2(x) = x - 1.$$

Check: $\quad q_1(1) = 1,\ q_1'(1) = 0; \qquad q_2(1) = 0,\ q_2'(1) = 1.$

Linear independence. Assume $a\, q_1(x) + b\, q_2(x) = 0$ for all x, i.e.

$$a + b(x - 1) = (a - b) + bx \equiv 0.$$

For this to hold identically, the coefficients must vanish: $b = 0$ and $a - b = a = 0$.
Thus the only solution is $a = b = 0$. Therefore, $\{1,\ x - 1\}$ is linearly independent and forms a basis of $\mathcal{P}_1(\mathbb{R})$.

c. Interpolating p in the basis $\{q_1, q_2\}$

Any polynomial $p \in \mathcal{P}_1(\mathbb{R})$ can be expressed as

$$p(x) = a\, q_1(x) + b\, q_2(x).$$

Since $q_1(1) = 1$, $q_1'(1) = 0$ and $q_2(1) = 0$, $q_2'(1) = 1$, it follows that

$$p(1) = a, \qquad p'(1) = b.$$

Thus the representation is

$$p(x) = p(1)\, q_1(x) + p'(1)\, q_2(x).$$

Substituting the given conditions $p(1) = 2$ and $p'(1) = 3$:

$$p(x) = 2\, q_1(x) + 3\, q_2(x) = 2 \cdot 1 + 3(x - 1).$$

Simplifying:

$$p(x) = 3x - 1.$$

In the standard basis $\{1, x\}$, this corresponds to

$$p(x) = c_0 + c_1 x, \qquad c_0 = -1,\ c_1 = 3,$$

which agrees with the result from part (a).