KeralaNotes

KTU STUDY MATERIALS

DISCRETE MATHEMATICAL STRUCTURES

MAT 203

Module 1

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FUNDAMENTALS OF COUNTING THEORY.

PIGEONHOLE PRINCIPLE

If n pigeon's are accomodated in m pigeon-holes, and $n > m$, then atleast one pigeonhole will contain two or more pigeons.

Equalently, if 'n' objects are put in m boxes, and $n > m$, then atleast one box will contain two or more objects.

⇒ GENERALISATION OF THE PIGEONHOLE PRINCIPLE :

If 'n' pigeon's are accomodated in 'm' pigeon holes, and $n > m$, then atleast one of the pigeon holes, must contain $\left\lfloor \frac{(n-1)}{m} \right\rfloor + 1$ pigeons.

where $\lfloor x \rfloor$ denote greatest integer less than or equal to x.

Proof

If possible, let each pigeonhole contain at most $\left\lfloor \frac{(n-1)}{m} \right\rfloor$ pigeons. Then then maximum number of pigeons in all the pigeon holes is

$$= m \left\lfloor \frac{(n-1)}{m} \right\rfloor \leq m \frac{(n-1)}{m} \quad \left\{ \because \left\lfloor \frac{(n-1)}{m} \right\rfloor \leq \frac{(n-1)}{m} \right.$$

ie the maximum number of pigeon's in all the pigeon holes $\leq (n-1)$.

This is against the assumption that there are n pigeon's.

Hence, one of the pigeonholes must contain atleast

$$\left\lfloor \frac{(n-1)}{m} \right\rfloor + 1$$

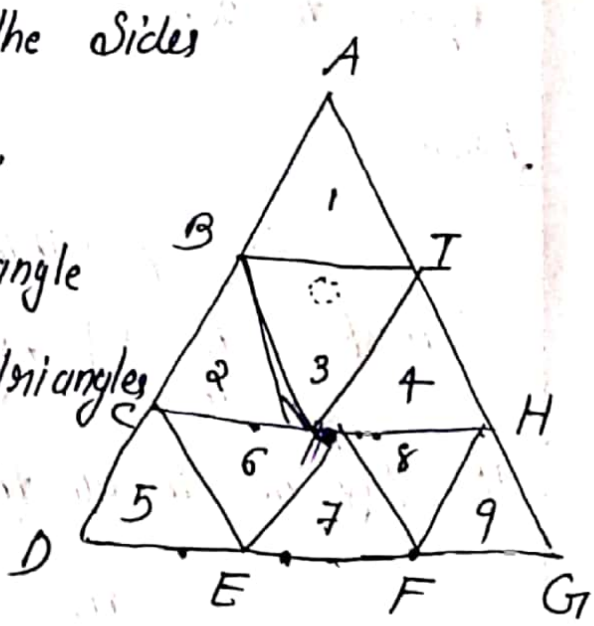
⇒ Example :

If we select 10 points in the interior of an equilateral triangle of side 1, Show that there must be atleast two points whose distance apart is less than $1/3$.

Proof

Let ADG be the given equilateral triangle. The pairs of points B, C, E, F and H I are the points of the trisection of the sides AD, DG and GA respectively,

we have divided the triangle ADG in to 9 equilateral triangles of each side $1/3$.



The 9 subtriangles may be regarded as 9 pigeon holes and 10 interior points may be regarded as 10 pigeons.

Then by the pigeon hole principle, atleast one subtriangle must contains 2 interior points.

The distance between any two interior points of any subtriangle cannot exceed the length of the side, namely $1/3$

Hence the result

Rule of Sum

The rule of Sum is a basic Counting principle, which states that, if there are n choices for one action and m choices for another action, and the two action's cannot be done at the same time, then there are n+m way's to choose one of these actions.

Examples :

⇒ Alice goes to the library to read a book, There are books different genres. She goes to the shelf which has, 7 Scince fictions, 5 mystery books, and 2 journals, Alice wants to read precisely 1 book Then, how many choices does she have?

→ 7 Sceince fictions
→ 5 mystery books } choices
→ 2 journals.

No: of choices is $= 7+5+2$
$= \underline{\underline{14}}$

⇒ Mary is wearing her lucky shirt today. and

She has to choose among 3 red skirts and 4 blue skirts to wear with the shirt. How many different outfit choices of one skirt does she have for the day?

* Since she can wear one of the 3 red skirts or one of the 4 blue skirts.

∴ There are $3+4=7$ different outfit choices.

→ Ravi goes to a pet shop and finds that the pet shop has 3 reptiles, 4 birds, 5 rabbits and 6 fish. If Ravi can only pick one animal as a pet, how many choices does Ravi have?

* There are 3 ways to select a reptile
* there are 4 ways to select a bird
* there are 5 ways to select a rabbit
* there are 6 ways to select a fish

By the rule of sum, there are $3+4+5+6$ $=18$ ways to select a pet.

Rule of product :

It States that if there are n way's of doing Some thing and m way's of doing another thing after that, there are $n \times m$ ways to perform both of these actions.

ie there are $n \times m$ ~~ways~~ different way's to do both actions.

Examples :

→ In a town of Germany, 8 news pappers and 4 magazines are printed. peter wants to subscribe to 1 news paper and 1 magazine. How many choices does he have?

He wants a news paper & a magazine

choices ~~are~~ $= 8 \times 4 = 32$.

→ 8 men and 6 women contest in an election. In how many way's can the people choose two leader, ~~one~~ a men & a women ?

No: of choices is $8 \times 6 = \underline{48}$

Permutation :

A permutation is an ordered arrangement of elements.

Example :

⇒ From a set $S = x, y, z$ by taking two at a time, all permutations are

$xy, yx, xz, zx, yz, zy.$

⇒ we have to form a permutation of three digit numbers from a set of numbers $S = \{1, 2, 3\}$

The different three digit numbers will be formed, when we arrange the digits

123, 132, 213, 231, 312, 321

⇒ **Number of permutations :**

The no: of permutations of 'n' different things taken r at a time is denoted by nP_r. ie $nP_r = \frac{n!}{(n-r)!}$

where $n! = n(n-1)(n-2)\ldots\ldots 3 \cdot 2 \cdot 1.$

Problems:

Q. Find the number of different 4 letter words, with or with out meaning's that can be formed from the letters of the word NUMBER?

A. There are 6 letters,

The no: of arrange ment of 6 letters, taken 4 at a time is

$$6P_4 = \frac{6!}{(6-4)!} = \frac{6\times5\times4\times3\times2\times1}{2\times1}$$

$$= \underline{\underline{360}}$$

Q. From a bunch of 6 cards, how many ways we can permut it

A As we are taking 6 cards, at a time, from a bunch of 6 cards, the permutation will be $6P_6 = 6!$

$$= \underline{\underline{720}}$$

Q. In how many way's can the letter of the word READER be arranged?

9. There are 6 letter's, ($2E$, $1A$, $1D$, $2R$) in the word READER,

The permutation will be,

$$\frac{6!}{2!\cdot 2!} = 180$$

Q. The no: of words, that can be formed by permuting the letters of the word MATHEMATICS. is ?

There are 11 letters,

Here the reppetion are $2M$, $2T$, $2A$

$\therefore$ the no: of permutations are

$$\frac{11!}{2!\cdot 2!\cdot 2!} = \frac{11!}{2\cdot 2\cdot 2} = \frac{11!}{8}$$

$$= \underline{\underline{4989600}}$$

Q. How many words can be formed with words, LUCKNOW, which has,

(1) No - restriction

(2) L as the first letter of the word

(3) L and W as the terminal letter

(4) All vowels together

(5) L always occuring before U

Q.

A. (1) The total no: of distinct letters = 7

(L U C K N O W)

So, the total no: of distinct letters is

$= 7! = 5040$

(2) L as the 1^{st} letter,

Now, we can arrange only 6 places,

$= 6! = 720.$

(3) Now, Here, we can arrange only 5 letters,

(L and W are placed)

So the total no: of arrangements is

$= 5!$

But the place of L and W can be interchange between themselves.

So, total no: of words that can be formed is $= 5! \times 2! = \underline{\underline{240}}$

a. There are 6 letter's.

(2E, 1A, 1D, 2R) in the word

READER,

The permutation will be,

$$\frac{6!}{2!\cdot 2!} = 180$$

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$$= 4989600$$

Q. How many words can be formed with words, LUCKNOW, which has, ① No - restriction

② L as the first letter of the word

③ L and W as the terminal letter

④ All vowels together

⑤ L always occurring before U

Q.

A. ① The total no: of distinct letters = 7

(L U C K N O W)

So, the total no: of distinct letters is

$= 7! = 5040$

$\boxed{LU}$CKNOW

② L as the 1st letter,

Now, we can arrange only 6 places,

$= 6! = 720.$

③ Now, Here, we can arrange only 5 letters,

(L and W are placed)

So the to-tal no: of arrangements is

$= 5!$

But the place of L and W can be interchange between themselves.

So, total no: of words that can be formed is $= 5! \times 2! = \underline{\underline{240}}$

(4) Vowels are U and O

U and O should be together. So, we will assume, these two letters, to be tied up with each other.

Now, we have 6 distinct things to be arranged

So, the no: of arrange ment is $6!$

But the place of U and O interchange b/w themselves, So, total no: of words can be formed is $6! \times 2! = \underline{\underline{1440}}$.

(5) $\frac{7!}{2} = \underline{\underline{2520}}$

Q. How many different words can be formed with letters of the word PENCIL when vowels occupy even places?

There are 6 letters in the word PENCIL
no letter is repeated.

Here Two vowels E and I

places are

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}

Even places are 2^{nd}, 4^{th} & 6^{th}

Now, there are 3 even ~~even~~ places for 2 letters,

$\therefore$ 3 vowels can be arranged is

$$3P_2 = 3! = 6 \text{ ways}$$

~~Four~~ ~~co~~ Four consonants can be arranged in remaining 4 places is $4! = 24$ ways.

$\therefore$ total no: of way's is

$$= 6 \times 24$$

$$= 144 \text{ way's}$$

COMBINATION :

A Combination is a Selection of Some given elements in which order does not matter.

The no: of all Combinations of n things, taken r at a time is

$$nC_r = \frac{n!}{r!\,(n-r)!}$$

* **Example :**

The cardinality of a ~~the~~ Set is 6, we have to choose 3 elements from the Set. Here the order does not matter, Hence the no: of Subsets will be

$$6C_3 = \frac{6!}{3!\,(6-3)!} = \underline{\underline{20}}$$

Q. There are 5 men and 5 women in a room, In howmany ways we can choose 3 men and 2 women from the room.?

A. The no of way's to choose 3 men out of 6 men is $6C_3$

and the no: of ways to choose 2 women from 5 women is $5C_2$

$\therefore$ the total no: of way's is $6C_3 \times 5C_2$

$= \underline{\underline{200}}$

Q. A group consist 4 girls and 7 boy's. In howmany way's can a team of 5 member's be Selected if the team has,

ⓐ No - girl

ⓑ At least one boy and one girl

A. ⓐ when no girls is to Selected

$$7C_5 = \frac{7!}{5!\ (7-5)!}$$

$= \underline{\underline{21}}$

② when atleast one boy and one girl is to be selected is

$$(4C_1 \times 7C_4) + (4C_2 \times 7C_3) + (4C_3 \times 7C_2) + (4C_4 \times 7C_1)$$

$$= 4 \times 35 + 6 \times 35 + 4 \times 21 + 1 \times 7$$

$$= \underline{\underline{441}}$$

Q. Ther are 6 men and 5 women, we have to form a Committee of 5 members, with the Condition, at most 2 women are there, Then, how many possibilities are there?

A. 3M & 2W

The possibilities are 3M & 2W or
4M & 1W or
5M & 0W

There fore the proper no: of possibilities is $6C_3 \times 5C_2 + 6C_4 \times 5C_1 + 6C_5 \times 5C_0$

=

Q. The no: of straight lines obtained by joining 16 points on a plane, no twice of the them being on the same line is

2 out of 16

ie $16C_2 = \frac{16!}{2!\,(16-2)!} = \frac{16!}{2!\,14!}$

$= \underline{\underline{120}}$

COMBINATION WITH REPETITION:

Assume that we have a Set A with n elements, Any selection of r objects from A, where each object can be selected more than once is called a combination of n objects taken r at a time, with repetition.

For example:

The Combination of the letters, a, b, c, d taken 3 at a time, with repetion are.

aaa, aab, aac, aad, abb, abc, abd, acc, acd, add bbb, bbc bbd, bcc, bcd, bdd, ccc, ccd, cdd, ddd.

So, the no: of Combinations of n objects, taken r at a time, with repetition is

$$^{n+r-1}C_r$$

Examples

Q. In a donut shop, there are 20 types of donuts. How many ways we select 12 different donuts to home?

There are 20 types of ~~20~~ donuts.

we want 12 types of donuts.

$\therefore$ Here $n = 20$

$r = 12$

$\therefore$ total no: of possibilities are,

$$^{n+r-1}C_r = {}^{20+12-1}C_{12}$$

$$= {}^{31}C_{12}$$

$$= \frac{31!}{12!\,(31-12)!} = 141120525$$

Imp

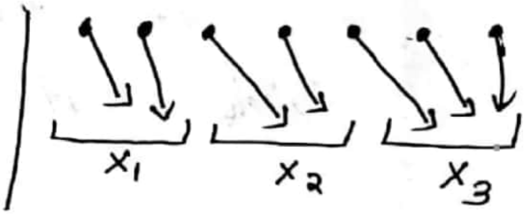
Q. Suppose $x_1 + x_2 + x_3 = 7$, where $x_1, x_2, x_3 \geq 0$

How many +ve integer solutions are there?

$x_1 + x_2 + x_3 = 7$

$2 + 3 + 2 = 7$

$2 + 4 + 1 = 7$



So, Here, we choose, the formula of Combination with repetition.

Here $\cancel{n} = \cancel{7}$

So, Suppose $x_1 + x_2 + x_3 + \cdots\cdots + x_n = r$, where $x_i \geq 0$,
then the total no: of Soluns are $^{n+r-1}C_r$

Here, $n=3, \ r=7$

$\therefore$ total no: of possibilities are

$$^{3+7-1}C_7 = {}^9C_7 = \frac{9!}{7!(9-7)!}$$

$$= \frac{9!}{7!(2!)} = \underline{\underline{36}}$$

Q. How many ways we put 10 identical balls in to 6 distinct bins ?

A. It is like $\underbrace{x_1 + x_2 + \dots + x_6}_{\text{bins}} = 10 \} $ total balls

$\therefore$ then the total no: of ways are,

$$^{n+r-1}C_r = {}^{6+10-1}C_{10}$$

$$= {}^{15}C_{10}$$

$$= \frac{15!}{10!(15-10)!} = \frac{15!}{10!(5!)}$$

$$= \underline{\underline{3003}}$$

BINOMIAL THEOREM

we know, $(x+y)^0 = 1$

$$(x+y)^1 = x+y$$

$$(x+y)^2 = x^2+2xy+y^2$$

$$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$$

They can be generalized by the formula, called binomial theorem.

ie $(x+y)^n = \sum_{k=0}^{n} {}^nC_k\, x^{n-k} y^k$

$$= {}^nC_0\, x^n + {}^nC_1\, x^{n-1} y + {}^nC_2\, x^{n-2} y^2 + \dots\dots + {}^nC_{n-1}\, x y^{n-1} + {}^nC_n\, y^n.$$

Here the coeffints ${}^nC_k$ are called binomial coefficient.

Problems:

Q. Expand $(x+y)^6$?

$$(x+y)^6 = 6C_0\, x^6 y^0 + 6C_1\, x^5 y + 6C_2\, x^4 y^2 + 6C_3\, x^3 y^3 + 6C_4\, x^2 y^4 + 6C_5\, x y^5 + 6C_6\, x^0 y^6$$

$$= 1 \cdot x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + 1y^6$$

$$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

Q. Expand $(x+y)^4$ by binomial thm

$$(x+y)^4 = x^4 + 4C_1 x^{4-1} y + 4C_2 x^{4-2} y^2 + 4C_3 x^{4-3} y^3 + y^4$$

$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Q. Expand by binomial thm $(a-1/a)^6$

$$(a-1/a)^6 = a^6 + 6C_1 a^{6-1}(-1/a)^1 + 6C_2 a^{6-2}(-1/a)^2$$
$$+ 6C_3 a^{6-3}(-1/a)^3 + 6C_4 a^{6-4}(-1/a)^4 + 6C_5 a^{6-5}(-1/a)^5$$
$$+ 6C_6 \cdot a^{6-6}(-1/a)^6$$

$\Rightarrow$ ~~$a^6 + 6a^5(-1/a) +$~~

$$\Rightarrow a^6 - 6a^4 + 15a^2 - 20 + \frac{15}{a^2} - \frac{6}{a^5} + 1/a^6$$

Q. Expand $\left(\frac{x^2}{2} - \frac{2}{x}\right)^4$

$$\left(\frac{x^2}{2} - \frac{2}{x}\right)^4 = \left(\frac{x^2}{2}\right)^4 + 4C_1\left(\frac{x^2}{2}\right)^{4-1}\left(-\frac{2}{x}\right) +$$
$$4C_2\left(\frac{x^2}{2}\right)^{4-2}\left(-\frac{2}{x}\right)^2 + 4C_3\left(\frac{x^2}{2}\right)^{4-3}\left(-\frac{2}{x}\right)^3$$
$$+ 4C_4\left(\frac{x^2}{2}\right)^{4-4}\left(-\frac{2}{x}\right)^4$$

$$\Rightarrow \frac{x^4}{16} + 4\left(\frac{x^2}{2}\right)^3 \cdot \left(\frac{-2}{x}\right) + \frac{4\cdot 3}{2\cdot 1}\left(\frac{x^2}{2}\right)^2\left(\frac{-2}{x}\right)^2$$

$$+ \frac{4\cdot 3\cdot 2}{3\cdot 2\cdot 1}\left(\frac{x^2}{2}\right)\left(\frac{-8}{x^3}\right) + \frac{16}{x^4}$$

$$= \frac{x^8}{16} - 4\cdot\frac{x^6}{8}\cdot\frac{2}{x} + 6\cdot\frac{x^4}{4}\cdot\frac{4}{x^2} - 4\frac{x^2}{2}\cdot\frac{8}{x^3} + \frac{16}{x^4}$$

$$= \frac{x^8}{16} - x^5 + 6x^2 - \frac{16}{x} + \frac{16}{x^4}$$

Q. Expand $(1.04)^5$ by the binomial formula and find its value to two decimal places.

A. $(1.04)^5 = (1+0.04)^5$

$$(1+0.04)^5 = 1^5 + 5C_1(1)^{5-1}(0.04) + 5C_2(1)^{5-2}(0.04)^2$$

$$+ 5C_3(1)^{5-3}(0.04)^3 + 5C_4(1)^{5-4}(0.04)^4 +$$

$$5C_5(1)^{5-5}(0.04)^5$$

$$= 1 + 0.2 + 0.016 + 0.00064 + 0.000128$$

$$= \underline{\underline{1.22}}$$

Q. Find the eigth term in the expansion of $\left(2x^2 - \frac{1}{x^2}\right)^{12}$?

A. we know, $(x+y)^n = \sum_{k=0}^{n} nC_k \, x^{n-k} y^k$

$$T_{k+1} = nC_k \, x^{n-k} y^k$$

Here, we want 8^{th} term,

ie T_8? $x = 2x^2$ $y = -1/x^2$ $n=12$, $k=7$

$$\therefore T_{7+1} = 12C_7 \left(2x^2\right)^{12-7} \left(-1/x^2\right)^7$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \left(2x^2\right)^5 \cdot \left(\frac{(-1)^7}{x^{14}}\right)$$

$$T_8 = 793 \times 32 x^{10} \left(\frac{-1}{x^{14}}\right)$$

$$= -\frac{25344}{x^4}$$

⇒ PRINCIPLE OF INCLUTION - EXCLUTION:

If A and B are finite subsets of a finite universal set U, then A

$$|A \cup B| = |A| + |B| - |A \cap B|$$

where $|A|$ denotes the no: of elements in the set A.

More over.

If A and B and C is any three finite sets, then $|A \cup B \cup C| = |A| + |A| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|$

* GENERALIZATION OF THE PRINCIPLE:

This principle, can be extended to a finite number of finite sets $A_1, A_2, \ldots \ldots A_n$.

ie $$|A_1 \cup A_2 \cup A_3 \cup \ldots \ldots \cup A_n| = \sum_i |A_i| - \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<k} |A_i \cap A_j \cap A_k| - \ldots \ldots + (-1)^{n+1} |A_1 \cap A_2 \cap \ldots \ldots \cap A_n|.$$

where the 1^{st} sum is over all i, and the second sum is over all pairs i, j with $i<j$, and the third sum is over all triples i, j, k with $i<j<k$ and so on.

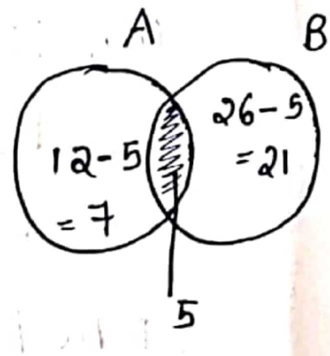
* Examples

Suppose a group of students are studying logic or mathematics. If 12 are studying logic, 26 are studying mathematics and 5 are studying both subjects. Then how many students are in the group?

A. Let A be the group of students studying logic

B " " " Mathematics

we have to find $|A \cup B|$



∴ total no: of ~~peoples~~ Students are $7+5+21 = 33$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 12 + 26 - 5 = \underline{\underline{33}}$

Q. There are 250 students in an engineering College of these 188 have taken a course in Fortran. 100 have taken a course in C and 35 have taken a course in Java. Further 88 have taken courses in both Fortran and C. 23 have taken courses in both C and Java. and 29 have taken course in both Fortran & Java. If 19 of these students have taken all the three courses. Then how many of these 250 students have not taken a course in any of these three prograthree programming languages?

n. Let F, C and J denote the students who have taken the languages Fortran, C & Java respe

Then $|F| = 188$

$|C| = 100$

$|J| = 35$

$|F \cap C| = 88$ $|C \cap J| = 23$ $|F \cap J| = 29$

& $|F \cap C \cap J| = 19$

Then the no: of students who have taken at least one of the three languages is given by

$$|F \cup C \cup J| = |F| + |C| + |J| - |F \cap C| - |C \cap J|$$

$$- |F \cap J| + |F \cap C \cap J|$$

$$= (188 + 100 + 35) - (88 + 23 + 29) + 19$$

$$= 323 - 140 + 19 = \underline{\underline{202}}$$

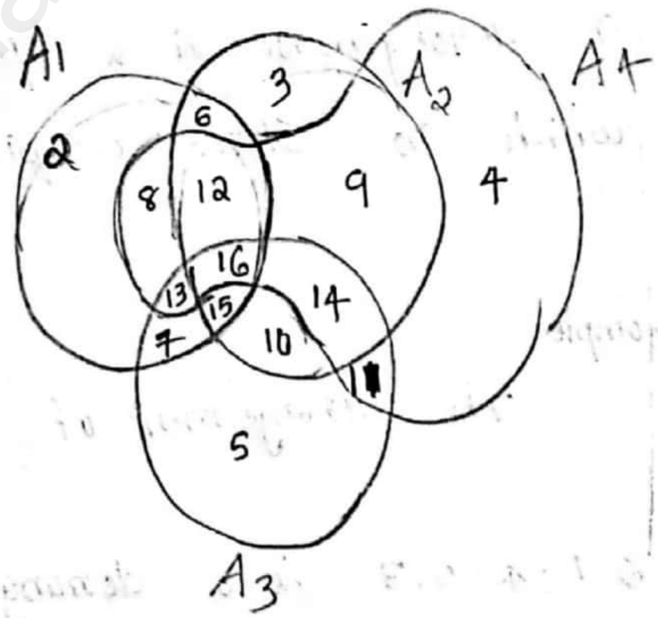
∴ No: of students who have not taken a course in any of these languages $= 250 - 202 = 48$

Q. A_1 A_2 A_3 A_4 are subsets of a set U containing elements with the following properties. Each subset contains 28 elements, the intersection of any of two subsets contains 12 elements. The intersection of any three of the subsets contains 5 elements. The intersection of all four subsets contains 1 element. ?

(a) How many elements belong to none of the four subsets?

(b) How many elements belongs to exactly one of the four subsets?

(c) How many elements belongs to exactly two of the four subsets?



(a) No of elements that belong to at least one of the four subsets

$= |A_1 \cup A_2 \cup A_3 \cup A_4|$

$= |A_1| + |A_2| + |A_3| + |A_4| - \{|A_1 \cap A_2| + |A_1 \cap A_3| + |A_1 \cap A_4|$

$+ |A_2 \cap A_3| + A_2 \cap A_4| + |A_3 \cap A_4|\} + \{|A_1 \cap A_2 \cap A_3| +$

$|A_1 \cap A_2 \cap A_4| + |A_1 \cap A_3 \cap A_4|\} + |A_2 \cap A_3 \cap A_4|\}$

$- |A_1 \cap A_2 \cap A_3 \cap A_4|$

$= [4 \times 28 - 6 \times 12 + 4 \times 5 - 1] = 59$

∴ the no: of elements that belong to none of the four subset $= 75 - 59 = 16$

DERANGEMENT

A derangement is a permutation of objects in which no object occupies its original position.

Example :

* The derangement of 1 2 3 are 231 & 312

* 2. 2·1·4·5.3 is a derangement of 12345

but 2 1 5 4 3 is not a derangement of 12345. Since 4 occupies its original position.

The total no: of derangements of a set of n elements is given by,

$$D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots\cdots + (-1)^n \frac{1}{n!}\right]$$

Q. Five gentle man A, B, C, D & E attend a party. where before joining the party, they leave their over coats, in a cloak room. After the party, the over coats get mixed up and are returned to the gentle man in a random manner. find the no: of permutations in which none gets his over coat.

A. ie $D_5 = 5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right]$

$$= 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} + \frac{1}{120}$$

$$= \frac{44}{5} = 44$$

Q Twenty people check their hats at a theatre. How many way's their hat's to be returned?

(a) No one receives his or her own hat?

(b) at least one person receives his or her own hat?

(a) No one receives his or her own hat

$$= D_{20} = \frac{1}{20!}\left[1 - \frac{1}{2!} + \frac{1}{3!} \cdots - \frac{1}{20!}\right]$$

$$=$$

(b) At least one person receives his or her own hat

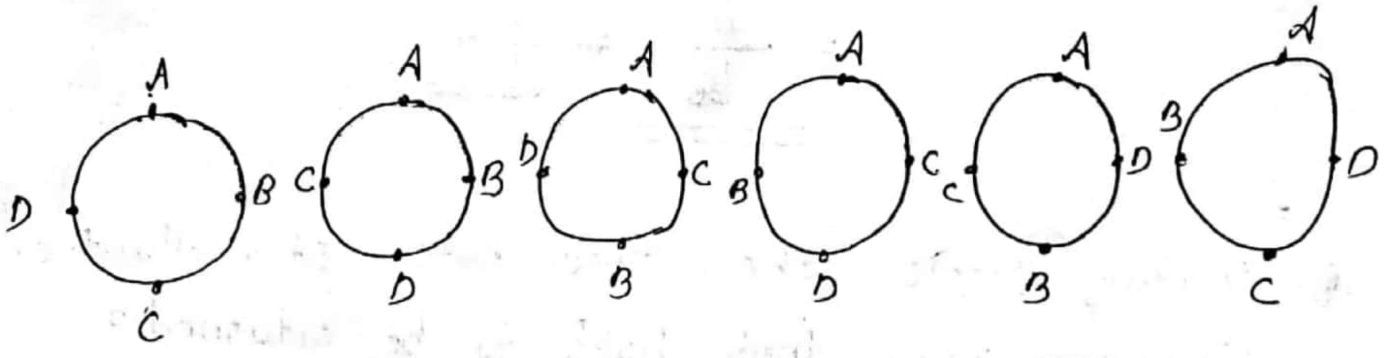
$$= 20! - D_{20}.$$

⟹ CIRCULAR PERMUTATION:

Here, the objects are arranged in a circle (or any closed curve), then we get circular permutation.

Example,

we can arrange 4 elements A, B, C, D in a circle as follows. we fix one of the element, say A, at the top point of circle. The other 3 elements B, C, D are permuted in all possible ways ie $6 = 3!$ different permutations are as follows.



From the example given above, we see that number of different circular arrangements of 4 elements is $(4-1)! = 6$

* The no: of different circular arrangements of n objects = $(n-1)!$

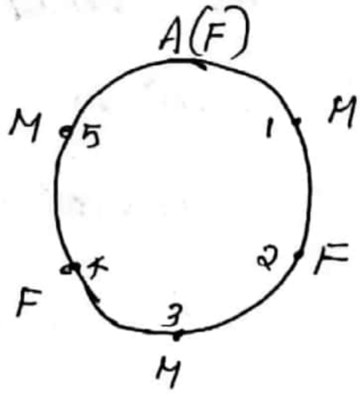
* If no distinction is made between clockwise & anti-clockwise direction, then the no: of different circular arrangements = $\frac{1}{2}(n-1)!$

Q. If 6 people A, B, C, D, E, F are seated about a round table. how many circular arrangements are possible? If arrangements are considered the same, if A, B, C are females and the others are males, in how many arrangements do the sexes alternate?

A. The no: of different circular arrangements of n objects is $(n-1)!$

∴ The required no: of circular arrangements is $(6-1)! = 5! = 120$

Since the rotation does not alter the circular arrangement. we can assume that A occupies the top position as shown in the figure



of the remaining places, positions, 1, 3, 5 occupied by 3 males,

This can be achieved in P(3,3)

$3P_3$ ways. $= 3! = 6$ ways

The remaining places 2 and 4 should be occupied by the remaining two females, This can be achieved in $2P_2$ ways $= 2! = 2$ ways

$\therefore$ Total no: of ways required circular arrangement is $= 6 \times 2 = 12$.

PROBLEMS

PERMUTATION

1. (a) Assuming that repetitions are not permitted, how many four digits number can be formed from the six digits 1, 2, 3, 5, 7, 8 ?

(b) How many of these numbers are less than 4000?

(c) How many of these numbers are even ?

(d) How many of these numbers are odd ?

A. (a) The four digit number can be considered to be formed by filling up 4 blank places with the available 6 digits. Hence the no: of 4 digit numbers is

$=$ no: of 4 permutations of 6 numbers

$= 6P_4 = \frac{6!}{(6-4)!} = 360$

(b) If a 4 digit number is to be less than 4000 the first digit must be 1, 2, or 3.

Hence the first place can be filled with 3 ways, corresponding to any of these 3 ways the remaining 3 spaces can be filled up with remaining 5 digits

ie $5P_3$ ways

$\therefore$ required no $= 3 \times 5P_3$

$= \underline{\underline{180}}$

(c) if the 4-digit no: is to be even, the last digit must be 2 or 8. Hence the last space can be filled up in 2 ways, Corresponding to any of these 2 ways, the remaining 3 spaces can be filled up with remaining 5 digits

ie $5P_3$ ways,

Hence required no $= 2 \times 5P_3$

$= \underline{\underline{120}}$

(d) Similarly the required no: of odd numbers

$= 4 \times 5P_3$

$= \underline{\underline{240 \text{ ways}}}$

The total no: of derangements of a set of n elements is given by,

$$D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots\cdots + (-1)^n \frac{1}{n!}\right]$$

Q. Five gentle man A, B, C, D & E attend a party. where before joining the party, they leave their overcoats, in a cloak room. After the party, the overcoats get mixed up and are returned to the gentle man in a random manner. Find the no: of permutations in which none gets his overcoat.

ie $D_5 = 5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right]$

$= 1 - 1 + 1/2 - 1/6 + \frac{1}{24} + \frac{1}{120}$

$= \frac{11}{30}$

Q Twenty people check their hats at a theatre. How many ways their hat's to be returned?

(a) No one receives his or her own hat?

(b) at least one person receives his or her own hat?

(a) No one receives his or her own hat

$$= D_{20} = \frac{1}{20!}\left[1 - \frac{1}{2!} + \frac{1}{3!} \cdots\cdots - \frac{1}{20!}\right]$$

$$=$$

(b) At least one person receives his or her own hat

$$= 20! - D_{20}.$$

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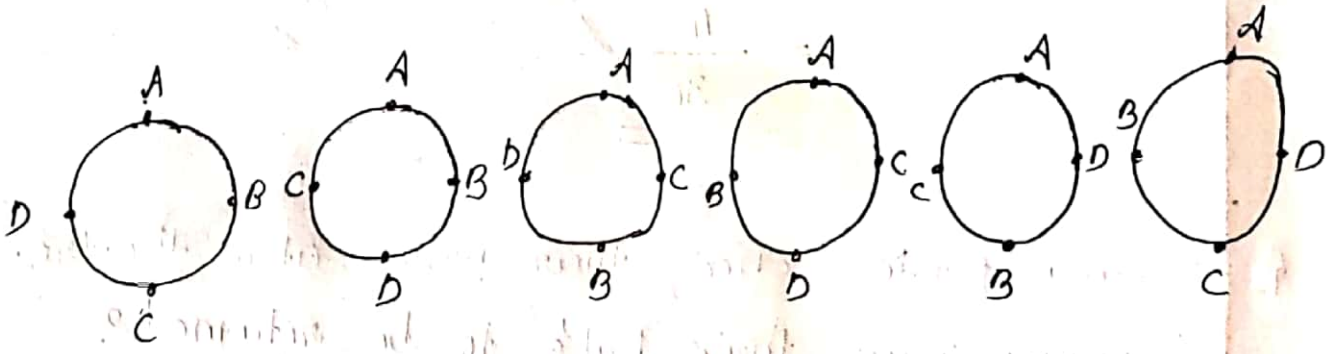

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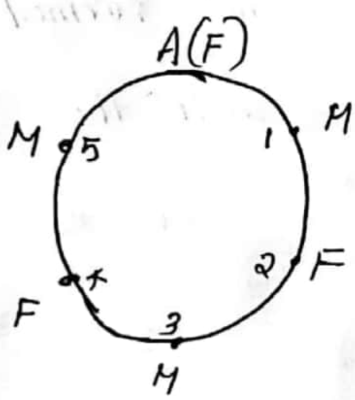

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∴ Total no: of ways required Circular arrangement is $= 6 \times 2 = 12$.

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$$\therefore \text{ required no} = 3 \times 5P_3$$
$$= \underline{\underline{180}}$$

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