KeralaNotes

KTU STUDY MATERIALS

LOGIC SYSTEM DESIGN

CST 203

Module 2

Related Link :

- KTU S3 STUDY MATERIALS
- KTU S3 NOTES
- KTU S3 SYLLABUS
- KTU S3 TEXTBOOK PDF
- KTU S3 PREVIOUS YEAR SOLVED QUESTION PAPER

Module : 2

- Logic Gate
- Boolean Algebra
- Kmap
- Tabulation method

Logic Gate

1. Basic Gate : (a) NOT
   (b) AND
   (c) OR

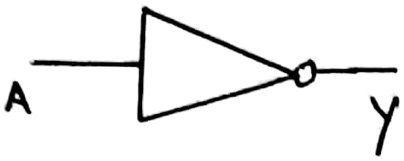
2. Universal Gate : (a) NAND
   (b) NOR

3. Arithmetic Gate : (a) XOR
   (b) XNOR

1. Basic Gate

(a) NOT Gate (inverter)

Symbol



Truth table

input A	output Y
0	1
1	0

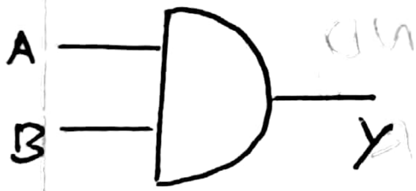
Boolean Expression

$\boxed{Y = \bar{A}}$

Buffer => Input = output

(b) AND Gate (multiplayer)

Symbol



Three input

Truth table

A	B	Y
0	×0	0
0	×1	0
1	×0	0
1	×1	1

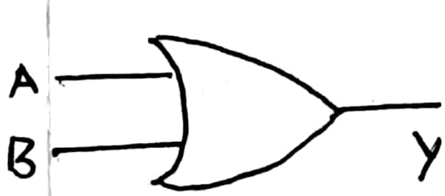
Boolean expression

$\boxed{Y = A \cdot B}$

and

(c) OR Gate

Symbol



Truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Expression

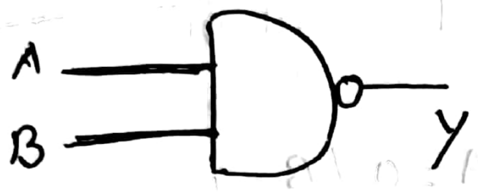
$\boxed{Y = A + B}$

2. Universal Gate

Universal because we can make any other gate by using NAND or NOR.

(a) NAND Gate

Symbol (AND+NOT)



Truth table

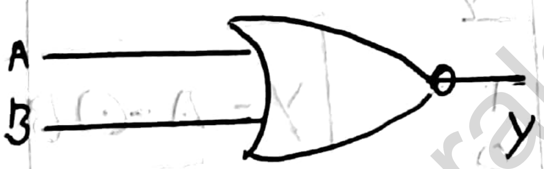
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Boolean Expression

$$\boxed{Y = \overline{AB}}$$

(b) NOR Gate

Symbol (OR+NOT)



Truth table

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

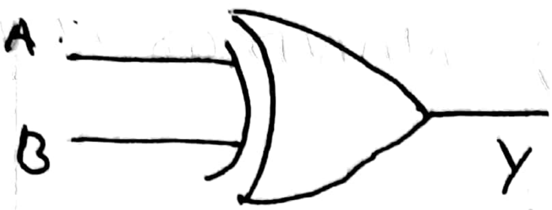
Boolean expression

$$\boxed{Y = \overline{A+B}}$$

3. Arithmetic Gate

(a) XOR (Exclusive OR)

Symbol



Truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Boolean Expression

$$Y = A \oplus B$$

$$Y = \bar{A}B + A\bar{B}$$

output is high (1) if input are differe

Eg: $A=0 / B=0$

$1 \cdot 0 + 0 \cdot 1 = 0$

$A = 0 / B = 1$

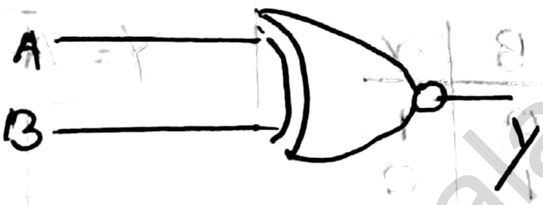
$1 \cdot 1 + 0 \cdot 0$

$= 1 + 0 = 1$

(b) XNOR (Exclusive XNOR)

Symbol

XOR + NOT



Truth table

A	B	Y
0	0	1
0	1	0
1	1	0
1	1	1

Boolean Expression

$$Y = A \odot B$$

$$\overline{\bar{A}B + A\bar{B}}$$

output is high (1) if input are same

Boolean Algebra

It is a set of rules used to simplify / minimize the Boolean expression without changing its functionality

1. Complement Rule : $\boxed{\bar{\bar{A}} = A}$

Eg: $A=0$, $\bar{A}=1$, $\bar{\bar{A}}=0$

2. AND Rule :

$$\boxed{\begin{aligned} A \cdot A &= A \\ A \cdot 0 &= 0 \\ A \cdot 1 &= A \\ A \cdot \bar{A} &= 0 \end{aligned}}$$

3. OR Rule :

$$\boxed{\begin{aligned} A+A &= A \\ A+0 &= A \\ * \boxed{A+1 = 1} \\ A+\bar{A} &= 1 \end{aligned}}$$

eg: $\underbrace{\bar{A}BC + ABC}_{=1} +$

$\underbrace{AB}_{=1}+AC+1$

4. Distributive Law:

$$\boxed{\begin{aligned} A \cdot (B+C) &= AB + \cancel{BC}\, AC \\ \boxed{A+(B \cdot C)} &\boxed{= (A+B)(A+C)} \end{aligned}}$$

Eg: $A+\bar{A}B \rightarrow (A+\bar{A}) \cdot (A+B)$

$1 \cdot (A+B) = A+B$

Eg: $B+\bar{B}C \rightarrow (B+\bar{B}) \cdot (B+C)$

$= 1 \cdot (B+C) = \underline{\underline{B+C}}$

5. Commutative law: $A+B = B+A$, $A \cdot B = B \cdot A$

6. Associative law: $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

7. Demorgan's law:
$$\overline{A+B} = \bar{A} \cdot \bar{B}$$
$$\overline{AB} = \bar{A} + \bar{B}$$

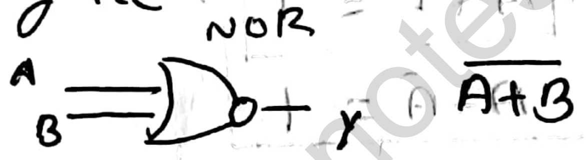
NOR ~~NAND~~ = Bubbled AND

NAND = Bubbled OR

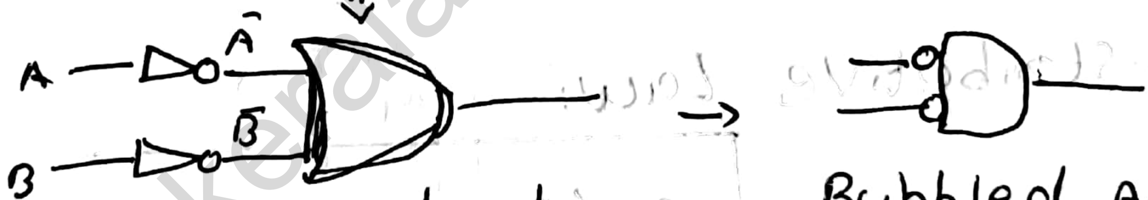
1. $\overline{A+B} = \bar{A} \cdot \bar{B}$

↓

NOR gate



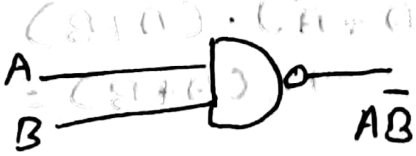
⇓



~~Bubbled and~~

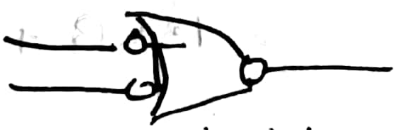
Bubbled AND

2. $\overline{AB} = \bar{A} + \bar{B}$



NAND

⇓



bubbled OR

Questions:

1. $A \cdot A = A$

2. $A + (BC) = (A+B)(A+C) = \cancel{A \cdot [B+C]}$

3. $A + 1 = 1$

4. $A + \hat{A} = 1$

sum of product (SOP)

$$Q = A \cdot B \cdot \bar{C} + A \cdot \bar{B} \cdot C + \bar{A} \cdot B \cdot C$$

product of sum (POS)

$$Q = (A+B) \cdot (\bar{B}+C) \cdot (A+1)$$

$$SOP = \underbrace{A \cdot B \cdot \bar{C}}_{or} + \underbrace{A\bar{B}C}_{or} + \bar{A}BC \quad (+, or)$$

$$POS \quad (A+B+C) \cdot (A+B+\bar{C}) \cdot (\bar{A}+B+\bar{C})$$

and and

(·, and)

Sum of product (SOP)

Considered parameter: $\boxed{1}$

	A	B	C	o/p
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

A: $0 \rightarrow \bar{A}$, $1 \rightarrow A$

Min term (m)

3rd min term, 4

$= (\bar{A}\cdot B\cdot C) + (A\cdot\bar{B}\cdot\bar{C}) + (A\cdot B\cdot\bar{C}) + (A\cdot B\cdot C)$

6, 7

$= m_3 + m_4 + m_6 + m_7$

$\Sigma_m(3,4,6,7)$

(sum of min term)

Product of sum (POS)

Considered parameter: $\boxed{0}$

	A	B	C	o/p
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

A: $0 \rightarrow A$, $1 \rightarrow \bar{A}$

Max term (M)

M_0, M_3, M_6, M_7

$= (A+B+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+\bar{B}+C)\cdot(\bar{A}+\bar{B}+\bar{C})$

$= M_0 \cdot M_3 \cdot M_6 \cdot M_7$

~~$= \Sigma_m(0,3,6,7)$~~

$\Pi_M(0,3,6,7)$

(product of max term)

	A	B	C	D	Min term (m) SOP $0 \to \bar{A}$, $1 \to A$	Max term (M) POS $0 \to A$, $1 \to \bar{A}$
0	0	0	0	0	$\bar{A}\bar{B}\bar{C}\bar{D}$ (m_0)	$A+B+C+D$ (M_0)
1	0	0	0	1	$\bar{A}\bar{B}\bar{C}D$ (m_1)	$A+B+C+\bar{D}$ (M_1)
2	0	0	1	0	$\bar{A}\bar{B}C\bar{D}$ (m_2)	$A+B+\bar{C}+D$ (M_2)
3	0	0	1	1	$\bar{A}\bar{B}CD$ (m_3)	$A+B+\bar{C}+\bar{D}$ (M_3)
4	0	1	0	0	$\bar{A}B\bar{C}\bar{D}$ (m_4)	$A+\bar{B}+C+D$ (M_4)
5	0	1	0	1	$\bar{A}B\bar{C}\bar{D}$ (m_5)	$A+\bar{B}+C+D$ (M_5)
6	0	1	1	0	$\bar{A}.BC\bar{D}$ (m_6)	$A+\bar{B}+\bar{C}+D$ (M_6)
7	0	1	1	1	$\bar{A}BCD$ (m_7)	$A+\bar{B}+\bar{C}+\bar{D}$ (M_7)
8	1	0	0	0	$A\bar{B}\bar{C}\bar{D}$ (m_8)	$\bar{A}+B+C+D$ (M_8)
9	1	0	0	1	$A\bar{B}\bar{C}D$ (m_9)	$\bar{A}+B+C+\bar{D}$ (M_9)
10	1	0	1	0	$A\bar{B}C\bar{D}$ (m_{10})	$\bar{A}+B+\bar{C}+D$ (M_{10})
11	1	0	1	1	$A\bar{B}CD$ (m_{11})	$\bar{A}+B+\bar{C}+\bar{D}$ (M_{11})
12	1	1	0	0	$AB\bar{C}\bar{D}$ (m_{12})	$\bar{A}+\bar{B}+C+D$ (M_{12})
13	1	1	0	1	$AB\bar{C}D$ (m_{13})	$\bar{A}+\bar{B}+C+\bar{D}$ (M_{13})
14	1	1	1	0	$ABC\bar{D}$ (m_{14})	$A+\bar{B}+\bar{C}+\bar{D}$ (M_{14})
15	1	1	1	1	$ABCD$ (m_{15})	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$ (M_{15})

Problem:

1. A, B, C, D, E find m_{18}, M_{18}

$$
\begin{array}{r|l}
2 & 18 \ (0) \\
2 & 9 \ (1) \\
2 & 4 \ (0) \\
2 & 2 \ (0) \\
 & 1
\end{array}
$$

$18 \rightarrow 1\ 0\ 0\ 1\ 0$

[Greater than 7 = 4 bit]

$m_{18} = A\bar{B}\bar{C}D\bar{E}$

$M_{18} = \bar{A} + B + C + \bar{D} + E$

$\Sigma_m (0, 3, 5, 7, 9)$

Here largest No is 9 it is a Four bit No

$m_0 = 0000 \quad \bar{A}\bar{B}\bar{C}\bar{D}$

$m_3 = 0011 \quad \bar{A}\bar{B}CD$

$m_5 = 0101 \quad \bar{A}B\bar{C}D$

$m_7 = 0111 \quad \bar{A}BCD$

$m_9 = 1001 \quad A\bar{B}\bar{C}D$

$= (\bar{A}\bar{B}\bar{C}\bar{D}) + (\bar{A}\bar{B}CD) +$
$(\bar{A}\bar{B}\bar{C}D) + (\bar{A}BCD) +$
$(A\bar{B}\bar{C}D)$

0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

2

$\pi M(0,5,7)$ Here loog it is a 3 bit NO

$$\begin{array}{ccc} 0 & 5 & 7 \\ \downarrow & \downarrow & \downarrow \\ 000 & 101 & 111 \end{array}$$

$M_0 = 000 = A+B+C$

$M_5 = 101 = \bar{A}+B+\bar{C}$

$M_7 = 111 = \bar{A}+\bar{B}+\bar{C}$

$= (A+B+C)\cdot(\bar{A}+\bar{B}+\bar{C})\cdot(\bar{A}+\bar{B}+\bar{C})$

Questions

1. Minterm of 20

$$\begin{array}{r|l} 2 & 20\ (0) \\ 2 & 10\ (0) \\ 2 & 5\ (1) \\ 2 & 2\ (0) \\ & 1 \end{array}$$

$20 \rightarrow 10100$

Minterm of 20 = $A \cdot \bar{B} C \bar{D} \bar{E}$

2. Maxterm of 56

$$\begin{array}{r|l} 2 & 56\ (0) \\ 2 & 28\ (0) \\ 2 & 14\ (0) \\ 2 & 7\ (1) \\ 2 & 3\ (1) \\ & 1 \end{array}$$

$= 111000$

$= \cancel{ABCDEF}$

$\bar{A} \cdot \bar{B} \bar{C} D E F$

3. Summation minterm $(2, 3, 5, 7)$

$m_2 = 010 = \bar{A}B\bar{C}$

$m_3 = 011 = \bar{A}BC$

$m_5 = 101 = A\bar{B}C$

$m_7 = 111 = ABC$

$= (\bar{A}B\bar{C}) + (\bar{A}BC) + (A\bar{B}C) + (ABC)$

Standard / canonical SOP

standard
↑
SOP $= ABC$ + $\bar{A}BC + AB\bar{C} + A\bar{B}\bar{C}$
OR

POS $= (A+\bar{B}+\bar{C}) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+C)$
↓
$(A+B+\bar{C})$ and

non standard
Eg: for SOP :

- $ABC + AB + B\bar{C}$

$\underset{\bar{C}}{C}$ $\underset{\bar{A}}{A}$ → missing

- $A + BC + \bar{A}\bar{B}C$

$\bar{B}\bar{C}$ A → missing
$\bar{B}C$ $\bar{A}$
BC
$B\bar{C}$

Eg of not standard POS:

- $(A+B+C)\cdot(\underbrace{\bar{A}+\bar{B}}_{\substack{C\\ \bar{C}}})\cdot(\underbrace{\bar{B}+\bar{C}}_{\substack{A\\ \bar{A}}})$ → missing

- $(\underbrace{A}_{\substack{\bar{B}\bar{C}\\ BC\\ BC\\ B\bar{C}}})+(\bar{A}+\bar{B}+\bar{C})$ → missing

Step 1 : write down all the terms

Step 2 : If one ~~ore~~ more variable are missing, expand value by multiplying with sum of each one of the missing ~~value~~ variable and its complement

Step 3 : Droup out Redundant terms

Examples :

Q convert the sop into standard sop

1. $\bar{A}BC + \bar{A}\bar{B} + \bar{B}C + A\bar{B}\bar{C}$

step 1:

$\bar{A}BC + \bar{A}\bar{B} + \bar{B}C + A\bar{B}\bar{C}$

$C \quad \bar{C} \qquad A \quad \bar{A}$

2. $\Rightarrow \bar{A}BC + \bar{A}\bar{B}(C+\bar{C}) + \bar{B}C(A+\bar{A}) + A\bar{B}\bar{C}$

$\Rightarrow \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + A\bar{B}\bar{C}$

3. Check repeat

$\Rightarrow \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}C$

2. $Y = A + \bar{B}C$ $\quad A, B, C$

BC

A	$\bar{B}C$
$\bar{B}\bar{C}$	A
$\bar{B}C$	$\bar{A}$
$B\bar{C}$	
BC	

$A(B+\bar{B})(C+\bar{C}) + \bar{B}C(A+\bar{A})$

~~$AB + A\bar{B}$~~ $A(BC + B\bar{C} + \bar{B}C + \bar{B}\bar{C}) +$

$\bar{B}CA + \bar{B}C\bar{A}$

ABC

$\Rightarrow ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} +$
$A\bar{B}C + \bar{A}C\bar{B}$

$\Rightarrow ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}C\bar{B}$

3. Convert the equation into standard POS

$(A+B+\bar{C})(\bar{A}+C)(\bar{A}+\bar{B})$

B. C

$\bar{B}$ $\bar{C}$

$= (A+B+\bar{C})(\bar{A}+\bar{C}+B)(\bar{A}+C+\bar{B})$
$(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$

arrange :

$= (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$
$\cancel{(\bar{A}+\bar{B}+C)} \cdot (\bar{A}+\bar{B}+\bar{C})$

$\rightarrow (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$
$(\bar{A}+\bar{B}+\bar{C})$

Content

NAND →
- not
- AND
- OR
- NOR

NOR →
- NOT
- AND
- OR
- NAND

⇒ **NAND** (NOT + AND)

Boolean expression

$$\boxed{Y = \overline{AB}}$$

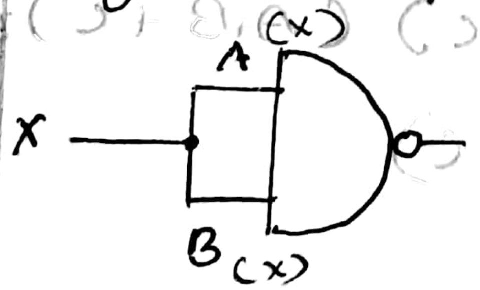
Truth table

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(i) **NOT Function**

NAND → NOT

An inverter can be made from NAND gate by connecting all the input together and creating, in effect a single common input



Proof

$Y = \overline{AB}$

puting $A = x$ $B = x$ [equal]

$Y = \overline{XX}$

By using Demorgan's ...

$Y = \bar{X} + \bar{X}$ [Reference : $A + A = A$]

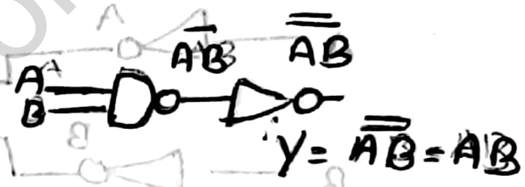
$\therefore$ $\boxed{Y = \bar{X}}$ output complement of input

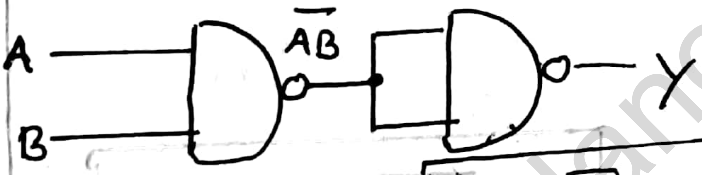
(ii) <u>AND Function</u> [Relation : Nand complement → AND]

NAND → AND

An AND function is generated by simply inverting the output of NAND gate

$\boxed{\text{NAND + NOT → AND}}$





$\boxed{Y = \overline{\overline{AB}} = AB}$

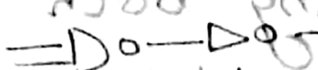
AND

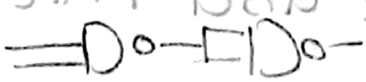
A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

A	B	$\overline{AB}$	$\overline{\overline{AB}}$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

$\overline{\overline{AB}} = AB$

we know how to connect NAND → NOT using here





(iii) OR Function

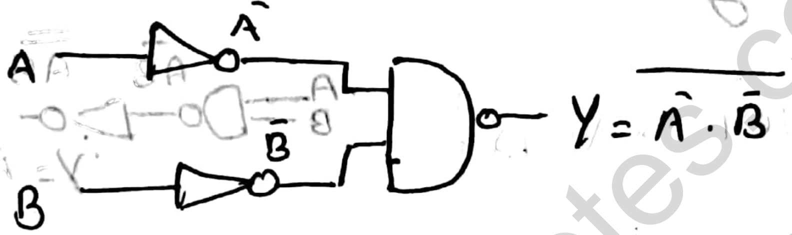
NAND → OR

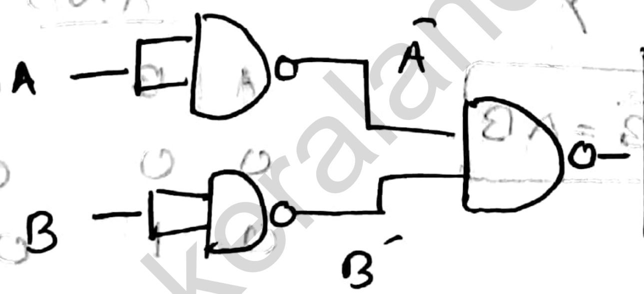
An OR function is generated by simply inverting both the inputs of NAND Gate

$$Y = A + B$$

$$= \bar{\bar{A}} + \bar{\bar{B}} \quad \text{[Demorgans law]} \quad [\bar{\bar{A}} = A]$$

$$= \overline{\bar{A} + \bar{B}}$$





$$\boxed{Y = \overline{\bar{A} \cdot B} = A' + B}$$

(iv) NOR Function [OR+NOT]

~~NAND → NOR~~
NAND → OR

If we connect a not at OR output we get NOR Function

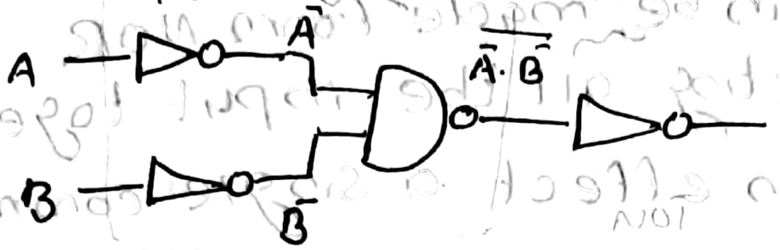
A NOR function is generated by simply inverting both the inputs of NAND Gate and invert the output

$Y = \overline{A+B}$

$= \bar{A} \cdot \bar{B}$ [Demorgans law] [$\bar{\bar{A}} = A$]

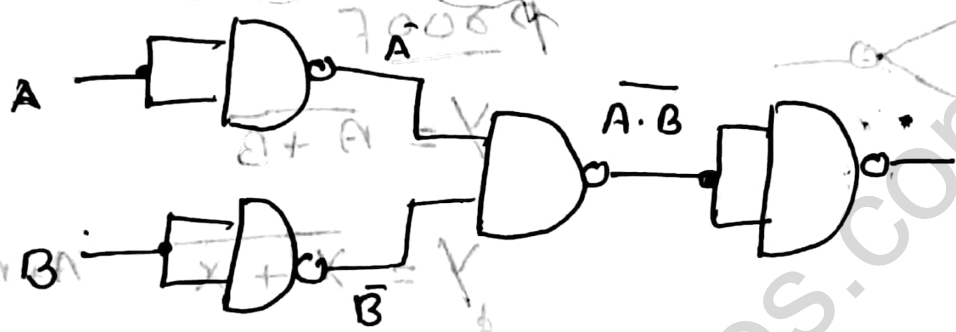
$= \overline{\overline{\bar{A} \cdot \bar{B}}}$

OR



$Y = \overline{\bar{A} \cdot \bar{B}}$

OR

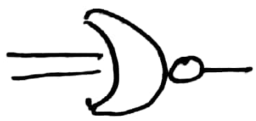


$\boxed{Y = \overline{\bar{A} \cdot \bar{B}} = \overline{A+B}}$

⇒ **NOR Gate** (NOT + OR)

→ NOT

Boolean Expression



$Y = \overline{A+B}$

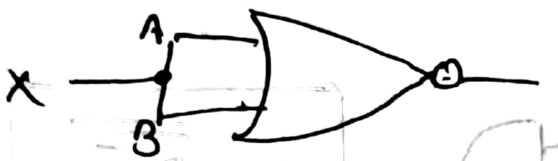
Truth table

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(i) NOT function

NOR → NOT

An inverter can be made from NOR gate by connecting all the input together and creating, in effect a single common output.



shorting NOR gate

proof

$Y = \overline{A + B}$

$Y = \overline{X + X}$

$A + A = A$

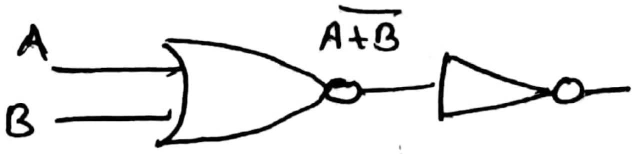
$$\boxed{Y = \overline{X}}$$

(ii) OR Function

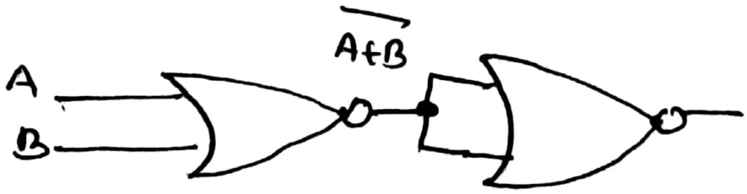
[NOR complement → OR]

NOR → OR

An OR function is generated by simply inverting the output of NOR gate.



$Y = \overline{\overline{A + B}} = A + B$



$$\boxed{Y = \overline{\overline{A + B}} = A + B}$$

A	B	A·B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	$\bar{A}+\bar{B}$	$\overline{\bar{A}+\bar{B}}$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

(iii) **AND function**

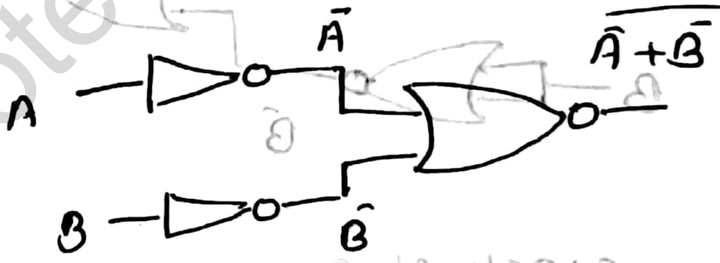
A·NOR → AND

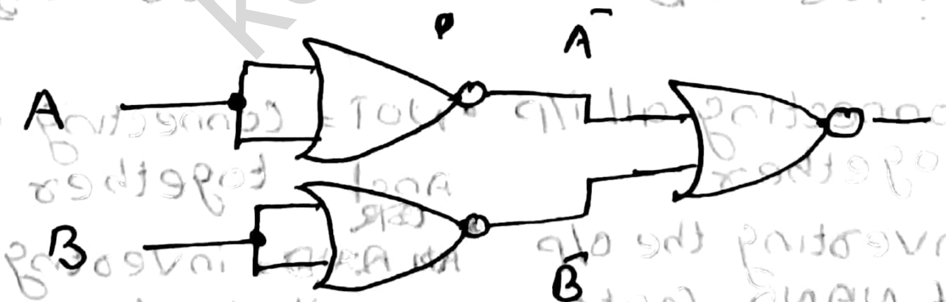
An AND function is generated by simply inverting both input of NOR gate

$Y = A \cdot B$

$= \bar{\bar{A}} \cdot \bar{\bar{B}}$ $\quad [\bar{\bar{A}} = A]$

$= \overline{\bar{A} + \bar{B}}$



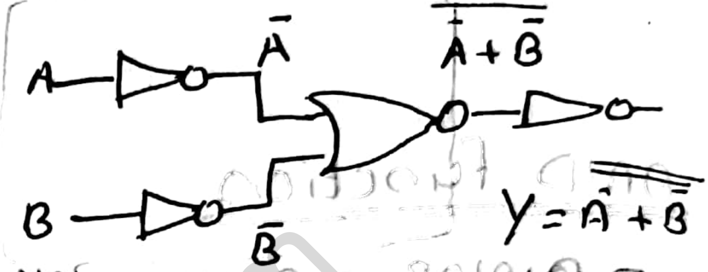


$\boxed{Y = \overline{\bar{A}+\bar{B}} = A \cdot B}$

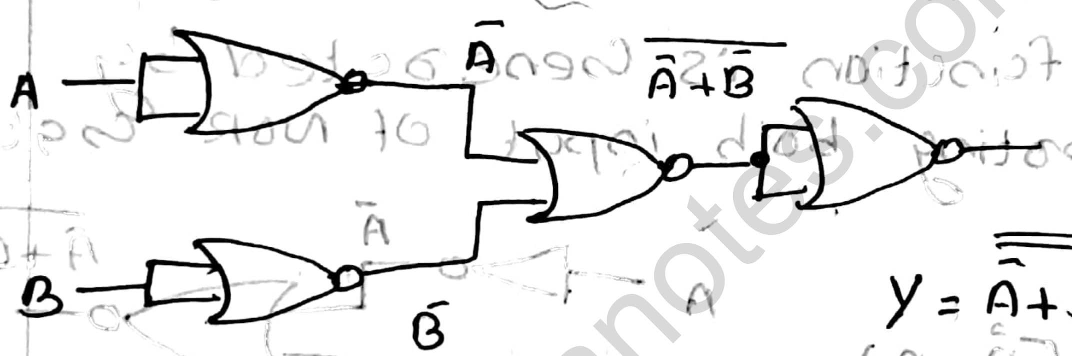
(iv) NAND function

An NAND function is generated by simply inverting both the inputs of NOR Gate and invert the output

$$Y = \overline{A \cdot B}$$
$$= \bar{A} + \bar{B} \quad [\bar{\bar{A}} = A]$$
$$= \overline{\overline{\bar{A} + \bar{B}}}$$



AND NOT



$$Y = \overline{\overline{\bar{A}+\bar{B}}} = \overline{A \cdot B}$$
$$= \overline{A \cdot B}$$

Conclusion

NAND Gate

- NOT = Connecting all i/p together
- AND = inverting the o/p of NAND gate
- OR = " i/p
- NOR = inverting the i/p of nand gate and invert the output

NOR Gate

- NOT = Connecting all i/p together
- AND = inverting the i/p of NOR Gate
- OR = " o/p
- NAND = inverting all i/p of NOR Gate and invert o/p

Karnaugh Map

We have

Karnaugh Map is introduced to simplification of Boolean function in a easy way.

2 variable : → SOP
→ POS

3 Variable : → SOP
→ POS

4 Variable : → SOP
→ POS

5 variable : → SOP
→ POS

2 variable KMAP

$\bar{A}B + AB + AB + \bar{A}\bar{B}$

$\boxed{2^n} = 2^2 = 4$

3 variable Kmap

$\bar{A}BC + AB\bar{C} + ABC + \bar{A}\bar{B}\bar{C}$

$2^3 = 8$

4 variable Kmap

$\bar{A}BCD + ABCD + AB\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D}$

$2^4 = 16$ cells

5 variable K map

A B \ CDE	000	001	011	010	110	111	101	100
00								
01								
11								
10								

5 variable Karnaugh map (Grey code)

$2^5 = 32$ cells

x————x

4 variable K map

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01				
AB 11				
$A\bar{B}$ 10				

Grey code

① [00
01] (2)
10
① [11

↓

00
01
11
10

↓

A, B, C, D

00 01 10 11

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	m_0 0000 $\bar{A}\bar{B}\bar{C}\bar{D}$	m_1 0001 $\bar{A}\bar{B}\bar{C}D$	m_3 0011 $\bar{A}\bar{B}CD$	m_2 0010 $\bar{A}\bar{B}C\bar{D}$
$\bar{A}B$ 01	m_4 0100 $\bar{A}B\bar{C}\bar{D}$	m_5 0101 $\bar{A}B\bar{C}D$	m_7 0111 $\bar{A}BCD$	m_6 0110 $\bar{A}BC\bar{D}$
AB 11	m_{12} 1100 $AB\bar{C}\bar{D}$	m_{13} 1101 $AB\bar{C}D$	m_{15} 1111 $ABCD$	m_{14} 1110 $ABC\bar{D}$
$A\bar{B}$ 10	m_8 1000 $A\bar{B}\bar{C}\bar{D}$	m_9 1001 $A\bar{B}\bar{C}D$	m_{11} 1011 $A\bar{B}CD$	m_{10} 1010 $A\bar{B}C\bar{D}$

0000 — 0
0001 — 1
0010 — 2
0011 — 3
0100 — 4
0101 — 5
0110 — 6
0111 — 7
1000 — 8
1001 — 9
1010 — 10
1011 — 11
1100 — 12
1101 — 13
1110 — 14
1111 — 15

3 variable K map

A, B, C

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	m_0 000 $\bar{A}\bar{B}\bar{C}$	m_1 001 $\bar{A}\bar{B}C$	m_3 011 $\bar{A}BC$	m_2 010 $\bar{A}B\bar{C}$
A 1	m_4 100 $A\bar{B}\bar{C}$	m_5 101 $A\bar{B}C$	m_7 111 ABC	m_6 110 $AB\bar{C}$

2 Variable kmap

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	m_0 00 $\bar{A}\bar{B}$	m_1 01 $\bar{A}B$
A 1	m_2 10 $A\bar{B}$	m_3 11 AB

Representation of Truth table on KMAP

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

A \ B	0	1
0	0	1
1	1	0

A \ B	$\bar{B}$	B
$\bar{A}$	0	1
A	1	0

like this we can also do with 4 variable

Representation of Boolean Expression on KMAP

➡ plot Boolean expression,

$Y = AB\bar{C} + ABC + \bar{A}\bar{B}C$ on the karnaugh map

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0 (0)	1 (1)	0 (3)	0 (2)
A	0 (4)	0 (5)	1 (7)	1 (6)

$\bar{A}\bar{B}C$

ABC $AB\bar{C}$

➡ $\sum m(1, 5, 7, 9, 15)$

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

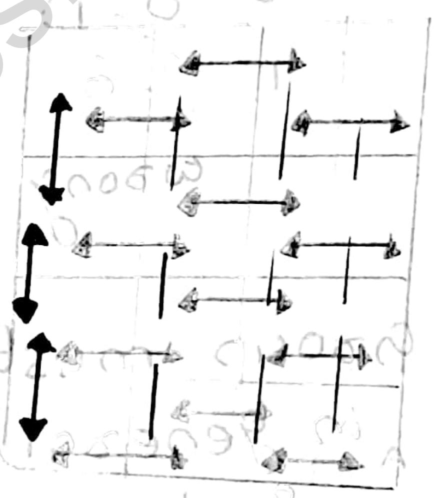
$\Sigma m(1,5,7,9,15)$

	1		
	1		
	1	1	
	1		

Rules of Kmap

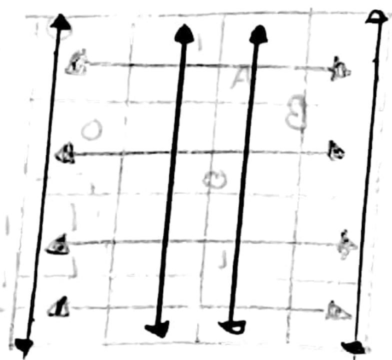
Neighbouring cells

(a) Neighbouring cells in the rows are adjacent



(b) Neighbouring cells in the colowns are adjacent

(c) Leftmost and corresponding adjacent rightmost cells are adjacent

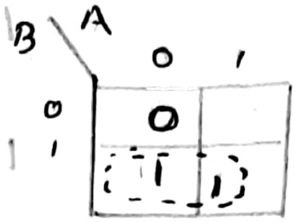


(d) Top and corresponding bottom cells are adjacent

RULES :

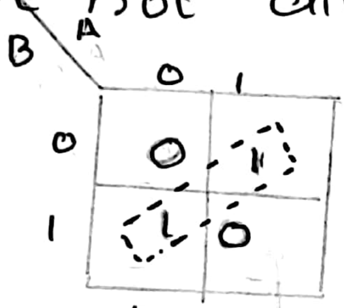
1. Group may not include any cells containing a zero

wrong

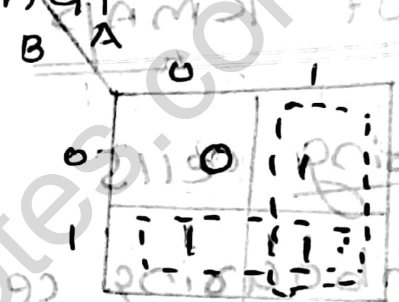


Right

2. Groups may be horizontal or vertical, but not diagonal

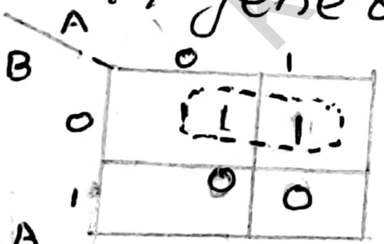


wrong

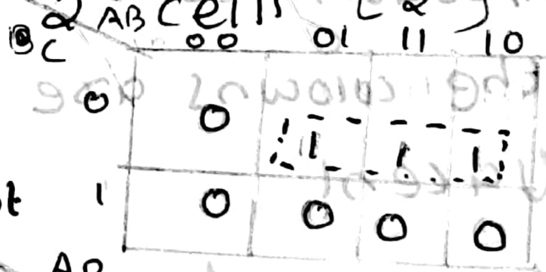


Right

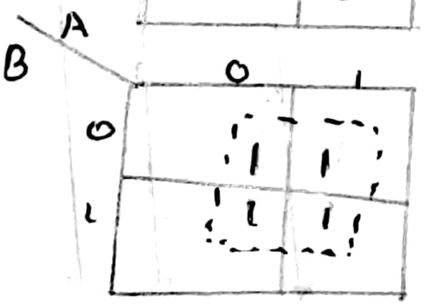
3. Group must contain 1, 2, 4, 8, 16 or in general 2^n cells [2^n]



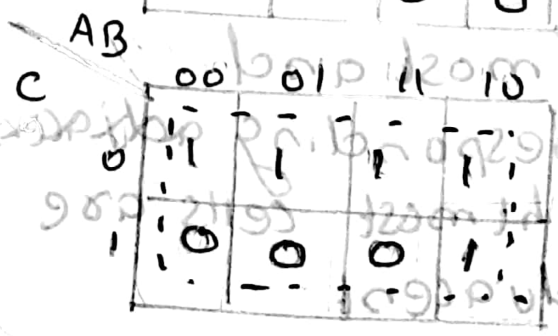
2 Right



group of 3 wrong



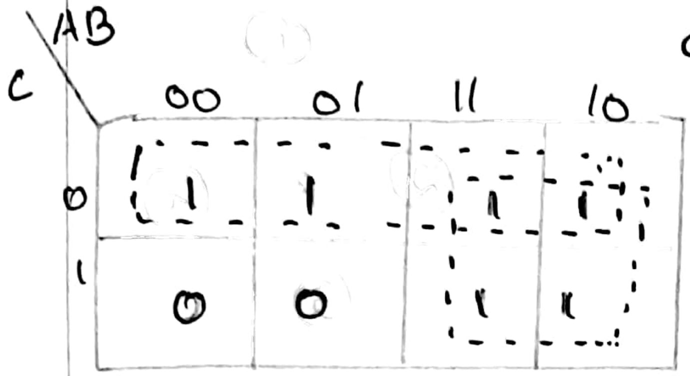
right group of 4



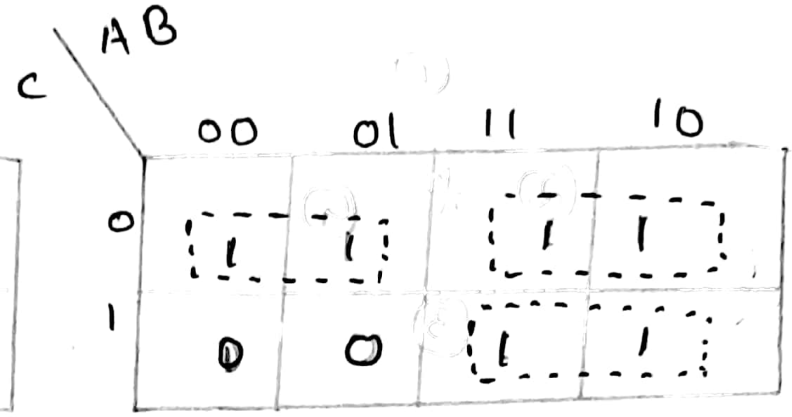
group of 5 wrong

16
8
4
2
1

4. Each group should be as large as possible

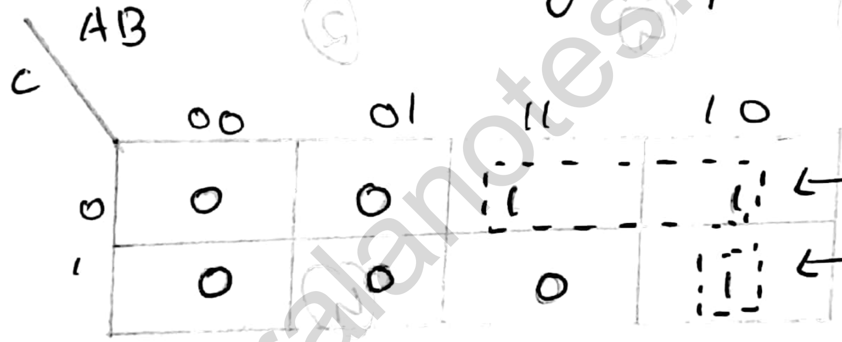


Right



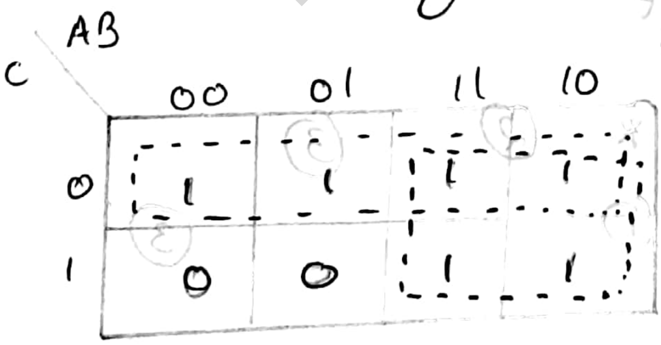
wrong

5. Each cell containing a one must be in at least one group

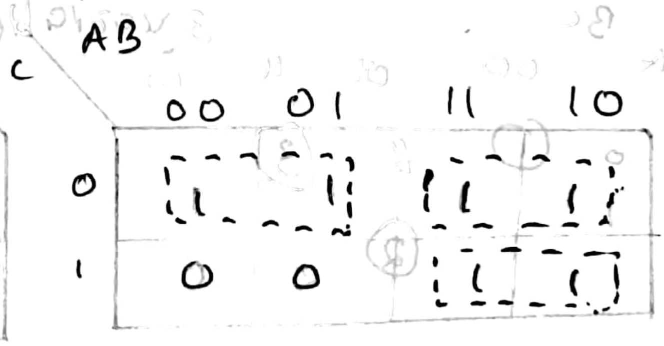


Group I
one present
Group II
atleast one group

6. Groups may overlaps



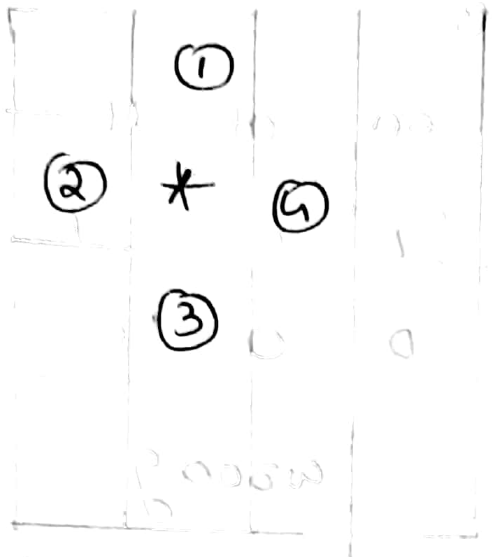
↑
groups overlaps
Right

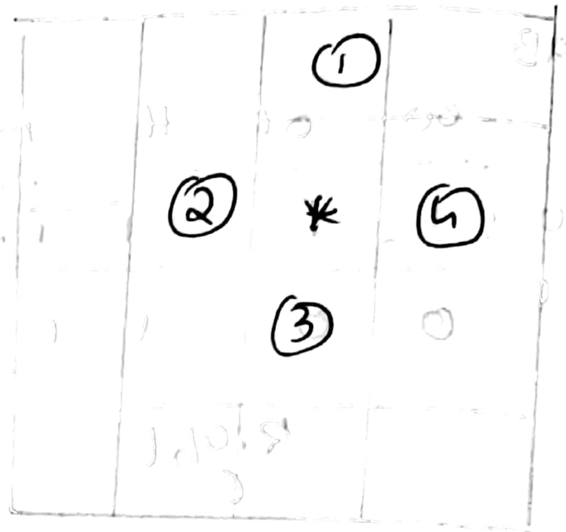


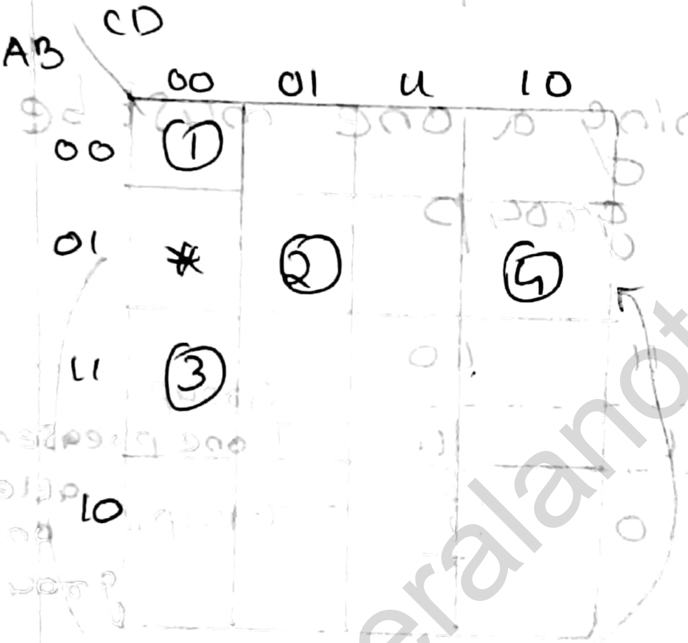
Groups not overlap
wrong

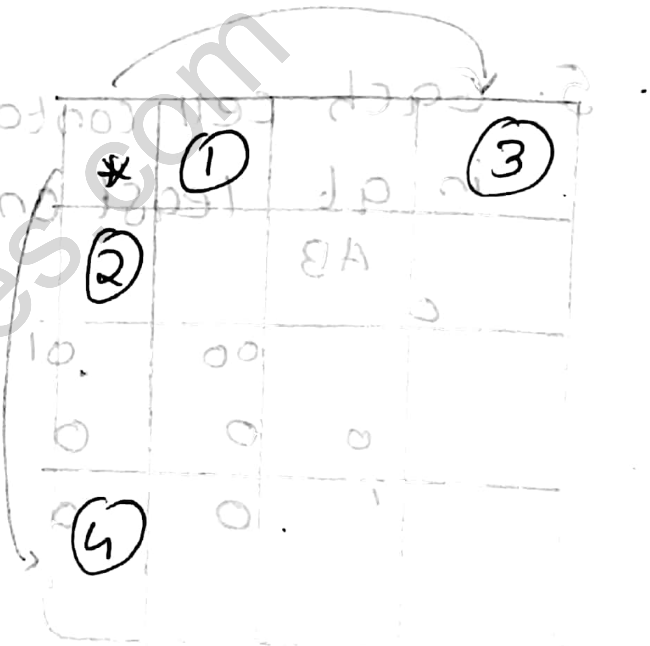
some examples:

4 variable

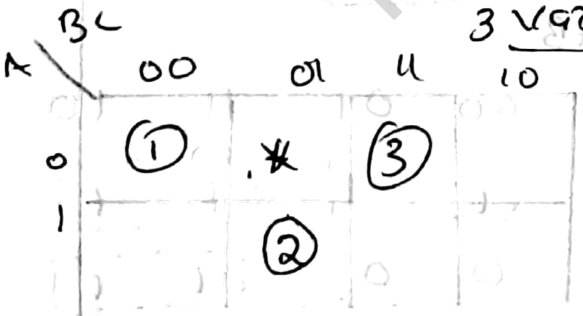


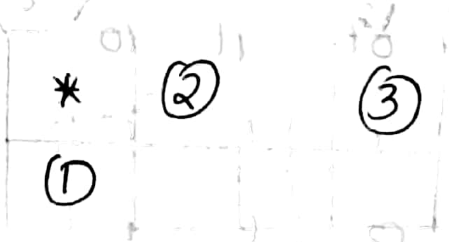


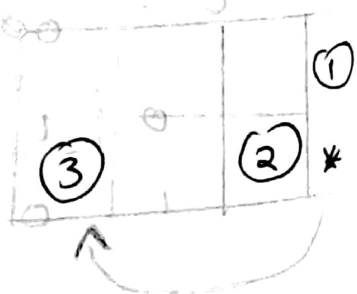




3 variable







part-2

→ Numerical problems : SOP KMAP [2, 3, 4 variables]

→ DON'T CARE conditions

→ NUMERICAL PROBLEMS : DON'T care conditions

Grouping of two adjacent once

Examples:

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	1	0
A 1	0	0	0	0

$\bar{A}C$

$\bar{A}C$

$\bar{A}$ → common in row

C → common in $\bar{B}C$, BC

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	0
A 1	1	0	0	1

$A\bar{C}$

row → A
common in $\bar{B}\bar{C}$ & $B\bar{C}$
⇒ $\bar{C}$

$A\bar{C}$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	1	0
A 1	0	0	1	0

→ BC

In row no common

$1 \times (BC) \Rightarrow BC$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	1	0
A 1	0	0	1	0

→ Group 1 → $\bar{A}C$

→ Group 2 → BC

$\Rightarrow \bar{A}C + BC$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	1	0
A 1	0	0	1	1

→ Group 1 → $\bar{A}C$

→ pair not required

→ Group 2 AB

$\Rightarrow \bar{A}C + AB$

Grouping of four adjacent ones

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	0
AB 1	1	1	1	1

A

$\Rightarrow A$

- coloumn not considerd because we use all coloumn so net effect = 1

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	1	0
$\bar{A}B$ 01	0	0	1	0
AB 11	0	0	1	0
$A\bar{B}$ 10	0	0	1	0

CD

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	0	0	0

B (rows $\bar{A}B$, AB)

BD

$Y = BD$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	1	0	0	1
$A\bar{B}$ 10	1	0	0	1

$\bar{D}$

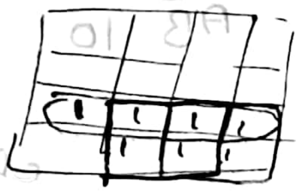
$Y = A\bar{D}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	0	0	1

$\bar{B}\bar{D}$

$Y = \bar{B}\bar{D}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	1	1	1	1
$A\bar{B}$ 10	0	1	1	1



Group 3

AC

Group 1

AB

Group 2 AD

$Y = AB + AD + AC$

Grouping of Eight adjacent...

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	1	1	1	1
AB 11	1	1	1	1
$A\bar{B}$ 10	0	0	0	0

B

$Y = B$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	1	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	0

D

$Y = D$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	1	1	1

$\bar{B}$

$Y = \bar{B}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	1	0	0	1
AB 11	1	0	0	1
$A\bar{B}$ 10	1	0	0	1

$\to \bar{D}$ $\quad Y = \bar{D}$

Simplify Boolean Expression

1. $Y = \bar{A}\bar{B}D + AB\bar{C}\bar{D} + \bar{A}BD + ABC\bar{D}$

Not standard SOP so:

C $\quad\quad C$

$\bar{C}$ $\quad\quad \bar{C}$

$Y = \bar{A}\bar{B}\overset{CD}{\cancel{Bc}} + \bar{A}\bar{B}\bar{C}D + AB\bar{C}\bar{D} +$

$\bar{A}BCD + \bar{A}B\bar{C}D + ABC\bar{D}$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		1	1	
$\bar{A}B$		1	1	
AB	1			1
$A\bar{B}$				

$\to \bar{A}D$

$\to AB\bar{D}$

$$\boxed{Y = AB\bar{D} + \bar{A}D}$$

2. $F(A,B,C,D) = \Sigma_m (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1		1
$\bar{A}B$	1	1		1
AB	1	1		1
$A\bar{B}$	1	1		

$\bar{A}\bar{D}$

$B\bar{D}$

$\bar{C}$

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

$$Y = \bar{C} + B\bar{D} + \bar{A}\bar{D}$$

Don't Care Terms

(incompletely specified Functions)

- In some logic circuits, certain input conditions never occur. Therefore the corresponding output never appear

- In such case output level is not defined, it can be either HIGH or LOW
- These output level are indicated by 'x' or 'd' in the truth tables and are called don't care conditions.

Eg:

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	X
1	1	1	X

} Don't care condition

- when forming groups of cell, treat the don't care cell as either a 1 or 0 or ignore the don't cares.
- This is helpful if it allows us to form a larger groups than would otherwise be possible without the don't cares. There is no requirement

to group all or any of the
. only use them in group if it simplifies the logic

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	0
A 1	0	1	x	x

out: $A\bar{B}C$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	0
A 1	0	1	x	x

out: AC

Find the Reduced SOP using KMAP

1. $F(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,4)$ don't care

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	x	1	1	x
$\bar{A}B$	x	0	1	0
AB	0	0	1	0
$A\bar{B}$	0	0	1	0

$\bar{A}\bar{B}$

CD

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

$Y = \bar{A}\bar{B} + CD$

2. $F(A,B,C) = \Sigma m(0,1,3,7) + d(4,5)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1	1	1	X
A	0	X	1	0

$\rightarrow \bar{A}$

C

$$y = \bar{A} + C$$

problems

1. Using K-map simplify Boolean function F as sum of products using the don't care condition d

$$F(w,x,y,z) = w'(x'y + x'y' + xyz) + x'z'(y+w)$$

$$d(w,x,y,z) = w'x(y'z + yz) + wyz$$

$\Rightarrow F(w,x,y,z) = \cancel{\bar{w}\bar{x}y +}$

$\cancel{= \bar{x}y\bar{w} + \bar{x}\bar{y}\bar{w} + xy}$

$$\Rightarrow \bar{w}\bar{x}y + \bar{w}\bar{x}\bar{y} + \bar{w}xyz + \bar{x}y\bar{z} + w\bar{x}\bar{z}$$

$\bar{w}\bar{x}y + w\bar{x}\bar{y} + wxyz + \bar{x}$

$z \quad z \quad w \quad y$

$\bar{z} \quad \bar{z} \quad \bar{w} \quad \bar{y}$

$F(w,x,y,z)$:

$\Rightarrow \bar{w}\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}z +$

$\bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}xyz + w\bar{x}y\bar{z} + \bar{w}\bar{x}y\bar{z} +$

$w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z}$

$\bar{x}y$

$d(w,x,y,z) \Rightarrow$

$\bar{w}x\bar{y}z + \bar{w}xyz + wyz$

x

$\bar{x}$

$\Rightarrow \bar{w}x\bar{y}z + \bar{w}xyz + wxyz + w\bar{x}yz$

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	1	1	1	1
$\bar{w}x$		1	1	
wx			X	
$w\bar{x}$	1		X	1

$\bar{x}\bar{z}$

$\bar{w}z$

$\bar{x}\bar{z}$

$\cancel{Y = \bar{w}\bar{x}\bar{z} + \bar{w}z + w\bar{x}\bar{z}}$

$\cancel{Y = \bar{w}z + \bar{x}\bar{z}}$

$Y = \bar{w}z + \bar{x}\bar{z}$

2. Reduce the following expression using kmap and implement the real minimal expression in universal logic

1. $F(A, B, C, D) = \Sigma m(0, 1, 2, 3, 5, 7, 8, 9, 10, 12, 13)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01		1	1	
AB 11	1	1		
$A\bar{B}$ 10	1	1		1

$\bar{A}D$

$A\bar{C}$

$\bar{B}\bar{D}$

$\bar{B}D$

$$Y = \bar{A}D + A\bar{C} + \bar{B}\bar{D}$$

3. Simplify the given Boolean function F
$F(w, x, y, z) = \Sigma(2, 3, 12, 13, 14, 15)$
(i) sum of product (ii) product of sum

SOP

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$			1	1
$\bar{w}x$				
wx	1	1	1	1
$w\bar{x}$				

$\bar{w}\bar{x}y$

wx

SOP $Y = \bar{w}\bar{x}y + wx$

POS

$wx \backslash yz$	$y+z$	$y+\bar{z}$	$\bar{y}+\bar{z}$	$\bar{y}+z$
$w+x$			0	0
$w+\bar{x}$				
$\bar{w}+\bar{x}$	0	0	0	0
$\bar{w}+x$				

$w+x+\bar{y}$

$\bar{w}+\bar{x}$

POS $Y = (w+x+\bar{y})\cdot(\bar{w}+\bar{x})$

4. Simplify the Boolean function
$F(w,x,y,z) = \Sigma m(0,5,7,8,9,10,11,14,15)$

$wx \backslash yz$	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	1			
$\bar{w}x$		1	1	
wx			1	1
$w\bar{x}$	1	1	1	1

$\bar{w}xz$

wy

$w\bar{x}$

$\bar{x}\bar{y}\bar{z}$

$Y = \bar{x}\bar{y}\bar{z} + \bar{w}xz + wy + w\bar{x}$

5.6. Reduce the following function using Karnaugh map techique and implement using basic gates.

$$F(A,B,C,D) = \bar{A}\bar{B}D + AB\bar{C}\bar{D} + \bar{A}BD + ABC\bar{D}$$

C / $\bar{C}$ C / $\bar{C}$

$\Rightarrow \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D + AB\bar{C}\bar{D} + ABC\bar{D} + \bar{A}BCD + \bar{A}B\bar{C}D$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		1	1	
$\bar{A}B$		1	1	
AB	1			1
$A\bar{B}$				

$\bar{A}D$

$AB\bar{D}$

$$Y = \bar{A}D + AB\bar{D}$$

6. Simplify the logic function sp… by the truth table using the karnaugh map method. Y is the output variable and A, B, and C are the input variable

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
A	1	0	1	0
$\bar{A}$	1	0	1	0

$\bar{B}\bar{C}$ $\quad$ BC

$$Y = \bar{B}\bar{C} + BC$$

7. Reduce the following function to to its minimum sum of product form:

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D$$

AB \ CD	$\bar C \bar D$	$\bar C D$	CD	$C\bar D$
$\bar A \bar B$		1		
$\bar A B$		1	1	
AB	1	1	1	1
$A\bar B$			1	

$\bar B \bar D +$
$A B \bar D +$
$\bar B C D$

$\cancel{\bar B D}$ $\quad$ $\cancel{\bar B D}$ $\quad$ $\cancel{AB\,B}$ $\quad$ $\cancel{A\bar B C\bar D}$ $\quad$ $\cancel{\bar B C \bar D}$

$\cancel{A\bar B \bar C \bar D}$ $\quad$ $\cancel{A\bar B CD}$

$Y = \bar A \bar C D + A B \bar C +$ $\quad$ $\bar B \bar C \bar D$ $\quad$ $\bar B C D$
$A C D + \bar A B C$

$+ BD$

$\cancel{Y =} \; A\cancel{BD} + \bar B \bar C D + \bar B [illegible] + \bar B [illegible]$

$\cancel{\bar B \bar C \text{ to } \bar B \bar D + AB\bar D}$

8. Reduce the following four variable function to the minimum sum of product form

$Y : \bar A \bar B C \bar D + A B C \bar D + A \bar B C \bar D + A \bar B C D +$
$A \bar B \bar C \bar D + A B \bar C \bar D + \bar A \bar B C D + \bar A \bar B \bar C \bar D$

AB \ CD	$\bar C \bar D$	$\bar C D$	CD	$C\bar D$
$\bar A \bar B$	1		1	1
$\bar A B$				
AB	1			1
$A\bar B$	1		1	1

$\bar B\, C$ $\quad$ $\cancel{\bar B C \bar D}$ $\quad$ $\bar B \bar D$

$A\bar D$

$\cancel{AB\bar D}$ $\quad$ $\cancel{AB\bar D}$

$\bar B \bar D /$
$AB\bar D$
$/\bar B C D$

$\bar B C D$

$\cancel{Y = AB\bar{D} + \bar{B}\bar{C}D + \bar{B}CD + \ldots}$

$\cancel{Y = \bar{B}C + \bar{B}\bar{D} + AB\bar{D}}$

$$Y = \bar{B}\bar{D} + \bar{B}C + A\bar{D}$$

part - 3

5 variable KMAP

- Requires 32 cells (2^5)
- The adjacent cells are difficult to identify on a single 32 cell K-MAP therefore two 16 cell K-map are generally used
- If the variables are A, B, C, D & E two identical 16 cells maps containing B, C, D, & E are connected.
- one map is used for A and other for $\bar{A}$

A=0 / $\bar{A}$

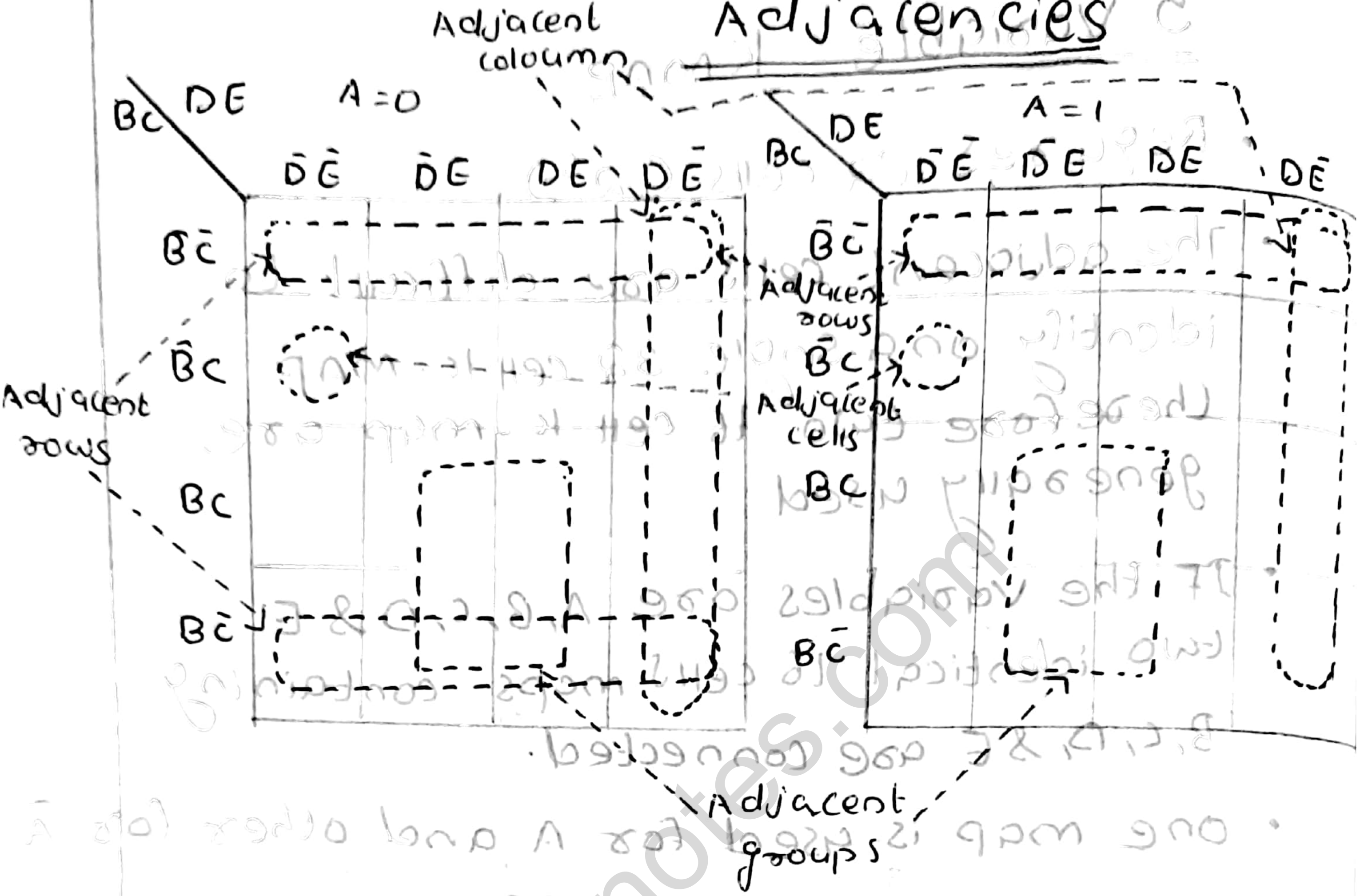
BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	0	1	3	2
$\bar{B}C$ 01	4	5	7	6
BC 11	12	13	15	14
$B\bar{C}$ 10	8	9	11	10

A=1 / A

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	16	17	19	18
$\bar{B}C$ 01	20	21	23	22
BC 11	28	29	31	30
$B\bar{C}$ 10	24	25	27	26

5 variable KMAP with example

Adjacencies



problems

1. $F(A,B,C,D,E) = \Sigma m(0,1,6,7,8,9,21,22,23,29,31)$

0 - 7 → 3 variable
0 - 15 → 4 variable
0 - 31 → 5 variable

A=0 / $\bar{A}$

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	1 (0)	1 (1)	(3)	(2)
$\bar{B}C$	(4)	(5)	1 (7)	1 (6)
BC	(12)	(13)	(15)	(14)
$B\bar{C}$	1 (8)	1 (9)	(11)	(10)

$\bar{A}\bar{C}\bar{D}$

$\bar{B}CD$

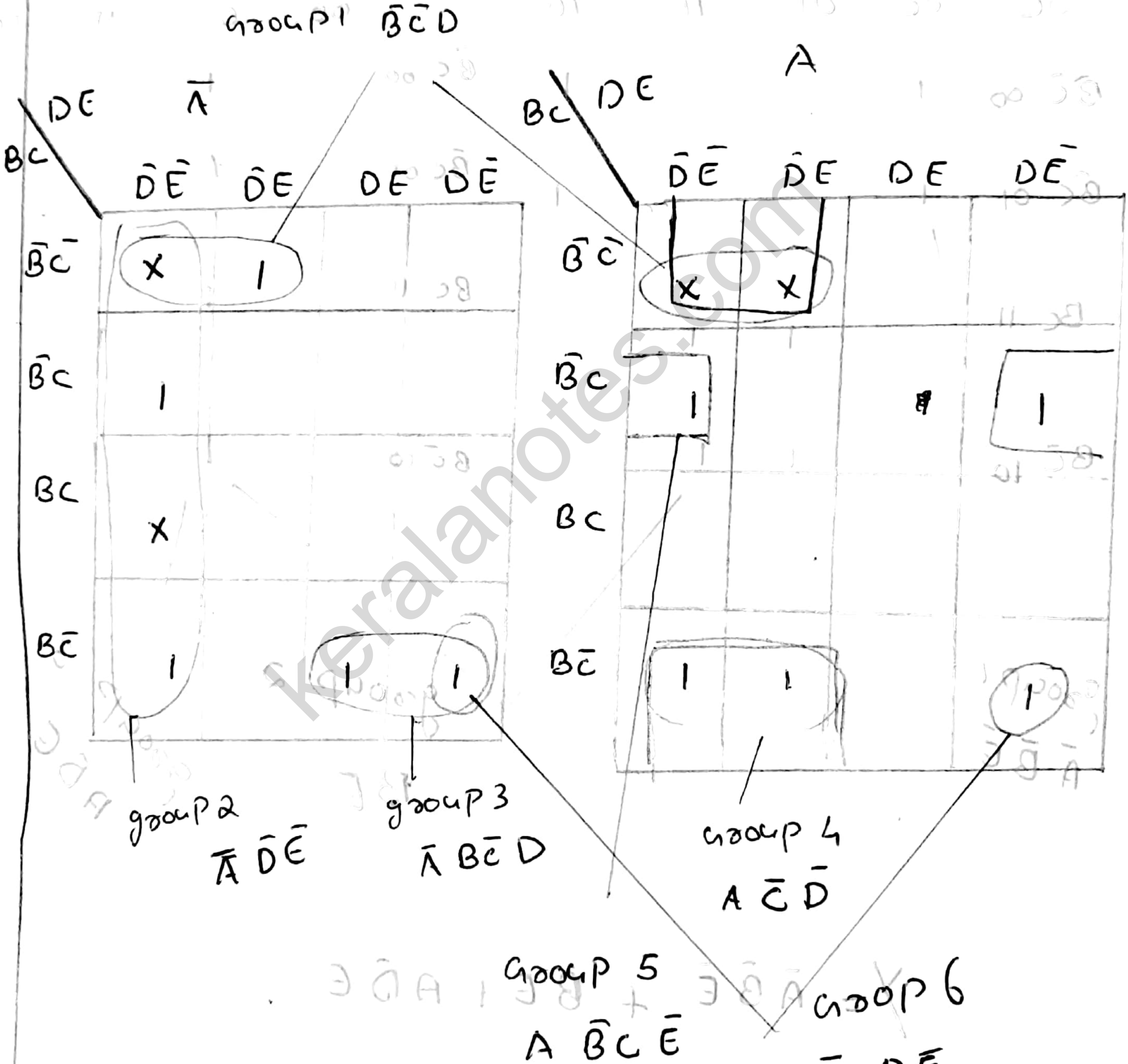
A=1 / A

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	(16)	(17)	(19)	(18)
$\bar{B}C$	(20)	1 (21)	1 (23)	1 (22)
BC	(28)	1 (29)	1 (31)	(30)
$B\bar{C}$	(24)	(25)	(27)	(26)

ACE

$$Y = \bar{A}\bar{C}\bar{D} + \bar{B}CD + AC\bar{E}$$

2 $$F(A,B,C,D,E) = \sum m(1,4,8,10,11,20,22,24,25,26) + d(0,12,16,17)$$



$$Y = \bar{B}\bar{C}\bar{D} + \bar{A}\bar{D}\bar{E} + \bar{A}B\bar{C}D + A\bar{C}\bar{D} + A\bar{B}C\bar{E} + B\bar{C}D\bar{E}$$

3 Simplify the boolean function,

$$F(A,B,C,D,E) = \Sigma m(0,2,4,6,9,11,13,15,17,21,25,27,29,31)$$

$\bar{A}$

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	1			1
$\bar{B}C$ 01	1			1
BC 11		1	1	
$B\bar{C}$ 10		1	1	

A

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00		1		
$\bar{B}C$ 01		1		
BC 11		1	1	
$B\bar{C}$ 10		1	1	

group 1
$\bar{A}\bar{B}\bar{E}$

group 2
BE

group 3
$A\bar{D}E$

$$Y = \bar{A}\bar{B}\bar{E} + BE + A\bar{D}E$$

4. $F(A,B,C,D,E) = \Sigma m(0, 5, 6, 8, 9, 10, 11, 16, 20, 24, 25, 26, 27, 29, 31)$

$\bar{A}$

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	1			
$\bar{B}C$ 01		1		1
BC 11				
$B\bar{C}$ 10	1	1	1	1

A

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	1			
$\bar{B}C$ 01	1			
BC 11		1	1	
$B\bar{C}$ 10	1	1	1	1

group 1 $(\bar{C}\bar{D}\bar{E})$

group 2 $\bar{A}\bar{B}C\bar{D}E$

group 3 $\bar{A}\bar{B}CD\bar{E}$

group 4 $B\bar{C}$

group 5 $A\bar{B}\bar{D}\bar{E}$

group 6 ABE

Simplification of POS Expression

(MAX TERM KMAP)

Steps to solve

1. plot the k-map and place 0's in those cells corresponding to the 0s in the TT or Maxterms in the product of Sum Expressions

2. check the kmap for adjacent 0's and encircle those 0's

3. Form the simplified pos Expression by taking pos terms of all the groups

AB \ CD	$C+D$ ~~A+B~~ 00	$C+\bar{D}$ ~~A+B~~ 01	$\bar{C}+\bar{D}$ ~~A+B~~ 11	$\bar{C}+D$ ~~A+B~~ 10
$A+B$ 00	$A+B+C+D$ M_0	$A+B+C+\bar{D}$ M_1	M_3	M_2
$A+\bar{B}$ 01	M_4	M_5	M_7	M_6
$\bar{A}+\bar{B}$ 11	M_{12}	M_{13}	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$ M_{15}	M_{14}
$\bar{A}+B$ 10	M_8	M_9	M_{11}	M_{10}

problems

1. $F(A,B,C,D) = \prod M\,(0,2,3,8,9,12,13,15)$

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0		0	0
$A+\bar{B}$ 01				
$\bar{A}+\bar{B}$ 11	0	0	0	
$\bar{A}+B$ 10	0	0		

$A+B+\bar{C}$

$A+B+D$

$\bar{A}+\bar{B}+\bar{D}$

$\bar{A}+C$

$$F = (\bar{A}+C)(\bar{A}+\bar{B}+\bar{D})(A+B+\bar{C})$$
$$(A+B+D)$$

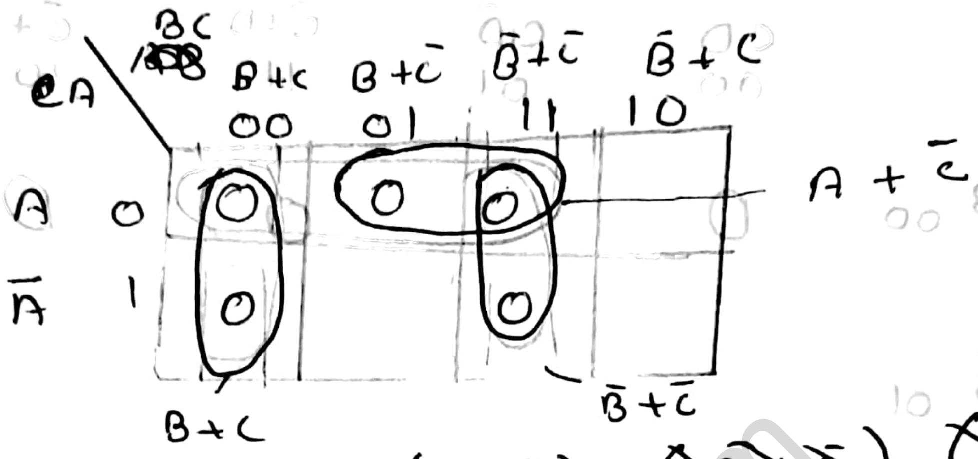
2. minimise ~~maximise~~ ~~minimize~~ the expression

$$Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})$$
$$(\bar{A}+B+C)(A+B+C)$$

ans: $(A+B+\bar{C}) = M_1$, $(A+\bar{B}+\bar{C}) = M_3$,

$(\bar{A}+\bar{B}+\bar{C}) = M_7$, $(\bar{A}+B+C) = M_4$,

$(A+B+C) = M_0$



~~$Y = A+\bar{A}+(B+C)\cdot(B+\bar{C})\cdot(A+$~~

~~$Y = (B+C)\cdot(B+C)\cdot(AB)$~~

$Y = (B+C)\cdot(\bar{B}+C)\cdot(A+\bar{C})$

Tabulation method or Quine McCluskey method

- Based on the concept of prime implicants
- prime implicant is a product or sum term

Q Simplify Boolean expression using Quine McCluskey method (Tabular method)

$F(A,B,C,D) = \Sigma_m (0,1,3,7,8,9, \ldots$

Step 0: Convert the decimal to Binary number

Decimal Number	Equivalent Binary Number		Minterm
0	0000	0	m_0
1	0001	1	m_1
3	0011	2	m_3
7	0111	3	m_7
8	1000	1	m_8
9	1001	2	m_9
11	1011	3	m_{11}
15	1111	4	m_{15}

Step 1: Arrange the minterm according to Number's of 1's

Group	Minterm no.	In Binary A	B	C	D
0	0	0	0	0	0
1	1	0	0	0	1
	8	1	0	0	0
2	3	0	0	1	1
	9	1	0	0	1
3	7	0	1	1	1
	11	1	0	1	1
4	15	1	1	1	1

Step 2: Compare each minterm in Group 'n' with each minterm in group (n+1) and identify the matched pairs

- A matched pair is a pair of minterms which differ only in one variable (Grey code)
- For the variable at differ place.

Group	minterm no	A	B	C	D
0	0	0	0	0	0
1	1	0	0	0	1
	8	1	0	0	0
2	3	0	0	1	1
	9	1	0	0	1
3	7	0	1	1	1
	11	1	0	1	1
4	15	1	1	1	1

Table 2

Group	minterm no	A	B	C	D
0	(0,1)	0	0	0	-
	(0,8)	-	0	0	0
1	(1,3)	0	0	-	1
	(1,9)	-	0	0	1
	(8,9)	1	0	0	-
2	(3,7)	0	-	1	1
	(3,11)	-	0	1	1
	(9,11)	1	0	-	1
3.	(7,15)	-	1	1	1
	(11,15)	1	-	1	1

Table 3

Step 3: Now compare all the minterms of table 3 with those in the adjacent groups. As shown in table 4

Groups	minterm no.	A	B	C	D
0	(0,1)	0	0	0	-
	(0,8)	-	0	0	0
1	(1,3)	0	0	-	1
	(1,9)	-	0	0	1
	(8,9)	1	0	0	-
2	(3,7)	0	-	1	1
	(3,11)	-	0	1	1
	(9,11)	1	0	-	1
3	(7,15)	-	1	1	1
	(11,15)	1	-	1	1

(In Binary: A B C D)

Table 3

Groups	minterm no.	A	B	C	D
1	0,1,8,9	-	0	0	-
	0,8,1,9	-	0	0	-
	(same so take any one)				
2	1,3,9,11	-	0	-	1
	1,9,3,11	-	0	-	1
	compare if there is no match pair stop the table				
3	3,7,11,15	-	-	1	1
	3,11,7,15	-	-	1	1

(In Binary: A B C D)

Table 4

Step 4: Repeat the procedure for grouping. If can group the quads of minterm in the adjacent groups of table 4 to obtain groups of eight minterms. There are no such matching.

• check again for mached pairs if not exist stop

Now prepare prime implicant table as shown in Table 5

Group	Minterms	Binary Representation A	B	C	D	SOP [1 - A / 0 - $\bar{A}$]
1	$m_0 - m_1 - m_8 - m_9$	-	0	0	-	$\bar{B}\bar{C}$
	$m_0 - m_8 - m_1 - m_9$	-	0	0	-	
2	$m_1 - m_3 - m_9 - m_{11}$	-	0	-	1	$\bar{B}D$
	$m_1 - m_9 - m_3 - m_{11}$	-	0	-	1	
3.	$m_3 - m_7 - m_{11} - m_{15}$	-	-	1	1	CD
	$m_3 - m_{11} - m_7 - m_{15}$	-	-	1	1	

Table 5

$\bar{B}\bar{C} + \bar{B}D + CD$ - prime implicant

Now we have to find essential prime implicant

Step 6: Find essential prime implicants

PI (prime implicant)	Minterms groups and Boolean Representation	0	1	3	7	8	9	11	15
✓	$\bar{B}\bar{C}$ 0, 1, 8, 9	X	X			X	X		
×	$\bar{B}D$. 1, 3, 9, 11		X	X			X	X	
✓	CD 3, 7, 11, 15			X	X			X	X

Rounding which have only on value (X)

Table 6

Here $\bar{B}D$ does not included in rounding so it is not PI

From table 6 prime implicants are

$\bar{B}\bar{C}$ and CD

$\bar{B}D$ is Not prime implicand

Required output

$$\boxed{Y = \bar{B}\bar{C} + CD}$$