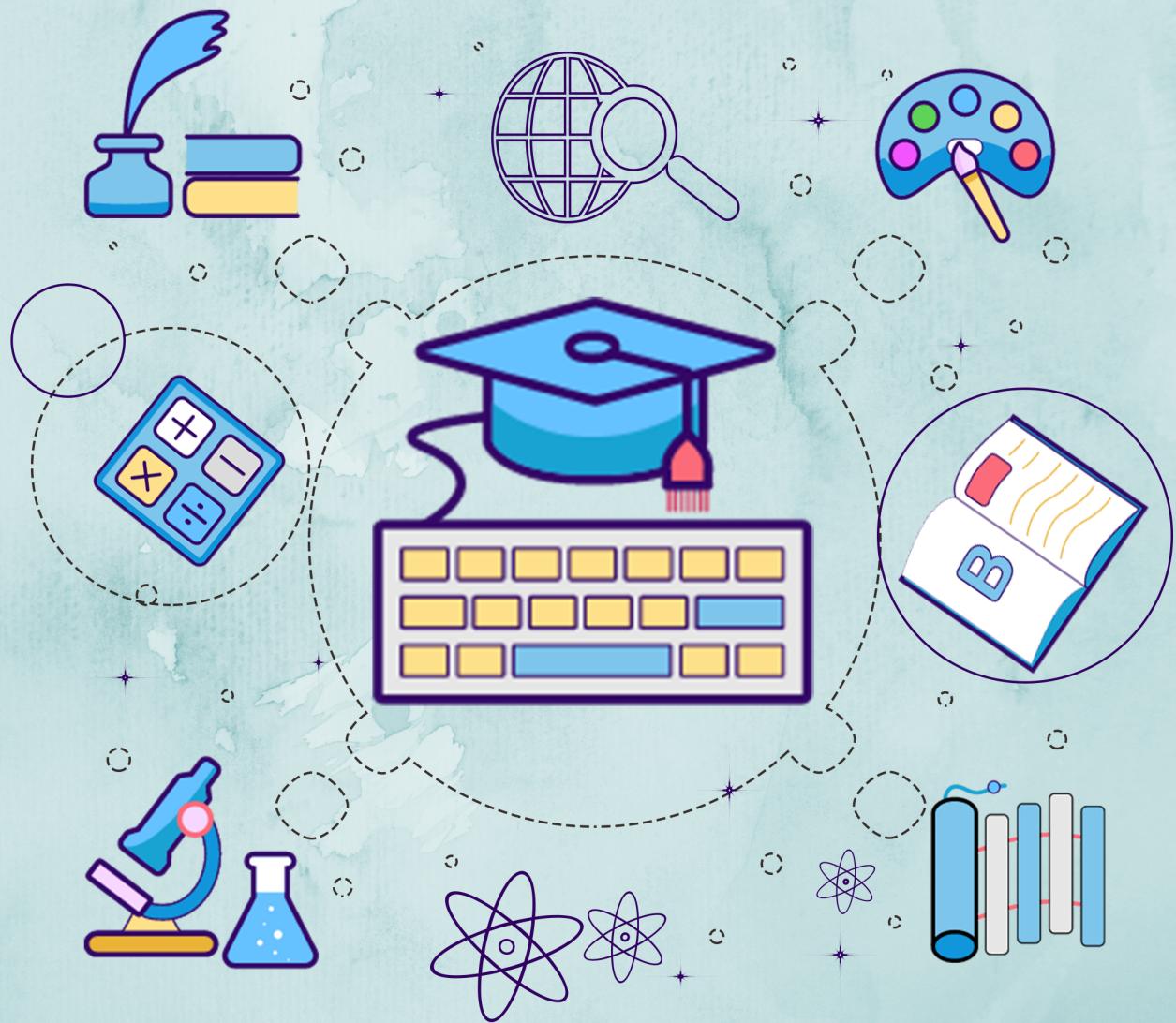


Kerala Notes



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KTU STUDY MATERIALS

LOGIC SYSTEM DESIGN

CST 203

Module 1

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SOLVED QUESTION PAPER

Module - I

Number system, operation and codes

Number system

- For counting various items
- Naming and representing numbers
- Mathematical notation - symbols etc.

Number system classification

- Binary number system
 - Base 2
 - 0, 1
- Decimal number system
 - Base 10
 - 0, 1, ..., 9
- Octal number system
 - Base 8
 - 0, 1, ..., 7
- Hexa decimal number system
 - Base 16
 - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Decimal number system

- Radix - 10 number system

- 10 different digit / symbol
0 ... 9

Eg: $5678.9 = 5000 + 600 + 70 + 8 + 0.9$

\downarrow \downarrow \downarrow \downarrow \downarrow

most significant bit (MSB) least significant bit (LSB)

$$10^3 \quad 10^2 \quad 10^1 \quad 10^0$$

\downarrow \downarrow \downarrow \downarrow

Binary number system

Radix - 2 number system

- 2 "0" or "1"

- powers of 2

$$2^3, 2^2, 2^1, 2^0, 2^{-1}, \dots$$

11010101

10101010

11010101

Octal number system

- Radix - 8 number system
- 0 ... 7
- powers of 8

Eg: 567

$$5 \times 8^2 + 6 \times 8^1 + 7 \times 8^0$$

567

Hexadecimal number system

- Radix - 16 number system
- 0 ... 9, A ... F
- powers of 16

Eg: 3F1D

$$3 \times 16^3 + F \times 16^2 + 1 \times 16^1 + D \times 16^0$$

3 F 1 D

Relationship b/w Decimal, Binary, Octal & Hexadecimal numbers

Decimal	Octal	Hexadecimal	Binary
0	0	0	0000
1	1	1	0001
2	2	2	0010
3	3	3	0011
4	4	4	0100
5	5	5	0101
6	6	6	0110
7	7	7	0111
8	10	8	1000
9	11	9	1001
10	12	A	1010
11	13	B	1011
12	14	C	1100
13	15	D	1101
14	16	E	1110
15	17	F	1111

Number system conversion

→ Decimal - Binary

$$1. \quad (39.25)_{10} \rightarrow (?)_2$$

decimal ↓
 $\underline{20 \cdot (\div)}$ fraction (x)

Binary base (2)

↳ 39

2	39	(1) - remainder
2	19	(0)
2	9	(1)
2	4	(0)
2	2	(0)
2	1	(1)

Bottom to top

Fraction: 0.25

$$100111$$

↓ ↓ ↓ ↓ ↓ ↓ ↓

$$= 0.25 \times 2 = 0.5$$

top to
bottom

$$0.5 \times 2 = 1.0$$

min 2 step

$$0.0 \times 2 = 0.0$$

$$(0.25)_{10} = (010)_2$$

$$= (100111.010)_2 \rightarrow \text{Binary}$$

* IF decimal no. only no need of fraction

Decimal to octal

(10)

(8)

$$(39.25)_{10} \rightarrow ()_8$$

$$8 \overline{)39 \quad 7}$$

47

$$0.25 \times 8 = 2.00$$

$$0.00 \times 8 = 0.00$$

20

$$= (47.20)_8 \rightarrow \text{octal.}$$

Decimal to Hexadecimal

(10)

(16)

$$0.1 = 8 \times 2.0$$

10 → A

11 → B

$$0.0 = 8 \times 0.0$$

12 → C

13 → D

$$(0.1)_{10} = (2.0)_{16}$$

14 → E

15 → F

$$(0.1)_{10} = (2.0)_{16}$$

1. (47.35)

$(47.35)_{10} \rightarrow (47.35)_{16}$

$$16 \overline{)47} \quad \textcircled{15} \rightarrow \textcircled{F}$$

$$\begin{array}{r} 60 \\ 16 \overline{)360} \\ 32 \\ \hline 60 \\ 48 \\ \hline 12 \\ 12 \\ \hline 0 \end{array} \quad \begin{array}{r} 16 \\ 16 \overline{)32} \\ 32 \\ \hline 0 \end{array} \quad \begin{array}{r} 35 \\ 16 \overline{)560} \\ 48 \\ \hline 80 \\ 80 \\ \hline 0 \end{array}$$

के लिए

2 F

$$0.35 \times 16 = \textcircled{5}.60$$

$$0.60 \times 16 = \textcircled{9}.60$$

$$= 59$$

$\Rightarrow (2F.59)_{16}$

$$\begin{array}{r} 2 \\ 2 \overline{)3} \\ 2 \\ \hline 1 \end{array}$$

Octal to Binary

(8)

(2)

3 bit

Eg: $(37.45)_8 \rightarrow (37.45)_2$

$$= [\underbrace{011}_3 \underbrace{111}_7 \cdot \underbrace{100}_4 \underbrace{101}_5]_2$$

$$0 \rightarrow 0000$$

$$1 \rightarrow 0001$$

$$2 \rightarrow 010$$

$$3 \rightarrow 011$$

$$4 \rightarrow 100$$

$$5 \rightarrow 101$$

$$6 \rightarrow 110$$

$$7 \rightarrow \underline{\underline{111}}$$

$$4 \rightarrow 0 \quad 4 \rightarrow 1$$

$$2 \rightarrow 0 \quad 2 \rightarrow 1$$

0, 1, 0, 1 ...

$$2 \overline{)40} = 4 = 0$$

(ii)

Binary - octal

Eg: $(10110.11)_2 \rightarrow (26.6)_8$

group of 3

$$\begin{array}{r} 010110 \\ \hline \text{---} \\ (26.6)_8 \end{array}$$

Q solve $(672.13)_8 \rightarrow (?)_2$

$$= (110\ 111\ 010.001\ 011)_2$$

Q $(10111011.100)_2 \rightarrow (?)_8$

$$\begin{array}{r} 10111011.100 \\ \hline \text{---} \\ 243.4 \end{array}$$

$$\therefore (243.4)_8$$

(i) Hexadecimal to binary

(16) \rightarrow (2)

0 0 0 0 \rightarrow 0

0 0 0 1 \rightarrow 1

0 0 1 0 \rightarrow 2

0 0 1 1 \rightarrow 3

0 1 0 0 \rightarrow 4

0 1 0 1 \rightarrow 5

0 1 1 0 \rightarrow 6

0 1 1 1 \rightarrow 7

1 0 0 0 \rightarrow 8

1 0 0 1 \rightarrow 9

1 0 1 0 \rightarrow 10 (A)

1 0 1 1 \rightarrow 11 (B)

1 1 0 0 \rightarrow 12 (C)

1 1 0 1 \rightarrow 13 (D)

1 1 1 0 \rightarrow 14 (E)

1 1 1 1 \rightarrow 15 (F)

8 \rightarrow 0, 8 \rightarrow 1

4 \rightarrow 0, 4 \rightarrow 1, 4 \rightarrow 0
4 \rightarrow 1

2 \rightarrow 0, 2 \rightarrow 1, 2 \rightarrow 0, 2 \rightarrow 1
0, 1, 0, 1, 0, 1, 0...

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	(A)
11	(B)
12	(C)
13	(D)
14	(E)
15	(F)

Eg: $(101.01)_2 = (101.01)_2 \cdot 10^0 + 0 \cdot 10^{-1} + 1 \cdot 10^{-2}$

1. $(259A)_{16} \rightarrow ()_2$

$$\left(\underbrace{0\ 010}_{2} \ \underbrace{010}_{5} \ \underbrace{1\ 001}_{9} \ \underbrace{1010}_A \right)$$

(ii) Binary - Hexadecimal

$$(\underline{1000} \underline{1001} \underline{11})_{82} \rightarrow (3)_{816}$$

00110001001.11 00

(189.c) ~~yes~~ ~~85-0001~~
~~spc-1001~~

$$Q \quad (\text{FeO}_2)_{16} \rightarrow (\text{FeO})_8$$

1111 1110 0006 0010

$$Q \quad (1100110.101)_2 \rightarrow (7)_{8,16}$$

000111001110.1010 0 - 800 P.M.

四

ICE · A

Octal to Hexa decimal

Hexa decimal to octal

- Octal \rightarrow Binary \rightarrow hexa decimal
- hexa decimal \rightarrow Binary \rightarrow octal

Eg 1. $(653)_8 \rightarrow (110101011)_2 \rightarrow (C3)_{16}$

$000(1|0|1|0|1|0|1)2 \rightarrow (C3)_{16}$

I A B

Octal to binary

$\begin{array}{r} 110 \quad 101 \quad 011 \\ \hline 6 \quad 5 \quad 3 \end{array} \rightarrow (C3)_{16}$

$001|10101011 \quad (0)6 \quad 5 \quad 3$

I A B $110 \quad 101 \quad 011$

$0001|10101011$

1 A B

(2) Binary (8) Octal (16) Hexadecimal } \rightarrow Decimal

$$a_3 \ a_2 \ a_1 \ a_0 \dots a_{-1} \ a_{-2} \dots$$

$$(a_3 \times \gamma^3) + (a_2 \gamma^2) + (a_1 \gamma^1) + (a_0 \gamma^0) \\ + (a_{-1} \gamma^{-1}) + (a_{-2} \gamma^{-2})$$

$\gamma \rightarrow$ Base of system $\gamma = 2$ - Binary

$= 8$ - Octal

$= 16$ - Hexadecimal

Examples:

b

Binary - Decimal

(2)	(10)	position
-----	------	----------

Eg: 1. (10101.11)₂ \rightarrow (2, 0)

1	0	1	0	1	.	1	1
covert into 10							

$$a_4 \ a_3 \ a_2 \ a_1 \ a_0 \ a_{-1} \ a_{-2} \\ \gamma^4 \ \gamma^3 \ \gamma^2 \ \gamma^1 \ \gamma^0 \ \gamma^{-1} \ \gamma^{-2}$$

Here $\alpha = 2$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$$

$$\begin{aligned}
 & (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\
 & (1 \times 2^{-1}) + (1 \times 2^{-2}) \\
 & = 16 + 0 + 4 + \frac{1}{4} \\
 & = 20.25 \\
 & = (21.75)_{10}
 \end{aligned}$$

$$2^{-1} = \frac{1}{2^1} = 0.5$$

We can check by
converting

Octal to Decimal

(8) \rightarrow 8(10)

$(57.4)_8 \rightarrow ()_{10}$

Here $\alpha = 8$

5	7	.	4
---	---	---	---

$8^3 8^2 8^1 8^0$

$8^{-1} 8^{-2} 8^{-3}$

$$\begin{aligned}
 8^0 &= 1 & 8^{-1} &= \frac{1}{8} \\
 && &\times \frac{1}{8} =
 \end{aligned}$$

$$(5 \times 8^3) + (7 \times 8^2) + (4 \times 8^1)$$

$$40 + 7 + 0.5$$

$$\underline{(47.5)_{10}}$$

Q $(4507.44)_8 \rightarrow (D)_{10}$

4	5	0	7	.	4	4
---	---	---	---	---	---	---

$$a_3 \quad a_2 \quad a_1 \quad a_0 \quad a_{-1} \quad a_{-2}$$

$$\gamma^3 \quad \gamma^2 \quad \gamma^1 \quad \gamma^0 \quad \gamma^{-1} \quad \gamma^{-2}$$

Here, $\gamma = 8$

$$(4 \times 8^3) + (5 \times 8^2) + (0 \times 8^1) + (7 \times 8^0) + \\ (4 \times 8^{-1}) + (4 \times 8^{-2})$$

$$= 2048 + 320 + 0 + 7 + 0.5 + 0.0625$$

$$\underline{2375.5625}$$

Hexadecimal to Decimal

(16)

(10)

$$(3A3B)_{16} = (3 \times 16^4) + (A \times 16^3) + (3 \times 16^2) + (B \times 16^1) + (3 \times 16^0)$$

$$(BAD \cdot 8)_{16} \rightarrow C \rightarrow_{10}$$

B	A	D	.	8
q_2	q_1	q_0	q_{-1}	q_{-2}
α^2	α^1	α^0	α^{-1}	α^{-2}

A - 10
 B - 11
 C - 12
 D - 13
 E - 14
 F - 15

$$\alpha = 16$$

$$\begin{aligned}
 & (11 \times 16^2) + (10 \times 16^1) + (13 \times 16^0) + \\
 & (8 \times 16^{-1})
 \end{aligned}$$

$$2816 + 160 + 13 + 0.5$$

$$= 2989.5$$

part 3

Decimal \rightarrow Binary

$$(\div 8 \times)$$

bottom vice versa
top

\rightarrow Octal

\rightarrow Hexadecimal

Binary \rightarrow octal (3 bit)
(8 bit)

\rightarrow Hexadecimal

Octal \rightarrow hexade

hexadecimal to
octal

Practice problems:

1. $(250.55)_{10} \rightarrow (D_{16}, D_8)$

$(250.55)_{10} \rightarrow (D_{16}, D_8)$

$$\begin{array}{r} 16 \\ | \\ \underline{250(10)} \rightarrow A \\ | \\ 15 \rightarrow F \end{array}$$

$0.55 \times 16 = 8.8$
 $0.8 \times 16 = 12.8$
~~15 + 0.8~~ ~~0.8~~
 $0.0 \times 16 = 0.0$

$(FA.8C0)_{16}$

$(250.55)_{10} \rightarrow (D_8)$

$$\begin{array}{r} 8 \\ | \\ \underline{250(2)} \\ | \\ 8 \\ | \\ \underline{31(2)} \\ | \\ 3 \end{array}$$

$0.55 \times 8 = 4.4$
 $0.4 \times 8 = 3.2$

$(372.4314)_8$

$0.2 \times 8 = 1.6$
 $0.6 \times 8 = 4.8$

2. $(357)_8 = ()_{10}, ()_{16}$

$$(357)_8 = ()_{10}$$

3	5	7
---	---	---

a₂ a₁ a₀

8² 8¹ 8⁰

Here $\delta = 8$

$$(3 \times 8^2) + (5 \times 8^1) + (7 \times 8^0)$$

~~$$(3 \times 192) + (5 \times 40) + 7$$~~

~~$$= (239)_8$$~~

~~$$(357)_8 \rightarrow ()_{16}$$~~

3	5	7
---	---	---

a₂ a₁ a₀

16² 16¹ 16⁰

~~$$(3 \times 16^2) + (5 \times 16^1) + (7 \times 16^0)$$~~

~~$$= 768 + 80 + 7$$~~

~~$$= (855)_{10}$$~~

$$(357)_8 \rightarrow (77)_{16}$$

$$(011\ 101\ 111)_2 \rightarrow (77)_{16}$$

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	1	1	0	1	1	1	1

$a_8\ a_7\ a_6\ a_5\ a_4\ a_3\ a_2\ a_1\ a_0$

Q. Here $\cancel{a_7 = 1}$

$$\cancel{(0 \times 2^8)} + (\cancel{1} \times 2^7) + (\cancel{1} \times 2^6) + (\cancel{1} \times 2^5) + \\ \cancel{(0 \times 2^4)} + (\cancel{1} \times 2^3) + (\cancel{1} \times 2^2) + (\cancel{1} \times 2^1) + \\ (\cancel{1} \times 2^0)$$

$$= \cancel{128} + \cancel{64} + \cancel{32} + \cancel{16} + \cancel{8} + \cancel{4} + \cancel{2} + \cancel{1} \\ \cdot (255)_{16}$$

$$0000|1110|111$$

$$\underline{(0\ E\ F)_{16}}$$

$$3. (110101 \cdot 1011)_2 = C_8, C_{16}$$

$$(110101 \cdot 1011)_2 = C_8$$

$110|101 \cdot |101|100$

$$(65 \cdot 54)_8 (010 \cdot 11100111)$$

$$(110101 \cdot 1011) = C_{16} (010 \cdot 11100111)$$

$$0011|0101 \cdot |1011$$

$$(35 \cdot 13)_{16}$$

$$4. (463 \cdot 25)_{10} = C_2, C_8$$

$$(463 \cdot 25)_{10} = C_2, C_8 (08 \cdot 011)$$

$$\begin{array}{r} 2 | 463(1) \\ 2 | 231(1) \\ 2 | 115(1) \\ 2 | 57(1) \\ 2 | 28(0) \\ 2 | 14(0) \\ 2 | 7(1) \\ 2 | 3(1) \\ \hline \end{array}$$

$$0.25 \times 2 = 0.5 \quad (1101.0101)$$

$$0.5 \times 2 = 1.0 \quad (1101.0101)$$

$$0.0 \times 2 = 0.0 \quad (001101.101011)$$

$$(111001111.010)_2 \quad (1101.0101)$$

$$(463.25)_{10} \rightarrow (28)_{10} \quad (1101.0101)$$

$$\begin{array}{r} 8 | 463 \quad (7) \\ 8 | 57 \quad (1) \\ 8 | 7 \quad (0) \\ \hline 0 \end{array} \quad \begin{array}{l} 1101.0101 \\ 0.25 \times 8 = 2.0 \\ 0.16 \times 8 = 0.0 \end{array}$$

$$= (717.20)_8 \quad (1101.0101)$$

$$(717.20)_8 \quad (1101.0101)$$

5. $(36.25)_{10} \rightarrow (28)_{10} \quad (1101.0101)$

3

$$(36.25)_{10} \rightarrow (28)_{10}$$

$$8 \overline{)36.25} \quad (4)$$

$$0.25 \times 8 = 2$$

$$0.0 \times 8 = 0$$

$$= 44$$

$$(44.20)_8$$

$$(36.25)_{10} - (25)_{16}$$

$$16 \overline{)36(4)}$$

$$0.25 \times 16 = 4$$

$$0.0 \times 16 = 0$$

$$= 24$$

$$= (24.40)_{16}$$

$$6. (455)_{10} - (28)_2, (45)_{10}, (25)_8$$

$$\begin{array}{r} 1 \\ 2 \overline{)455(1)} \\ 2 \overline{)227(1)} \\ 2 \overline{)113(1)} \\ 2 \overline{)56(0)} \\ 2 \overline{)28(0)} \\ 2 \overline{)14(0)} \\ 2 \overline{)7(1)} \\ 2 \overline{)3(1)} \\ 1 \end{array}$$

$$(111000111)_2$$

$$(455)_{10} \rightarrow C_5$$

$$4 \overline{)455(3)}$$

$$4 \overline{)113(1)}$$

$$4 \overline{)28(6)}$$

$$4 \overline{)7(3)}$$

$$\underline{\quad 1}$$

$$(13013)_4$$

$$(455)_{10} \rightarrow C_8$$

$$8 \overline{)455(7)}$$

$$8 \overline{)56(0)}$$

$$\underline{\quad 7}$$

$$(707)_8$$

6. $(12.0625)_{10} - C_{16}, C_2$

~~12.0625~~

$$(12.0625)_{10} - C_{16}$$

$$(11000111)$$

16 12(12) \rightarrow 10

$$0.625 \times 16 = 10.0$$

$$0.0 \times 16 = 0.0$$

$$= (C.100)_{16}$$

$$(12.0625)_{10} \rightarrow (C.12)_{16}$$

$$8 \quad \begin{array}{r} 12 \\ \hline 10 \end{array} \times 8 = 0.5$$

$$0.625 \times 8 = 5.0$$

$$14.000 + 5.0 = 19.0$$

$$0.0 \times 8 = 0.0$$

$$(14.500)_{10} \quad (0.0 \times 8 = 0.0)$$

$$(14.040)_{8}$$

$$8. \quad (37.F_c)_{16} - C_2, C_8, C_{10}$$

$$(37.F_c)_{16} - C_2$$

$$(0011\ 0111\ .\ 1111100)_2$$

$$(37.F_c)_{16} - C_8$$

$$00011\ 0111\ .\ 11111000$$

$$(067.770)_{28}$$

$C_{37} \cdot Fc \rightarrow C_{210}$

3	7	.	F	c	12
---	---	---	---	---	----

$a_1 \quad a_0 \quad a_{-1} \quad a_{-2}$

$\alpha^1 \quad \alpha^0 \quad \alpha^{-1} \quad \alpha^{-2}$

Here $\alpha = 16$

$$e. (a \ 3 \times 16^1) + (7 \times 16^0) + (15 \times 16^{-1}) +$$

$$(12 \times 16^{-2})$$

$$48 + 7 + 0.9375 + 0.046875$$

$$(55.984375)$$

55.984375

$$55.984375 - 31(57.08) = 8$$

$$\frac{1}{x} \times 55.984375 = 31(57.08)$$

$$(0.011111 \cdot 110.8100)$$

$$0.011111 \cdot 110.8100$$

$$0.011111 \cdot 110.8100$$

Addition	{	Binary	Octal	Hexa decimal
Subtraction				
Multiplication	{			
Division		Binary	Octal	Hexa decimal

Binary addition

(2)

* one bit

$$\begin{array}{r} 1 \\ + \\ 1 \\ \hline 0 \end{array}$$

↓

$$\begin{array}{r} 1 \\ 0 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 1 \\ 0 \\ \hline 1 \\ 0 \\ \hline 1 \end{array}$$

↓

$$\begin{array}{r} 1 \\ 0 \\ \hline 1 \\ 0 \\ \hline 1 \end{array}$$

→ two bit

$$\begin{array}{r} 1 \\ 1 \\ + \\ 1 \\ \hline 1 \\ 0 \\ \hline \end{array}$$

sum → carry

Sum	carry
0 + 0 = 0	0
0 + 1 = 1	0
1 + 0 = 1	0
1 + 1 = 0	1

$$\begin{array}{r} 1 \\ 0 \\ + \\ 1 \\ \hline 1 \\ 0 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ + \\ 1 \\ \hline 1 \\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ + \\ 1 \\ \hline 1 \\ 1 \\ \hline \end{array}$$

c carry - sum

$$\begin{array}{r} 1 \\ 1 \\ + \\ 1 \\ \hline 1 \\ 1 \\ \hline \end{array}$$

Eg: $\begin{array}{r} \textcircled{1} \quad 1 \ 1 \ 0 \\ + \quad 1 \ 0 \ 1 \\ \hline 1 \ 0 \ 1 \ 1 \end{array}$

Eg: $\begin{array}{r} \textcircled{1} \ 0 \ 0 \ 1 \\ + \quad 1 \ 1 \ 0 \ 1 \\ \hline 1 \ 1 \ 0 \ 1 \ 0 \end{array}$

Octal addition

Eg: (8)

$$\begin{array}{r} \textcircled{1} \ 0 \ 1 \\ 1 \ 7 \ 7 \ 6 \\ + \quad 1 \ 3 \ 4 \ 5 \\ \hline 3 \ 3 \ 4 \ 3 \end{array}$$

Decimal (10)

$$\begin{array}{r} 1 \ 8 \ 5 \ 7 \\ + \quad 9 \ 9 \ 8 \ 9 \\ \hline 1 \ 7 \ 8 \ 4 \ 6 \end{array}$$

$(7+9) \rightarrow 16$

$$16 = 1 \times 10 + 6$$

\downarrow \downarrow

carry sum

$$(6+5) = 11 = (1 \times 8) + 3$$

$$12 = (1 \times 8) + 4$$

$$11 = (1 \times 8) + 3$$

$$44 \rightarrow 1 \times 10 + 4$$

$$18 \rightarrow 1 \times 10 + 8$$

$$17 \rightarrow 1 \times 10 + 7$$

Eg:

$$\begin{array}{r}
 & \textcircled{1} & \textcircled{1} & \textcircled{1} \\
 & 7 & 7 & 7 \\
 \textcircled{1} & 6 & 6 & 6 \\
 + & & & \\
 \hline
 & 1 & 6 & 6 & 5
 \end{array}$$

$$13 = (1 \times 8) + 5$$

$$14 = (1 \times 8) + 6$$

$$15 = (1 \times 8) + 7$$

Hexadecimal addition

Eg:

$$\begin{array}{r}
 & \textcircled{1} & \textcircled{1} \\
 A & D & D \\
 \textcircled{1} & D & A & D \\
 + & & & \\
 \hline
 & (6 \times 8) & A & E & 8
 \end{array}$$

$$A = 10$$

$$B = 11$$

$$C = 12$$

$$D = 13$$

$$E = 14$$

$$F = 15$$

$$C_{26} = (1 \times 16) + 10$$

Position (AD)

$$24 = (1 \times 16) + 8$$

$$\begin{array}{r}
 & \textcircled{1} & \textcircled{0} \\
 & 0 & 1 & 0 \\
 & 0 & 1 & 0 \\
 & 0 & 1 & 0 \\
 + & 0 & 1 & 0 \\
 \hline
 & 1 & 0 & 1 & 0
 \end{array}$$

Hexadecimal

(1) (2)

F F F

E 0 F +

G ↓ 1 6 A

1 F 6 8

A - 10
B - 11
C - 12
D - 13
E - 14
F - 15

Octal

② ②

7 7 7

② 6 6 6 +

↓ 7 7 7

26 6 4

=

$$20 = (1 \times 8) + 12 \times$$

$$21 = (2 \times 8) + 5$$

$$22 = (2 \times 8) + 6$$

23

$$20 = (2 \times 16) + 4$$

$$23 = (1 \times 16) + 7$$

$$31 = (1 \times 16) + 15$$

↓ F

Binary Multiplication

Eg:

1010 ×

101

—————

1 0 1 0
0 0 0 0
1 0 1 0

1 0 1 0 0 1 0

1 1 0 0 1 0

Binary		Sum	Carry
0	+ 0	0	0
1	+ 0	1	0
0	+ 1	1	0
1	+ 1	0	1

Binary Subtraction

1. General method

2. 1's complement method

3. 2's complement method

1. General method of subtraction

$$0 - 0 = 0$$

$$1 - 1 = 0$$

$$1 - 0 = 1$$

$$0 - 1 = 1 \text{ (borrow 1)}$$

with borrow

Binary $0 - 0, 1$

Eg:

①

$$\begin{array}{r} 11011 \\ - 10110 \\ \hline 00101 \end{array}$$

Subtraction of 10110 from 11011

(5) i's and 2's complement method

i's complement \rightarrow subtract each bit by 1
 (invert the number)
 [0 \rightarrow 1, 1 \rightarrow 0 or sub with 1]

2's complement \rightarrow i's comp + 1

Example:

$$1. \quad 1000 \quad (a) \text{ i's complement}$$

$$(b) \quad 2^{\text{'}} \text{ s complement}$$

$$(a) \quad 0111 - \text{i's comp}$$

$$(b) \quad \begin{array}{r} 0111 \\ + 1 \\ \hline 1000 \end{array} \rightarrow 2^{\text{'}} \text{ s}$$

$$2. \quad 1010 \rightarrow \text{i's complement}$$

$$(a) \quad 0101 - \text{carry}$$

$$(b) \quad \begin{array}{r} 0101 \\ + 1 \\ \hline 0110 \end{array} \rightarrow 2^{\text{'}} \text{ s complement}$$

Binary Subtraction using 1's Complement

Steps:

1. Find the 1's complement of the number to be subtracted
2. Perform Addition [Add first number and 1's complement of subtrahend]
3. • If carry is generated, Add the carry with the result.
• If there is no carry, take the 1's complement of the sum and assign negative sign.

Examples:

1. $B \rightarrow B$'s complement

2. $A + B$'s 1's complement

$$(1100)_2 - (0101)_2$$

1100

0+0=0
0+1=1
1+0=1
1+1=0
with carry 1

1. 1's complement of 0101 (B)
 $= 1010$

2. $\begin{array}{r} 1100 \\ 1010 \\ \hline 0110 \end{array}$ → Give A
→ B's 1's complement
CARRY 4 bit

$$\begin{array}{r}
 01101 \\
 + 01101 \\
 \hline
 01101
 \end{array}$$

Basic

$$\Rightarrow 0111$$

{in decimal} $\rightarrow 7$

$$(1100)_2 - (0101)_2$$

$$12 - 5 = 7$$

$$2. (0101)_2 - (1100)_2$$

$$1. 0011$$

$$2. \begin{array}{r}
 0100 \\
 + 0011 \\
 \hline
 1000
 \end{array}$$

$$1000 \text{ No carry}$$

$$3. 0111$$

so it's complement of 1000

$$0111$$

Putting negative sign = $- \underline{0111} (-)$

$$1000 - 0111 = 1001$$

$$1000 - 0111 = 1001$$

$$1000 - 0111 = 1001$$

3. Binary subtraction using 2's complement

Steps:

1. Find the 2's complement of the number to be subtracted
2. Perform Addition (Add first number and 2's complement of the subtrahend)
3. • If carry is generated, Discard the carry.
• If there is no carry, take the 2's complement of the sum and assign negative sign

Example:

$$\begin{array}{r} (1001)_2 \\ - (0100)_2 \\ \hline \end{array}$$

1. 2's complement of 0100
2's complement = 1011
$$\begin{array}{r} 1011 \\ + 0100 \\ \hline 1111 \end{array}$$

$$2. \quad \begin{array}{r} 1001 \\ - 1100 \\ \hline 1001 \end{array}$$

carry we can neglect carry

$$= \underline{\underline{0101}}$$

$$2. \quad (0110) - (1011)$$

1. 2's complement of 1011

$$\text{Ans. } 1011 + 0101 = \underline{\underline{0101}}$$

$$2. \quad \begin{array}{r} 0110 \\ + 0101 \\ \hline \end{array}$$

$$\text{Ans. } \cancel{1000} \quad \text{No carry so} \\ 1011$$

3. Take 2's complement of result of
 $(0010) - (0011) = 1011$

$$\text{is complement} \rightarrow \underline{\underline{0100}}$$

$$\text{2's complement } 0100 + \\ 1011 = \underline{\underline{0101}}$$

Octal Subtraction

1. General method
2. 7's complement method
3. 8's complement method

1. General Method

(a)

→ Decimal subtraction $(_{10})$

$$100 - 48 = 964 - 488$$

8	5	10	9	5	10
9	6	4	8	6	8
4	8	8	8	8	8
4	7	6			

$$10 + 4 - 8 = 6$$

$$10 + 5 - 8 = 7$$

→ Octal subtraction

6	3	8
7	6	3
5	6	4

$$8 + 3 - 4 = 7$$

$$8 + 3 - 6 = 5$$

($10 + 3 - 6 = 7$)

$$\begin{array}{r}
 5 \ 7 \\
 6 \ 2 \ 8 \\
 \hline
 2 \ 6 \ 5 \\
 \hline
 3 \ 4 \ 3
 \end{array}$$

$$8 + 2 - 6 = 4$$

(b) 7's and 8's complement of octal number

7's complement

Subtract each and every octal digit from Number 7

Eg: 7's complement of 65 is 12

$$[77 - 65 = 12]$$

Eg: 7's complement of 423 is 354

$$(777 - 423 = 354)$$

8's complement

7's complement + 1

Eg: 8's complement of 27 is 51

$$[77 - 27 + 1 = 51]$$

$$321 \text{ is } 457 \quad (777 - 321 + 1 \\ = 457)$$

$\Rightarrow 7's$ - ⊕ sub each bit with 7
 $\Rightarrow 8's$ - $7's \text{ comp} + 1$

eg: 6075

$$7's \rightarrow \begin{array}{r} 1 \\ - 6075 \\ \hline 8887 \\ \hline 1702 \end{array}$$

$$8's \rightarrow \begin{array}{r} 1 \\ - 1702 \\ \hline 1703 \end{array}$$

Q EG: Find 8's complement of 3056

$$7's \rightarrow \begin{array}{r} 1 \\ - 3056 \\ \hline 4721 \end{array}$$

$$8's \rightarrow 4722$$

Q eg: $605 \cdot 535$

$$7's \rightarrow 777 \cdot 777$$

$$\begin{array}{r} 605 \cdot 535 \\ \hline \end{array}$$

$$\begin{array}{r} 172 \cdot 242 \\ \hline \end{array}$$

$$8's \rightarrow 173 \cdot 242$$

$$\begin{array}{r} 172 \cdot 242 \\ \hline \end{array}$$

$$\begin{array}{r} 172 \cdot 242 \\ \hline 172 \cdot 243 \\ \hline \end{array}$$

+ add with
19st bit

2. Octal Subtraction using 7's complement

Steps:

1. find the 7's complement of the number to be subtracted
2. perform Addition [Add first number and 7's complement of subtrahend)
3. • IF carry is generated, Add the carry with the result
• IF carry is not there, take 7's

compliment of the sum and assign negative sign.

Eg: $(412)_8 - (263)_8$ (These octal so max 7 [0...7])

1. 263 7's complement \rightarrow

$$\begin{array}{r} 777 \\ 263 \\ \hline \end{array}$$

2. $\begin{array}{r} 412 \\ 514 \\ \hline 1026 \end{array}$ carry 1
Here $4+5=9$, it is more than 7

$$\begin{array}{r} 326 \\ + 1 \\ \hline 1027 \end{array}$$

Eg: $(263)_8 - (412)_8$

1. 7's compliment of 412 = 777

$$\begin{array}{r} 263 \\ + 365 \\ \hline 650 \end{array}$$

$8 = 1 \times 8 + 0$
 $13 = 1 \times 8 + 5$

No carry

7's complement of 650

$$\begin{array}{r} 777 \\ - 650 \\ \hline 127 \end{array}$$

give -ve = -127

3. Octal's subtraction using 8's complement

Step:

complement

- Find the 8's complement of the number to be subtracted
- Perform addition [Add first number and 8's complement of subtrahend]
- If carry is generated, Discard the carry.
 - If there is no carry, Take the 8's complement of the sum and assign negative sign.

Eg: $(372)_8 - (144)_8$

1. $144_8 \rightarrow 8$'s complement \rightarrow

7's + 1

$$\begin{array}{r} 777 \\ 144 \\ \hline 633 \end{array} \rightarrow 633 + 1 = \underline{\underline{634}}$$

2.

$$\begin{array}{r} 1 \\ ① | 3 \quad 7 \quad 2 \quad + \\ 6 \quad 3 \quad 4 \\ \hline ① | 2 \quad 2 \quad 6 \end{array}$$

$$10 = 1 \times 8 + 2$$

$$10 = 1 \times 8 + 2$$

3. Discarding carry $8 = 0226$

Eg. $(144)_8 - (372)_8$

1. 8's complement of 372 , 7's + 1

$$\begin{array}{r} 144 \\ + 406 \\ \hline 552 \end{array}$$

$$10 = 1 \times 8 + 2$$

$$\begin{array}{r} 372 \\ + 406 \\ \hline 778 \end{array}$$

3. No carry

so 8's complement \rightarrow ~~1000~~ 052

$$\begin{array}{r}
 \begin{array}{c} 777 \\ \hline 552 \\ + 225 \\ \hline 1111 \\ \hline 226 \end{array}
 \end{array}$$

Give negative sign

$$\Rightarrow -226 //$$

Q Compute 8's complement of octal

number 672.237

= 7's comp + 1 \Rightarrow

$$777.77 -$$

$$\underline{672.23}$$

$$105.56 +$$

$$105.56$$

Q Perform the octal number of subtraction (417) - (237) using 7's and 8's complement method

(a) 7's complement method

$$1. \quad \begin{array}{r} 777 \\ - 237 \\ \hline 540 \end{array}$$

$$2. \quad \begin{array}{r} 417 \\ + 540 \\ \hline 1157 \end{array}$$

$$3. \quad \begin{array}{r} 157 \\ + 158 \\ \hline 160 \end{array}$$

$$\begin{array}{r} 157 \\ + 160 \\ \hline 1x8+0 \end{array}$$

(b) 8's complement method

$$1. \quad \begin{array}{r} 777 \\ - 237 \\ \hline 540 \end{array} \rightarrow \begin{array}{r} 540 \\ + 541 \\ \hline 1081 \end{array}$$

$541 \rightarrow 8's \text{ complement}$

$$2. \quad \begin{array}{r} 417 \\ + 541 \\ \hline 1158 \end{array} \quad q = 1 \times 8 + 1$$

$$\begin{array}{r} 417 \\ + 541 \\ \hline 160 \end{array}$$

Discard carry

$$= \underline{\underline{158}}$$

$$\underline{\underline{160}}$$

Hexadecimal Subtraction

1. General method
2. 15's complement method
3. 16's complement method
4. General method

$$\begin{array}{r}
 16 \quad 16 \\
 8 \quad 6 \quad 3 \quad 16 \\
 9 \uparrow \quad 6 \quad B \\
 \hline
 5 \quad 8 \quad 7 \quad C
 \end{array}$$

$$\begin{array}{r}
 3 \quad E \quad 9 \quad F \\
 = \quad C
 \end{array}$$

$$\begin{array}{r}
 + 121 \\
 \hline
 0 \quad 21
 \end{array}$$

$A \rightarrow 10$
 $B \rightarrow 11$
 $C \rightarrow 12$
 $D \rightarrow 13$
 $E \rightarrow 14$
 $F \rightarrow 15$

$$\begin{aligned}
 & 16 + B - C \\
 & 16 + 11 - 12 \\
 & 27 - 12 = 15
 \end{aligned}$$

$$16 + 6 - 8$$

$$22 - 8 = 14$$

15's and 16's complement of

Hexadecimal subtraction

→ 15's complement - Subtract every each and every HD digit from number $F\ 0\ 1\ 5$

eg: A 86 → $\begin{array}{r} 888 \\ - 000 \\ \hline 888 \end{array}$ - $\begin{array}{r} F\ F\ F \\ - A\ 8\ C \\ \hline 5\ 7\ 9 \end{array}$

eg: $E21A.F2$

+ 15's

$\begin{array}{r} \text{FFF F. FF} \\ \text{E21 A} \\ \hline \end{array}$

$\begin{array}{r} \text{E21 A} \\ \text{F2} \\ \hline \end{array}$

$\begin{array}{r} \text{IDE S. OD} \\ \hline \end{array}$

$\begin{array}{r} \text{IDE S. OD} \\ + \\ \hline \end{array}$

$\begin{array}{r} \text{IDE S. OD} \\ + \\ \hline \end{array} \rightarrow 16's \text{ complement}$

2. Hexadecimal subtraction using 15's complement

Step:

1. Find the 15's complement of the number to be subtracted

2. perform Addition (Add first number and 15's complement of subtrahend)

3. • IF carry is generated, Add with the result

• If carry is not generated, take the 15's complement of the sum and assign VR 01.

$$1. (B01)_{16} + (98F)_{16}$$

1.

F	F	F	-	
9	8	F		
<hr/>				
6			7	0

2.

B	0	1	+	
①	<hr/>			
6			7	0
<hr/>			1	7

carry 10 add 10 to 10 = 20
B+6=17 = 1x16+1

3.

B	0	0	1	7	1	+		
<hr/>						1		
<hr/>						1	7	2

2. $(C69A)_{16} + (C13)_{16}$

1.

-	F	F	F		
C	1	3	-		
<hr/>			3	E	C

2.

6	9	A
3	E	C
<hr/>		

$$A+C=22=1\times 16+6$$

$$1+9+E=25=1\times 16+9$$

3. No carry

$$\begin{array}{r}
 \text{A} \quad \text{FFF} \\
 - \text{A} \ 8 \ 6 \\
 \hline
 \text{5} \ 7 \ 9
 \end{array}
 \quad - \underline{\underline{579}}$$

Hexadecimal Subtraction using

Step: 1. Find the 16's complement of the number to be subtracted

1. Find the 16's complement of the number to be subtracted
2. perform Addition (Add first number and 16's complement of subtrahend)
3. • IF carry is generated, discard the carry
 • If there is no carry, Take the 16's compliment of the sum and assign negative sign.

Example:

$$1. (971)_{8,16} - (CB1)_{16}$$

$$\begin{array}{r} F\ F\ F \\ \text{---} \\ C\ B\ 1 \\ \hline 3\ 4\ E \end{array}$$

$$16's \text{ complement} \rightarrow \frac{3\ 4\ E}{\underline{\underline{3\ 4\ F}}}$$

$$2. \begin{array}{r} 9\ 7\ 1 \\ + 3\ 4\ F \\ \hline C\ C\ 0 \end{array} \quad l+F = 16 \rightarrow 1 \times 16 + 0$$

3. No carry

$$\begin{array}{r} 9\ F\ F \\ - C\ C\ 0 \\ \hline 3\ 3\ F \end{array} \quad X = 45 - 4 \times 15$$

$$3. 3\ 3\ F + \quad 16 = 1 \times 16 + 0$$

$$3. 3\ 4\ 0 \rightarrow 16's \text{ complement}$$

$$\cancel{-3\ 4\ 0} \Rightarrow -\underline{\underline{3\ 4\ 0}}$$

$$2. (CB1)_{16} - (971)_{16}$$

1.

FFFF

$$\begin{array}{r} 971 \\ \hline 68E \end{array}$$

 $\rightarrow 15's$

$$\begin{array}{r} 68E \\ \hline 68F \end{array}$$

 $\rightarrow 16's$

2.

CB1

$$\begin{array}{r} 1 | 68F \\ , 1 | 340 \end{array}$$

07.0 APR

70.820

$16 = 1 \times 16 + 0$

$20 = 1 \times 16 + 4$

$19 = 1 \times 16 + 3$

3.

discarding carry $\Rightarrow 340$

Self marking (Q1) - (3Ari)

Q1. 16's complement of hexa decimal number FAOC.2B

number FAOC.2B

A - 10

B - 11

C - 12

D - 13

E - 14

F - 15

$$\begin{array}{r} FFFF.FF \\ FAOC.2B \\ \hline \end{array}$$

$$05F3.B4 \rightarrow 15's$$

$$\begin{array}{r} 5F34.B4 \\ \hline 1 + \end{array}$$

$$5 + 5F34.B5$$

$$0 + 81x5 = 11$$

Q2. 15's and 16's complement of 34L. To

16's complement -

$$16's \text{ complement} \rightarrow \begin{array}{r} \cancel{3CA.87} \\ + CS3.0F \\ \hline \cancel{3CA.88} \\ CS3.10 \end{array}$$

43.

Qd. (3AF) - (4Ac) perform the

Subtraction using 1's and 10's complement method

(G) 1's complement method

(3AF) - (4AC)

$$\begin{array}{r}
 & F & F & F \\
 & 4 & A & C \\
 \hline
 B & S & 3
 \end{array}$$

$$2. \quad \begin{array}{r} 1 \\ 3 A F \\ + \\ B S 3 \\ \hline F O 2 \end{array}$$

A-10
B-11
C-12
D-13
E-14
F-15

$$18 = 1 \times 16 + 2$$

3. NO carry (\Rightarrow)

$$\begin{array}{r} \text{F F F} \\ \text{---} \\ \text{O F D} \end{array} \quad \text{FD}$$

$$= -\text{FD}$$

(b) 16's complement method

$$(3AF) - (4AC)$$

$$\begin{array}{r} \text{F F F F} \\ \underline{- 4 A C} \\ \text{B 5 3} \end{array} \quad \rightarrow 15's \text{ of } 4AC$$

$$\text{B 5 4} \rightarrow 16's$$

$$\begin{array}{r} (1) \text{ 11} \\ 3AF + 19 = 1 \times 16 + 3 \\ \hline \text{B 5 4} \\ (2) \text{ 11} \\ 0 3 \\ \hline \text{carry} \end{array} \quad \begin{array}{l} 19 = 1 \times 16 + 3 \\ 16 = 1 \times 16 + 0 \\ 17 = 1 \times 16 + 1 \end{array}$$

$$\begin{array}{r} 3. \Rightarrow 103 \\ \hline \text{0 1 0 0} \end{array}$$

$$\begin{array}{r} 1100 \\ - 1100 \\ \hline 0000 \end{array}$$

Q4. (F236) - (E26D) compute

hexadecimal subtraction using
normal method.

(F236) - (GC2E)

$$\begin{array}{r}
 & 16 & 2 & 16 \\
 F & 1 & 2 & 8 & 6 \\
 & 1 & 2 & 8 & 6 \\
 \hline
 E & C & 2 & E \\
 \hline
 D & 6 & 0 & 8 \\
 \hline
 \end{array}$$

$$16 + 6 - E$$

$$22 - E,$$

$$16 + 2 - C, \quad C)$$

Binary coded Decimal

1. Representation
2. BCD Addition
3. BCD Subtraction

In this code each digit is represented by 4 bit binary number

Decimal	Binary	BCD
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111

8	1000	1000
9	1001	1001
10	1010	<u>0001</u> <u>0000</u>
11	1011	<u>0001</u> <u>0001</u>
12	1100	0001 0010
13	1101	0001 0011
14	1110	0001 0100
15	1111	0001 0101

Eg: BCD of 156

$$\begin{array}{r}
 1 \ 5 \ 6 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1
 \end{array}$$

Eg: BCD of 908

1001 0000 0111

BCD → Decimal

Eg: 10100 → Decimal

000 10100 \Rightarrow 14100

Ans: 14100

BCD Addition

Steps:

- (1) sum ≤ 9 ; carry 0 \rightarrow Answer is correct
- (2) sum ≤ 9 ; carry 1 \rightarrow Answer is incorrect
- (3) If sum > 9 ~~0011~~ \rightarrow invalid BCD

invalid BCD

Add 6 (0110)

to

convert valid

Examples:

BCD 2 + BCD 6

$$\begin{array}{r}
 2 + 6 = 8 \rightarrow \\
 \begin{array}{r}
 0 0 1 0 \\
 0 1 1 0 \\
 \hline
 1 0 0 0 \rightarrow 8
 \end{array}
 \end{array}$$

18 in < 9 and carry 1

Here condition is 1, Answer is correct

2. BCD 3 + BCD 7

$$\begin{array}{r}
 3 \rightarrow 0 0 1 1 \\
 7 \rightarrow 0 1 1 \\
 \hline
 1 0 1 0 \rightarrow 10 \rightarrow A > 9
 \end{array}$$

invalid \Rightarrow 100100
 Adding with 6 \rightarrow 0110

$$\begin{array}{r}
 0110 \\
 + 1000 \\
 \hline
 10110
 \end{array}$$

 $\Rightarrow 00010010110$

3. BCD 8 + BCD 9

$$\begin{array}{r} 8 \rightarrow 10000 \\ 9 \rightarrow 16001 \\ \hline 10001 \\ \text{carry } 1 \end{array}$$

if carry is invalid

$$\begin{array}{r} \text{So } \Rightarrow 10001 + \\ 0110 \\ \hline 1011\textcircled{0}1 \end{array}$$

$\Rightarrow 001 @ \textcircled{0} 111$ (17)

4.

 $BCD\ 57 + BCD\ 26$ $\downarrow \downarrow$ ~~5000~~

$$5 \rightarrow 0101 \quad | \quad 0111 +$$

$$2 \rightarrow 0010 \quad | \quad 0110 \rightarrow 6$$

$$\underline{0111 \quad | \quad 1101} \quad C D \rightarrow 13$$

(7)

Invalid

Valid

 \Rightarrow

$$\begin{array}{r} 1 \ 1 \ 1 \\ 0 \ 1 \ 1 \ 1 \\ \hline 1101 + \\ 0110 \rightarrow C6 \end{array}$$

$$\underline{1000 \ 0011}$$

 \downarrow

8

 $\downarrow 600 \rightarrow (8.3)$

30110

 $\Rightarrow 1000 \ 0011$

(7) This is the answer

BCD Subtraction (9's complement)

Step:

- Take 9's complement of number to be subtracted

- ADD it to the result

- If the result is invalid BCD, add 6 (0110)

- Shift the carry to next bit

- If end around carry is generated, add it to the result

9's complement \rightarrow eg: 186 \Rightarrow 999
186

$$1. \quad 987.6 - 678.9 \quad \text{BCD}$$

$$1. \quad \begin{array}{r} 9991.9 \\ - 678.9 \\ \hline 321.0 \end{array}$$

$$2. \quad \begin{array}{r} 987.6 \\ \downarrow \downarrow \swarrow \uparrow \uparrow \\ 1001 \ 1000 \ 0111 \cdot 0110 + \\ 0011 \ 0010 \ 0001 \cdot 0000 \\ \hline 1100 \ 1010 \ 1000 \ 0110 \end{array}$$

1001	1000	0111	0110
0011	0010	0001	0000
1100	0010	1000 → (8)	0110 → (6)
0010	0010	(10) A	

1	1000	1010	1000	0110		
1	0110	0110			(0110) 3 + 1000	
1001010000001000000110	<hr/>					
1001010000001000000110						

4. shifting the carry to next bit

if no
candy
we can
simply
wait

0010 0000 1000 · 0110 6
| 000 | 0010 | +

0011 0000 1000 101110
~ ~ ~ ~
3 0 8 7

$$\Rightarrow 308 \cdot 7$$

Q. ~~988~~ 98.3 - 81.2

Step 1

$$\begin{array}{r}
 99.9 \\
 (99.9) \\
 - 81.2 \\
 \hline
 18.7
 \end{array}$$

Step 2

$$\begin{array}{r}
 98.3 \\
 0000.000 \\
 1001 \quad 1000 \quad .001 \\
 0001 \quad 1000 \quad .011 \\
 \hline
 01010000 \quad 01010 - (10)
 \end{array}$$

Step 3:

not valid valid
 not valid invalid
 [≤ 9 but
 carry 1]

$$\begin{array}{r}
 10'1' \\
 0001 \\
 \hline
 1011 - (11)
 \end{array}
 \quad
 \begin{array}{r}
 1000 \\
 1000 \\
 \hline
 0000 - (0)
 \end{array}
 \quad
 \begin{array}{r}
 0011 \\
 0111 \\
 \hline
 1010 - (10)
 \end{array}$$

not valid valid invalid

1010	10000	1010
0110	0110	0110 +
<hr/>		
10000	10110	10000

↓ ↓ ↓

0000 0110 0000

1111 1111 1111

0001 0111 0001

87

000 - 0101 00001010

0000 = 117.1

117.1 117.1

117.1 117.1

117.1 117.1
117.1 117.1

000 - 0101 0001 1101

117.1 117.1

117.1 117.1

Q1. Subtract BCD number 837 using 9's and 10's complement method.

837

$$9's \rightarrow \begin{array}{r} 999 \\ - 837 \\ \hline 162 \end{array}$$

$$10's \text{ comp} \rightarrow \begin{array}{r} 163 \\ - 837 \\ \hline 162 \end{array}$$

Q2. Subtract BCD number 866 - 170 using 9's complement method

$$\begin{array}{r} 999 \\ - 170 \\ \hline 829 \end{array}$$

$$\begin{array}{r} 866 \\ - 170 \\ \hline 196 \end{array}$$

$$2. \quad 366 + 829$$

$$\begin{array}{r} 0011 \\ 1000 \\ \hline 1011 \end{array} \quad \begin{array}{r} 0110 \\ 0010 \\ \hline 1000 \end{array} \quad \begin{array}{r} 0111 \\ 1001 \\ \hline 1000 \end{array} + \begin{array}{r} 0000 \\ 0001 \\ \hline 0001 \end{array}$$

+ ~~0001~~

invalid | valid | valid

~~211~~
~~1011 1000 0000~~
~~0001 0001 0001~~
~~invalid invalid invalid~~

3.

KeralaNotes

1011 + 10000 = 101000

$$\begin{array}{r} 4. \quad 0000 \quad 1000 \quad 0000 \\ - 051-3360 \quad 6911 \quad 818110512002 \quad 67 \\ \hline 0000 \quad 1000 \quad 0000 \quad 1 \\ 0000 \quad 1000 \quad 0000 \quad 1 \\ \hline 0 \quad 9 \quad 1 \quad - \text{PPP} \quad .1 \\ 051 \\ \hline 051 \\ \hline \end{array}$$

366 + 829 = 895 100%
6150

acetate HIO Otto HIO

0 0 11 | 0 11 0 | 0 11 0
1 0 00 | 0 0 10 | 0 1 0 0 1 0 +

~~1011100011100011~~

invalid 14 | valid 15 | invalid 15

1010	1000	10110
0110	0100	001001
100001	1000	010101
0	1	1 +
0010	1001	0100
0001		
1	9	6 \Rightarrow 196

0011	0010	0110
1000	0010	1001
1011	1000	100000
0110		0110
10001	1000	00110
		1001
		1001
		0110
		+

3. Convert BCD number 1000 1110 into decimal number

1000 1110

814

Types of code

Special code



(i) weighted code

Each position of the number is represented a specific weight

e.g. in Decimal code, if number is 567
then weight of 5 is 100
6 is 10
7 is 1

TYPES OF WEIGHTED CODE

(a) 8421 code \rightarrow

$a^3 \ a^2 \ a^1 \ 0$

(b) 2421 code \rightarrow

1010

2421

(ii) Non weighted code

Non weighted code are not assigned with any weight to each digit position. That is each digit position within the number is not assigned a fixed value.

TYPE OF WEIGHTED CODE

(a) Excess 3 code

(b) Gray code

Error detecting and correcting code

(Hamming code / parity code)

- code which allow error detection

\rightarrow error detection code

- code which allow error detection and as well as correction

\rightarrow error detecting and correcting ^{code} code

Hamming code

7 bit

4 Bit → Data

3 Bit → parity [error detection]

Parity two type:

- 1 (i) odd parity: odd no. of ones
- 0 (ii) even parity: even no. of ones



$$P_1 = D_3 \oplus D_5 \oplus D_7$$

$$P_2 = D_3 \oplus D_6 \oplus D_7$$

$$P_4 = D_5 \oplus D_6 \oplus D_7$$

$$2^0 = \text{parity} = 1$$

$$2^1 = " = 2$$

$$2^2 = " = 4$$

$$2^3 = " = 8$$

NO need because up to 7

ODD → 1

EVEN → 0

Q Eg: Data bits are 1011, find code

D_7	D_6	D_5	D_4	D_3	P_4	P_2	P_1
1	0	1		1			

so $P_1 = D_3 D_5 D_7 \Rightarrow \overbrace{111}^{\text{odd}} \rightarrow \text{odd parity}$

$P_2 = D_3 D_6 D_7 \Rightarrow 101 = \text{even} = 0$

$P_4 = D_5 D_6 D_7 \Rightarrow 101 = 0$
even

Hamming code: 1010101

Q Eg: 1110101 Find no. of errors in the hamming code

D_7	D_6	D_5	D_4	D_3	P_2	P_1
1	1	1	0	1	0	1

$P_1 = D_3 D_5 D_7 = 111$
(error) = 1
odd

$P_2 = D_3 D_6 D_7 = 111$
(error) = 1 \rightarrow error

$P_4 = D_5 D_6 D_7 = 111$
(error) = 1 \rightarrow error

two errors

Q1. Write the hamming code of the data bits 1010

D_7	D_6	D_5	D_3	P_4	P_2	P_1
1	0	1	1	0	0	1

$$P_1 = D_3 \oplus D_5 \oplus D_7 \Rightarrow 0 \oplus 0 \oplus 1 = 1$$

(even) (odd)

$$P_2 = D_3 \oplus D_6 \oplus D_7 \Rightarrow 0 \oplus 0 \oplus 1 = 1$$

(even) (odd)

~~P₃~~

$$P_4 = D_5 \oplus D_6 \oplus D_7 \Rightarrow 1 \oplus 0 \oplus 1 = 0$$

(even)

Hamming code: 1010010

Q2. 7 bit Hamming code will be 1110010 find the number of errors.

1	1	1	0	0	1	0
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$$P_1 = D_3 \oplus D_5 \oplus D_7 = 0 \oplus 1 \oplus 1 = 0$$

(even)

two errors

$$P_2 = D_3 \oplus D_6 \oplus D_7 = 0 \oplus 1 \oplus 1 = 0 \rightarrow \text{error}$$

(even)

$$P_4 = D_5 \oplus D_6 \oplus D_7 = 1 \oplus 0 \oplus 1 = 0 \rightarrow \text{error}$$

(even)

Excess 3 code

2421 code

ASCII code

EBCDIC code

Gray code

Reflective Code

(Self complement code)

- A code is said to be reflective when the
- code 9 is the complement of code 0,
- code 8
- code 7
- code 6
- code 5

Type of Reflective code

- (i) 2421 code (ii) Excess 3 code

Excess 3 code

- Excess 3 code is a modification of BCD numbers.
- Excess 3 can be derived from natural BCD code by adding 3 to each coded number.

Eg: Decimal 12 can be represented in BCD as 0001 0010

Now adding 3 to each digit we get excess 3 code as 0100 0101

(BCD+3)

	BCD	Excess
0	0000	0011
1	0001	0101
2	0010	0110
3	0011	0111
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

BCD

Excess [BCD]

0011 [0000+0011]

0100

0101

0110

0111

[0100+0011] - 1000

0110

1001

0111

1010

1000

1011

1001

1100

Ex: 156 → BCD & Excess

$$\begin{array}{cccc} 1000 & 0001 & 0101 & 0110 \\ \text{BCD} = & 0001 & 0101 & 0110 \\ 1100 & & & \end{array}$$

$$\text{Excess} = 0100 \ 1000 \ 1001$$

- In Excess 3 code we get 9's complement of a number by just complement each bit. Due to this excess 3 code are called self complementing code.

Reflective code

$$\begin{array}{l} q_{\text{comp}} \rightarrow 0 \\ s \rightarrow 1 \end{array}$$

8421 code

- The weights of this code are 8, 4, 2, 1.
- This code has all positive weights. So it is positively weighted code.
- It is an unnatural BCD code. Sum of weight of unnatural BCD code are equal to 9. [8+4+2+1=9]
- It is a self-complementing code. Self complementing code provide the 9's complement of a decimal number, just by interchanging 1's and 0's in its equivalent 8421 representation.

Decimal	8421 BCD	2421 BCD
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	1011
6	0110	1100
7	0111	1101
8	1000	1110
9	1001	1111

$$\text{Q4 Q1} = 6$$

$$\begin{array}{r} \text{Q4 Q1} \\ \hline 1100 \end{array}$$

KeralaNotes

$$\begin{array}{r} \text{Q4 Q1} \\ \hline 1011 \end{array} \rightarrow 24 + 1 \text{ BCN}$$

Gray code

- Gray code is a special case in unit distance code
- In unit distance code, bit patterns for two consecutive numbers differ only in one bit position. These code are also called cyclic code.

<u>Binary</u>	<u>Gray</u>	→ two consecutive numbers differ only one position
0 → 0000	0 → 0000	
1 → 0001	1 → 0001	
2 → 0010	2 → 0011	
3 → 0011	3 → 0010	
4 → 0100	4 → 0010	
5 → 0101	5 → 0111	
6 → 0110	6 → 0110	
7 → 0111	7 → 0100	
8 → 1000	8 → 1100	
9 → 1001	9 → 1101	
10 → 1010	10 → 1111	

Alpha Numeric codes

[Characters code]

The code which consists of letters, digits and various special characters are called Alpha numeric code.

Types:

(i) ASCII CODE

(ii) EBCDIC CODE

ASCII CODE

American standard code for information interchange is

a 7 bit code

The standard ASCII character set consist of 128 decimal numbers ($2^7 = 128$)

A - Z \rightarrow 65 - 90

a - z \rightarrow 97 - 122

Space \rightarrow 32

• \rightarrow 46

° \rightarrow 48

Example:

I. Computer

$$A = 65$$

$$= 64 + 1$$

$$= 97 + 1$$

~~64~~

$$C \rightarrow 64 + 3 = 67 \rightarrow Z = 96 + 26$$

$$O @ \rightarrow 96 + 15 = 111 \rightarrow \text{Binary (7 bit)}$$

$$M \rightarrow 96 + 13 = 109$$

$$P \rightarrow 96 + 16 = 112$$

$$U \rightarrow 96 + 21 = 117$$

$$T \rightarrow 96 + 20 = 116$$

$$E \rightarrow 96 + 5 = 101$$

$$W @ \rightarrow 96 + 18 = 114$$

Shortcut:

E	J	O	T	Y
5	10	15	20	25

$$A \rightarrow 64 + 1$$

$$a \rightarrow 96 + 1$$

Q How Old Are You

$$H \rightarrow 64 + 8 = 72$$

$$O \rightarrow 96 + 15 = 111$$

$$W \rightarrow 96 + 23 = 119$$

space $\rightarrow 32$

$$O \rightarrow 64 + 15 = 111$$

$$I \rightarrow 96 + 12 = 108$$

$$D \rightarrow 96 + 4 = 100$$

space $\rightarrow 32$

$$A \rightarrow 96 + 1 = 97$$

$$S \rightarrow 96 + 18 = 114$$

$$C \rightarrow 96 + 5 = 101$$

space $\rightarrow 32$

$$Y \rightarrow 64 + 25 = 121$$

$$O \rightarrow 96 + 15 = 111$$

$$U \rightarrow 96 + 21 = 117$$

EBCDIC code

- The Extended Binary coded Decimal interchange code is a standard 8 bit code developed by IBM.
- The standard EBCDIC code character set consist of 256 decimal numbers ($2^8 = 256$)
- The code is rarely used. Used for IBM mainframes only.

0000 0000	0000 0001	0000 0010	0000 0011	0000 0100	0000 0101	0000 0110	0000 0111	0000 1000	0000 1001	0000 1010	0000 1011	0000 1100	0000 1101	0000 1110	0000 1111
0000 0000	0000 0001	0000 0010	0000 0011	0000 0100	0000 0101	0000 0110	0000 0111	0000 1000	0000 1001	0000 1010	0000 1011	0000 1100	0000 1101	0000 1110	0000 1111
0000 0000	0000 0001	0000 0010	0000 0011	0000 0100	0000 0101	0000 0110	0000 0111	0000 1000	0000 1001	0000 1010	0000 1011	0000 1100	0000 1101	0000 1110	0000 1111
0000 0000	0000 0001	0000 0010	0000 0011	0000 0100	0000 0101	0000 0110	0000 0111	0000 1000	0000 1001	0000 1010	0000 1011	0000 1100	0000 1101	0000 1110	0000 1111
0000 0000	0000 0001	0000 0010	0000 0011	0000 0100	0000 0101	0000 0110	0000 0111	0000 1000	0000 1001	0000 1010	0000 1011	0000 1100	0000 1101	0000 1110	0000 1111