Omnifocus Imaging with Pupil based Scheimpflug Camera Model

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1. INTRODUCTION

[TO DO] Write introduction to the topic

2. GEOMETRIC MODEL OF SCHEIMPFLUG IMAGING

A. Geometric Image for Tilted Lens and Sensor

1. Transfer of chief rays’ direction cosine from entrance to exit pupil

The schematic in Fig. 1 represents a camera in terms of the paraxial pupil planes of a lens along the image plane. The lens is pivoted the point . We have overloaded the notation to also represent the coordinate frame of the camera. The pivot of the lens (equivalently, the optical axis) is the origin of . The centers of paraxial entrance and exit pupils—represented by and —lie along the optical axis at distances and respectively from . The diameters of entrance and exit pupils are represented as and respectively. The image plane is pivoted about at the point in the camera frame . This point, represented by is the origin of the two-dimensional image coordinate frame, also represented by . The figure also depicts two rays from the object space to the image space that are fundamental to geometric optics modeling—the chief ray and the marginal ray.

The chief ray which originates at the object point in the object space with direction cosine passes through the center of the entrance pupil , reemerges from the exit pupil with direction cosine , and intersects the image plane at . Are the input and out direction cosine vectors and equal? In other words, suppose the chief ray in the object and image space makes angles and with the optical axis, then is and have the same absolute value? To answer this question, we consider the marginal rays and the pupils. Suppose, the marginal ray in the object space, which originates from the base (projection) of the object point on the optical axis and travels to the edge of the paraxial entrance pupil at height , makes an angle with the optical axis. In the image space, let us suppose, the marginal ray from the edge of the exit pupil at height to the base of the image point on the optical axis makes an angle with the optical axis. Then, if and (generally the case in macroscopic imaging), then we obtain:

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Note although the image point , by definition, lie on the image plane, its projection on the the optical axis lie on the image plane only in the special case when the optical axis is perpendicular to the image plane.

Now, , the ratio of the paraxial exit pupil height to the entrance pupil height is defined as the *pupil magnification* [ref]. Further, according to the *Lagrange invariant* property [ref] of the two rays (the chief ray and the marginal ray) the transverse magnification () is reciprocal to the angular magnification (). Therefore, Eq. (1) reduces to:

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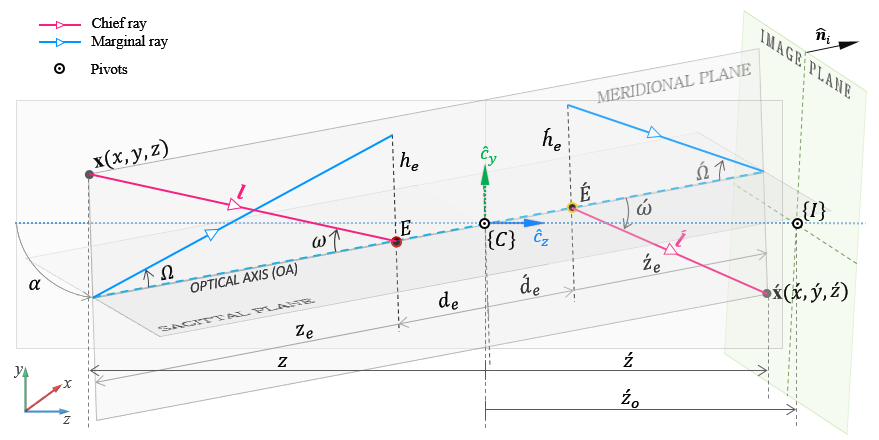
Equation (2), also derived in [ref] using a different approach, implies that the angle of emergence of the pencil of chief rays from the exit pupil towards the image plane depends on the pupil magnification.

To derive the relation between the object and image space direction cosine of the chief ray— and —let us first suppose that the optical axis is coincident with the z-axis of . Consequently, the zenith angle of all chief rays in the object space and all chief rays in the image space are and respectively. For any specific chief ray if the azimuthal angles in the object and image space are and respectively, and if and , then we obtain:

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Following few algebraic steps using Eq. (2), Eq. (3) and the fact that the chief ray in the object and image space is confined to the same meridional plane (i.e., ) [ref], we obtain:

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 Fig. 1. Schematic of the general optical system with the lens pivoted at and the image plane pivoted at .

We can write Eq. (4) compactly as:

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where, . Further, we can safely drop the negative sign in Eq. (5) since the ray emerging from the exit pupil travels in the direction of positive z-axis towards the image plane.

To derive the general expression for the transfer of chief ray’s direction cosine, we first introduce —the rotation matrix applied to the optical axis to rotate the lens about the pivot . We also introduce a new coordinate frame, sharing its origin with , but fixed to the lens such that the z-axis of is coincident with the optical axis. The pupils and the frame rotate along with the optical axis. As before, we represent the input direction cosine of the chief ray in frame as . The vector in frame becomes . As a result,, the z-component of , becomes , where is the third column of . Using Eq. (5) we obtain the output direction cosine of the chief ray in frame as:

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Finally, we obtain the output direction cosine of the chief ray, in the camera frame , that emerges from the exit pupil as :

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where, .

We expect the direction cosine to have unit magnitude. It is indeed straightforward to show the -Norm of is equal to one, and is the normalizing term. Note that if the pupil magnification of the lens is equal to one, then , which implies that the opening angles of the image and object space perspective cones are equal irrespective of the orientation of the optical axis. In terms of geometric optics, also implies that the paraxial entrance and exit pupil planes are coincident with the front and rear principal planes respectively. Such lenses in which are called symmetric lenses.

2. Expression for image coordinate for arbitrary orientation of lens and sensor planes

[TO DO] Write short introduction for this section.

If the entrance and exit pupils are located at distances and from the pivot (origin of ) along the optical axis, then following rotation of the optical axis, the locations of the entrance and exit pupils in the frame is andrespectively. Further, we express the chief ray emerging from the exit pupil as , where the parameter determines the length of the ray. Substituting Eq. (7) for we obtain:

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We would like to determine the expression for for which . Let be the perpendicular distance of the image plane, with surface normal , from the origin of . Then, represents the equation of the image plane in in Hessian normal form. Therefore, when , we obtain:

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Further, if we represent the orientation of the image plane by , then . Also, since the point lies on the image plane, we can write .

Substituting Eq. (9) into Eq. (8) and using we obtain the expression for the image point as:

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Let the location of the entrance pupil in be We express in terms of and as . Substituting into Eq. (10) yields:

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The Eq. (11) expresses the image point in the the camera frame. It is more useful to represent in the two-dimensional image frame . If , be the image coordinates in camera frame , and be the equivalent image coordinate in frame , then . Therefore, the expression for the image point in the two-dimensional sensor coordinates when the lens and sensor planes are free to rotate about their own pivots follows as:

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B. Object, Lens and Image Plane Relationships for Focusing using Scheimpflug Camera

Hitherto, we have expressed the coordinates of the image point corresponding to an object point for when the lens and sensor planes are free to rotate about their respective pivots. However, we did not apply any constraints on the orientations of the object, lens and sensor planes such that points on the object plane are brought to focus (geometric) on the sensor plane. To that effect, we use a variant of the Gaussian lens formula for the ubiquitous parallel plane imaging configuration that relates the object-plane-to-entrance-pupil distance , exit-pupil-to-image-plane distance , pupil magnification and focal length [ref] as shown below:

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In Eq. (13) we specify the directed distances and along the optical axis. Let us suppose that object plane is pivoted at in the camera frame . Also, we represent the orientation of the object plane using the rotation matrix . Then, the object plane normal, following rotation, is the vector . Now, suppose the orientations of the three planes are such that points in the arbitrarily tilted object plane form focused images on the arbitrarily tilted image plane. Then, the projection of the chief ray in the object space from to on the optical axis and the projection of the chief ray in the image space from to on the optical axis must satisfy Eq. (13).

Following similar formulation of the chief ray as in § [2.A.2](#Sec_2_A_2), we obtain **,** the length of the chief ray from to as:

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and , the length of the chief ray from to as:

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The ray vector of length and direction in the object space is. The projection of this ray vector on the optical axis () is, and the corresponding directed distance (from towards ) is Similarly, the projection of ray in the image space on the optical axis (and the corresponding directed distance) is. Substituting and into Eq. (13), and using Eq. (7) we obtain:

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Following some algebraic manipulations, especially noting that is equivalent to because is a diagonal matrix and is a rotation matrix, we obtain:

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The -Norm of the direction cosine equals one, and , in general , cannot be perpendicular to the vector. Therefore, we obtain:

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Further, we can simplify Eq. (18) if we let and. Then, after factoring and out of the denominator terms, we can write Eq. (18) as:

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This expedient simplification from Eq. (18) to Eq. (19) is possible because we can describe the unit normal vectors and using only the components along - and -axes. In other words, if we know the - and - components of the normal, we can determine the - component uniquely because planes are limited to rotations between and about both - and -axes (one of the assumptions in this model).

Eq. (19) is most general in the sense that it readily yields the specific formulae for special cases such as focusing with sensor tilt, focusing with lens tilt, or focusing with both sensor and lens tilts.

3. VERIFICATION OF SCHEIMPFLUG IMAGING MODEL

A. Verification of the Imaging Equation in Zemax

We verified the accuracy of the imaging equation Eq. (12) by comparing the numerically computed values of image points (intersection of chief ray with a tilted image plane) using Eq. (12) with the corresponding image points obtained by tracing chief rays from a grid of points belonging to a tilted object plane. Fig. 2 shows the layout plot of the optical system modeled in Zemax showing (1) an object plane, (2) an ideal lens made from two paraxial surfaces and pivoted about a point away from the entrance pupil (), and (3) an image plane pivoted about the image plane pivot along the -axis.

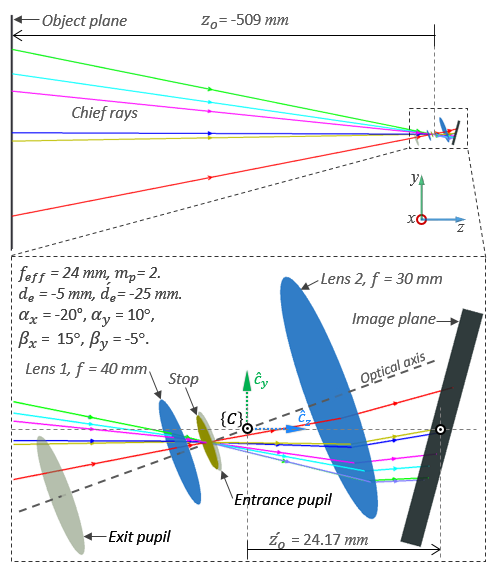


Fig. 2. Chief rays traced from a grid of points in the object plane through an ideal lens tilted about a point away from the entrance pupil along the optical axis to the tilted image plane.

The results of the simulation are tabulated in Table 1, which shows the set of object points, the numerically computed image points, the ray traced image points, and the absolute difference between the numerically computed and ray traced image points. We observe that the numerically computed and ray traced values of the image points are very close; the small difference in their values can be attributed to the error associated with floating point operations. This comparison demonstrates that the analytically derived expression (Eq. (12)) representing geometric relationship between a three-dimensional object point and its image point in the absence of optical aberrations is accurate.

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| Table 1. Comparison of numerically computed image and ray traced (Zemax) image points for the optical system shown in Fig. 2. | | | |
| Object point | Computed image points | Ray-traced image points | Absolute difference |
| (0.0, 0.0, -509.0) | (-0.3108, -0.6291, 0.0) | (-0.3108, -0.6291, 0.0) | (1.8E-09, 3.1E-09, 7.5E-15) |
| (10.0, -10.0, -509.0) | (-0.8003, -0.0863, 0.0) | (-0.8003, -0.0863, 0.0) | (2.1E-09, 2.7E-09, 3.0E-15) |
| (-50.0, 50.0, -509.0) | (2.1291, -3.3352, 0.0) | (2.1291, -3.3352, 0.0) | (1.2E-09, 3.2E-09, 2.9E-15) |
| (70.71, 70.71, -509.0) | (-4.2013, -5.0221, 0.0) | (-4.2013, -5.0221, 0.0) | (2.6E-09, 5.1E-09, 4.7E-15) |
| (100.0, 0.0, -509.0) | (-5.5251, -1.0101, 0.0) | (-5.5251, -1.0101, 0.0) | (1.3E-09, 8.4E-09, 3.1E-15) |
| (0.0, 100.0, -509.0) | (-0.6031, -6.4387, 0.0) | (-0.6031, -6.4387, 0.0) | (2.2E-09, 4.0E-09, 2.2E-16) |
| (100.0, 100.0, -509.0) | (-5.8238, -6.8542, 0.0) | (-5.8238, -6.8542, 0.0) | (5.6E-10, 2.5E-10, 2.2E-15) |

B. Verification of Equation for Focusing on Tilted planes in Zemax

While several object, lens and image plane relationships can be derived from Eq. (19) corresponding to the specific cases of Scheimpflug imaging configurations, here we show and verify the relationships for focusing on an object plane tilted about the -axis by rotating a thick lens about the center of its entrance pupil. For this configuration we obtain the following two relationships—expression for the image plane pivot distance and the object tilt angle —starting from Eq. (19):

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Table 2 enumerates the results of our test. To verify the above equations, we implemented a thick lens model of focal length in Zemax using two paraxial surface (to simulate aberration-free, geometric imaging) having pupil magnification . The lens surfaces were grouped within two coordinate break surfaces that allowed the lens to be tilted about the entrance pupil. The object plane surface was placed at from (and from the entrance pupil). For every object plane orientation (*col.* 1), the appropriate lens tilt angle (*col.* 2) and image plane distance (*col.* 3) were obtained using Zemax’s optimization function, to minimize spot radius across the field. Following optimization for every , the value of obtained from Zemax (along with the values of , , ) was used to numerically compute (*col.* 4) and (*col.* 5) using the derived equations Eq. (20) and Eq. (21). We can observe that the values of and obtained numerically using the derived equations are very closely matched.

It must be noted that while Eq. (21) is useful in finding the value of the object plane tilt angle for a given value of lens tilt angle , obtaining the inverse function for evaluating in terms of is not straightforward. However, a simple iterative algorithm, which starts from an initial estimate of by setting , can be used to estimate the required lens tilt angle required for focusing on a tilted object surface.

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| Table 2. Verification of equations Eq. (20) and Eq. (21) for focusing on a tilted object plane by tilting a lens about the entrance pupil. | | | | |
| (Zemax)1 | (Zemax)2 | (Zemax)3 | (numerical)4 | (numerical)5 |
| 0.0° | 0.0° | 29.17073 *mm* | -2.2E-15° | 29.17073 *mm* |
| -10.0° | -0.46989° | 29.17145 *mm* | -10.0° | 29.17145 *mm* |
| 25.0° | 1.24249° | 29.17572 *mm* | 25.0° | 29.17572 *mm* |
| -40.0° | -2.23504° | 29.18687 *mm* | -40.0° | 29.18687 *mm* |
| 65.0° | 5.69682° | 29.27607 *mm* | 65.0° | 29.27607 *mm* |
| -80.0° | -14.79587° | 29.90304 *mm* | -80.0° | 29.90304 *mm* |
| 1. Object plane tilt about the -axis set in Zemax. 2. Lens plane tilt about the -axis obtained through optimization using ray-tracing in Zemax. 3. Image plane distance obtained through optimization using ray-tracing in Zemax. 4. Object plane tilt computed numerically using Eq.(21) the value of in column 2. 5. Image plane distance computed numerically using the value of in column 2. | | | | |

4. APPLICATION OF THE MODEL FOR OMNIFOCUS IMAGING USING LENS TILT

A. Basic Idea for Synthesizing Omnifocus Image using Lens Tilts

We can infer several insights about the geometric properties of the image formed in a Scheimpflug camera from Eq. (12). In this section, we use one such interesting consequence of Eq. (12) that is useful for synthesizing an omnifocus image by selectively blending multiple images captured while rotating a lens about its entrance pupil.

An omnifocus image has everything in the close foreground to far background in sharp focus [ref]. Lenses can focus only on a single surface—usually, the plane of sharp focus—as dictated by the laws of physics. Consequently, objects fore and aft the plane of sharp focus gradually become out of focus and appear blurry in the image. This interplay of light and lenses leads to the limited depth of field (DOF) problem. Several methods have been proposed to circumvent this problem, for example, depth dependent image deconvolution, wavefront coding, plenoptic imaging, Scheimpflug imaging, focus stacking, etc.

In Scheimpflug imaging the lens or the sensor or both are rotated, which induces a rotation of the plane of sharp focus allowing scenes with significant depths (or object planes that are tilted) to be in focus at the image plane [ref].

In focus stacking (or z-stacking), a number of images are captured at multiple focus depths by changing either the focal length or the image plane distance. Consequently, regions of the scene that are a definite distance from the lens are in focus only in a single image. Collectively, however, the stack contains the all or most regions of scene in focus distributed amongst the images. An omnifocus image is created by registering the images, followed by identifying and blending the in-focus regions [ref].

The DOF region in Scheimpflug imaging is still limited to a small region (approximately a wedge) around the plane of sharp focus. In focus stacking, significant portions of each DOF region extends perpendicular to the optical axis of the lens and beyond the field-of-view of the camera, resulting in suboptimal utilization.

Our analysis of Eq. (12) suggest that we can borrow the central ideas of Scheimpflug imaging and focus stacking methods to device a simple technique for creating omnifocus images while bypassing the above shortcomings of either methods. Our technique relies on capturing multiple images of the scene while rotating a lens about the entrance pupil. In particular, we show that the proposed method is simplest if the pupil magnification of the lens equals one (i.e., a symmetric lens).

A critical step in the synthesis of an omnifocus image from a stack of images is registration, which is the process of spatially aligning the images in the stack to a reference image by applying a mapping function—either known *a priori* from the model or estimated from the images. The degree of accuracy of image registration directly influences the quality of the synthesized image.

In general, a rotation of the lens about a pivot located along the optical axis results in a complex depth-dependent warping of the image field. Specifically, the extent of distortion of the points in the image is a function of points’ depth in the object space—a phenomenon known as parallax. [write about such registration and point out accuracy problems … local operation, iterative algorithms etc. … feature based aligning will have problem because of invalirable blurring in parts of the image]

If, however, the lens is rotated about its entrance pupil, then the warping of the image field is independent of the scene depth, and purely from a geometric standpoint, the images in the stack are isomorphic through a mapping , where is the difference angle of the lens’ orientation between and . Further, we can derive this mapping, called the *inter-image homography* [ref], from Eq. (19) allowing us to analytically register the images in the sequence. As a result, the registration process is efficient (not requiring any iterative algorithm) and exact.

The specific structure of the inter-image homography matrix, depends on the pupil magnification . Interestingly, if the pupil magnification equals one (a perfectly symmetric lens), the inter-image homography between the image obtained under a lens tilt of about the -axis and the reference image (), reduces to a simple similarity transformation consisting only scaling and translation components. The mapping between and is shown below:

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where, is the image point in the reference image (), is the corresponding image point obtained under lens rotation , is the distance of the exit pupil center from the entrance pupil center (the pivot point in this case) along the optical axis, and is the location of the image plane’s pivot

B. Simulation of Omnifocus Imaging using Zemax and PyZDDE

5. DISCUSSION AND CONCLUSION

Discuss the structure of the inter-image homography if the pupil magnification is not one. (parallax) …. And discuss a possible strategy for registration.

Discuss the situation if we need to increase the DOF of scheimpflug imaging (within a small zone).

[write about what happens if parameters d and z are not known …. In equation 22]

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