



## Department of Electronics and Communication Engineering



# **MARRI LAXMAN REDDY INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

(AN AUTONOMOUS INSTITUTION)

(Approved by AICTE, New Delhi & Affiliated to JNTUH, Hyderabad)

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# DIGITAL LOGIC DESIGN AND COMPUTER ORGANIZATION

- UNIT-II: Combination circuits, binary adder & subtracter

Presentation By  
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Assoc. Prof.  
Department of ECE



**MARRI LAXMAN REDDY**  
**INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

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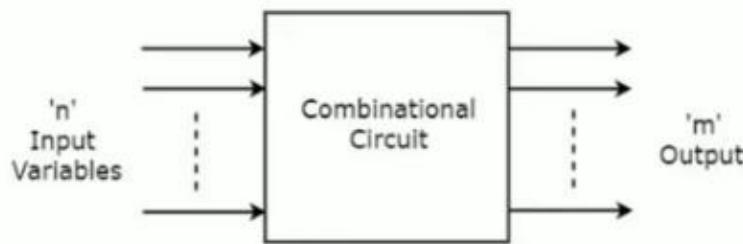
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# Digital Circuits

## Combinational Circuits

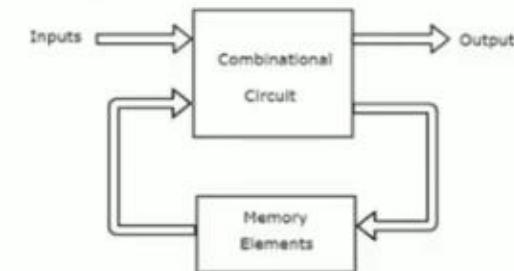
- Outputs of combinational circuit depend on the combination of present **inputs only**



- No Feedback
- No Memory
- Ex. Adder, Subtractor, Code Converter, Encoder, Decoder, Multiplexer and Demultiplexer

## Sequential Circuits

- Outputs of Sequential circuit depend on the combination of **present inputs** and **previous (Past) Output**.



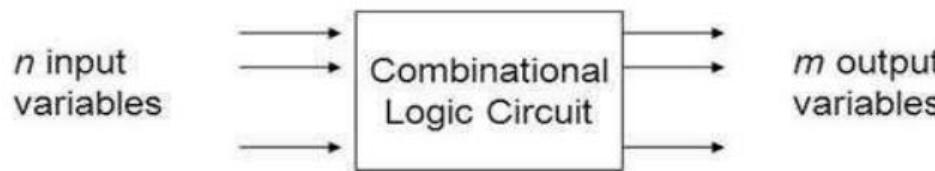
- Feedback is present
- Memory is present

# Combinational Circuits

- Combinational circuit is a circuit in which we combine the different gates in the circuit, for example encoder, decoder, multiplexer and demultiplexer.  
Some of the characteristics of combinational circuits are following:
- The output of combinational circuit at any instant of time, depends only on the levels present at input terminals.
- The combinational circuit do not use any memory. The previous state of input does not have any effect on the present state of the circuit.
- A combinational circuit can have an  $n$  number of inputs and  $m$  number of outputs.

# COMBINATIONAL CIRCUITS

- **Block diagram:**  
 $2^n$  possible combinations of input values.



- Specific functions :of combinational circuits  
Adders,subtractors,multiplexers,comprators,encoder,Decoder.  
MSI Circuits and standard cells

# ANALYSIS PROCEDURE

## Analysis procedure

To obtain the output Boolean functions from a logic diagram, proceed as follows:

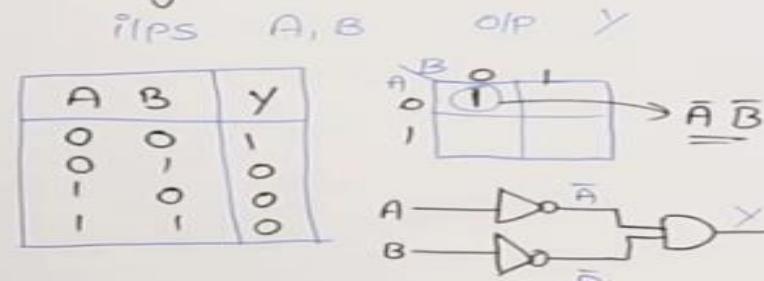
1. Label all gate outputs that are a function of input variables with arbitrary symbols. Determine the Boolean functions for each gate output.
2. Label the gates that are a function of input variables and previously labeled gates with other arbitrary symbols. Find the Boolean functions for these gates.
3. Repeat the process outlined in step 2 until the outputs of the circuit are obtained.

# DESIGN PROCEDURE

## Design Procedure

1. The problem is stated
2. The number of available input variables and required output variables is determined.
3. The input and output variables are assigned letter symbols.
4. The truth table that defines the required relationship between inputs and outputs is derived.
5. The simplified Boolean function for each output is obtained.
6. The logic diagram is drawn.

Example: Design a combinational circuit with two inputs, which produce output as logic 0 when any one input is one.



# Combinational circuits

## Example:

Design a combinational logic circuit with three input variables that will produce a logic 1 output when more than one input variable are logic 1.

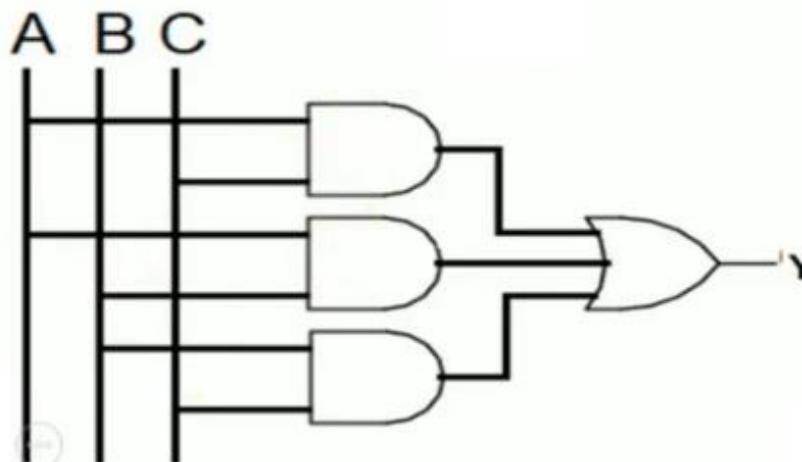
Truth Table for Circuit

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

# Combinational circuits

| $A$            | $\overline{B}\overline{C}$ | $\overline{B}C$ | $BC$ | $B\overline{C}$ | $Y$ |
|----------------|----------------------------|-----------------|------|-----------------|-----|
| $\overline{A}$ | 0                          | 0               | 1    | 0               |     |
| $A$            | 0                          | 1               | 1    | 1               |     |

$$Y = AC + AB + BC$$



Truth Table for Circuit

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

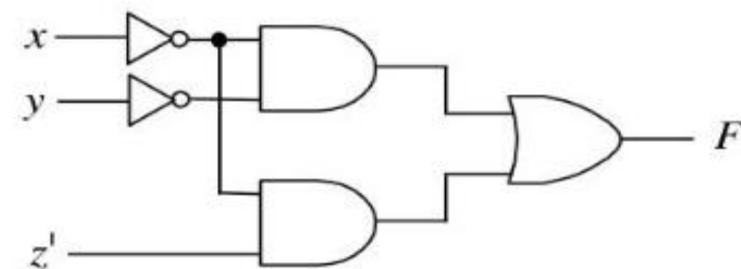
# DESIGN OF COMBINATIONAL LOGIC

Example: Design a combinational circuit with three inputs and one output. The output is a 1 when the binary value is less than three. The output is 0 otherwise.

| $x$ | $y$ | $z$ | $F$ |
|-----|-----|-----|-----|
| 0   | 0   | 0   | 1   |
| 0   | 0   | 1   | 1   |
| 0   | 1   | 0   | 1   |
| 0   | 1   | 1   | 0   |
| 1   | 0   | 0   | 0   |
| 1   | 0   | 1   | 0   |
| 1   | 1   | 0   | 0   |
| 1   | 1   | 1   | 0   |

| $x$ | $y$ | $z$ | 00 | 01 | 11 | 10 |
|-----|-----|-----|----|----|----|----|
| 0   | 1   | 1   | 1  | 1  |    | 1  |
| 1   |     |     |    |    |    |    |

$$F = x' y' + x' z'$$

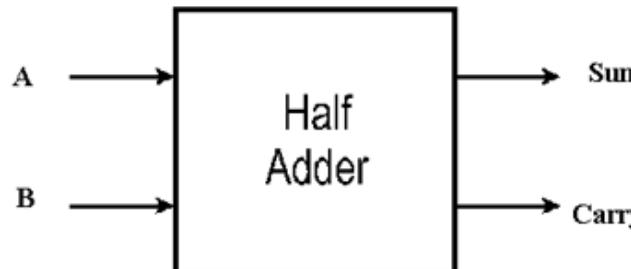


# BINARY ADDERS

## ADDERS

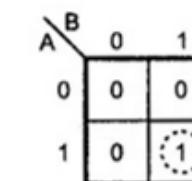
### Half Adder

A Half Adder is a combinational circuit with two binary inputs (augends and addend bits) and two binary outputs (sum and carry bits.) It adds the two inputs (A and B) and produces the sum (S) and the carry (C) bits.



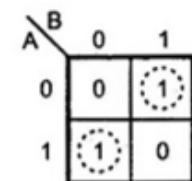
| A | B | Sum | Carry |
|---|---|-----|-------|
| 0 | 0 | 0   | 0     |
| 0 | 1 | 1   | 0     |
| 1 | 0 | 1   | 0     |
| 1 | 1 | 0   | 1     |

For Carry

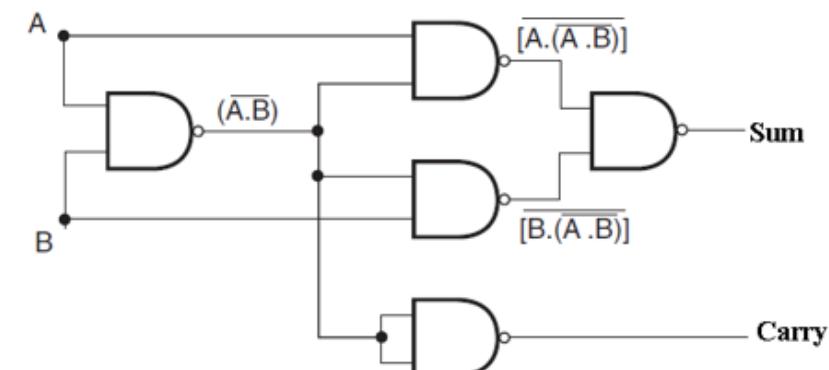
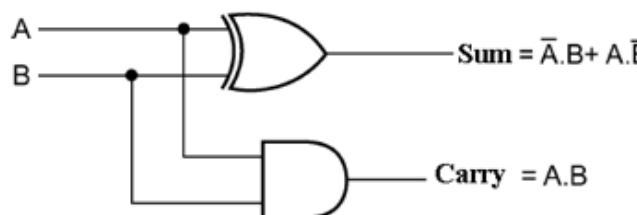


$$\text{Carry} = AB$$

For Sum



$$\begin{aligned} \text{Sum} &= AB + A.B \\ &= A \oplus B \end{aligned}$$



# BINARY ADDERS

## Full Adder

The full-adder adds the bits A and B and the carry from the previous column called the carry-in  $C_{in}$  and outputs the sum bit S and the carry bit called the carry-out  $C_{out}$ .

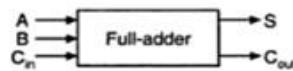


Fig: block diagram

| Inputs |   |          | Outputs |       |
|--------|---|----------|---------|-------|
| A      | B | $C_{in}$ | Sum     | Carry |
| 0      | 0 | 0        | 0       | 0     |
| 0      | 0 | 1        | 1       | 0     |
| 0      | 1 | 0        | 1       | 0     |
| 0      | 1 | 1        | 0       | 1     |
| 1      | 0 | 0        | 1       | 0     |
| 1      | 0 | 1        | 0       | 1     |
| 1      | 1 | 0        | 0       | 1     |
| 1      | 1 | 1        | 1       | 1     |

Fig:Truth table

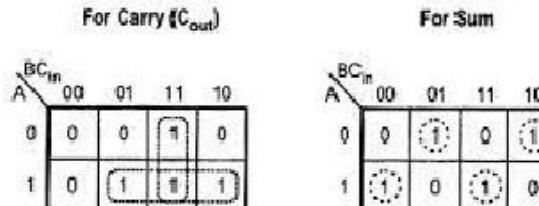
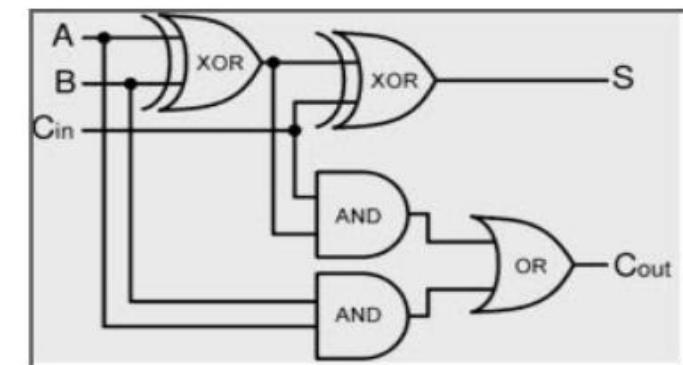


Fig. Maps for full-adder

$$\begin{aligned}
 S &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC \\
 &= A(\bar{B}\bar{C} + BC) + \bar{A}(B\bar{C} + \bar{B}C) \\
 &= A(B \text{ XNOR } C) + \bar{A}(B \text{ XOR } C) \\
 &= A \text{ XOR } B \text{ XOR } C
 \end{aligned}$$

$$\begin{aligned}
 C_{out} &= AB\bar{C} + ABC + \bar{A}BC + ABC + A\bar{B}C + ABC \\
 &= ABC + ABC + \bar{A}BC + A\bar{B}C + (ABC + ABC) \\
 &= AB(\bar{C} + C) + C(AB + A\bar{B}) + ABC \\
 &= AB + C(A \text{ XOR } B) + ABC \\
 &= AB(1+C) + C(A \text{ XOR } B) \\
 &= AB + C(A \text{ XOR } B)
 \end{aligned}$$



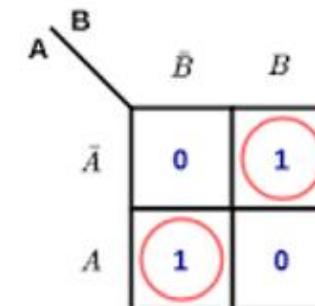
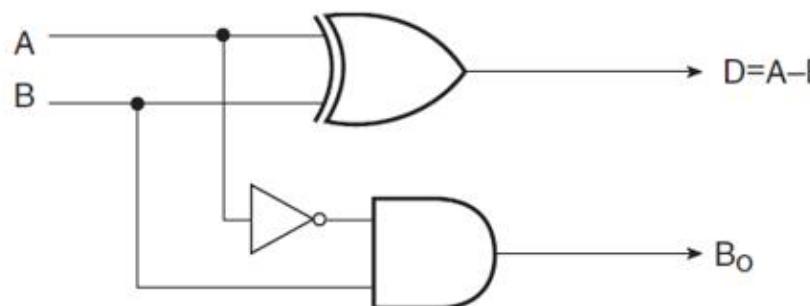
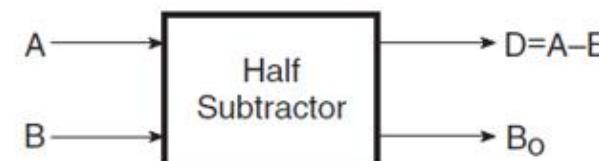
Full Adder Logical Diagram

# BINARY SUBTRACTORS

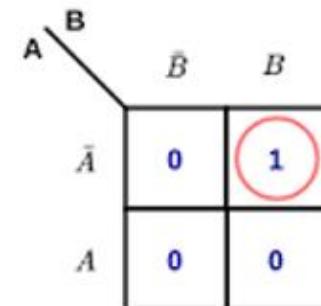
## Half Subtractor

A Half-subtractor is a combinational circuit with two inputs A and B and two outputs difference and barrow.

| A | B | D | $B_0$ |
|---|---|---|-------|
| 0 | 0 | 0 | 0     |
| 0 | 1 | 1 | 1     |
| 1 | 0 | 1 | 0     |
| 1 | 1 | 0 | 0     |



$$D = A'B + AB' = A \oplus B$$



$$B_0 = \bar{A} \cdot B$$

# BINARY SUBTRACTORS

## Full subtractor

The full subtractor performs subtraction of three input bits: the minuend, subtrahend, and borrow in and generates two output bits difference and borrow out.

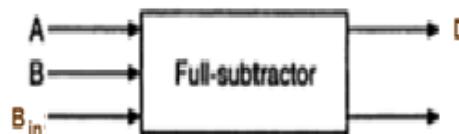
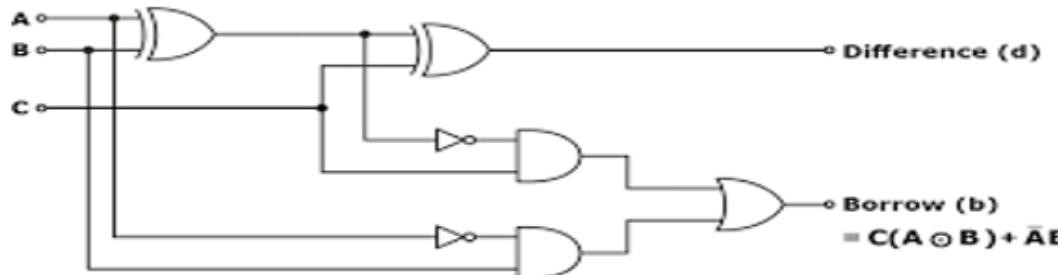


Fig : Block diagram

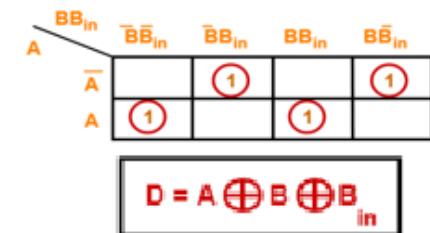


Logic diagram of full subtractor in SOP form

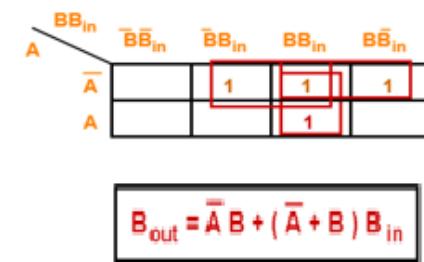
| Inputs |   |          | Difference | Borrow |
|--------|---|----------|------------|--------|
| A      | B | $B_{in}$ | D          | $B$    |
| 0      | 0 | 0        | 0          | 0      |
| 0      | 0 | 1        | 1          | 1      |
| 0      | 1 | 0        | 1          | 1      |
| 0      | 1 | 1        | 0          | 1      |
| 1      | 0 | 0        | 1          | 0      |
| 1      | 0 | 1        | 0          | 0      |
| 1      | 1 | 0        | 0          | 0      |
| 1      | 1 | 1        | 1          | 1      |

Fig : Truth table

For D:



For  $B_{in}$ :





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# DIGITAL LOGIC DESIGN AND COMPUTER ORGANIZATION

## UNIT-II: Decimal adder, binary multiplier, Magnitude comparator

Presentation By

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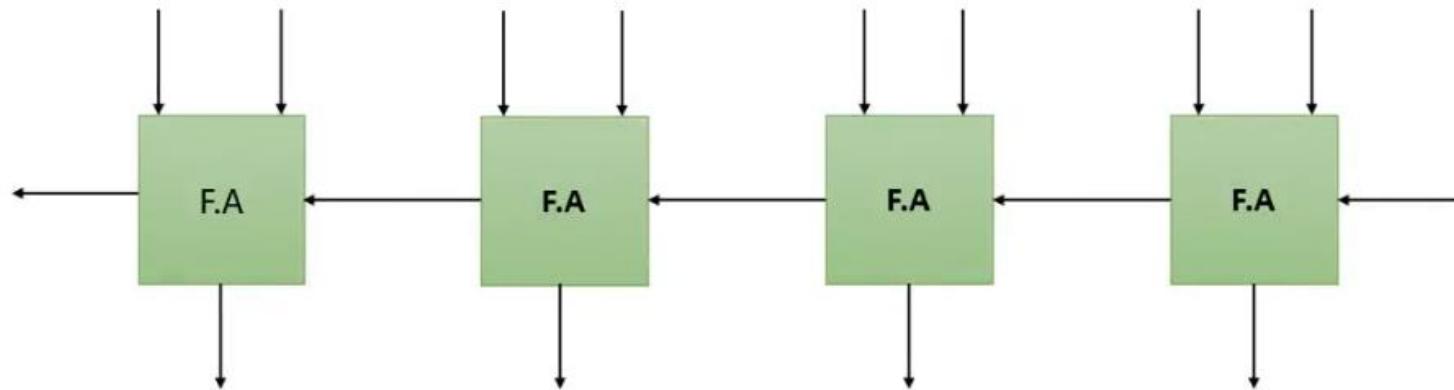
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## Binary Parallel Adder

- By the use of single “Full-Adder”, we are only capable of adding “two one-bit numbers” and an “input carry”. So to add binary numbers that consist more than one bit , we have to use Parallel-Adder.
- The Binary Parallel adder consists of “Full-adders” connected in cascade, with the output carry from each full adder connected to the input carry of the next full adder.

Block Diagram of Four-Bit Parallel Adder



## Binary Parallel Adder

- By the use of single "Full-Adder", we are only capable of adding "two one-bit numbers" and an "input carry". So to add binary numbers that consist more than one bit, we have to use Parallel-Adder.
- The Binary Parallel adder consists of "Full-adders" connected in cascade, with the output carry from each full adder connected to the input carry of the next full adder.

multibits

$$\begin{array}{r} A = \overbrace{\quad 0 \quad 1 \quad 0 \quad 1} \\ B = \quad 1 \quad 0 \quad 1 \quad 0 \\ \hline A + B \end{array}$$

Binary parallel Adder

Single F.A = two one-bit no + input carry

A =  $\begin{cases} 1 \\ 0 \end{cases}$  } one bit no

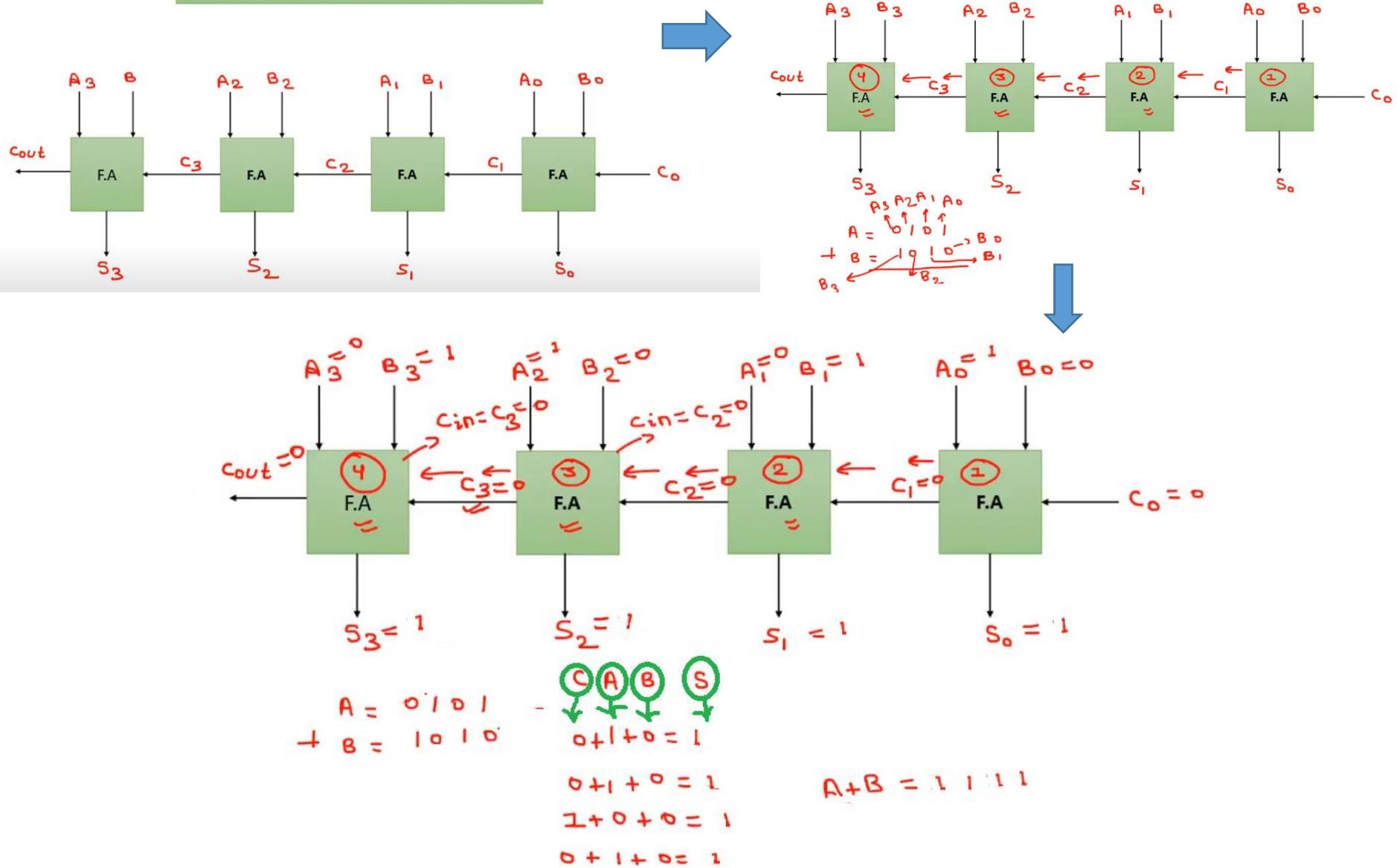
B =  $\begin{cases} 1 \\ 0 \end{cases}$  } one bit no

$A + B + Cin \rightarrow$  single

Full Adder

$\rightarrow$  cascade connection  $\rightarrow$  Full Adder

Block Diagram of Four-Bit Parallel Adder



# Designing of BCD Adder

- A BCD adder adds two BCD digits and produces BCD digit.
- BCD cannot be greater than 9.

$$\begin{array}{r} 7 \\ + 1 \\ \hline 8 \end{array} \quad \begin{array}{r} 0111 \\ + 0001 \\ \hline 1000 \end{array}$$

↓

$$\begin{array}{r} 7 \\ + 4 \\ \hline 11 \end{array} \quad \begin{array}{r} 0111 \\ + 0100 \\ \hline 1011 \end{array}$$
$$\begin{array}{r} + 0110 \\ \hline 10001 \end{array}$$

↓

$$\begin{array}{r} 9 \\ + 8 \\ \hline 17 \end{array} \quad \begin{array}{r} 1001 \\ + 1000 \\ \hline 10001 \end{array}$$
$$+ 0110$$

↓

## **BCD ADDER:**

A BCD adder should perform the following

- Add two 4-bit BCD number using straight Binary Addition.
- If the sum of two numbers is equal to or less than 9, then the sum is valid BCD number and no correction is required.
- If the sum of two numbers is greater than 9 or carry is generated from the sum, then the sum is not valid BCD number. Then add 0110 (6) to the sum, the result will be valid BCD number. If further a carry is generated then add 0110 to the result.

## DESIGN:

- In fig. 1  $B_3B_2B_1B_0$  and  $A_3A_2A_1A_0$  are the BCD inputs.  $S_3S_2S_1S_0$  and  $C_{out}$  is the output of Adder 1.
- Checked the output of Adder 1, whether it is greater than or less than 9.
- If the sum of Adder 1 is greater than 9 then the output of combinational circuit should be 1 (i.e  $C_{out}$  should be high ) and correction is required.
- Write the truth table and K-Map for combinational circuit.
- The Boolean Expression from K-Map  $Y= S_3S_2+S_3S_1$
- The output of combinational circuit  $Y_2$  is connected to  $B_2B_1$  of Adder 2 and  $B_3B_0$  are connected to the ground. Therefore  $B_3=B_0=0$
- The output sum of Adder 1 is connected to Adder 2. the output of Adder 2 is the final result of BCD addition with Carry which can be ignored if any.

## Correction:

BCD:

| Decimal | 8 | 4 | 2 | 1 |
|---------|---|---|---|---|
| 0       | 0 | 0 | 0 | 0 |
| 1       | 0 | 0 | 0 | 1 |
| 2       | 0 | 0 | 1 | 0 |
| 3       | 0 | 0 | 1 | 1 |
| 4       | 0 | 1 | 0 | 0 |
| 5       | 0 | 1 | 0 | 1 |
| 6       | 0 | 1 | 1 | 0 |
| 7       | 0 | 1 | 1 | 1 |
| 8       | 1 | 0 | 0 | 0 |
| 9       | 1 | 0 | 0 | 1 |
| 10      | 1 | 0 | 1 | 0 |
| 11      | 1 | 0 | 1 | 1 |
| 12      | 1 | 1 | 0 | 0 |
| 13      | 1 | 1 | 0 | 1 |
| 14      | 1 | 1 | 1 | 0 |
| 15      | 1 | 1 | 1 | 1 |

$$\begin{array}{r}
 \text{BCD}_1 + \text{BCD}_2 \\
 \downarrow \\
 \begin{array}{r}
 \textcolor{red}{1} \textcolor{red}{0} \textcolor{red}{1} \textcolor{red}{0} \\
 + 0 1 1 0 \\
 \hline
 \textcolor{teal}{0} \textcolor{teal}{0} \textcolor{teal}{0} \textcolor{teal}{1} \textcolor{teal}{0} \textcolor{teal}{0} \textcolor{teal}{0} \\
 \textcolor{teal}{1} \qquad \qquad \qquad \textcolor{teal}{0}
 \end{array}
 \end{array}$$

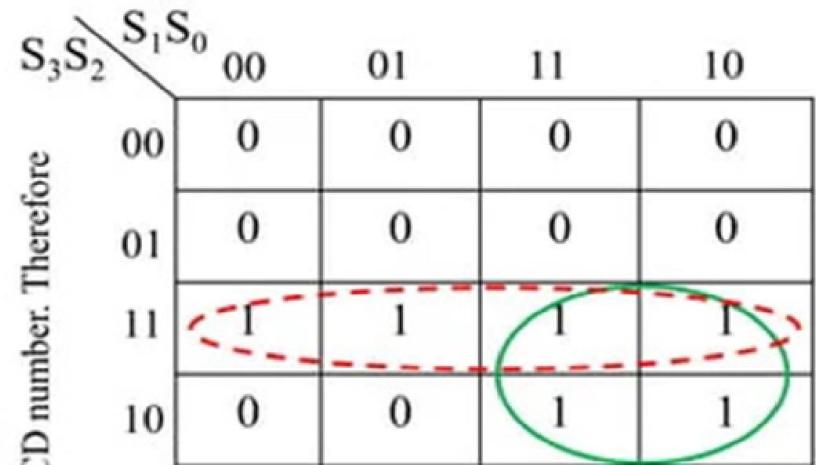
**TRUTH TABLE**

| INPUTS         |                |                |                | OUTPUT S       |
|----------------|----------------|----------------|----------------|----------------|
| S <sub>3</sub> | S <sub>2</sub> | S <sub>1</sub> | S <sub>0</sub> | Y <sub>1</sub> |
| 0              | 0              | 0              | 0              | 0              |
| 0              | 0              | 0              | 1              | 0              |
| 0              | 0              | 1              | 0              | 0              |
| 0              | 0              | 1              | 1              | 0              |
| 0              | 1              | 0              | 0              | 0              |
| 0              | 1              | 0              | 1              | 0              |
| 0              | 1              | 1              | 0              | 0              |
| 0              | 1              | 1              | 1              | 0              |
| 1              | 0              | 0              | 0              | 0              |
| 1              | 0              | 0              | 1              | 0              |
| 1              | 0              | 1              | 0              | 1              |
| 1              | 0              | 1              | 1              | 1              |
| 1              | 1              | 0              | 0              | 1              |
| 1              | 1              | 0              | 1              | 1              |
| 1              | 1              | 1              | 0              | 1              |
| 1              | 1              | 1              | 1              | 1              |

Sum is valid BCD number. Therefore Y=0

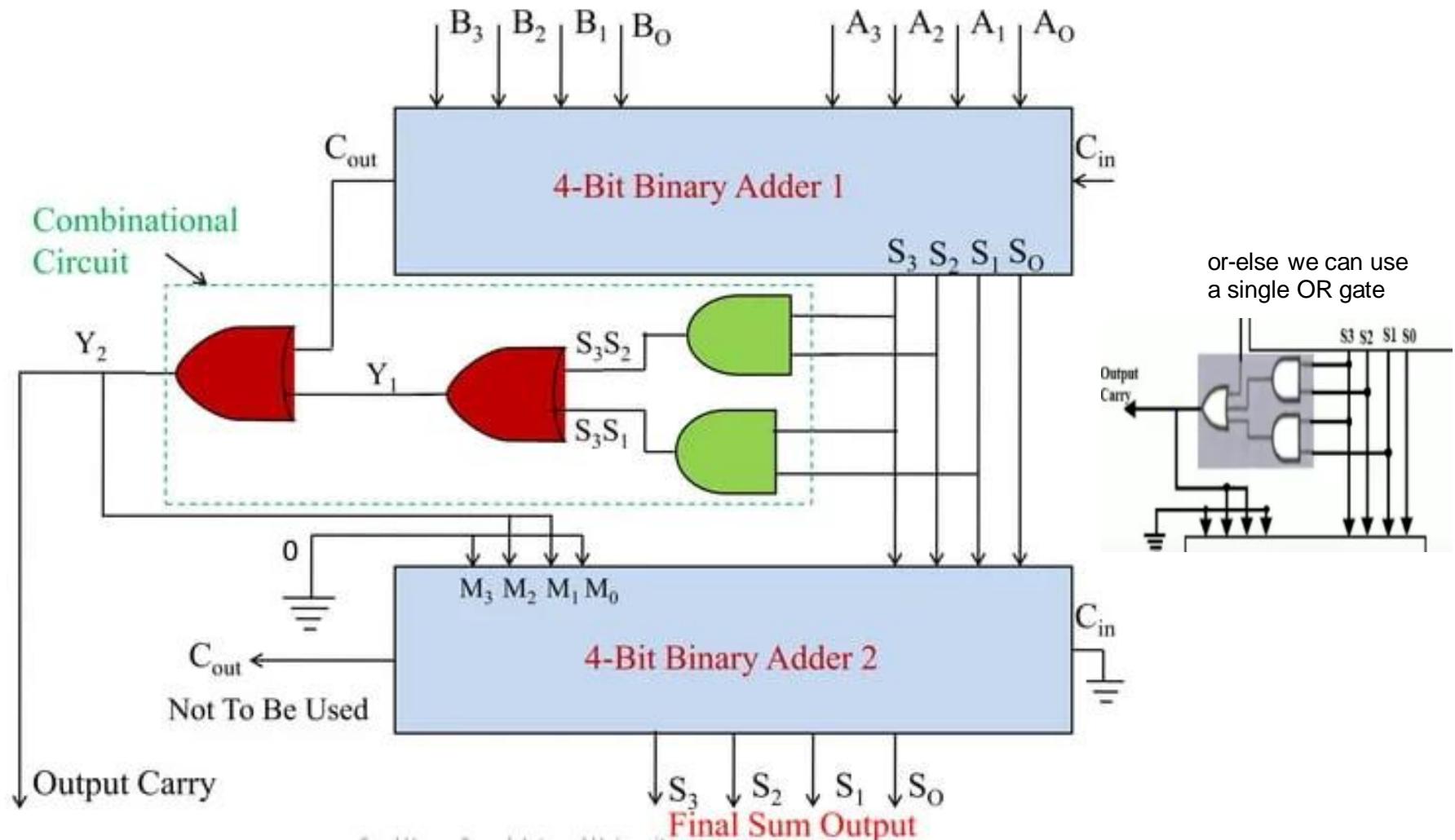
Sum is valid BCD number. Therefore Y=1

Sum is valid BCD number. Therefore Y=0

**K-Map**


$$Y_1 = S_3S_2 + S_3S_1$$

Fig. 1: BLOCK DIAGRAM OF BCD ADDER



**CASE 1: Sum equal to or less than 9 with carry 0**

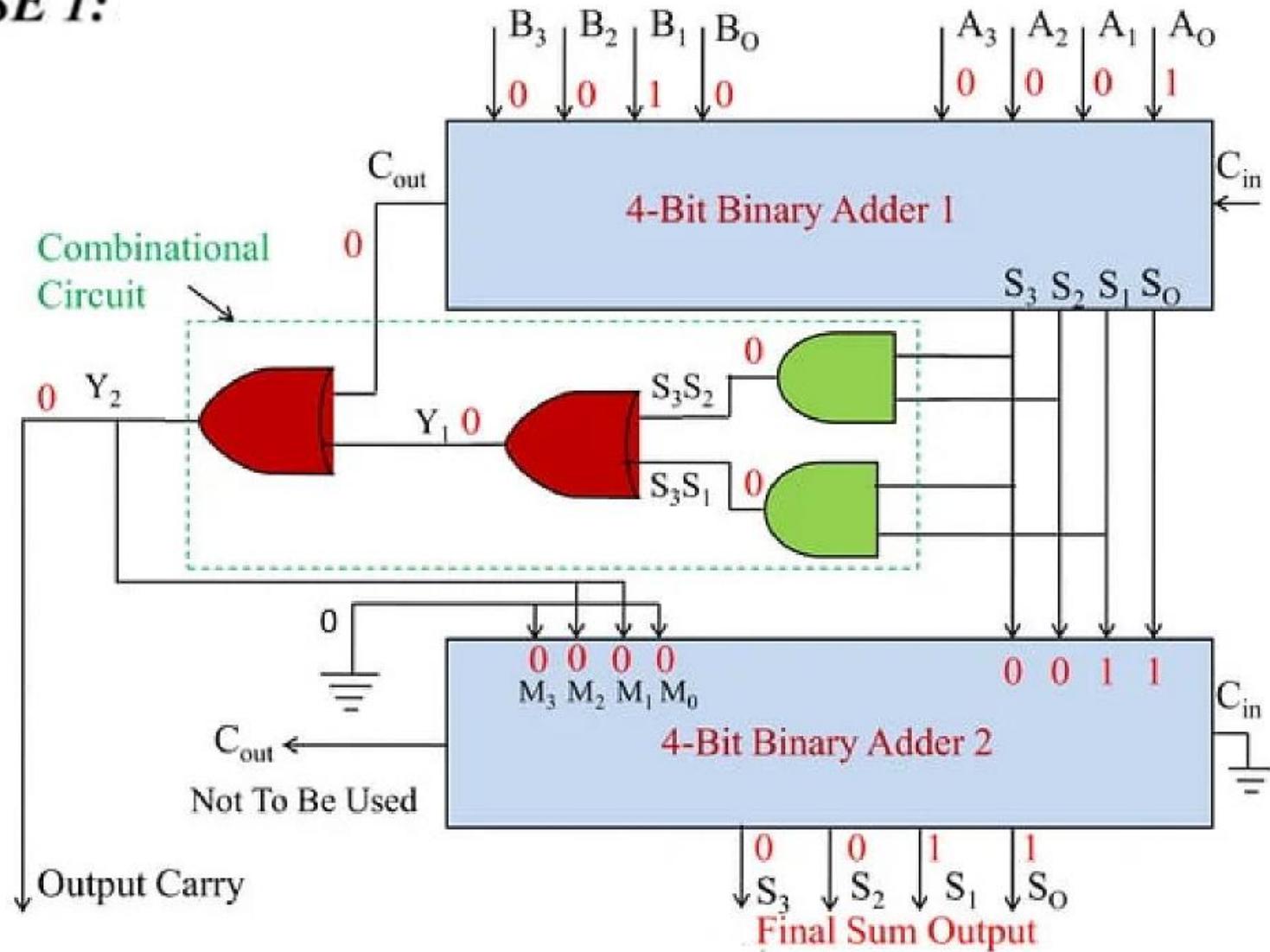
| DECIMAL      | BCD  |
|--------------|--|
| 2            | 0 0 1 0                                    |
| + 1          | 0 0 0 1                                    |
| —————        |  |
| Carry      0 |  |
| SUM      3   | 0 0 0 1 1                                  |
|              | <i>Final Carry</i> <i>Valid BCD Number</i> |

Output of combinational circuit  $Y_2 = 0$ ,  $M_3 M_2 M_1 M_0 = 0$

Output of Adder 2 is same as the output of Adder 1

## CASE 1:

Fig. 1: BLOCK DIAGRAM OF BCD ADDER



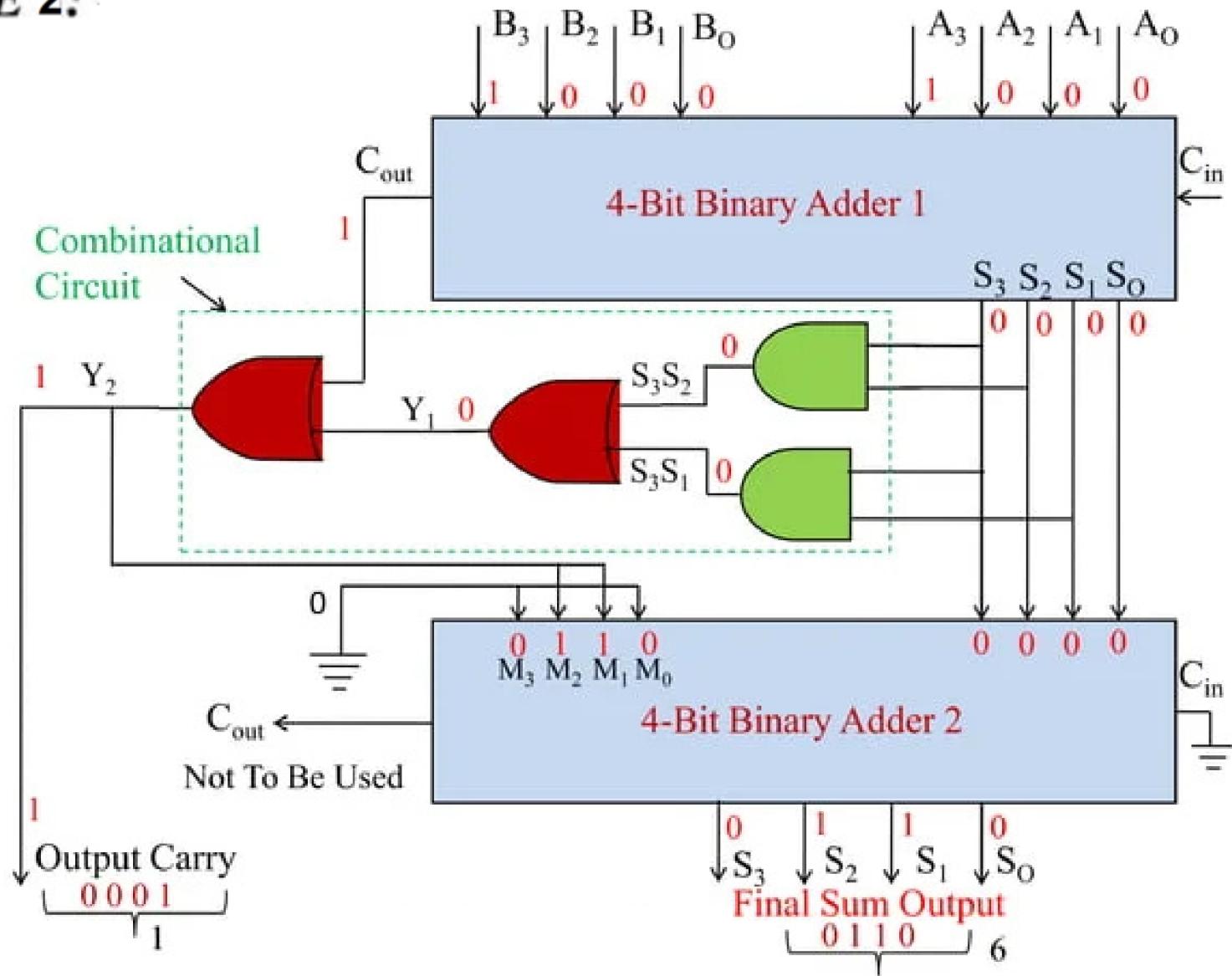
## *CASE 2: Sum is equal to or less than 9 with carry 1*

| DECIMAL                   | BCD  |                                      |
|---------------------------|--|--------------------------------------|
| 8                         | 1 0 0 0  |                                      |
| + 8                       | 1 0 0 0  |                                      |
| <hr/> Carry               | <hr/> <span style="color: red; font-size: 2em;">1</span>   |                                      |
| SUM    16                 | 1 0 0 0 0  | <i>Invalid BCD Number</i>            |
|                           | 0 1 1 0  | <i>Add 6 for correction</i>          |
| <hr/> <i>Final result</i> | <i>Final Carry</i> ↗ <span style="color: red; font-size: 2em;">1</span> 0 <span style="color: red; font-size: 2em;">1</span> 1 0<br><hr/> <span style="font-size: 2em;">0</span> <span style="font-size: 2em;">0</span> <span style="font-size: 2em;">0</span> <span style="font-size: 2em;">1</span> ↘<br>1 | <i>Valid BCD number with carry 1</i> |
|                           | <hr/> <span style="font-size: 2em;">0</span> <span style="font-size: 2em;">1</span> <span style="font-size: 2em;">1</span> 0 ↘<br>6  |                                      |

Output of combinational circuit  $Y_2 = 1$ ,  $M_3 M_2 M_1 M_0 = 0 1 1 0$

**CASE 2:**

**Fig. 1: BLOCK DIAGRAM OF BCD ADDER**



## Binary Multiplier

- ❑ A binary-multiplier is used multiply two binary numbers.
- ❑ It is built using “binary adders”. The two numbers are known as “Multiplicand” and “Multiplier” and the result is known as “Product”.

$$\begin{array}{l} A \rightarrow \text{multiplicand} \\ \times B \rightarrow \text{multiplier} \\ \hline \underline{A \times B} \rightarrow \text{product} \end{array}$$

Binary multiplication:  
 $0 \times 0 = 0$        $1 \times 0 = 0$   
 $0 \times 1 = 0$        $1 \times 1 = 1$   
Shift and Add principle

$$\begin{array}{r} 011 \\ \times 110 \\ \hline 1000 \\ 1011 \times \leftarrow \text{left shift} \\ 011 \times \times \rightarrow \text{left shift} \\ \hline \underline{\underline{10010}} \rightarrow \text{product} \end{array}$$

## 2\*2 Multiplier

Multiplicand  $A = A_1, A_0$

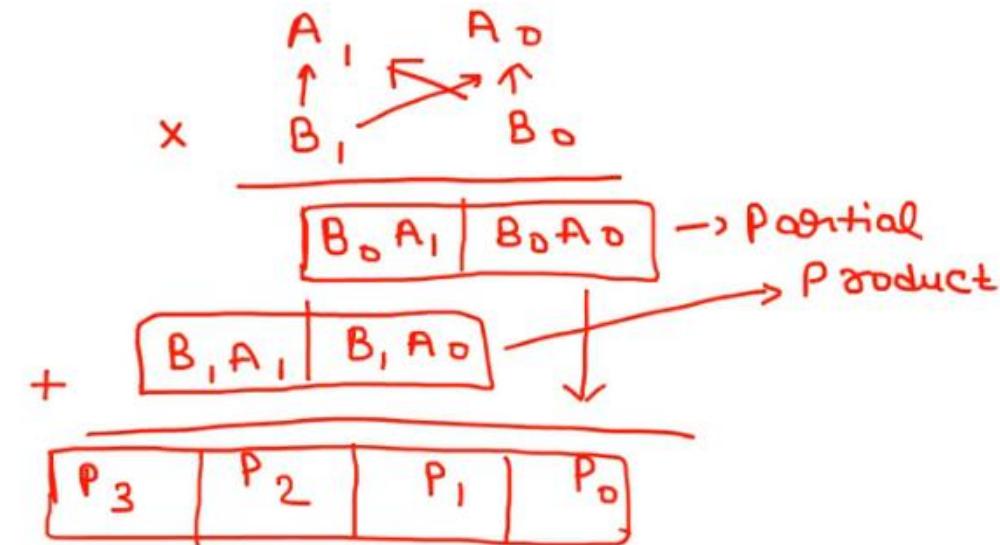
Multiplier  $B = B_1, B_0$

$$P_0 = B_0 A_0$$

$$P_1 = \underbrace{B_0 A_1 + B_1 A_0}$$

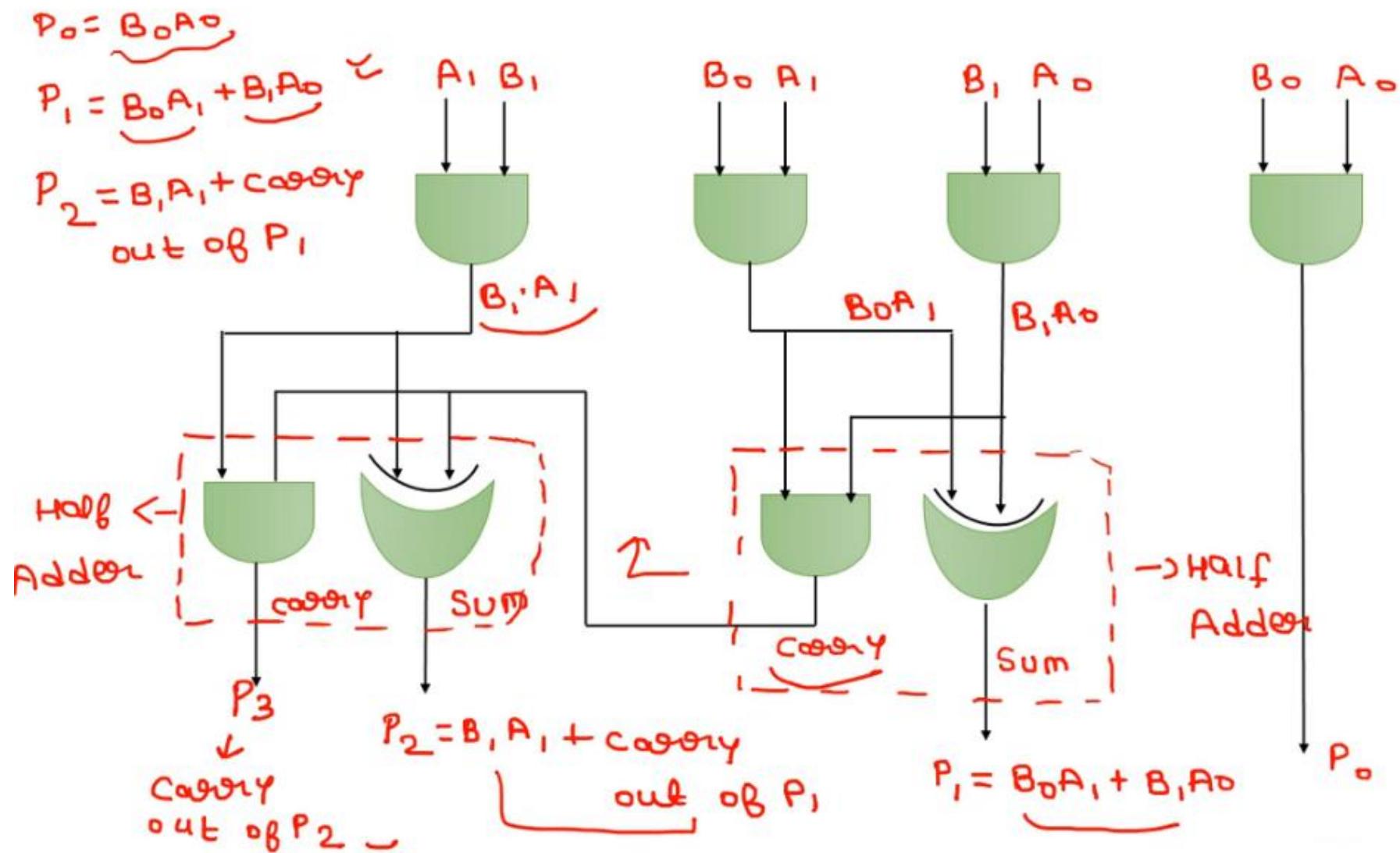
$$P_2 = B_1 A_1 + \text{carry out of } P_1$$

$$P_3 = \text{carry out of } P_2$$



Partial product  $\rightarrow$  Using AND Gate

Partial product Add  $\rightarrow$  Full Adder/ Half Adder



# MAGNITUDE COMPARATOR

Magnitude comparator takes two numbers as input in binary form and determines whether one number is greater than, less than or equal to the other number.

## 1-Bit Magnitude Comparator

A comparator used to compare two bits is called a single bit comparator.

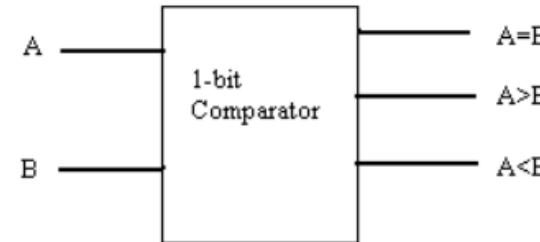


Fig :1 Block diagram

# 1-bit Comparator

Truth Table

| A | B | $A = B$ | $A > B$ | $A < B$ |
|---|---|---------|---------|---------|
| 0 | 0 | 1       | 0       | 0       |
| 0 | 1 | 0       | 0       | 1       |
| 1 | 0 | 0       | 1       | 0       |
| 1 | 1 | 1       | 0       | 0       |

$$\text{Output} = \overline{\overline{A} \overline{B}} + AB$$



Truth Table

| A | B | $A = B$ | $A > B$ | $A < B$ |
|---|---|---------|---------|---------|
| 0 | 0 | 1       | 0       | 0       |
| 0 | 1 | 0       | 0       | 1       |
| 1 | 0 | 0       | 1       | 0       |
| 1 | 1 | 1       | 0       | 0       |

$$(A > B) = A \overline{B}$$



Truth Table

| A | B | $A = B$ | $A > B$ | $A < B$ |
|---|---|---------|---------|---------|
| 0 | 0 | 1       | 0       | 0       |
| 0 | 1 | 0       | 0       | 1       |
| 1 | 0 | 0       | 1       | 0       |
| 1 | 1 | 1       | 0       | 0       |

$$(A < B) = \overline{A} B$$



$$\text{Output} = \overline{\overline{A} \overline{B}} + AB$$



$$(A > B) = A \overline{B}$$

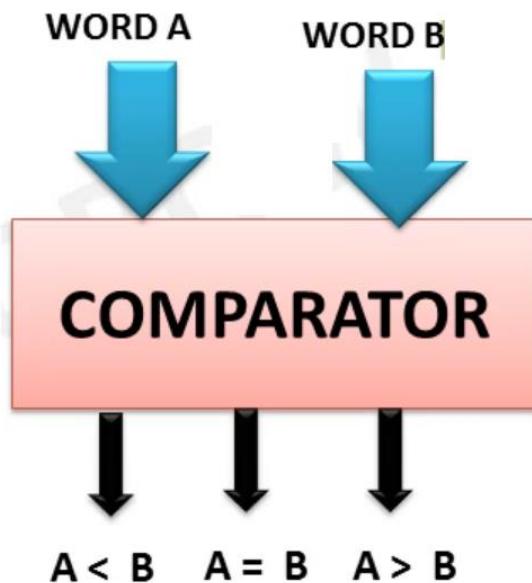


$$(A < B) = \overline{A} B$$



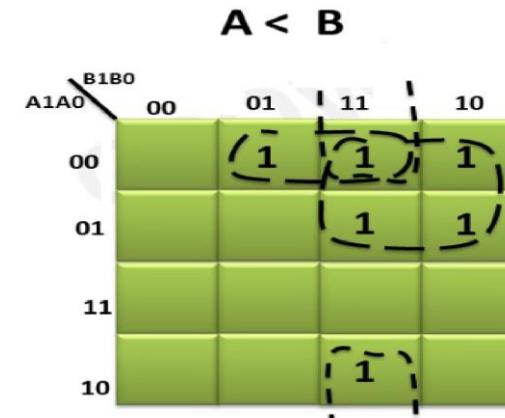
**2 BIT COMPARATOR**

| A1 | A0 | B1 | B0 | A < B | A = B | A > B |
|----|----|----|----|-------|-------|-------|
| 0  | 0  | 0  | 0  | 0     | 1     | 0     |
| 0  | 0  | 0  | 1  | 1     | 0     | 0     |
| 0  | 0  | 1  | 0  | 1     | 0     | 0     |
| 0  | 0  | 1  | 1  | 1     | 0     | 0     |
| 0  | 1  | 0  | 0  | 0     | 0     | 1     |
| 0  | 1  | 0  | 1  | 0     | 1     | 0     |
| 0  | 1  | 1  | 0  | 1     | 0     | 0     |
| 0  | 1  | 1  | 1  | 1     | 0     | 0     |
| 1  | 0  | 0  | 0  | 0     | 0     | 1     |
| 1  | 0  | 0  | 1  | 0     | 0     | 1     |
| 1  | 0  | 1  | 0  | 0     | 1     | 0     |
| 1  | 0  | 1  | 1  | 1     | 0     | 0     |
| 1  | 1  | 0  | 0  | 0     | 0     | 1     |
| 1  | 1  | 0  | 1  | 0     | 0     | 1     |
| 1  | 1  | 1  | 0  | 0     | 0     | 1     |
| 1  | 1  | 1  | 1  | 0     | 1     | 0     |

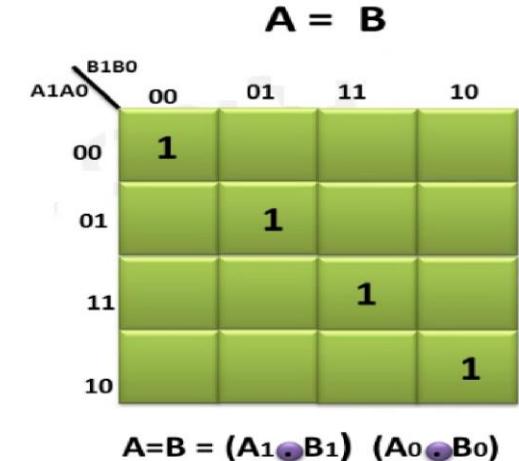


## 2 BIT COMPARATOR

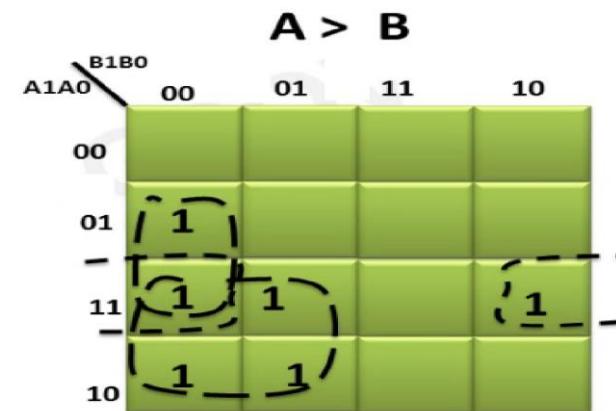
| A1 | A0 | B1 | B0 | A < B | A = B | A > B |
|----|----|----|----|-------|-------|-------|
| 0  | 0  | 0  | 0  | 0     | 1     | 0     |
| 0  | 0  | 0  | 1  | 1     | 0     | 0     |
| 0  | 0  | 1  | 0  | 1     | 0     | 0     |
| 0  | 0  | 1  | 1  | 1     | 0     | 0     |
| 0  | 1  | 0  | 0  | 0     | 0     | 1     |
| 0  | 1  | 0  | 1  | 0     | 1     | 0     |
| 0  | 1  | 1  | 0  | 1     | 0     | 0     |
| 0  | 1  | 1  | 1  | 1     | 0     | 0     |
| 1  | 0  | 0  | 0  | 0     | 0     | 1     |
| 1  | 0  | 0  | 1  | 0     | 0     | 1     |
| 1  | 0  | 1  | 0  | 0     | 1     | 0     |
| 1  | 0  | 1  | 1  | 1     | 0     | 0     |
| 1  | 1  | 0  | 0  | 0     | 0     | 1     |
| 1  | 1  | 0  | 1  | 0     | 0     | 1     |
| 1  | 1  | 1  | 0  | 0     | 0     | 1     |
| 1  | 1  | 1  | 1  | 0     | 1     | 0     |



$$A < B = A_1'B_1 + A_1'A_0'B_0 + A_0'B_1B_0$$



$$A = B = (A_1 \oplus B_1) \cdot (A_0 \oplus B_0)$$



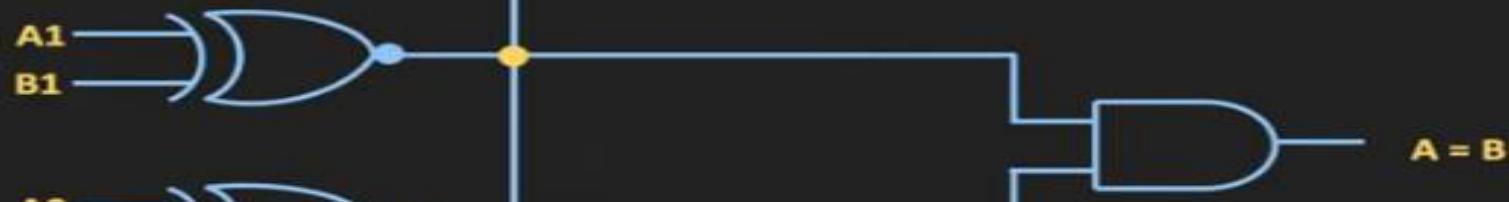
$$A > B = A_1B_1'B_0' + A_1A_0B_0' + A_1B_1'$$

## 2-bit Comparator

$$(A > B) = (A_1 \overline{B_1}) + (A_1 \odot B_1)(A_0 \overline{B_0})$$



$$(A = B) = (A_1 \odot B_1)(A_0 \odot B_0)$$



$$(A < B) = (\overline{A_1} B_1) + (A_1 \odot B_1)(\overline{A_0} B_0)$$





## Department of Electronics and Communication Engineering



# **MARRI LAXMAN REDDY INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

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(Approved by AICTE, New Delhi & Affiliated to JNTUH, Hyderabad)

Accredited by NBA and NAAC with 'A' Grade & Recognized Under Section 2(f) & 12(B) of the UGC act, 1956

# DIGITAL LOGIC DESIGN AND COMPUTER ORGANIZATION

- UNIT-II: Decoders, Encoders, Multiplexers

Presentation By

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Assoc. Prof.

Department of ECE



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## DECODER

- A decoder is a combinational circuit.
- A decoder accepts a set of inputs that represents a binary number and activates only that output corresponding to the input number. All other outputs remain inactive.
- Fig. 1 shows the block diagram of decoder with ‘N’ inputs and ‘M’ outputs.
- There are  $2^N$  possible input combinations, for each of these input combination only one output will be HIGH (active) all other outputs are LOW
- Some decoder have one or more ENABLE (E) inputs that are used to control the operation of decoder.

## BLOCK DIAGRAM OF DECODER

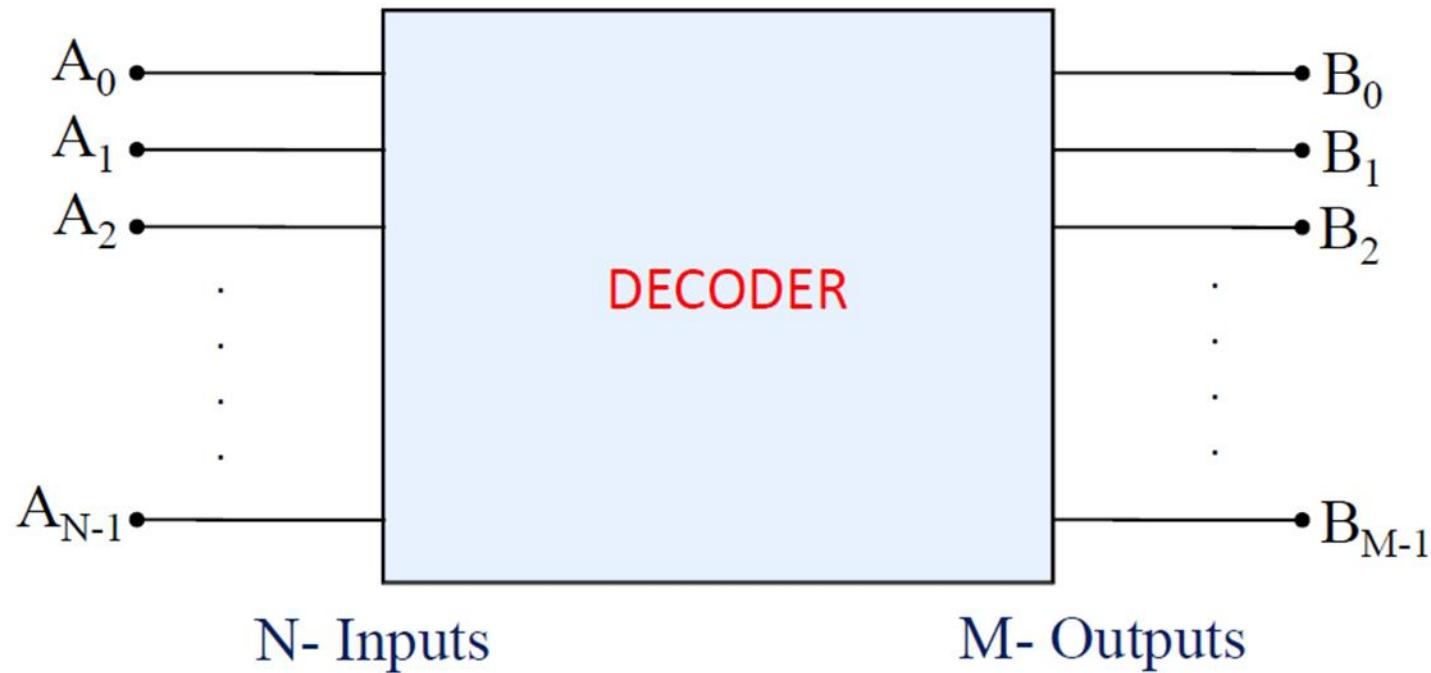


Fig. 1

*Only one output is High for each input*

## 2 to 4 Line Decoder:

- Block diagram of 2 to 4 decoder is shown in fig. 2
- A and B are the inputs. ( No. of inputs =2)
- No. of possible input combinations:  $2^2=4$
- No. of Outputs :  $2^2=4$ , they are indicated by  $D_0$ ,  $D_1$ ,  $D_2$  and  $D_3$
- From the Truth Table it is clear that each output is “1” for only specific combination of inputs.

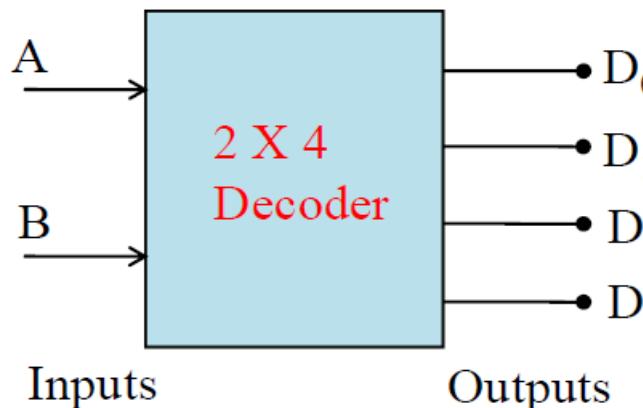


Fig. 2

TRUTH TABLE

| INPUTS |   | OUTPUTS |       |       |       |
|--------|---|---------|-------|-------|-------|
| A      | B | $D_0$   | $D_1$ | $D_2$ | $D_3$ |
| 0      | 0 | 1       | 0     | 0     | 0     |
| 0      | 1 | 0       | 1     | 0     | 0     |
| 1      | 0 | 0       | 0     | 1     | 0     |
| 1      | 1 | 0       | 0     | 0     | 1     |

## BOOLEAN EXPRESSION:

From Truth Table

$$D_0 = \overline{A} \overline{B}$$

$$D_2 = A \overline{B}$$

$$D_1 = \overline{A} B$$

$$D_3 = AB$$

## LOGIC DIAGRAM:

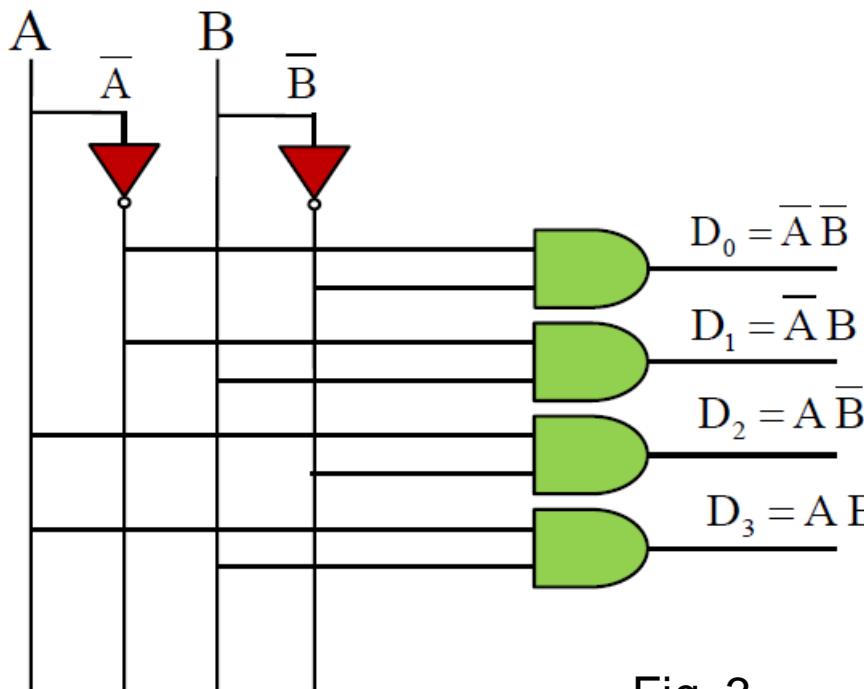


Fig. 3

TRUTH TABLE

| INPUTS |   | OUTPUTS |       |       |       |
|--------|---|---------|-------|-------|-------|
| A      | B | $D_0$   | $D_1$ | $D_2$ | $D_3$ |
| 0      | 0 | 1       | 0     | 0     | 0     |
| 0      | 1 | 0       | 1     | 0     | 0     |
| 1      | 0 | 0       | 0     | 1     | 0     |
| 1      | 1 | 0       | 0     | 0     | 1     |

### 3 to 8 Line Decoder:

- Block diagram of 3 to 8 decoder is shown in fig. 4
- A , B and C are the inputs. ( No. of inputs =3)
- No. of possible input combinations:  $2^3=8$
- No. of Outputs :  $2^3=8$ , they are indicated by  $D_0$  to  $D_7$
- From the Truth Table it is clear that each output is “1” for only specific combination of inputs.

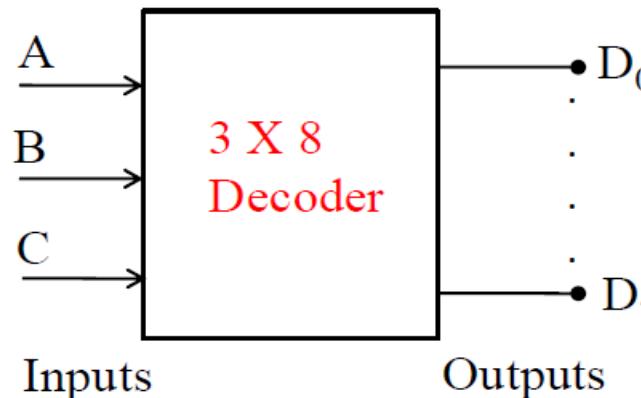


Fig. 4

TRUTH TABLE FOR 3 X 8 DECODER:

| INPUTS |   |   | OUTPUTS |    |    |    |    |    |    |    |  |
|--------|---|---|---------|----|----|----|----|----|----|----|--|
| A      | B | C | D0      | D1 | D2 | D3 | D4 | D5 | D6 | D7 |  |
| 0      | 0 | 0 | 1       | 0  | 0  | 0  | 0  | 0  | 0  | 0  | $D_0 = \overline{A} \overline{B} \overline{C}$ |
| 0      | 0 | 1 | 0       | 1  | 0  | 0  | 0  | 0  | 0  | 0  | $D_1 = \overline{A} \overline{B} C$            |
| 0      | 1 | 0 | 0       | 0  | 1  | 0  | 0  | 0  | 0  | 0  | $D_2 = \overline{A} B \overline{C}$            |
| 0      | 1 | 1 | 0       | 0  | 0  | 1  | 0  | 0  | 0  | 0  | $D_3 = \overline{A} B C$                       |
| 1      | 0 | 0 | 0       | 0  | 0  | 0  | 1  | 0  | 0  | 0  | $D_4 = A \overline{B} \overline{C}$            |
| 1      | 0 | 1 | 0       | 0  | 0  | 0  | 0  | 1  | 0  | 0  | $D_5 = A \overline{B} C$                       |
| 1      | 1 | 0 | 0       | 0  | 0  | 0  | 0  | 0  | 1  | 0  | $D_6 = A B \overline{C}$                       |
| 1      | 1 | 1 | 0       | 0  | 0  | 0  | 0  | 0  | 0  | 1  | $D_7 = A B C$                                  |

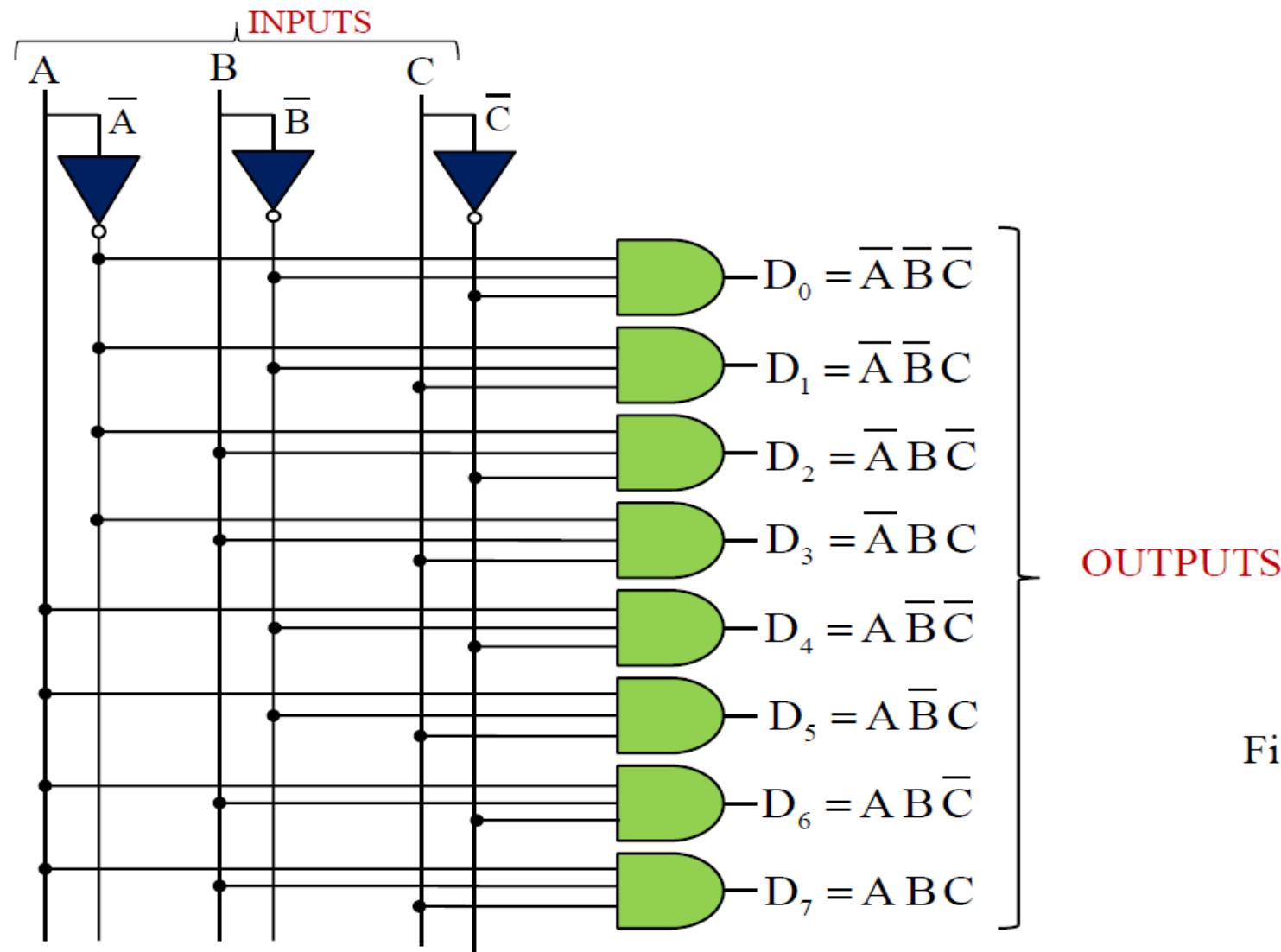
LOGIC DIAGRAM OF 3 X 8 DECODER:

Fig. 5

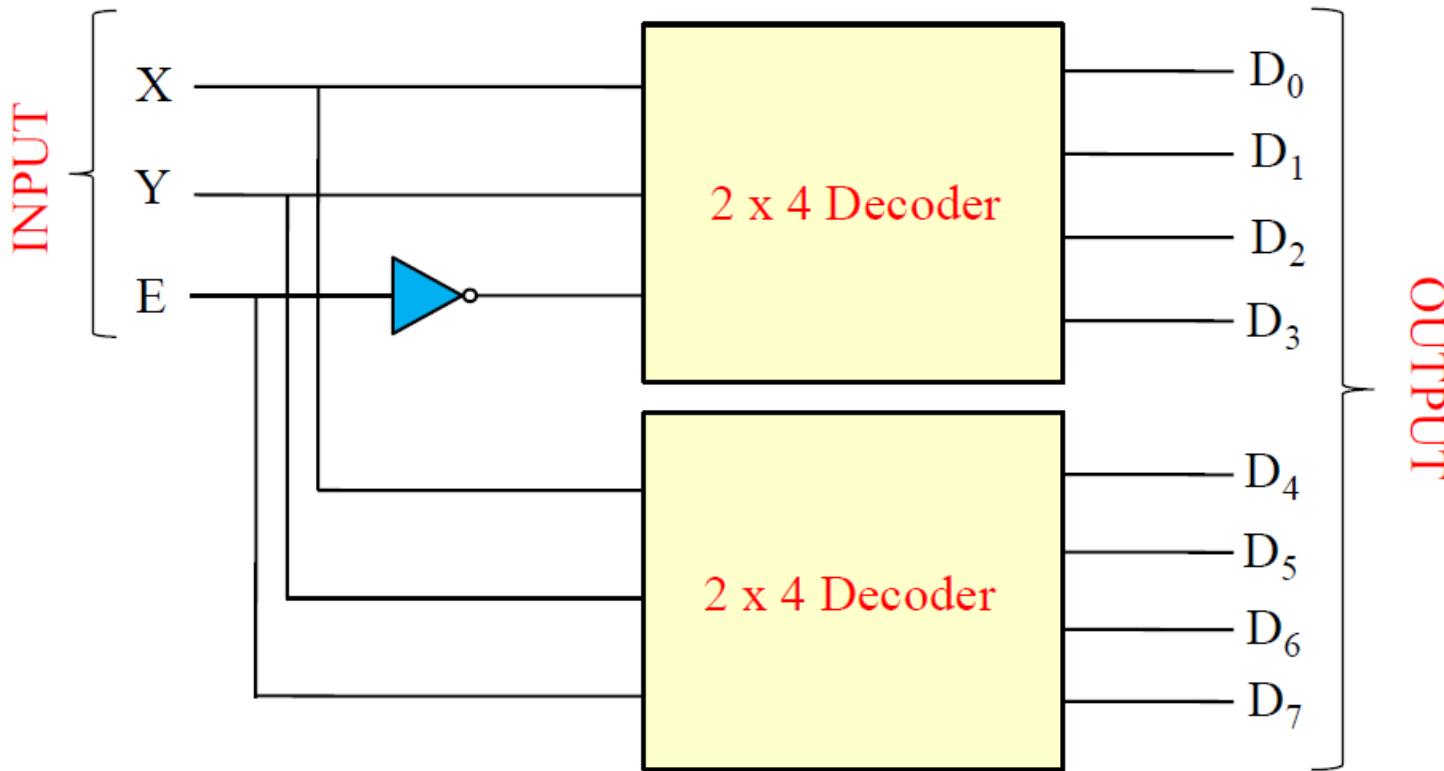
3 x 8 Decoder From 2 x 4 Decoder:

Fig. 6

**Example:** Implement the following multiple output function using a suitable Decoder.

$$f_1(A, B, C) = \sum m(0, 4, 7) + d(2, 3)$$

$$f_2(A, B, C) = \sum m(1, 5, 6)$$

$$f_3(A, B, C) = \sum m(0, 2, 4, 6)$$

**Solution:**  $f_1$  consists of don't care conditions. So we consider them to be logic 1.

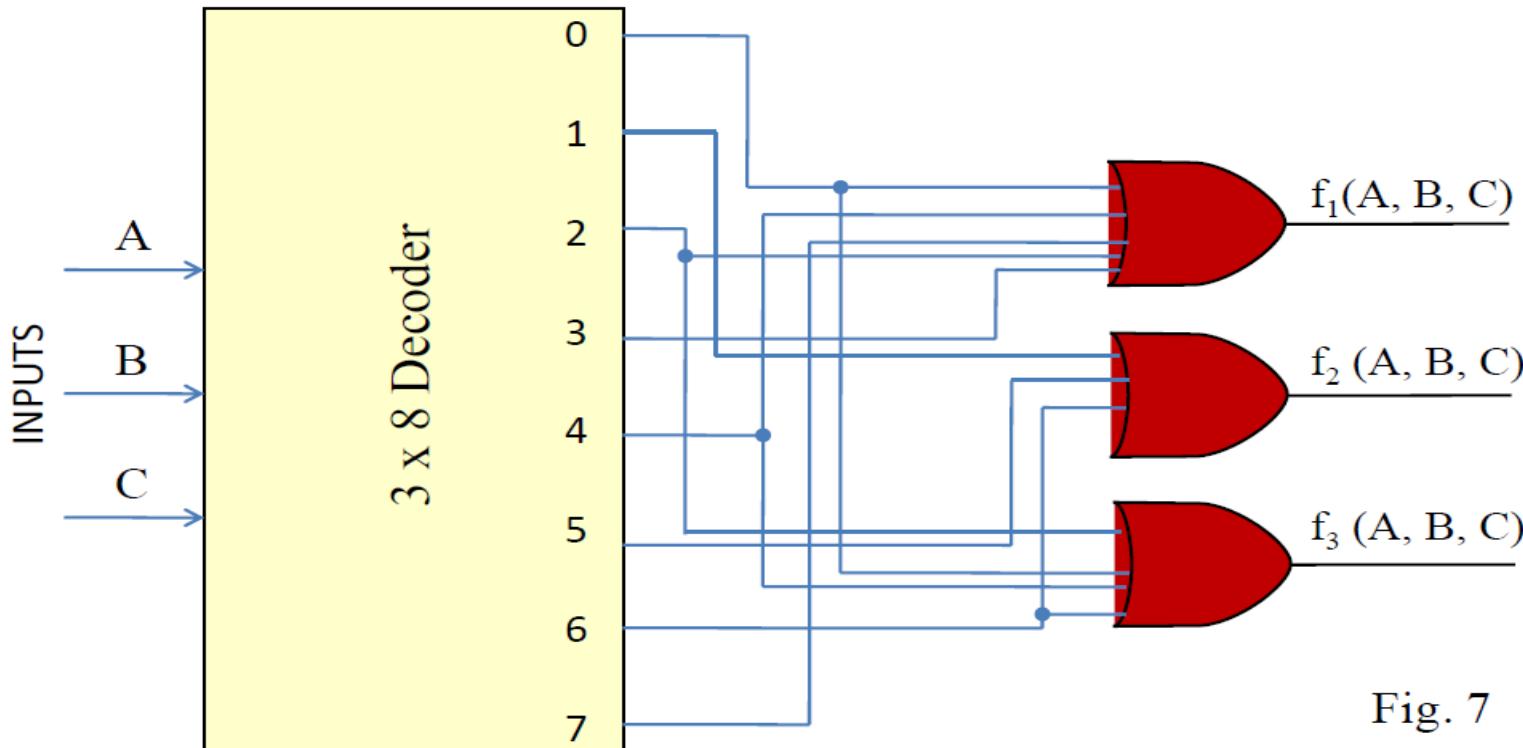


Fig. 7

**EXAMPLE:** A combinational circuit is defined by the following Boolean function. Design circuit with a Decoder and external gate.

$$F_1(x, y, z) = \overline{x} \overline{y} \overline{z} + x z \quad (\text{UPTU, 2004-05})$$

$$F_2(x, y, z) = x y \overline{z} + \overline{x} z$$

**SOLUTION:** STEP 1: Write the given function  $F_1$  in SOP form

$$F_1(x, y, z) = \overline{x} \overline{y} \overline{z} + (y + \overline{y}) x z$$

$$F_1(x, y, z) = \overline{x} \overline{y} \overline{z} + x y z + x \overline{y} \overline{z}$$

$$F_1(x, y, z) = \Sigma m(0, 5, 7)$$

$$F_2(x, y, z) = x y \overline{z} + \overline{x} z$$

$$F_2(x, y, z) = x y \overline{z} + (y + \overline{y}) \overline{x} z$$

$$F_2(x, y, z) = x y \overline{z} + \overline{x} y z + \overline{x} \overline{y} z$$

$$F_2(x, y, z) = \Sigma m(1, 3, 6)$$

## Boolean Function using Decoder:

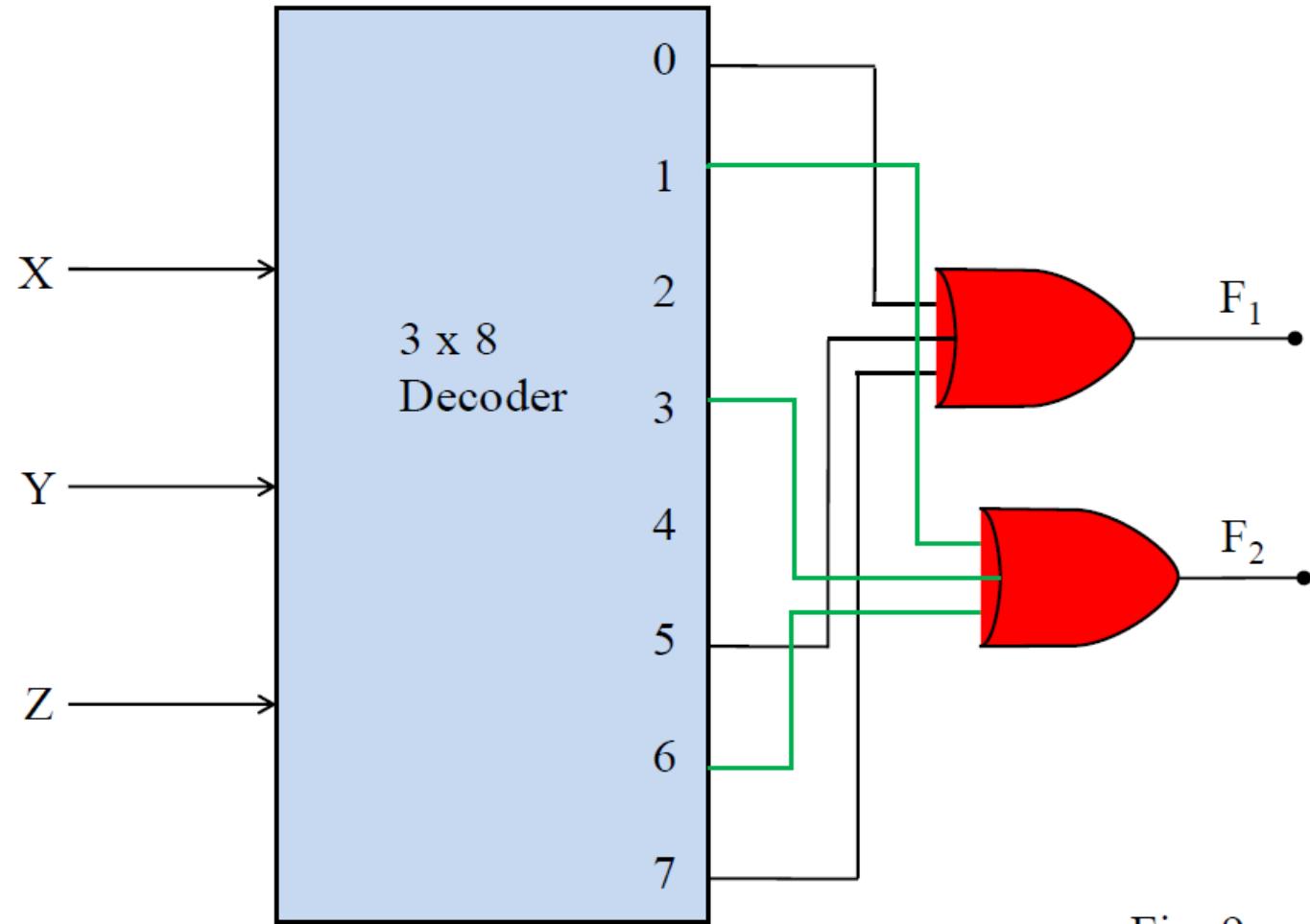


Fig. 9

## ENCODER

- An Encoder is a combinational logic circuit.
- It performs the inverse operation of Decoder.
- The opposite process of decoding is known as Encoding.
- An Encoder converts an active input signal into a coded output signal.
- Block diagram of Encoder is shown in Fig.10. It has ‘M’ inputs and ‘N’ outputs.
- An Encoder has ‘M’ input lines, only one of which is activated at a given time, and produces an N-bit output code, depending on which input is activated.

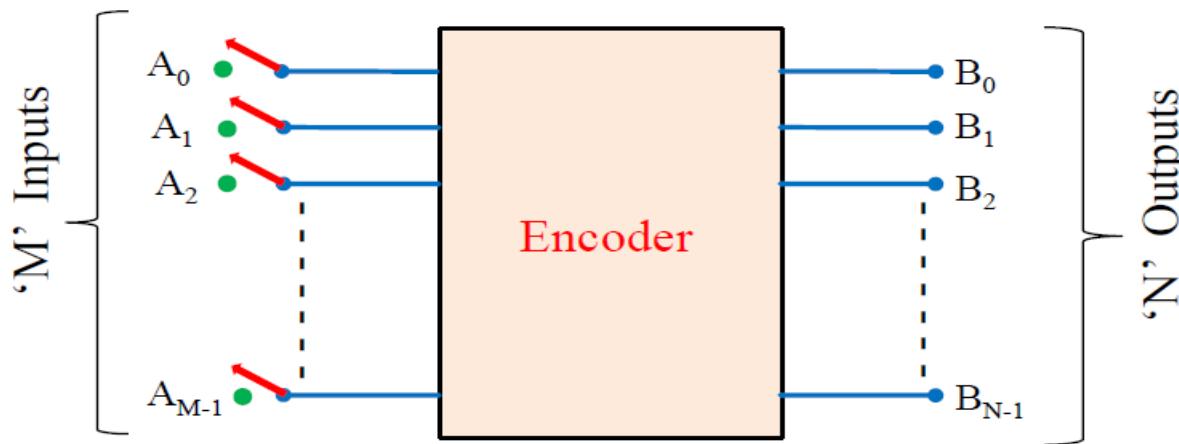


Fig. 10

- Encoders are used to translate the rotary or linear motion into a digital signal.
- The difference between Decoder and Encoder is that Decoder has Binary Code as an input while Encoder has Binary Code as an output.
- Encoder is an Electronics device that converts the analog signal to digital signal such as BCD Code.
- **Types of Encoders**
  - i. Priority Encoder
  - ii. Decimal to BCD Encoder
  - iii. Octal to Binary Encoder
  - iv. Hexadecimal to Binary Encoder

## ENCODER

M=4

M=2<sup>2</sup>

M=2<sup>N</sup>

'M' is the input and

'N' is the output

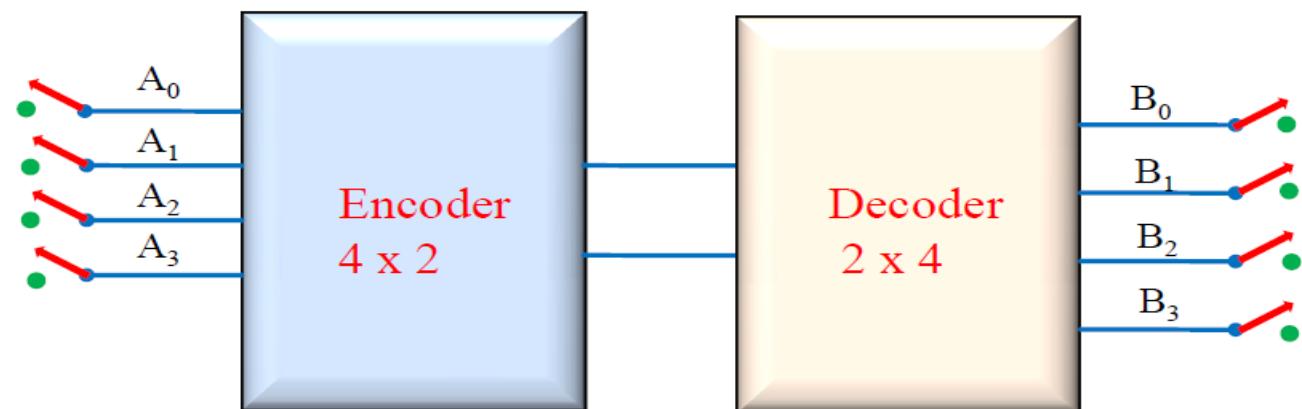


Fig. 11

## ENCODER

M=4

M=2<sup>2</sup>

M=2<sup>N</sup>

'M' is the input and

'N' is the output

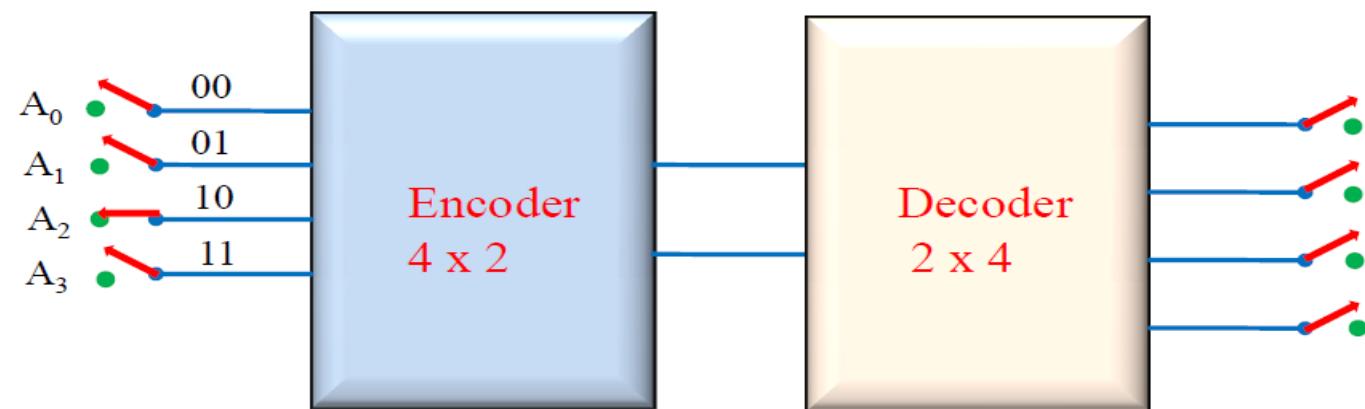


Fig. 12

## ENCODER

M=4

M=2<sup>2</sup>

M=2<sup>N</sup>

‘M’ is the input and  
‘N’ is the output

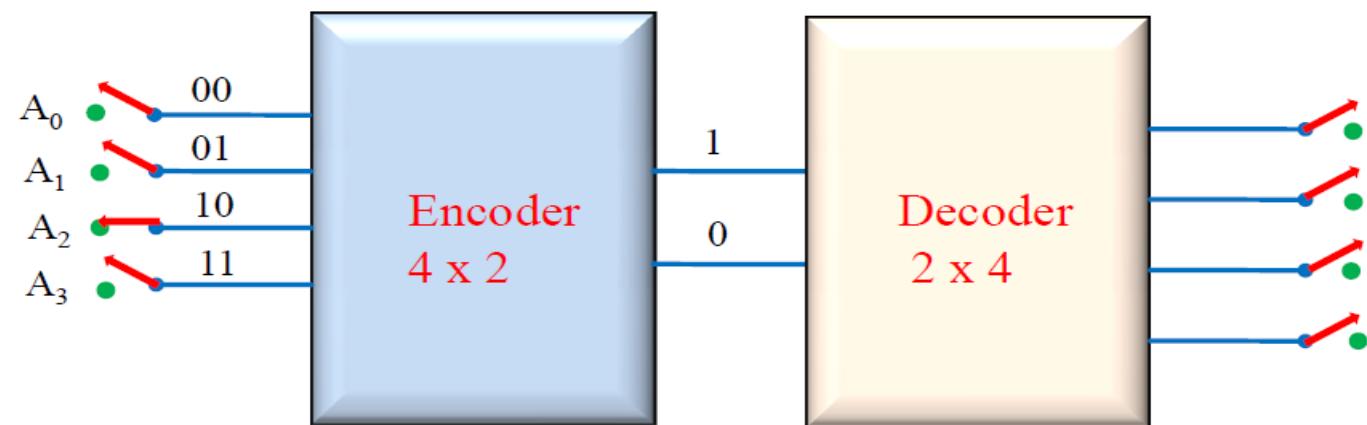


Fig. 13

## ENCODER

$M=4$

$M=2^2$

$M=2^N$

'M' is the input and  
'N' is the output

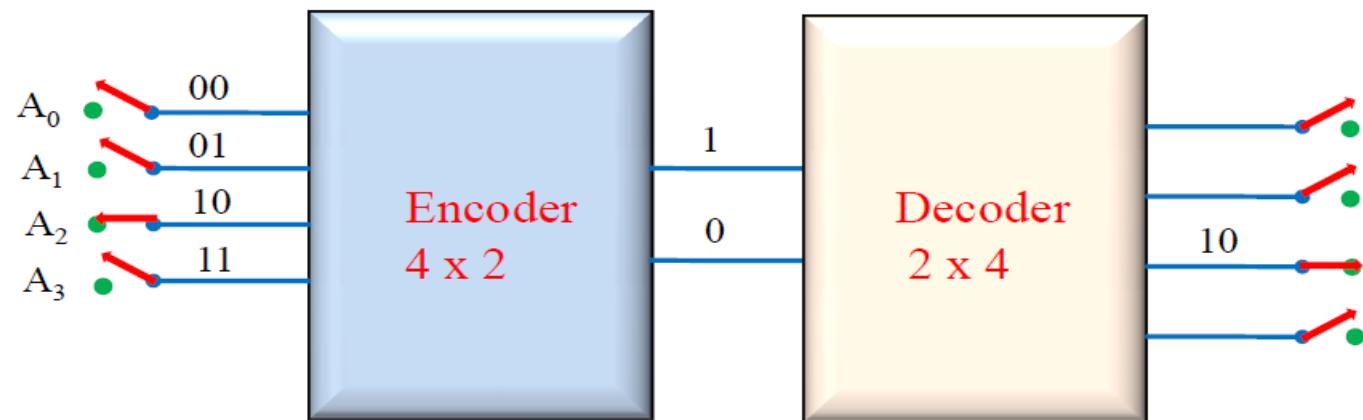
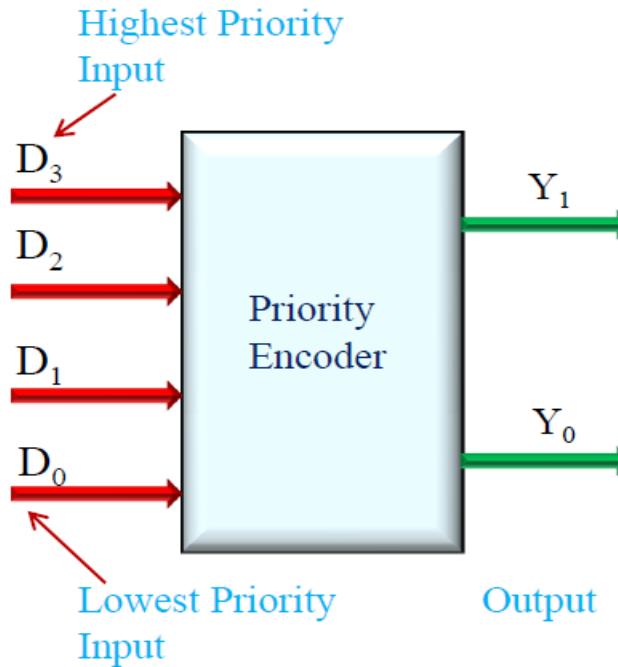


Fig. 14

## PRIORITY ENCODER:

- As the name indicates, the priority is given to inputs line.
- If two or more input lines are high at the same time i.e 1 at the same time, then the input line with high priority shall be considered.
- Block diagram and Truth table of Priority Encoder are shown in fig.15



*Block Diagram of Priority Encoder*

### TRUTH TABLE:

| INPUTS |       |       |       | OUTPUTS |       | V |
|--------|-------|-------|-------|---------|-------|---|
| $D_3$  | $D_2$ | $D_1$ | $D_0$ | $Y_1$   | $Y_0$ |   |
| 0      | 0     | 0     | 0     | x       | x     | 0 |
| 0      | 0     | 0     | 1     | 0       | 0     | 1 |
| 0      | 0     | 1     | x     | 0       | 1     | 1 |
| 0      | 1     | x     | x     | 1       | 0     | 1 |
| 1      | x     | x     | x     | 1       | 1     | 1 |

Fig.15

- There are four inputs  $D_0, D_1, D_2, D_3$  and two outputs  $Y_1$  and  $Y_2$ .
- $D_3$  has highest priority and  $D_0$  is at lowest priority.
- If  $D_3=1$  irrespective of other inputs then output  $Y_1 Y_0=11$ .
- $D_3$  is at highest priority so other inputs are considered as don't care.

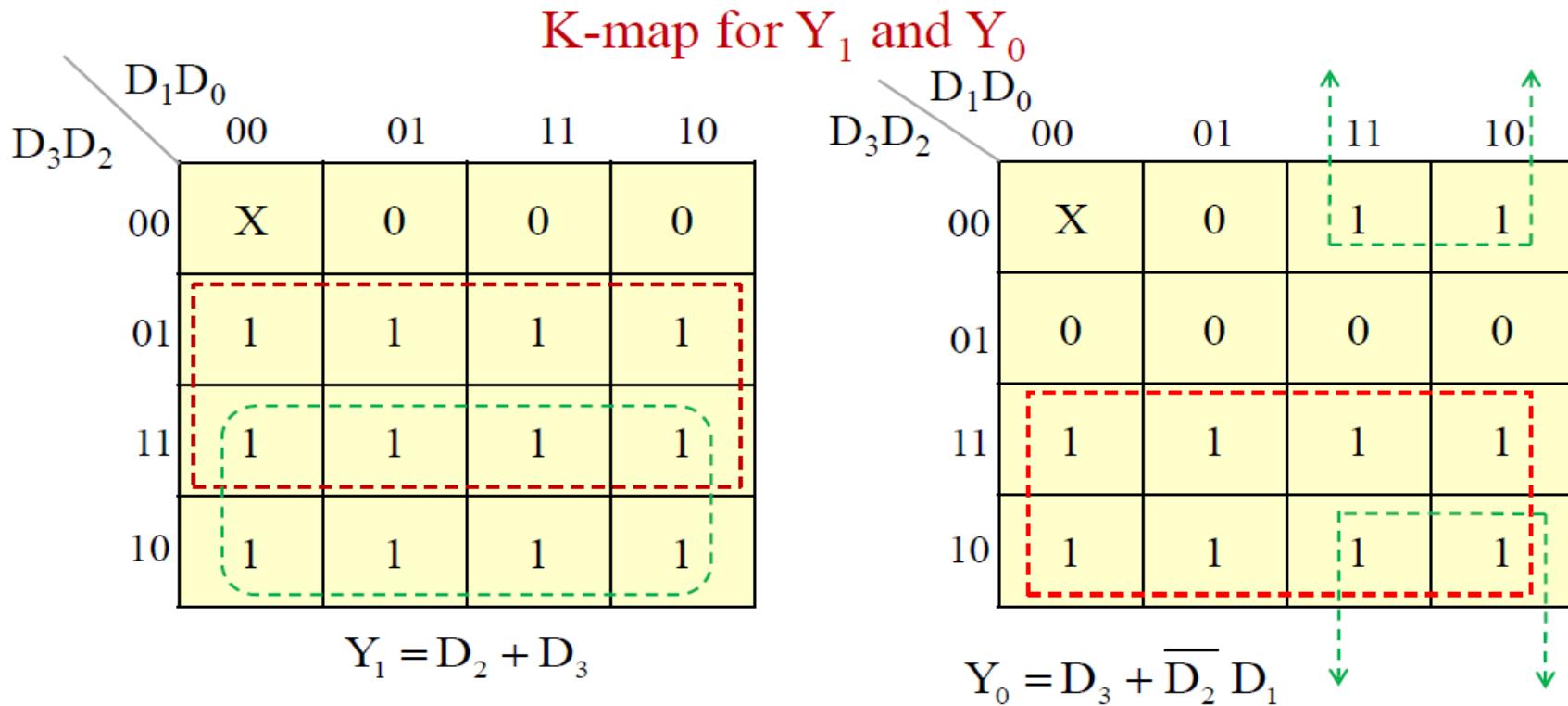


Fig. 16

## LOGIC DIAGRAM OF PRIORITY ENCODER:

$$Y_1 = D_2 + D_3$$

$$Y_0 = D_3 + \overline{D}_2 D_1$$

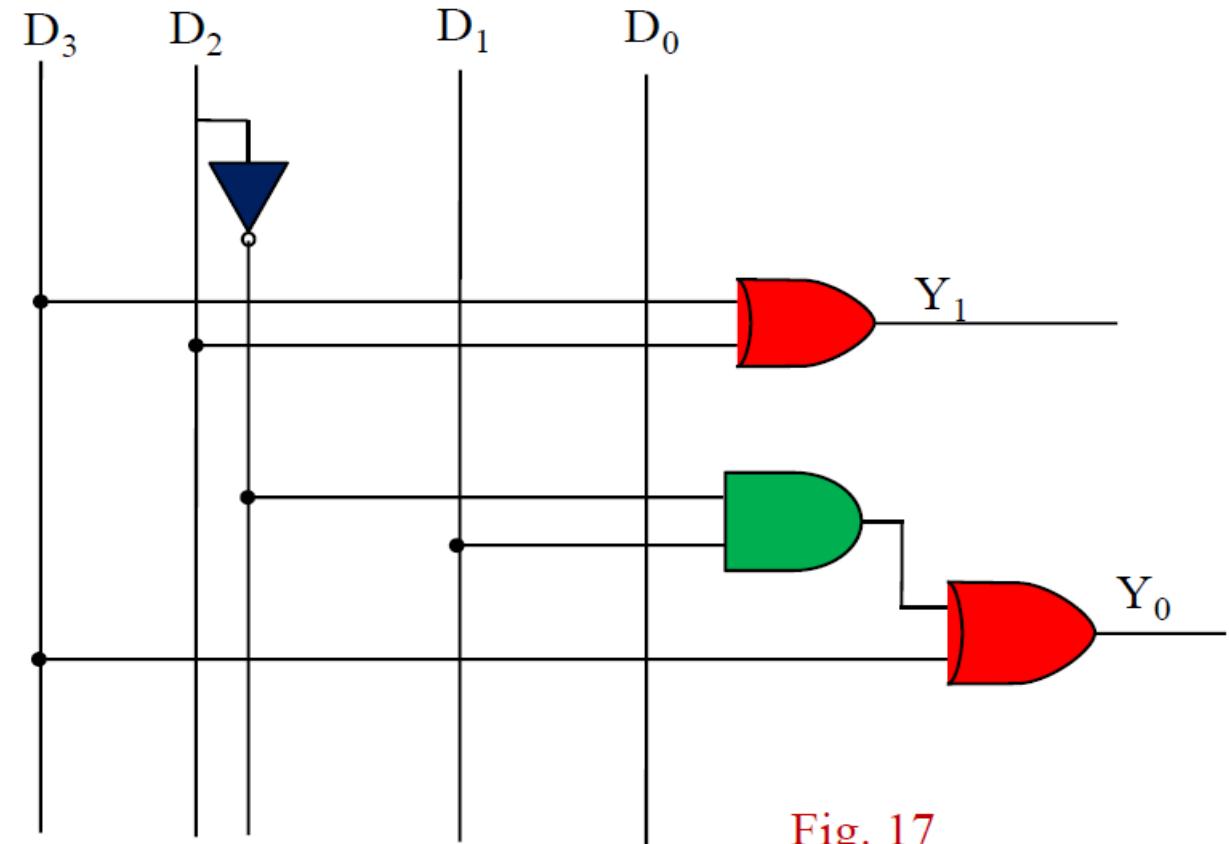


Fig. 17

## DECIMAL TO BCD ENCODER:

- It has ten inputs corresponding to ten decimal digits (from 0 to 9) and four outputs (A,B,C,D) representing the BCD.
- The block diagram is shown in fig.18 and Truth table in fig.19

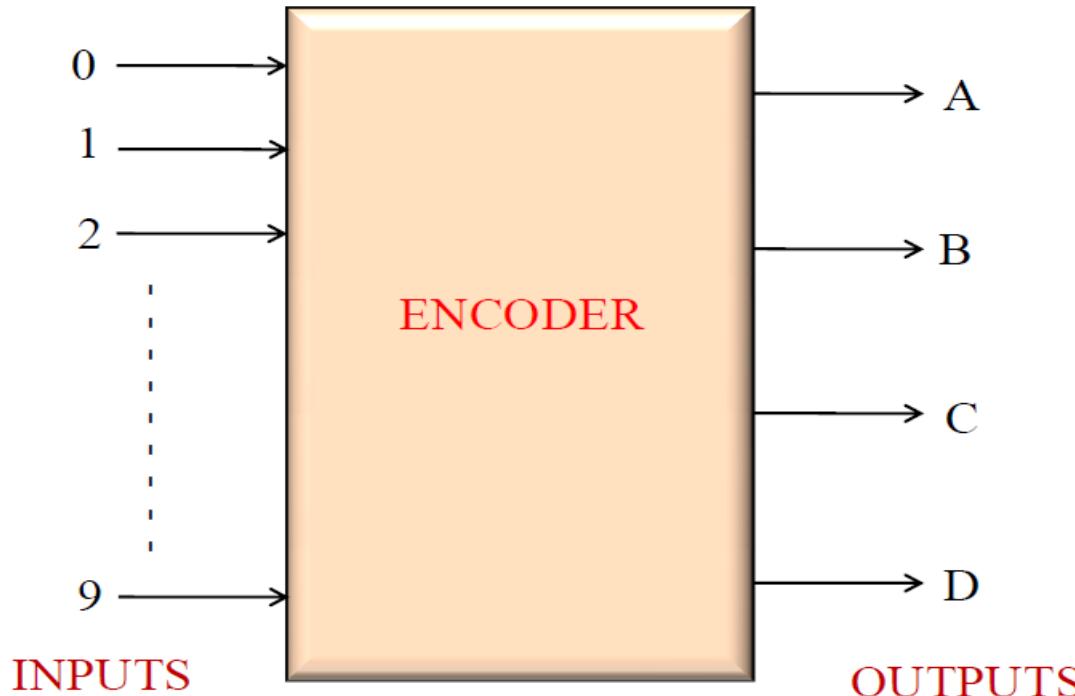


Fig. 18

Truth table:

| INPUTS |   |   |   |   |   |   |   |   |   | BCD OUTPUTS |   |   |   |
|--------|---|---|---|---|---|---|---|---|---|-------------|---|---|---|
| 0      | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A           | B | C | D |
| 1      | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0           | 0 | 0 | 0 |
| 0      | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0           | 0 | 0 | 1 |
| 0      | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0           | 0 | 1 | 0 |
| 0      | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0           | 0 | 1 | 1 |
| 0      | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0           | 1 | 0 | 0 |
| 0      | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0           | 1 | 0 | 1 |
| 0      | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0           | 0 | 1 | 0 |
| 0      | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0           | 1 | 1 | 1 |
| 0      | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0           | 1 | 0 | 0 |
| 0      | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1           | 1 | 0 | 1 |

Fig. 19

- From Truth Table it is clear that the output A is HIGH when input is 8 OR 9 is HIGH  
Therefore  $A=8+9$
- The output B is HIGH when 4 OR 5 OR 6 OR 7 is HIGH  
Therefore  $B=4+5+6+7$
- The output C is HIGH when 2 OR 3 OR 6 OR 7 is HIGH  
Therefore  $C=2+3+6+7$
- Similarly  $D=1+3+5+7+9$   
Logic Diagram is shown in fig.20

## DECIMAL TO BCD ENCODER

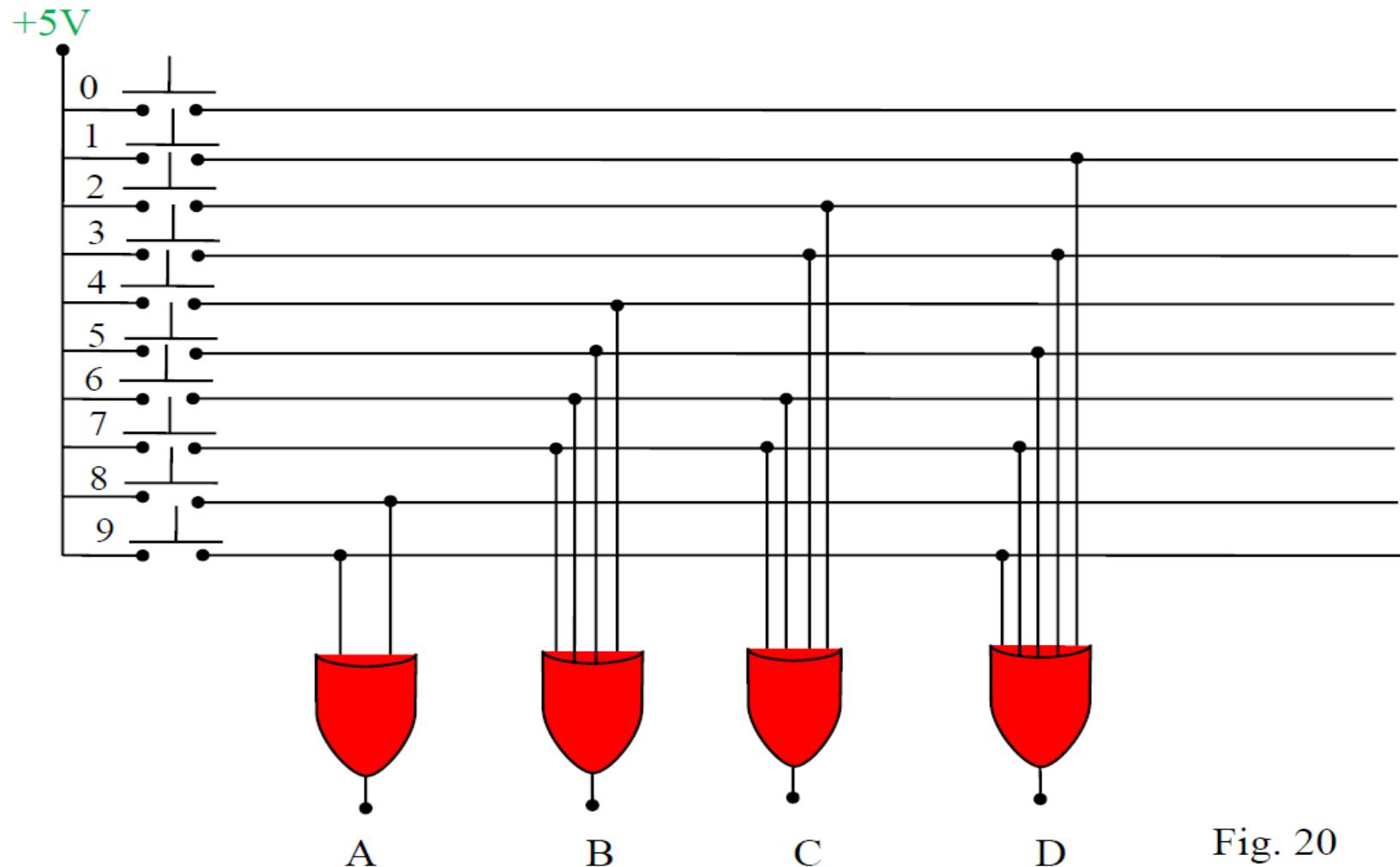


Fig. 20

## OCTAL TO BINARY ENCODER:

- Block Diagram of Octal to Binary Encoder is shown in Fig. 21
- It has eight inputs and three outputs.
- Only one input has one value at any given time.
- Each input corresponds to each octal digit and output generates corresponding Binary Code.

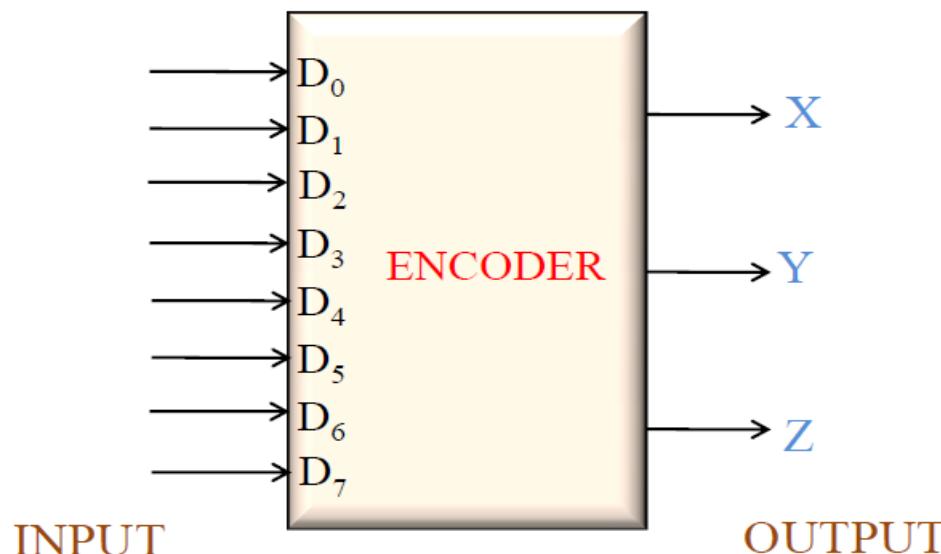


Fig. 21

TRUTH TABLE:

Fig. 22

| INPUT          |                |                |                |                |                |                |                | OUTPUT |   |   |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|--------|---|---|
| D <sub>0</sub> | D <sub>1</sub> | D <sub>2</sub> | D <sub>3</sub> | D <sub>4</sub> | D <sub>5</sub> | D <sub>6</sub> | D <sub>7</sub> | X      | Y | Z |
| 1              | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0      | 0 | 0 |
| 0              | 1              | 0              | 0              | 0              | 0              | 0              | 0              | 0      | 0 | 1 |
| 0              | 0              | 1              | 0              | 0              | 0              | 0              | 0              | 0      | 1 | 0 |
| 0              | 0              | 0              | 1              | 0              | 0              | 0              | 0              | 0      | 1 | 1 |
| 0              | 0              | 0              | 0              | 1              | 0              | 0              | 0              | 1      | 0 | 0 |
| 0              | 0              | 0              | 0              | 0              | 1              | 0              | 0              | 1      | 0 | 1 |
| 0              | 0              | 0              | 0              | 0              | 0              | 1              | 0              | 1      | 1 | 0 |
| 0              | 0              | 0              | 0              | 0              | 0              | 0              | 1              | 1      | 1 | 1 |

From Truth table:

$$X = D_4 + D_5 + D_6 + D_7$$

$$Y = D_2 + D_3 + D_6 + D_7$$

$$Z = D_1 + D_3 + D_5 + D_7$$

- It is assumed that only one input is HIGH at any given time. If two outputs are HIGH then undefined output will be produced. For example if  $D_3$  and  $D_6$  are HIGH, then output of Encoder will be 111. This output is neither equivalent code corresponding to  $D_3$  nor to  $D_6$ .
- To overcome this problem, priorities should be assigned to each input.
- From the truth table it is clear that the output X becomes 1 if any of the digit  $D_4$  or  $D_5$  or  $D_6$  or  $D_7$  is 1.
- $D_0$  is considered as don't care because it is not shown in expression.
- If inputs are zero then output will be zero. Similarly if  $D_0$  is one, the output will be zero.

$$X = D_4 + D_5 + D_6 + D_7$$

$$Y = D_2 + D_3 + D_6 + D_7$$

$$Z = D_1 + D_3 + D_5 + D_7$$

### LOGIC DIAGRAM:

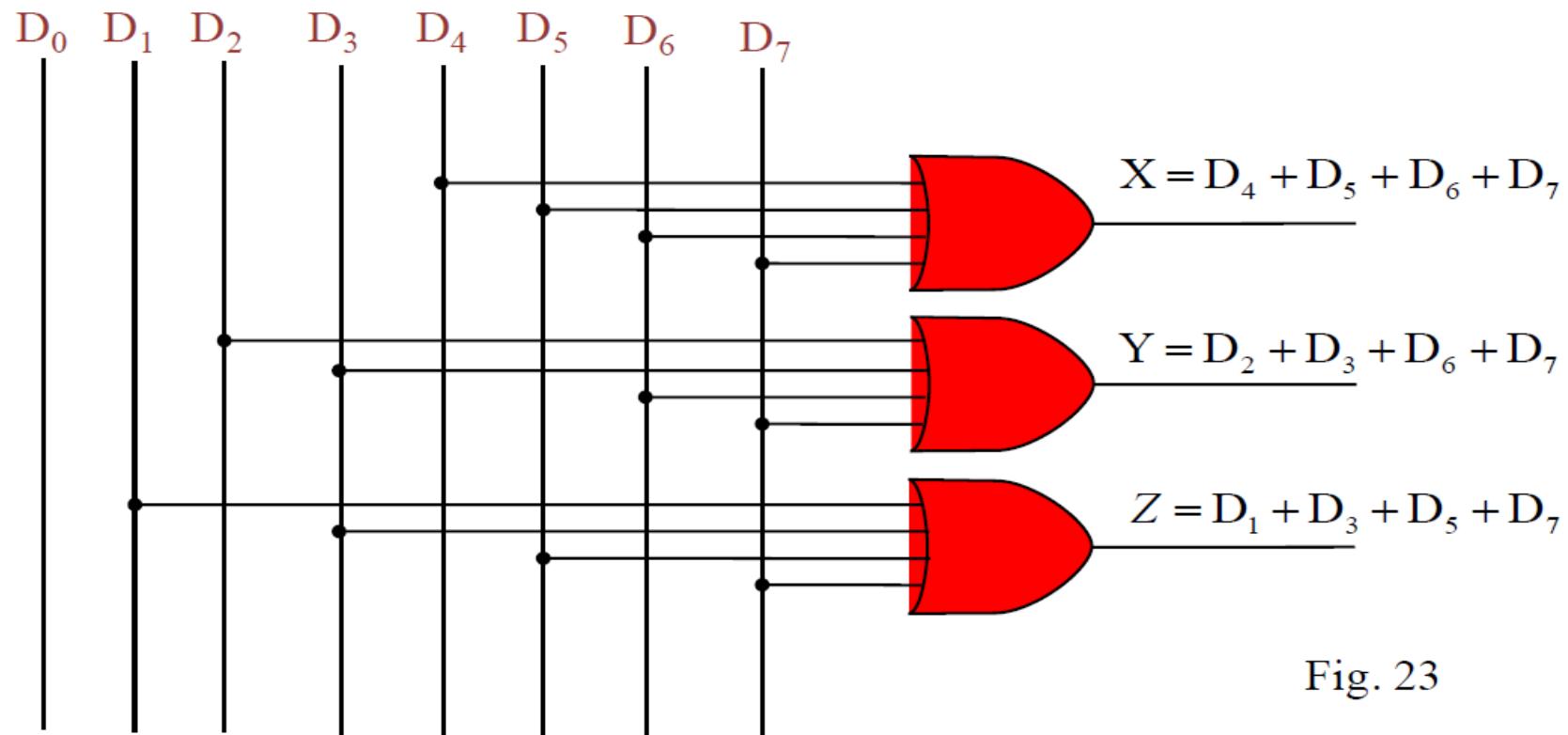


Fig. 23

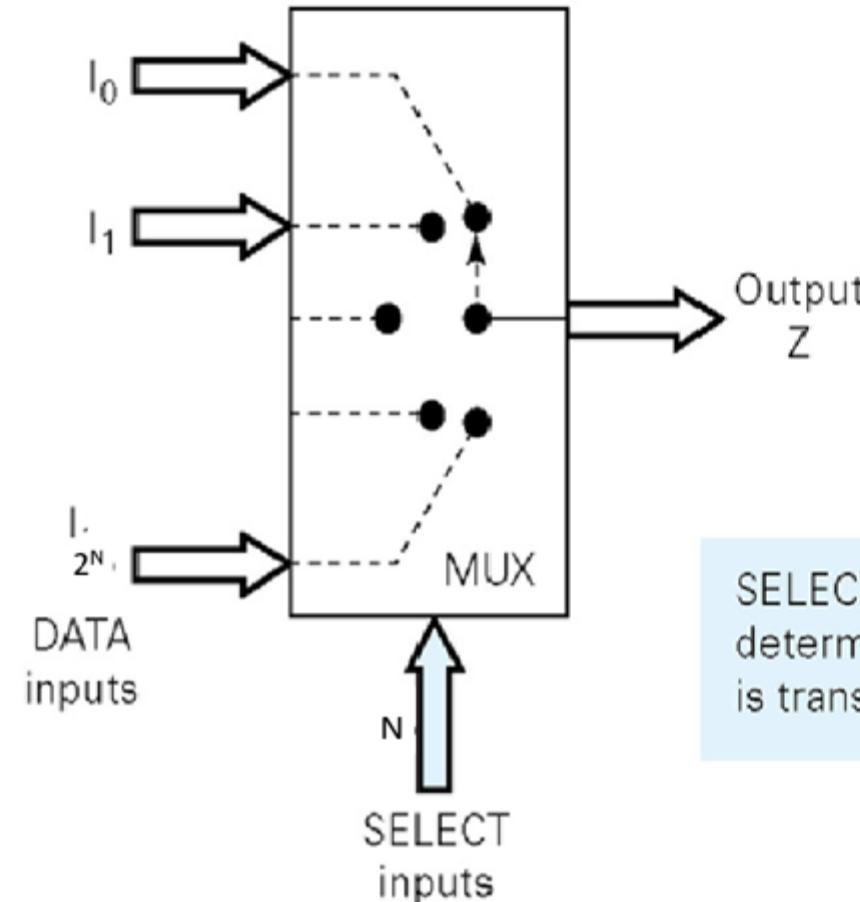
# Multiplexers

A multiplexers (MUX) is a device that allows digital information from several sources to be routed onto a single line for transmission over that line to a common destination.

The basic multiplexers has several data input lines and a single output line. It also has data-select inputs, which permit digital data on any one of the inputs to be switched to the output line.

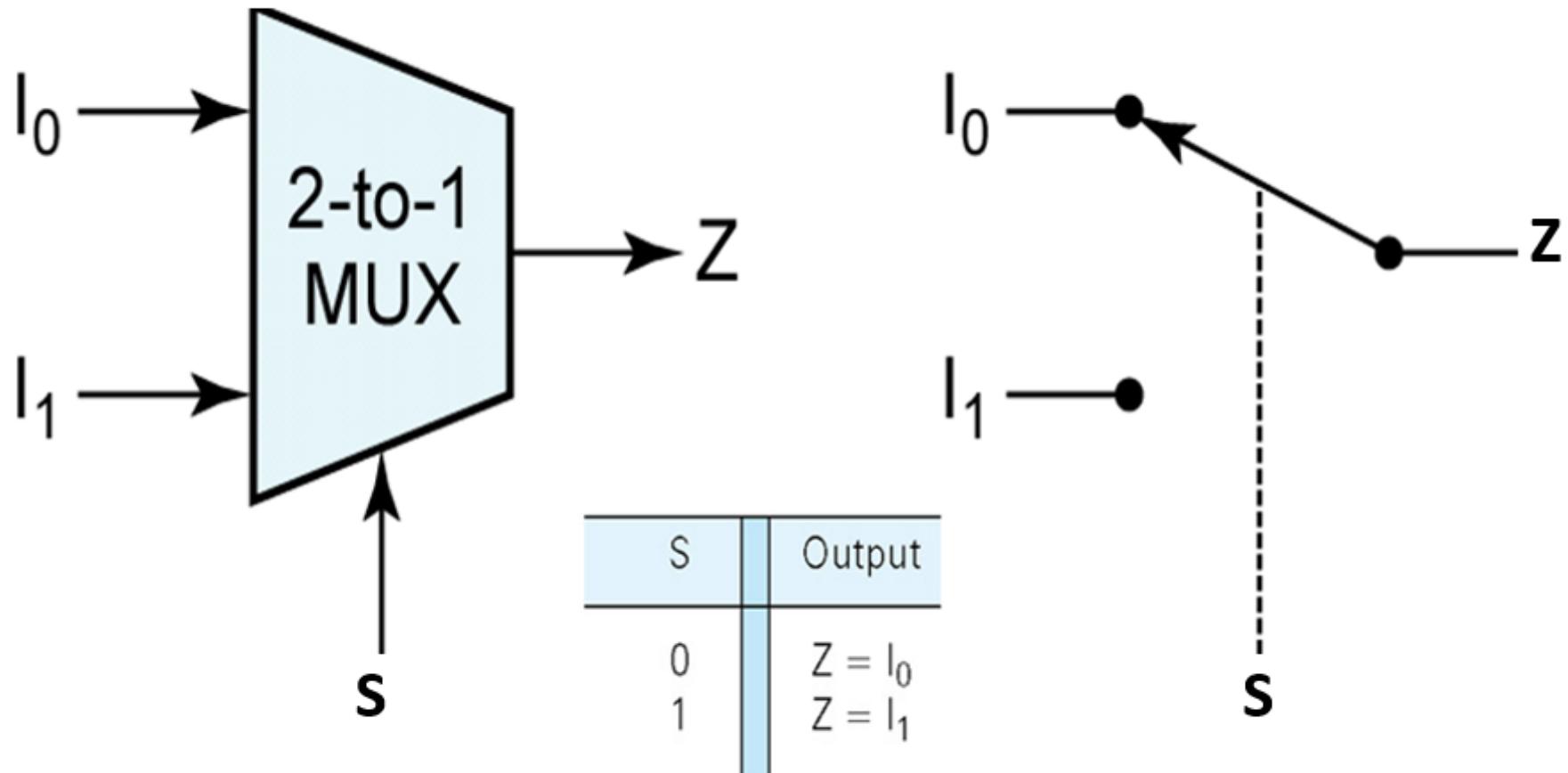
# Block Diagram

- A multiplexer has
  1.  $N$  control/select inputs
  2.  $2^N$  data inputs
  3. 1 output
- A multiplexer routes (or connects) the selected data input to the output.
- The value of the control inputs determines the data input that is selected.



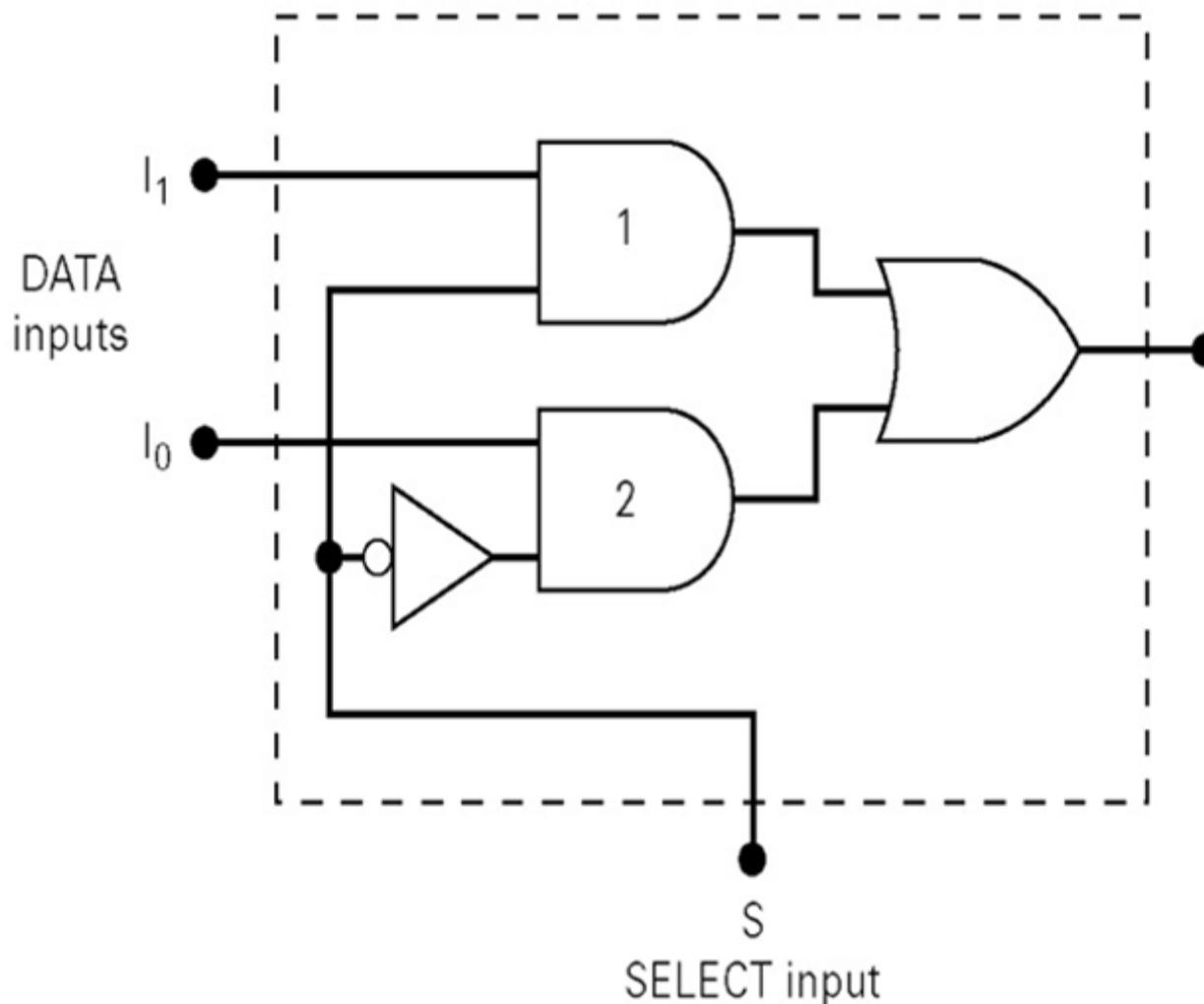
SELECT input code  
determines which input  
is transmitted to output Z.

## 2 to 1 mux



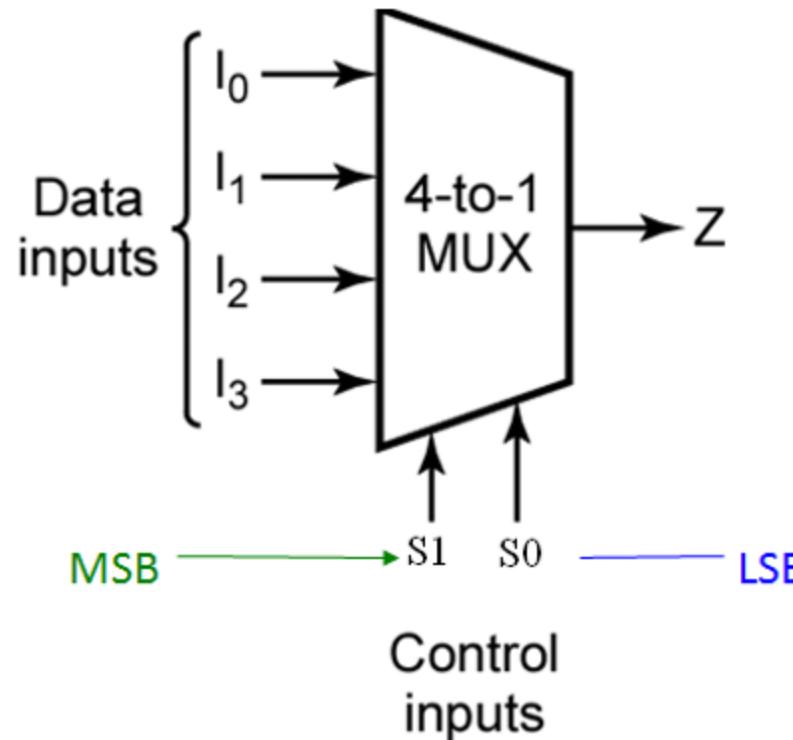
$$Z = S' \cdot I_0 + S \cdot I_1$$

# 2 to 1 multiplexer



$$Z = I_0 \cdot \bar{S} + I_1 \cdot S$$

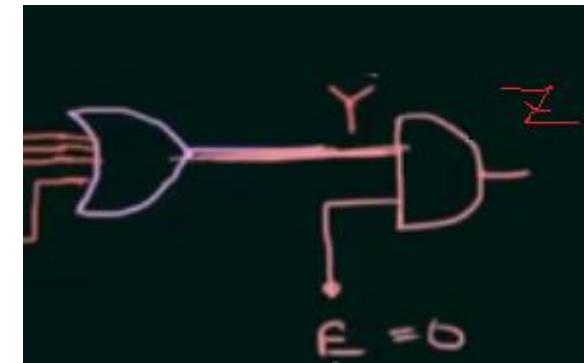
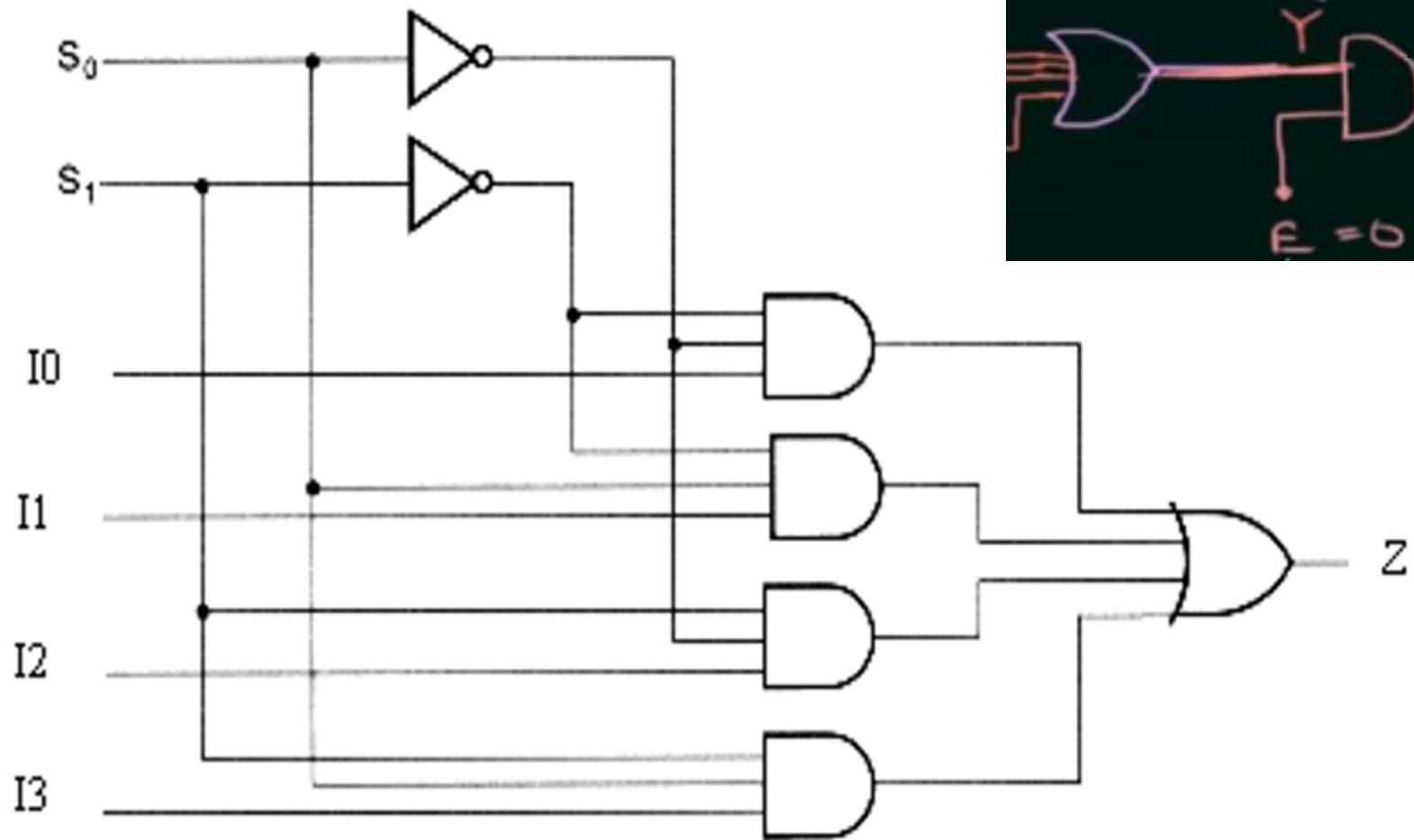
# 4 to 1Multiplexer



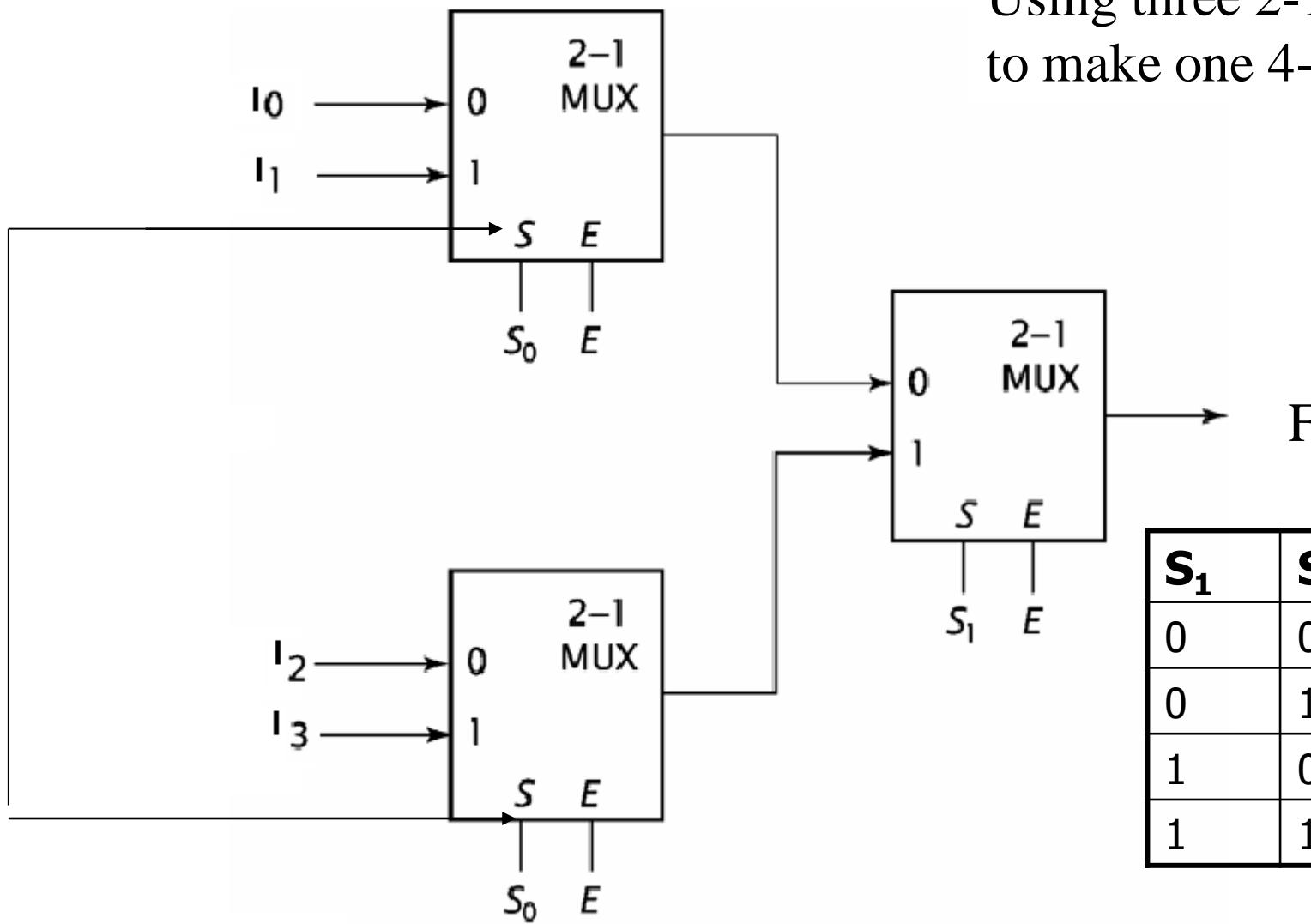
| S1 | S0 | Z     |
|----|----|-------|
| 0  | 0  | $I_0$ |
| 0  | 1  | $I_1$ |
| 1  | 0  | $I_2$ |
| 1  | 1  | $I_3$ |

$$Z = S_1' \cdot S_0' \cdot I_0 + S_1' \cdot S_0 \cdot I_1 + S_1 \cdot S_0' \cdot I_2 + S_1 \cdot S_0 \cdot I_3$$

# 4 to 1 multiplexer



# Cascading multiplexers

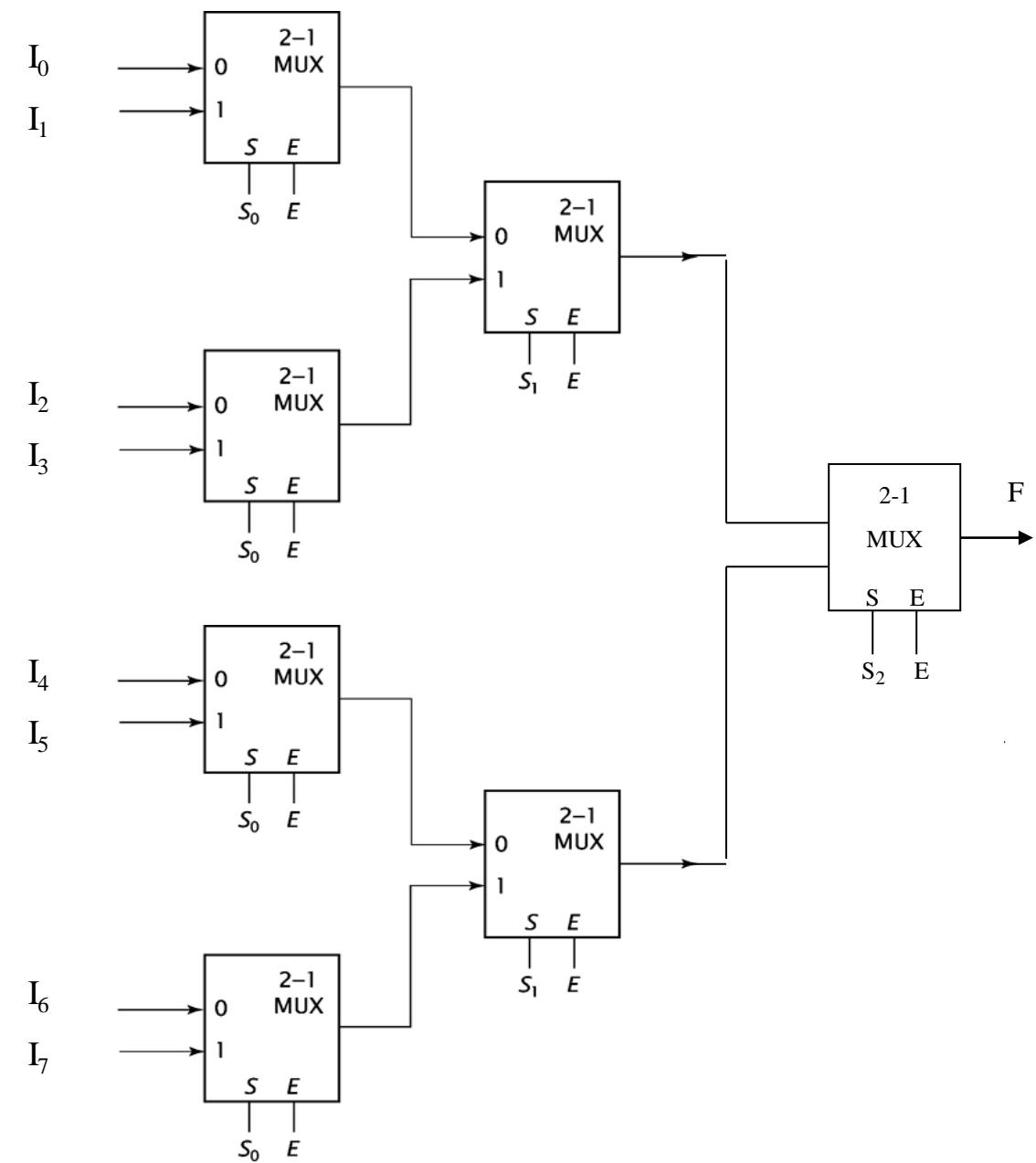


Using three 2-1 MUX  
to make one 4-1 MUX

| <b><math>S_1</math></b> | <b><math>S_0</math></b> | <b><math>F</math></b> |
|-------------------------|-------------------------|-----------------------|
| 0                       | 0                       | $I_0$                 |
| 0                       | 1                       | $I_1$                 |
| 1                       | 0                       | $I_2$                 |
| 1                       | 1                       | $I_3$                 |

8-to-1 multiplexer using  
2-to-1 multiplexers.

| <b>S<sub>2</sub></b> | <b>S<sub>1</sub></b> | <b>S<sub>0</sub></b> | <b>F</b>       |
|----------------------|----------------------|----------------------|----------------|
| 0                    | 0                    | 0                    | I <sub>0</sub> |
| 0                    | 0                    | 1                    | I <sub>1</sub> |
| 0                    | 1                    | 0                    | I <sub>2</sub> |
| 0                    | 1                    | 1                    | I <sub>3</sub> |
| 1                    | 0                    | 0                    | I <sub>4</sub> |
| 1                    | 0                    | 1                    | I <sub>5</sub> |
| 1                    | 1                    | 0                    | I <sub>6</sub> |
| 1                    | 1                    | 1                    | I <sub>7</sub> |



## LOGIC FUNCTION GENERATION USING MUX

Implement the Boolean function  $F=x'y'z + x'yz' + xyz' + xyz$  using a 4 to 1 MUX

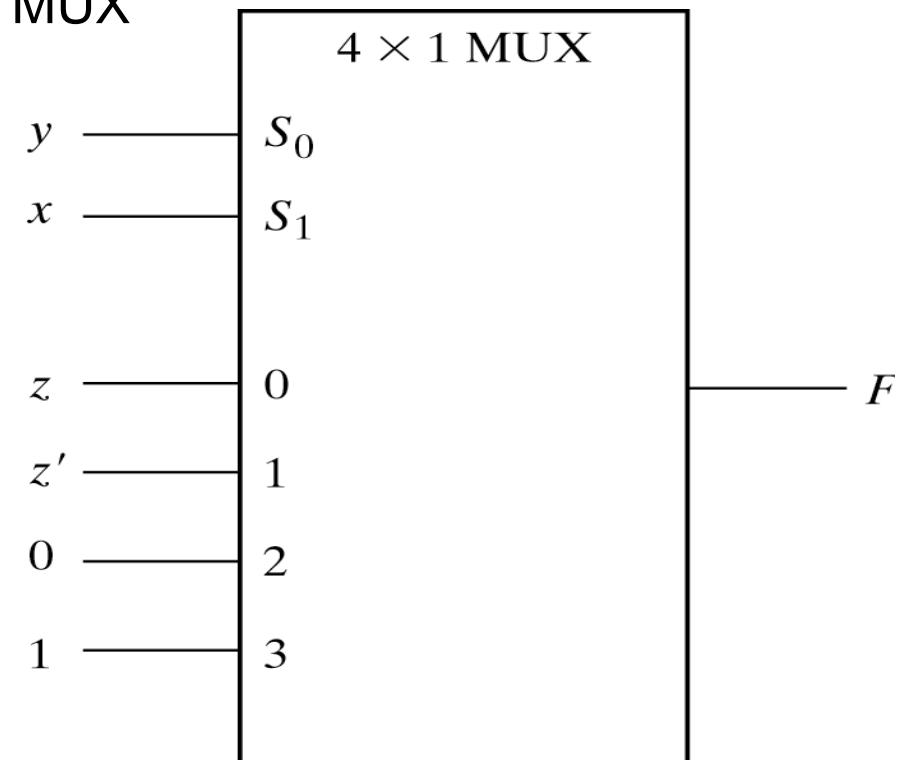
| $x$ | $y$ | $z$ | $F$ |
|-----|-----|-----|-----|
| 0   | 0   | 0   | 0   |
| 0   | 0   | 1   | 1   |
| 0   | 1   | 0   | 1   |
| 0   | 1   | 1   | 0   |
| 1   | 0   | 0   | 0   |
| 1   | 0   | 1   | 0   |
| 1   | 1   | 0   | 1   |
| 1   | 1   | 1   | 1   |

0

1

2

3



(a) Truth table

(b) Multiplexer implementation

## Applications of Multiplexer

- Multiplexers are used in various fields where multiple data need to be transmitted using a single line. Following are some of the applications of multiplexers
- Communication system
- Telephone network
- Computer memory
- Transmission from the computer system of a satellite

## Applications of De-Multiplexer

- De-multiplexer is used to connect a single source to multiple destinations. The main application area of de-multiplexer is communication system where multiplexers are used.
- Communication System
- ALU (Arithmetic Logic Unit)
- Serial to parallel converter



## Department of Electronics and Communication Engineering



# **MARRI LAXMAN REDDY INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

(AN AUTONOMOUS INSTITUTION)

(Approved by AICTE, New Delhi & Affiliated to JNTUH, Hyderabad)

Accredited by NBA and NAAC with 'A' Grade & Recognized Under Section 2(f) & 12(B) of the UGC act, 1956

# DIGITAL LOGIC DESIGN AND COMPUTER ORGANIZATION

- UNIT-II: Sequential circuits, latches, Flip-Flops

Presentation By

**Dr. Bapi Debnath**

Assoc. Prof.

Department of ECE



**MARRI LAXMAN REDDY  
INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

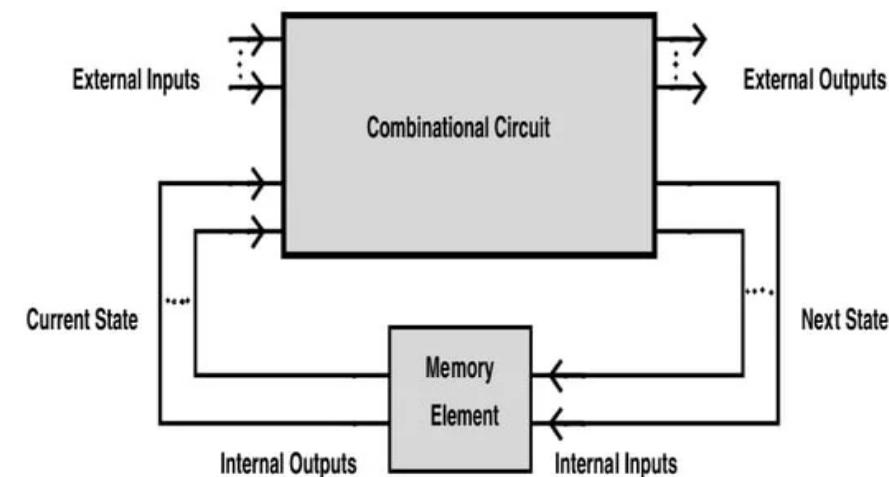
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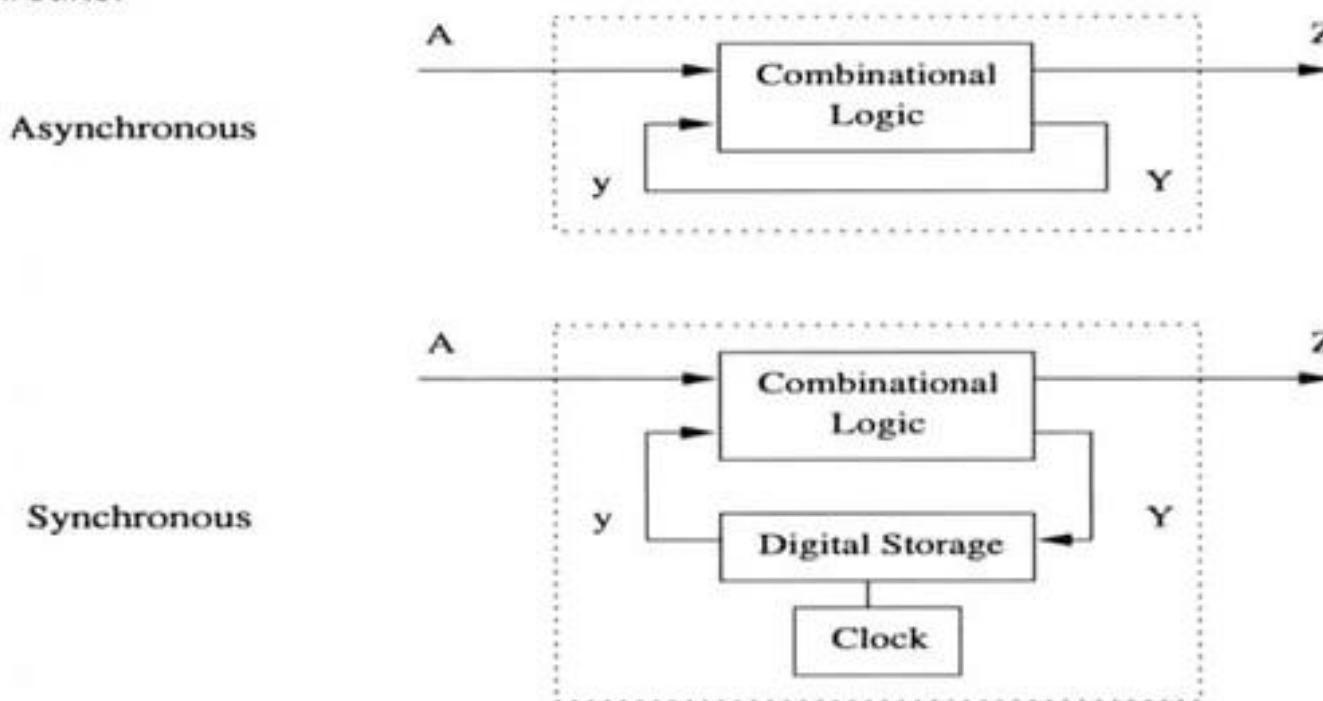
# Sequential Circuits

- Output depends on current as well as past inputs
- Depends on the history
- Have “memory” property
- Sequential circuit consists of
  - » Combinational circuit
  - » Feedback circuit
- Past input is encoded into a set of state variables
  - » Uses feedback (to feed the state variables)
    - Simple feedback
    - Uses flip flops

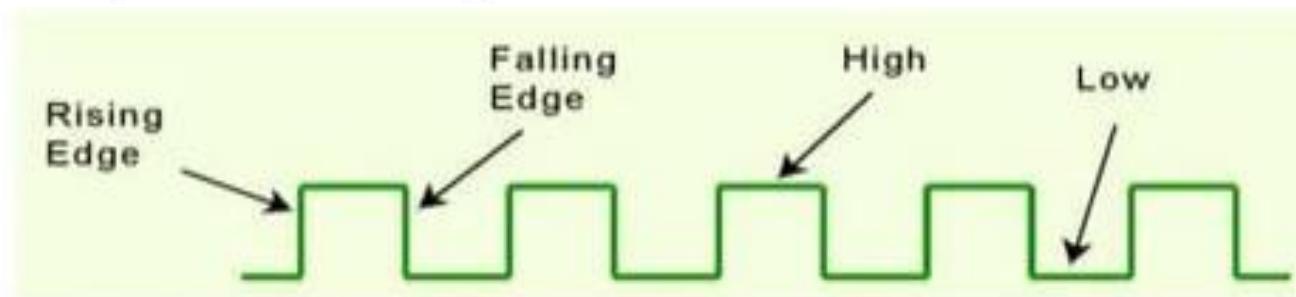


# Types of Sequential Circuits

In Asynchronous sequential circuits the output of the logic circuit can change state at any time, as soon as any input changes its state whereas in the case of synchronous systems a signal namely clock signal is used to determine/control the exact time at which any output can change its state. These are also called as clocked sequential circuits.



- The **memory element** is a device which can remember value indefinitely, or change value on command from its inputs.
- The memory element also has a clock input which provides timing for changing states.
- The feedback path is required for the circuit to have memory.
- State changes are controlled by clocks.  
A “clock” is a special circuit that sends electrical pulses through a circuit.
- Clocks produce electrical waveforms in generally some form of square wave
- Circuits can change state on the rising edge, falling edge, or when the clock pulse reaches its highest voltage.



# STORAGE ELEMENTS:

## LATCHES

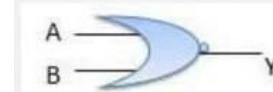
- are storage elements that operate with **signal levels** (rather than signal transitions)
- useful for storing binary information and for the design of asynchronous sequential circuits
- are the building blocks of flip-flops

## SR LATCHES

- The SR latch is a circuit with **two cross-coupled** NOR gates or two cross-coupled NAND gates,
- The two inputs are labeled **S** for set and **R** for reset.

### Quick Revision

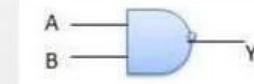
#### NOR gate & Truth Table:



| Inputs |   | Output           |
|--------|---|------------------|
| A      | B | $\overline{A+B}$ |
| 0      | 0 | 1                |
| 0      | 1 | 0                |
| 1      | 0 | 0                |
| 1      | 1 | 0                |

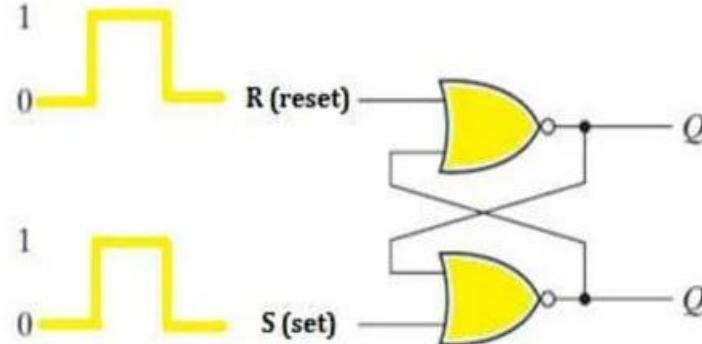
### Quick Revision

#### NAND gate & Truth Table:

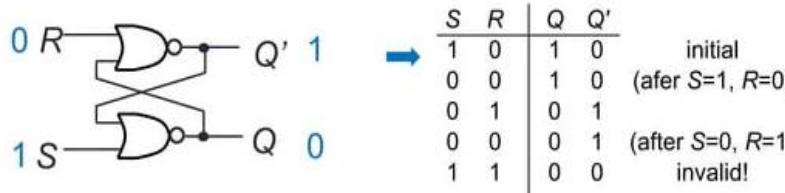


| Inputs |   | Output          |
|--------|---|-----------------|
| A      | B | $\overline{AB}$ |
| 0      | 0 | 1               |
| 0      | 1 | 1               |
| 1      | 0 | 1               |
| 1      | 1 | 0               |

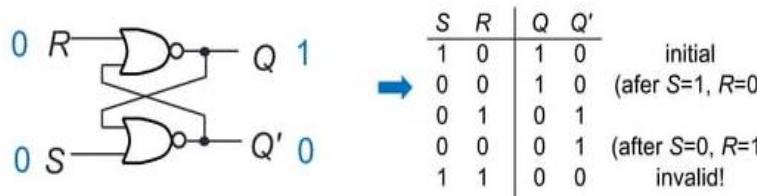
1


**Logic Diagram**

- 2
- when  $Q = 1$  and  $Q' = 0$ , the latch is said to be in the set state
  - $S=HIGH$  (and  $R=LOW$ )

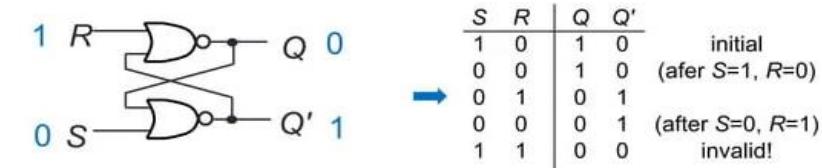


- 3
- The inputs must go back to their normal conditions ( $S=0, R=0$ ) before any changes occur



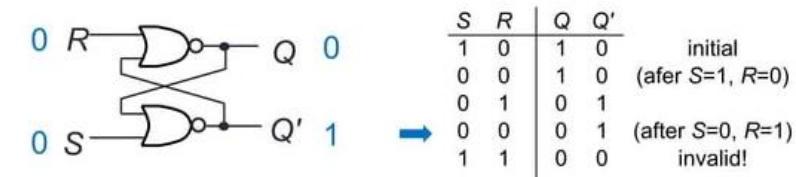
4

- When  $Q = 0$  and  $Q' = 1$ , it is in the reset state
- $S=LOW$  (and  $R=HIGH$ )



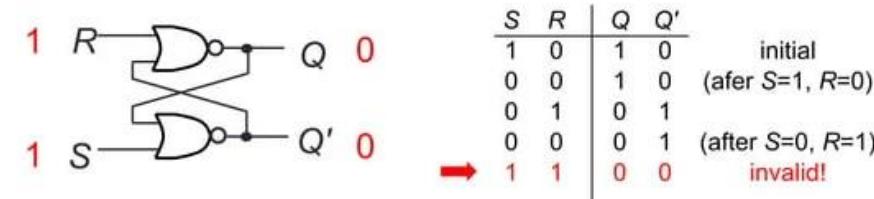
5

- The inputs must go back to their normal conditions ( $S=0, R=0$ ) before any changes occur



6

- If both inputs HIGH  $\Rightarrow$  Q and  $Q'$  both LOW (invalid)!



## SR Latch with NOR gate (Conclusion)

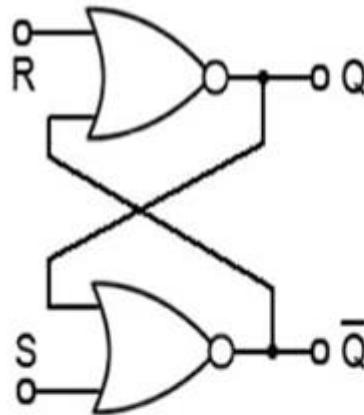


Table:

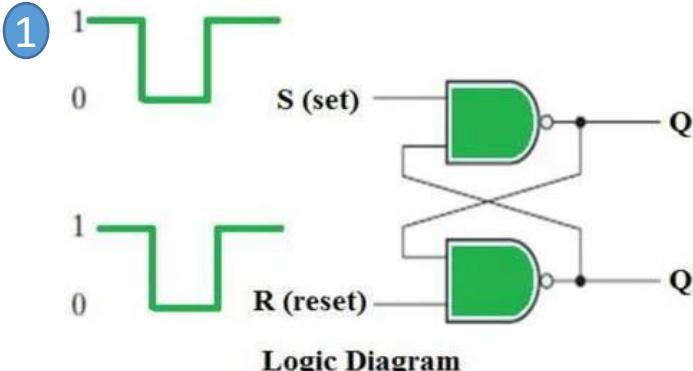
|    | R | S | Q | $\bar{Q}$ | Comments  |
|----|---|---|---|-----------|---|
| 1. | 0 | 1 | 1 | 0         | Q is set to 1 by 1 on S   |
| 2. | 0 | 0 | 1 | 0         | No Change after set   |
| 3. | 1 | 0 | 0 | 1         | Q is reset to 0 by 1 on R   |
| 4. | 0 | 0 | 0 | 1         | No Change after reset   |
| 5. | 1 | 1 | 0 | 0         | Not allowed (both outputs at 0)                                   |
| 6. | 0 | 0 | ? | ?         | If both inputs change from 11 to 00 outputs will be INDETERMINATE |

• **When S = 0 and R = 0:** If we assume Q = 1 and Q' = 0 as initial condition, then output Q after input is applied would be  $Q = (R + Q')' = 1$  and  $Q' = (S + Q)' = 0$ . Assuming Q = 0 and Q' = 1 as initial condition, then output Q after the input applied would be  $Q = (R + Q')' = 0$  and  $Q' = (S + Q)' = 1$ . So it is clear that when both S and R inputs are LOW, the output is retained as before the application of inputs. (i.e. there is no state change).

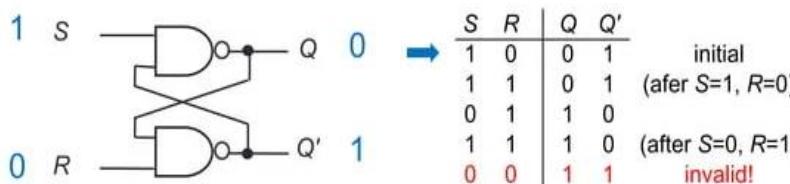
• **When S = 1 and R = 0:** If we assume Q = 1 and Q' = 0 as initial condition, then output Q after input is applied would be  $Q = (R + Q')' = 1$  and  $Q' = (S + Q)' = 0$ . Assuming Q = 0 and Q' = 1 as initial condition, then output Q after the input applied would be  $Q = (R + Q')' = 1$  and  $Q' = (S + Q)' = 0$ . So in simple words when S is HIGH and R is LOW, output Q is HIGH.

• **When S = 0 and R = 1:** If we assume Q = 1 and Q' = 0 as initial condition, then output Q after input is applied would be  $Q = (R + Q')' = 0$  and  $Q' = (S + Q)' = 1$ . Assuming Q = 0 and Q' = 1 as initial condition, then output Q after the input applied would be  $Q = (R + Q')' = 0$  and  $Q' = (S + Q)' = 1$ . So in simple words when S is LOW and R is HIGH, output Q is LOW.

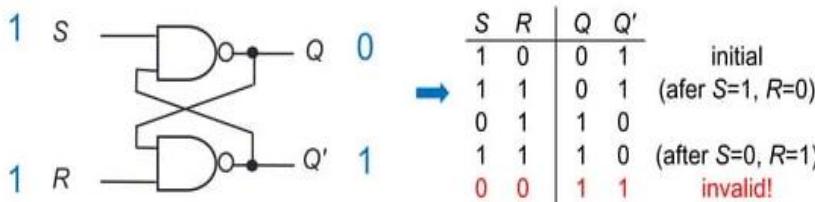
• **When S = 1 and R = 1 :** No matter what state Q and Q' are in, application of 1 at input of NOR gate always results in 0 at output of NOR gate, which results in both Q and Q' set to LOW (i.e.  $Q = Q'$ ). LOW in both the outputs basically is wrong, so this case is invalid



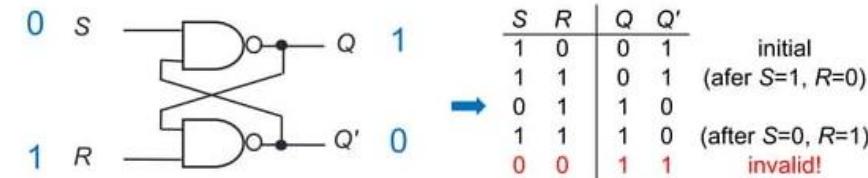
- 2
- when  $Q = 0$  and  $Q' = 1$ , the latch is said to be in the set state
  - $R=LOW$  (and  $S=HIGH$ )



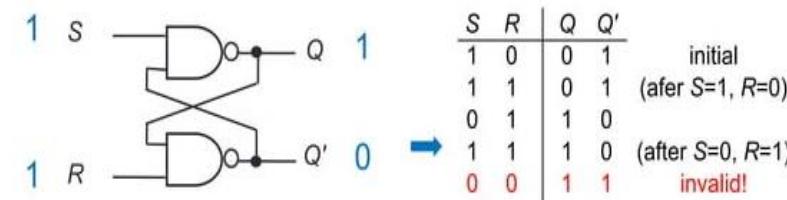
- 3
- The inputs must go back to their normal conditions ( $S=1, R=1$ ) before any changes occur



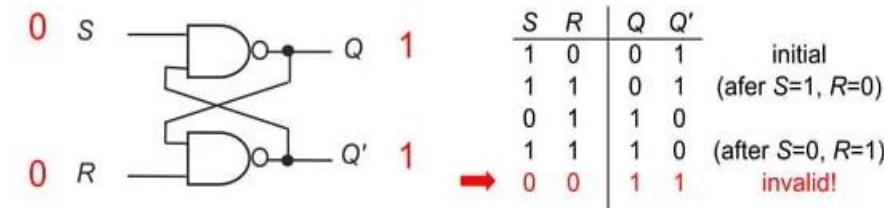
- 4
- When  $Q = 0$  and  $Q' = 1$ , it is in the reset state
  - $R=LOW$  (and  $S=HIGH$ )



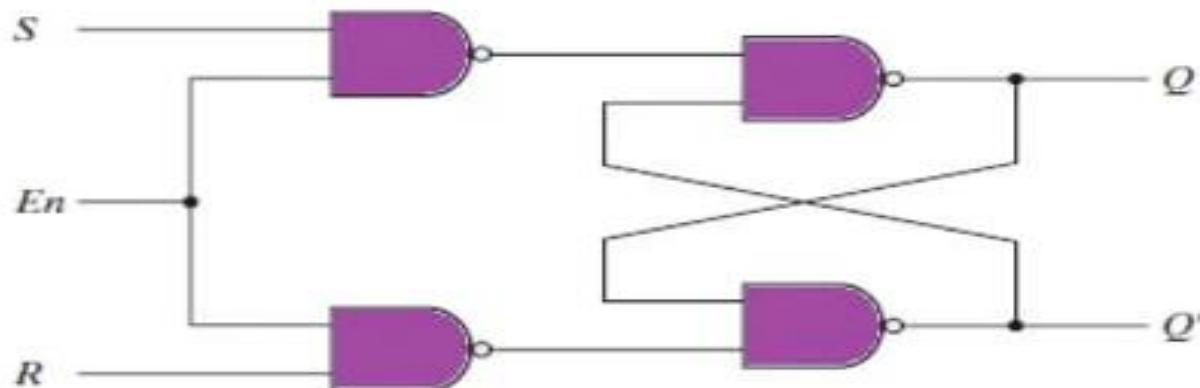
- 5
- The inputs must go back to their normal conditions ( $S=1, R=1$ ) before any changes occur



- 6
- If both inputs LOW  $\Rightarrow$  Q and Q' both LOW (invalid)!

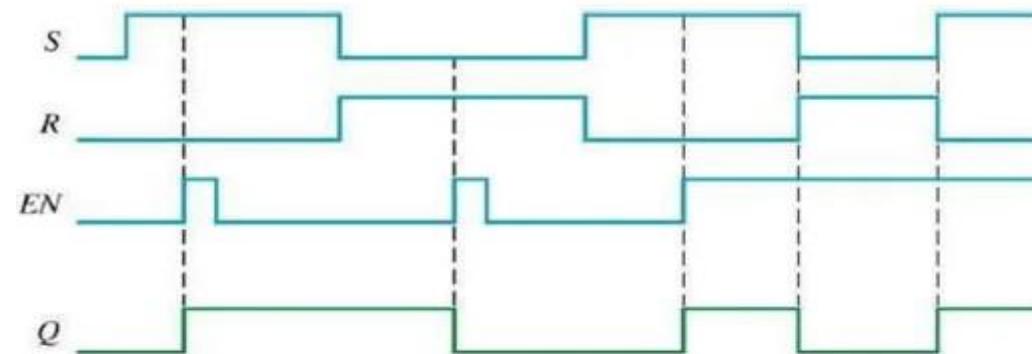


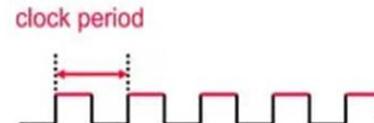
# SR LATCHES WITH CONTROL INPUT



**Logic Diagram**

| $En$ | $S$ | $R$ | Next state of $Q$     |
|------|-----|-----|-----------------------|
| 0    | X   | X   | No change             |
| 1    | 0   | 0   | No change             |
| 1    | 0   | 1   | $Q = 0$ ; reset state |
| 1    | 1   | 0   | $Q = 1$ ; set state   |
| 1    | 1   | 1   | Indeterminate         |





### Clock:-

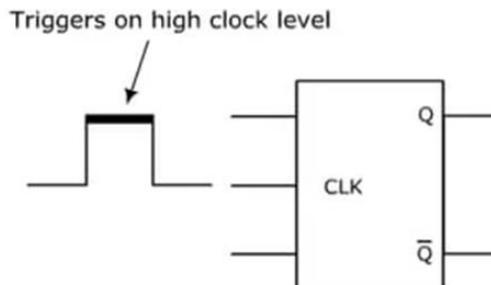
- A **clock** is a special device that whose output continuously alternates between 0 and 1.
- The time it takes the clock to change from 1 to 0 and back to 1 is called the **clock period**, or **clock cycle time**.
- The **clock frequency** is the inverse of the clock period. The unit of measurement for frequency is the **hertz**.
- Clocks are often used to synchronize circuits.

### Triggering

- Sequential circuits are dependant on clock pulses applies to their inputs.
- The result of flip-flop responding to a clock input is called **clock pulse triggering**, of which there are four types. Each type responds to a clock pulse in one of four ways :-
  1. High level triggering
  2. Low level triggering
  3. Positive edge triggering
  4. Negative edge triggering

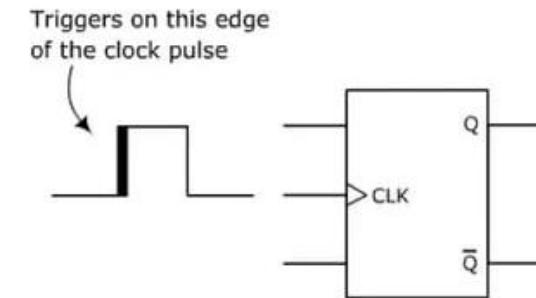
## High Level Triggering

One type of flip-flop responds to a clock signal during the time at which it is in the logic High state. This type is identified by a straight lead at the clock input, as shown below.



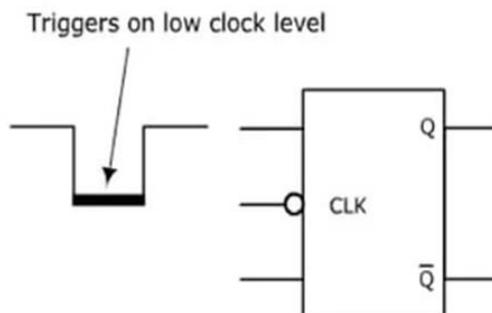
## Positive Edge Triggering

A third type of flip-flop responds to a clock signal during the low-to-high transition of a clock pulse. This type is identified by a clock input lead with a triangle, as shown below.



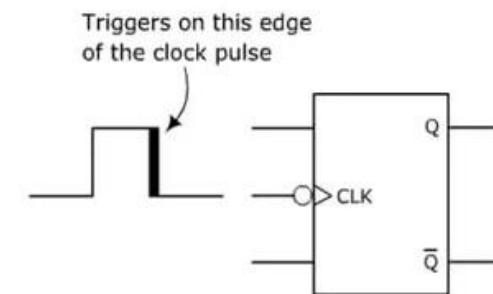
## Low Level Triggering

Another type of flip-flop responds to a clock signal during the time at which it is in the logic Low state. This type is identified by a clock input lead with a low-state indicator bubble, as shown below.



## Negative Edge Triggering

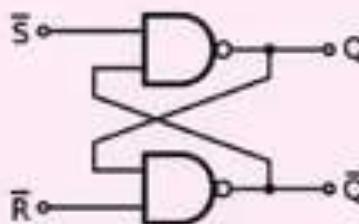
The fourth type of flip-flop responds to a clock signal during the high-to-low transition of a clock pulse. This type is identified by a clock input lead with a low-state indicator and triangle, as shown below.



# Difference between both....?

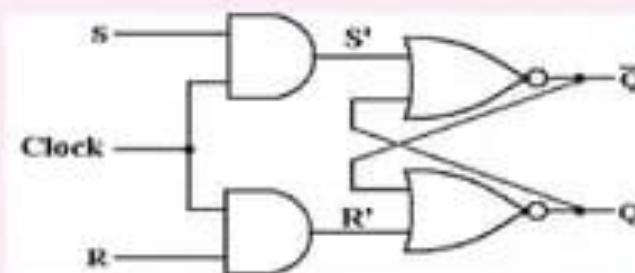
## Latches..

- ❖ Both are same but there is a little difference between both.
- ❖ Latches are the building blocks of sequential circuits.
- ❖ latches can be built from gates.
- ❖ latch does not have a *clock signal*.



## Flip Flop..

- ❖ flip-flops are also the building blocks of sequential circuits.
- ❖ Flip-flops can be built from latches.
- ❖ A flip-flop always has a *clock Signal*

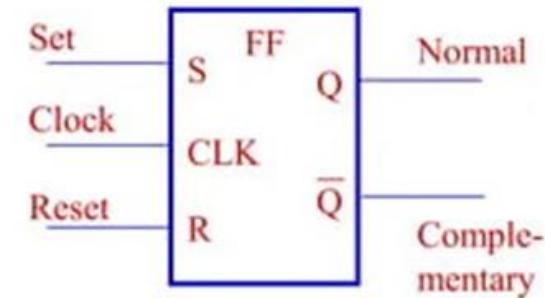
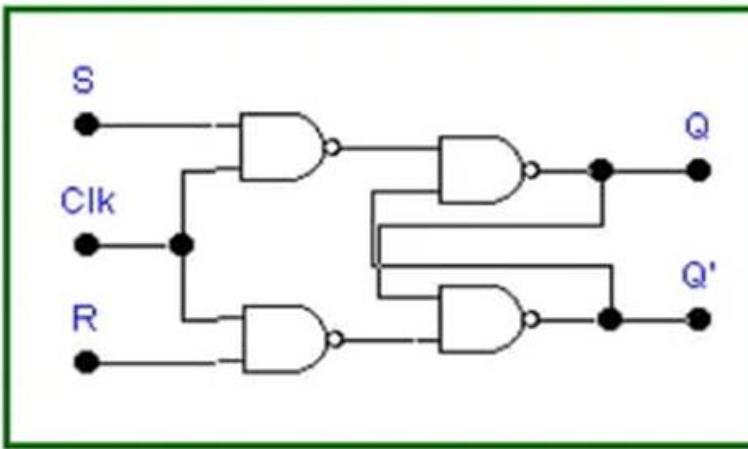


## Flip-flops

- ❑ A flip-flop is a bi-stable device, with inputs, that remains in a given state as long as power is applied and until input signals are applied to cause its output to change.
- ❑ There are four basic different types of flip-flops:
  - SR
  - D
  - JK
  - T

## CLOCKED R-S FLIP-FLOP

Symbols:

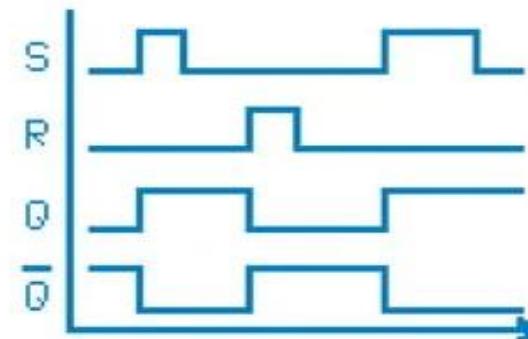
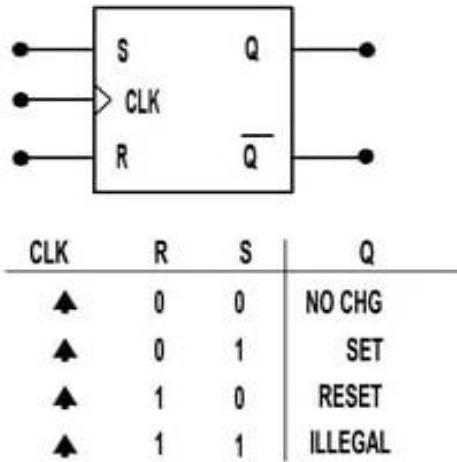


Truth Table:

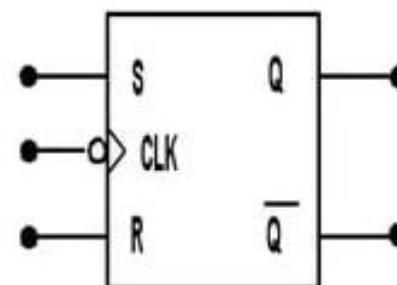
| Clk | S | R | $Q_n$       | $\bar{Q}_n$ |
|-----|---|---|-------------|-------------|
| 1   | 0 | 0 | No change   |             |
| 1   | 0 | 1 | 0           | 1           |
| 1   | 1 | 0 | 1           | 0           |
| 1   | 1 | 1 | Not Allowed |             |

## POSITIVE EDGE TRIGGERED R-S FLIP-FLOP

TIMING DIAGRAMS



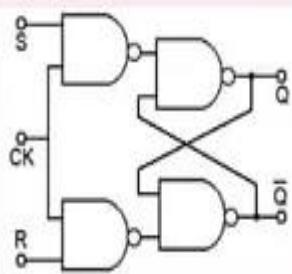
## NEGATIVE EDGE TRIGGERED R-S FLIP-FLOP



| CLK | R | S | Q       |
|-----|---|---|---------|
| ▼   | 0 | 0 | NO CHG  |
| ▼   | 0 | 1 | SET     |
| ▼   | 1 | 0 | RESET   |
| ▼   | 1 | 1 | ILLEGAL |

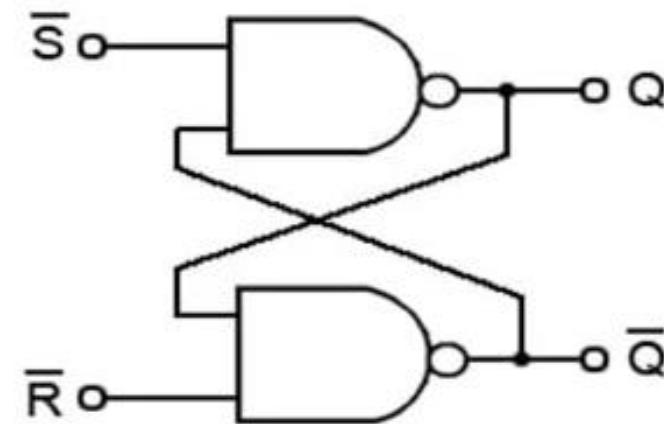
# CLOCKED R-S FLIP-FLOP or SR Flip-flop

## Logic Diagram:



## Truth Table:

| Clk | S | R | Q | Q' |              |
|-----|---|---|---|----|--------------|
| 1   | 1 | 1 | 1 | 1  | Not Allowed  |
| 1   | 0 | 1 | 0 | 1  | Reset        |
| 1   | 1 | 0 | 1 | 0  | Set          |
| 1   | 0 | 0 | Q | Q' | Memory state |
| 0   | 0 | 0 | 1 | 0  | Memory state |



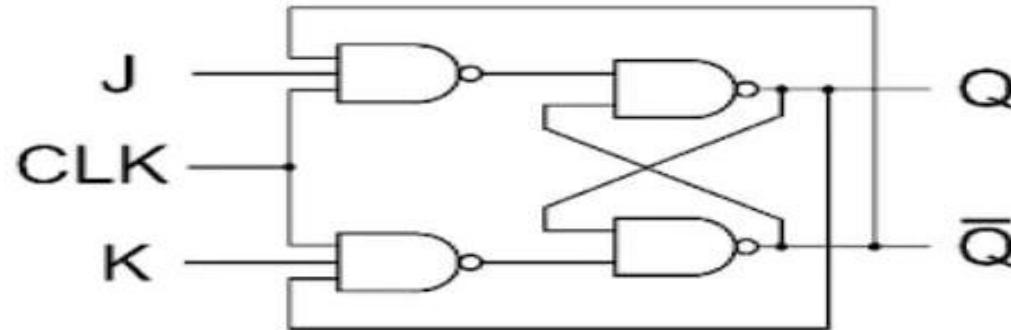
## SR Flip-flop

- The SR (Set-Reset) flip-flop is one of the simplest sequential circuits and consists of two gates connected .
- The output of each gate is connected to one of the inputs of the other gate.
- The circuit has two active low inputs marked S' and R', as well as two outputs, Q and Q'.

| Table |           |           |   |           |  |
|-------|-----------|-----------|---|-----------|--|
|       | $\bar{S}$ | $\bar{R}$ | Q | $\bar{Q}$ | Comments   |
| 1.    | 0         | 1         | 1 | 0         | Q is set to 1 by 0 on $\bar{S}$  |
| 2.    | 1         | 1         | 1 | 0         | No change, (1 on Q is remembered)                                      |
| 3.    | 1         | 0         | 0 | 1         | Q is reset to 0 by 0 on $\bar{R}$                                      |
| 4.    | 1         | 1         | 0 | 1         | No change, (0 on Q is remembered)                                      |
| 5.    | 0         | 0         | 1 | 1         | Both inputs at 0 – both outputs are at 1 (Non-allowed state)           |
| 6.    | 1         | 1         | ? | ?         | Inputs change from 0,0 to 1,1 together - outputs will be INDETERMINATE |

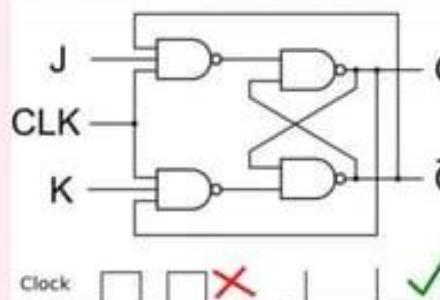
# JK Flip Flop

- One of the most useful and versatile flip flop is the JK flip flop the unique features of a JK flip flop are:
  - If the J and K input are both at 1 and the clock pulse is applied, then the output will change state, regardless of its previous condition.
  - If both J and K inputs are at 0 and the clock pulse is applied there will be no change in the output.
  - There is no indeterminate condition, in the operation of JK flip flop i.e. it has no ambiguous state.
- When J = 0 and K = 0,**  
These J and K inputs disable the NAND gates, therefore clock pulse have no effect on the flip flop. In other words, Q returns its last value.
- When J = 0 and K = 1,**  
The upper NAND gate is disabled the lower NAND gate is enabled if Q is 1 therefore, flip flop will be reset ( $Q = 0, Q' = 1$ ) if not already in that state.
- When J = 1 and K = 0**  
The lower NAND gate is disabled and the upper NAND gate is enabled if J is at 1, As a result we will be able to set the flip flop ( $Q = 1, Q' = 0$ ) if not already set.
- When J and K are both high,** the clock pulses cause the JK flip flop to toggle.



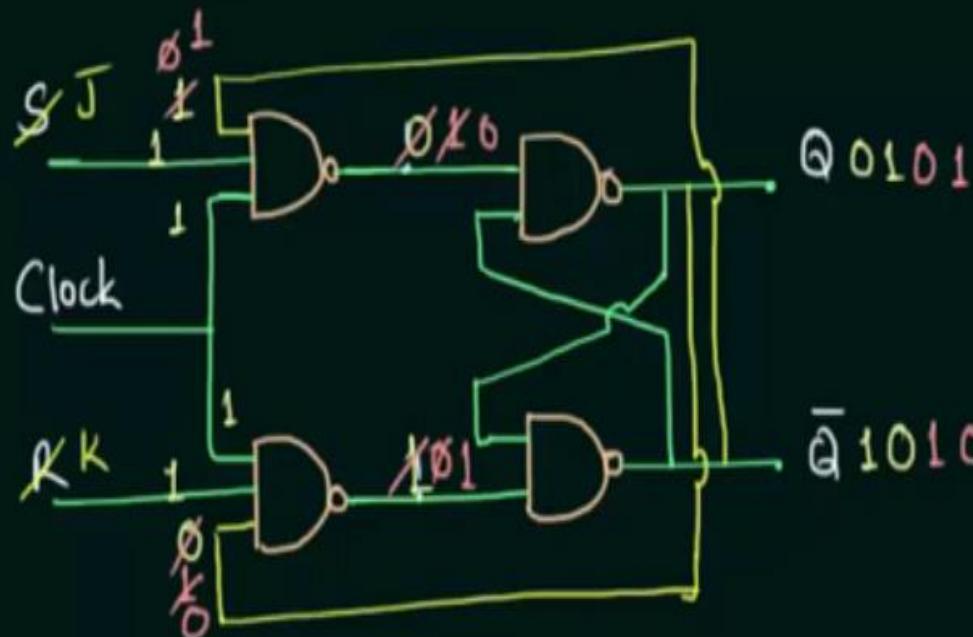
| Same as for SR Latch | Clock             | Input |   | Output |    | Description      |
|----------------------|-------------------|-------|---|--------|----|------------------|
|                      | Clk               | J     | K | Q      | Q' |                  |
| X                    | 0                 | 0     | 1 | 0      | 1  | Memory no change |
| X                    | 0                 | 0     | 0 | 1      | 0  | Reset Q>>0       |
| $\neg \downarrow$    | 0                 | 1     | 1 | 0      | 1  | Set Q>>1         |
| X                    | 0                 | 1     | 0 | 1      | 0  |                  |
| $\neg \downarrow$    | 1                 | 0     | 0 | 1      | 0  |                  |
| X                    | 1                 | 0     | 1 | 0      | 1  |                  |
| Toggle action        | $\neg \downarrow$ | 1     | 1 | 0      | 1  | Toggle           |
|                      | $\neg \downarrow$ | 1     | 1 | 1      | 0  |                  |

Circuit Diagram:



| Clk | J | K | Q | Q' | State              |
|-----|---|---|---|----|--------------------|
| 1   | 0 | 0 | Q | Q' | No change in state |
| 1   | 0 | 1 | 0 | 1  | Resets Q to 0      |
| 1   | 1 | 0 | 1 | 0  | Sets Q to 1        |
| 1   | 1 | 1 | - | -  | Toggles            |

## Racing explained

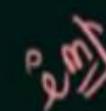


$Clk = 0$  Memory

$Clk = 1 \quad J = 1, K = 0, Q = 1, \bar{Q} = 0$

$Clk = 1 \quad J = 0, K = 1, Q = 0, \bar{Q} = 1$

| $Clk$ | $S$ | $R$ | $Q_{n+1}$      |
|-------|-----|-----|----------------|
| 0     | x   | x   | $Q_n$ (memory) |
| 1     | 0   | 0   | $Q_n$ (memory) |
| 1     | 0   | 1   | 0              |
| 1     | 0   |     | 1              |
| 1     | 1   |     | Not used.      |



$Clk = 1 \quad J = 1, K = 1$

assume  $Q = 0 \& \bar{Q} = 1$

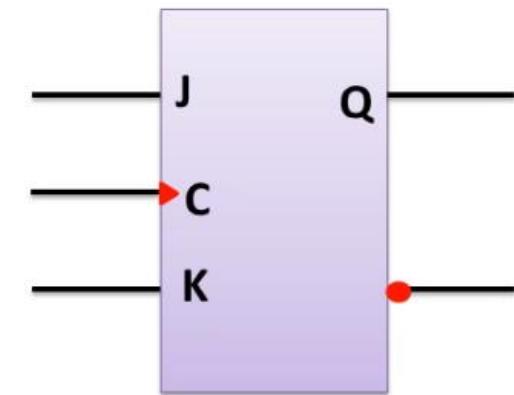
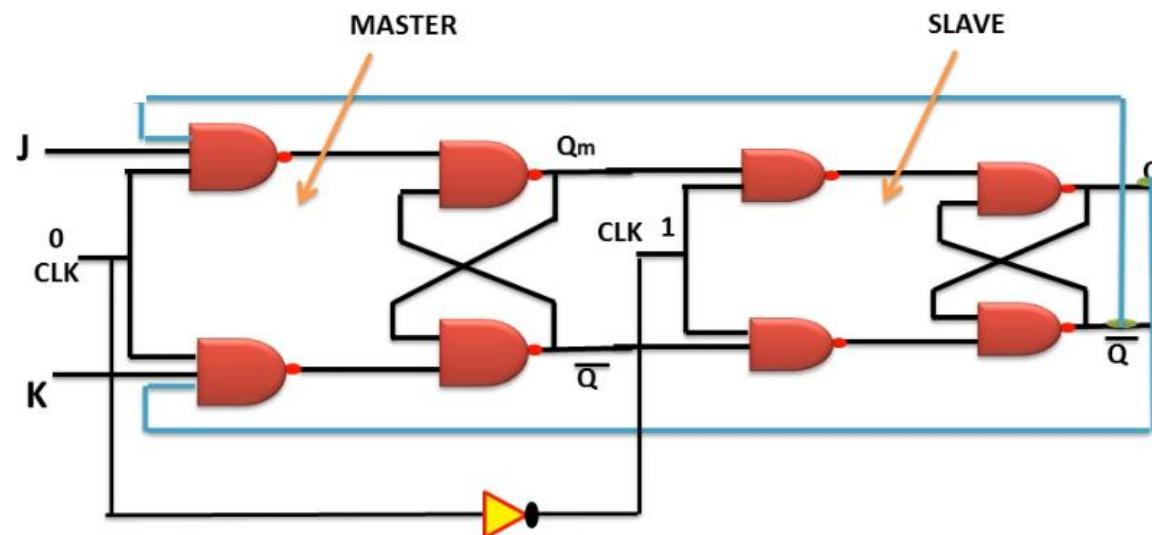
$Q_{n+1} = 0$

$Q = 0, 1, 0, 1 \dots$

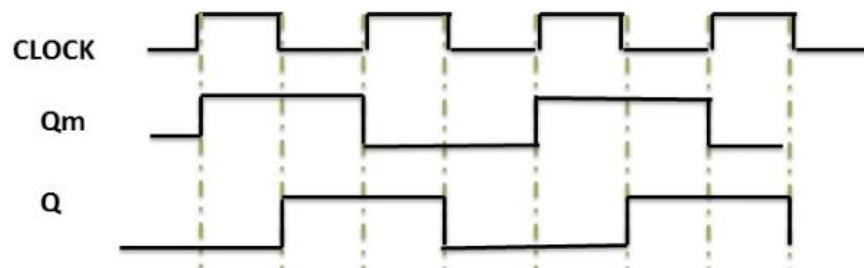
$Q_n \quad \bar{Q}_n \quad 1 \quad 0$

$\bar{Q} = 1, 0, 1, 0 \dots$

## JK OR MASTER SLAVE FLIP FLOP



**Graphic symbol**

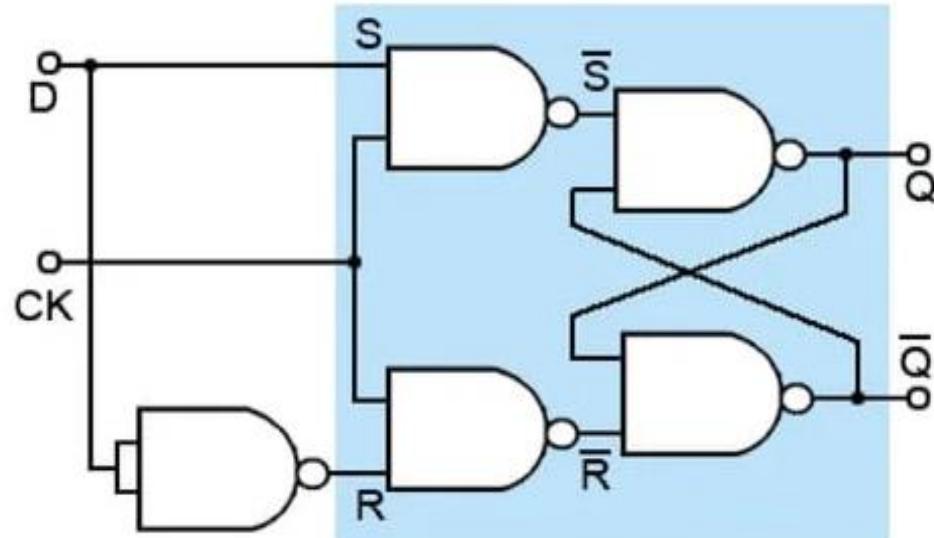


Master slave configuration is mainly used to eliminate the race around the condition and get rid of unstable oscillation in the flip flop.

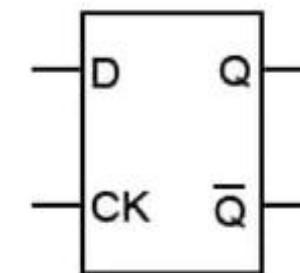
| J | K | Q(t+1)           |
|---|---|------------------|
| 0 | 0 | $Q(t)$ no change |
| 0 | 1 | 0 clear to 0     |
| 1 | 0 | 1 set to 1       |
| 1 | 1 | $Q'(t)$          |

# D-Flip Flop

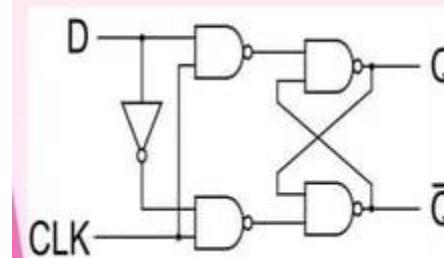
- The D type flip-flop has one data input 'D' and a clock input. The circuit edge triggers on the clock input. The flip-flop also has two outputs Q and Q' (where Q' is the reverse of Q).
- Such type of flip flop is a modification of clocked RS flip flop gates from a basic Latch flip flop and NOR gates modify it in to a clock RS flip flop.
- The D input goes directly to S input and its complement through NOT gate, is applied to the R input.
- When the clock is low, both AND gates are disabled, therefore D can change values without affecting the value of Q.
- When the clock is high, both AND gates are enabled. In this case, Q is forced equal to D
- When the clock again goes low, Q retains or stores the last value of D



| Inputs |   | Outputs   |           |
|--------|---|-----------|-----------|
| CK     | D | Q         | $\bar{Q}$ |
| 0      | X | No change |           |
| 1      | 0 | 0         | 1         |
| 1      | 1 | 1         | 0         |



Logic Diagram:

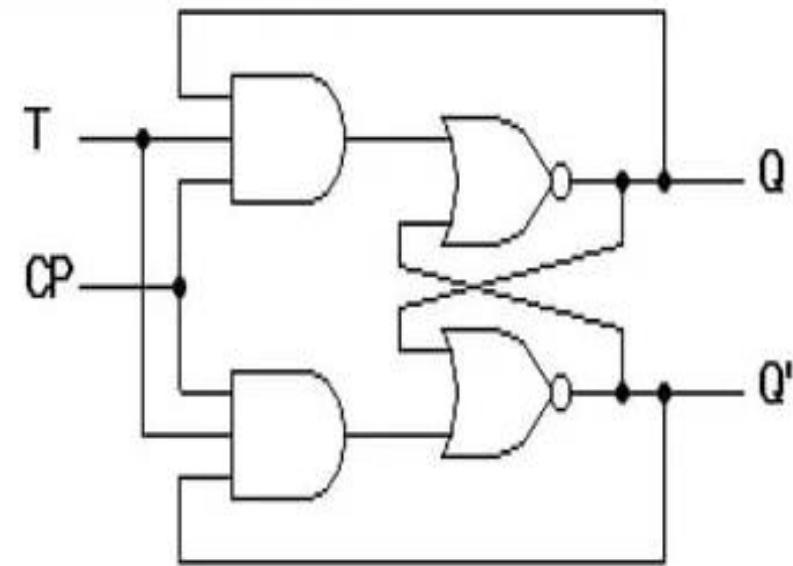
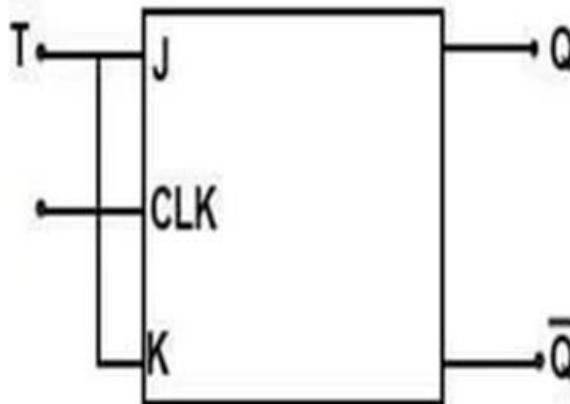


Truth Table:

| Cp | D | Q            | $Q'$ |  |
|----|---|--------------|------|--|
| 0  | 0 | Memory state |      |  |
| 1  | 1 | Set state    |      |  |
| 1  | 0 | Reset state  |      |  |

# Toggle Flip Flop or T-Flip Flop

- The T flip flop has only the Toggle and Hold **or NO Change** Operation.
- If Toggle mode operation. The output will toggle from 1 to 0 or vice versa.



| clk | T | Q         | $\bar{Q}$ | comments  |
|-----|---|-----------|-----------|-----------|
| ↑   | 0 | Q         | $\bar{Q}$ | No change |
| ↑   | 1 | $\bar{Q}$ | Q         | toggle    |

## EXCITATION TABLE SR FLIP FLOP

Characteristics table of flip flop show the next state when input and present state are known.

Characteristics table

|   |   | Q(n+1)                |
|---|---|-----------------------|
| S | R |                       |
| 0 | 0 | <b>Q(n) no change</b> |
| 0 | 1 | <b>0 clear to 0</b>   |
| 1 | 0 | <b>1 set to 1</b>     |
| 1 | 1 | <b>invalid</b>        |

| Q(n) | S | R | Q(n+1) |
|------|---|---|--------|
| 0    | 0 | 0 | 0      |
| 0    | 0 | 1 | 0      |
| 0    | 1 | 0 | 1      |
| 0    | 1 | 1 | x      |
| 1    | 0 | 0 | 1      |
| 1    | 0 | 1 | 0      |
| 1    | 1 | 0 | 1      |
| 1    | 1 | 1 | x      |

$Q(n)$  = previous state

$Q(n+1)$  = next state

Clock = 1

## EXCITATION TABLE SR FLIP FLOP

**Characteristics table**

| $Q(n)$ | $S$ | $R$ | $Q(n+1)$ |
|--------|-----|-----|----------|
| 0      | 0   | 0   | 0        |
| 0      | 0   | 1   | 0        |
| 0      | 1   | 0   | 1        |
| 0      | 1   | 1   | x        |
| 1      | 0   | 0   | 1        |
| 1      | 0   | 1   | 0        |
| 1      | 1   | 0   | 1        |
| 1      | 1   | 1   | x        |

**Excitation table**

| $Q(n)$ | $Q(n+1)$ | $S$ | $R$ |
|--------|----------|-----|-----|
| 0      | 0        | 0   | x   |
| 0      | 1        | 1   | 0   |
| 1      | 0        | 0   | 1   |
| 1      | 1        | x   | 0   |

| $S$ | $R$ | $Q(n+1)$         |
|-----|-----|------------------|
| 0   | 0   | $Q(n)$ no change |
| 0   | 1   | 0 clear to 0     |
| 1   | 0   | 1 set to 1       |
| 1   | 1   | invalid          |

$Q(n) = \text{previous state}$   
 $Q(n+1) = \text{next state}$   
 $\text{Clock} = 1$

## EXCITATION TABLE OF JK OR MASTER SLAVE FLIP FLOP

Characteristics table of flip flop show the next state when input and present state are known.

Characteristics table

| C | J | K | Q(n+1)            | Q(n) | J | K | Q(n+1) |
|---|---|---|-------------------|------|---|---|--------|
| 0 | 0 | 0 | Q(n)              | 0    | 0 | 0 | 0      |
| 1 | 0 | 0 | Q(n)              | 0    | 1 | 0 | 0      |
| 1 | 0 | 1 | 0                 | 0    | 1 | 1 | 1      |
| 1 | 1 | 0 | 1                 | 1    | 0 | 0 | 1      |
| 1 | 1 | 1 | $\overline{Q(n)}$ | 1    | 0 | 1 | 0      |
|   |   |   |                   | 1    | 1 | 0 | 1      |
|   |   |   |                   | 1    | 1 | 1 | 0      |

$Q(n)$  = previous state

$Q(n+1)$  = next state

Clock = 1

## EXCITATION TABLE OF JK OR MASTER SLAVE FLIP FLOP

| Characteristics table |   |   |          |
|-----------------------|---|---|----------|
| $Q(n)$                | J | K | $Q(n+1)$ |
| { 0                   | 0 | 0 | 0 }      |
| 0                     | 0 | 1 | 0 }      |
| { 0                   | 1 | 0 | 1 }      |
| 0                     | 1 | 1 | 1 }      |
| { 1                   | 0 | 0 | 1 }      |
| { 1                   | 0 | 1 | 0 }      |
| { 1                   | 1 | 0 | 1 }      |
| { 1                   | 1 | 1 | 0 }      |

Excitation table

| $Q(n)$ | $Q(n+1)$ | J | K |
|--------|----------|---|---|
| 0      | 0        | 0 | X |
| 0      | 1        | 1 | X |
| 1      | 0        | X | 1 |
| 1      | 1        | X | 0 |

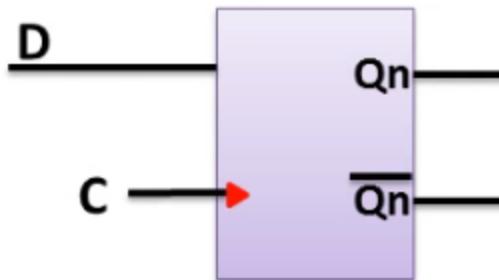
$Q(n)$  = previous state

$Q(n+1)$  = next state

Clock = 1

## EXCITATION TABLE OF D FLIP FLOP

Characteristics table of flip flop show the next state when input and present state are known.



| C | D | $Q(n+1)$ |
|---|---|----------|
| 0 | X | $Q(n)$   |
| 1 | 0 | 0        |
| 1 | 1 | 1        |

$Q(n)$  = previous state

$Q(n+1)$  = next state

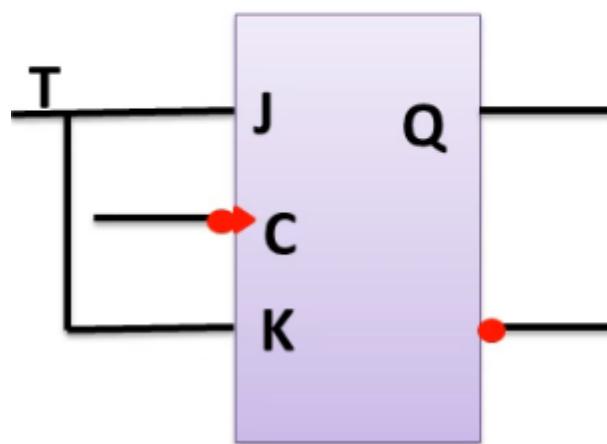
Clock = 1

Characteristics table

| $Q_n$ | D | $Q(n+1)$ |
|-------|---|----------|
| 0     | 0 | 0        |
| 0     | 1 | 1        |
| 1     | 0 | 0        |
| 1     | 1 | 1        |

Excitation table

| $Q_n$ | $Q(n+1)$ | D |
|-------|----------|---|
| 0     | 0        | 0 |
| 0     | 1        | 1 |
| 1     | 0        | 0 |
| 1     | 1        | 1 |



| C | T | $Q(n+1)$          |
|---|---|-------------------|
| 0 | X | $Q(n)$            |
| 1 | 0 | $Q(n)$            |
| 1 | 1 | $\overline{Q(n)}$ |

Characteristics table

| $Q_n$ | T | $Q(n+1)$ |
|-------|---|----------|
| 0     | 0 | 0        |
| 0     | 1 | 1        |
| 1     | 0 | 1        |
| 1     | 1 | 0        |

ODD ONE  
DETECTOR

$$Q(n+1) = Q_n \oplus T$$

Excitation table

| $Q_n$ | $Q(n+1)$ | T |
|-------|----------|---|
| 0     | 0        | 0 |
| 0     | 1        | 1 |
| 1     | 0        | 1 |
| 1     | 1        | 0 |



## Department of Electronics and Communication Engineering



# **MARRI LAXMAN REDDY INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

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# DIGITAL LOGIC DESIGN AND COMPUTER ORGANIZATION

- **UNIT-II:** Analysis of clocked sequential circuits, State Reduction and Assignment, Design Procedure.

Presentation By

**Dr. Bapi Debnath**

Assoc. Prof.

Department of ECE



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# Analysis of Clocked Sequential Circuit

Outputs & state of a clocked Sequential circuit is function of inputs and previous state.

Analysis of **clocked sequential logic circuit** consists of obtaining

- **state table:** Also called transition table consists of four sections; inputs, present state, next state and output.
- **state diagram:** Graphical representation of state table. A state is represented by a circle and transition between states is represented by a directed line. Input and output values of the transition is put on the directed line
- **state equation:** Specifies next state in terms of present state and inputs. Output equation is also described in terms of present state and inputs

# Analysis Steps

## State & output equation

- Assign variable names to input, outputs, flip flops input and output
- Determine FF's input equations in terms of input variable and present state.
- Determine next state equation of flip flop's using FF characteristic equation (state equation). Determine circuit's output if any, in terms of input variable/ present state (output equation).

## State Table

- List all binary combination of inputs and present states
- List next state of FF's and outputs using state equation and output equation.

## State Diagram

- Mark FF states and connect them with directed lines to next state with input / output values on the line. Each row of state table correspond to a directed line.

# Analysis : Example-1

Example circuit has one input X, two D FF with outputs A & B and one output Y.

Input to FF's;

$$D_A = AX + BX$$

$$D_B = \bar{A}X$$

Using characteristic equation of D FF;  $Q_{n+1} = D$

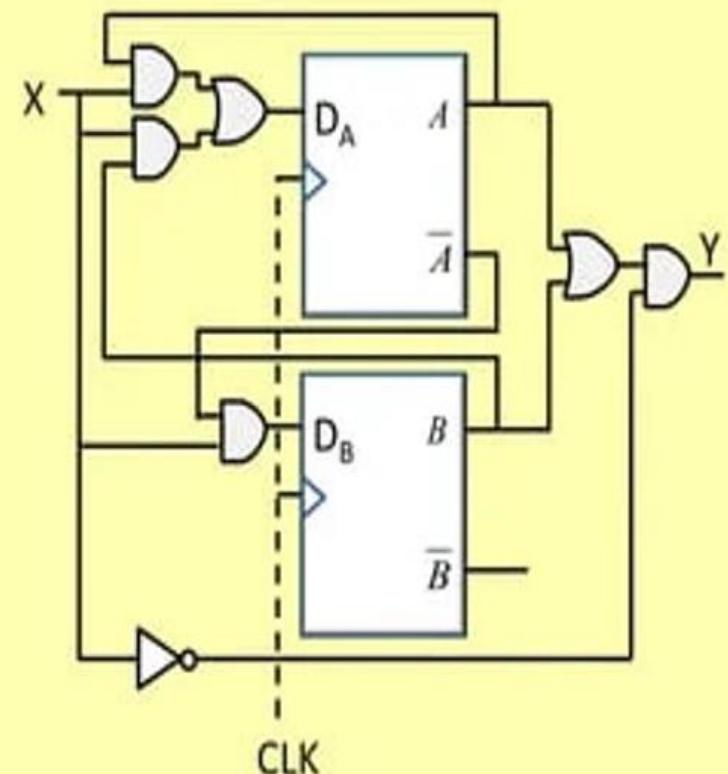
State equation;

$$A_{n+1} = AX + BX$$

$$B_{n+1} = \bar{A}X$$

Output equation;

$$Y = (A + B)\bar{X}$$



## Analysis: example -1

State/ output equation;

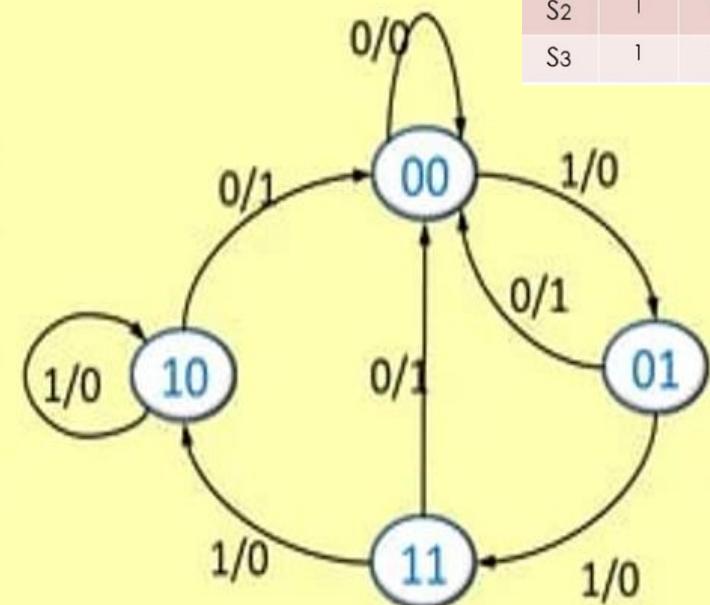
$$A_{n+1} = (A+B)X$$

$$B_{n+1} = \bar{A}X$$

$$Y = (A+B)\bar{X}$$

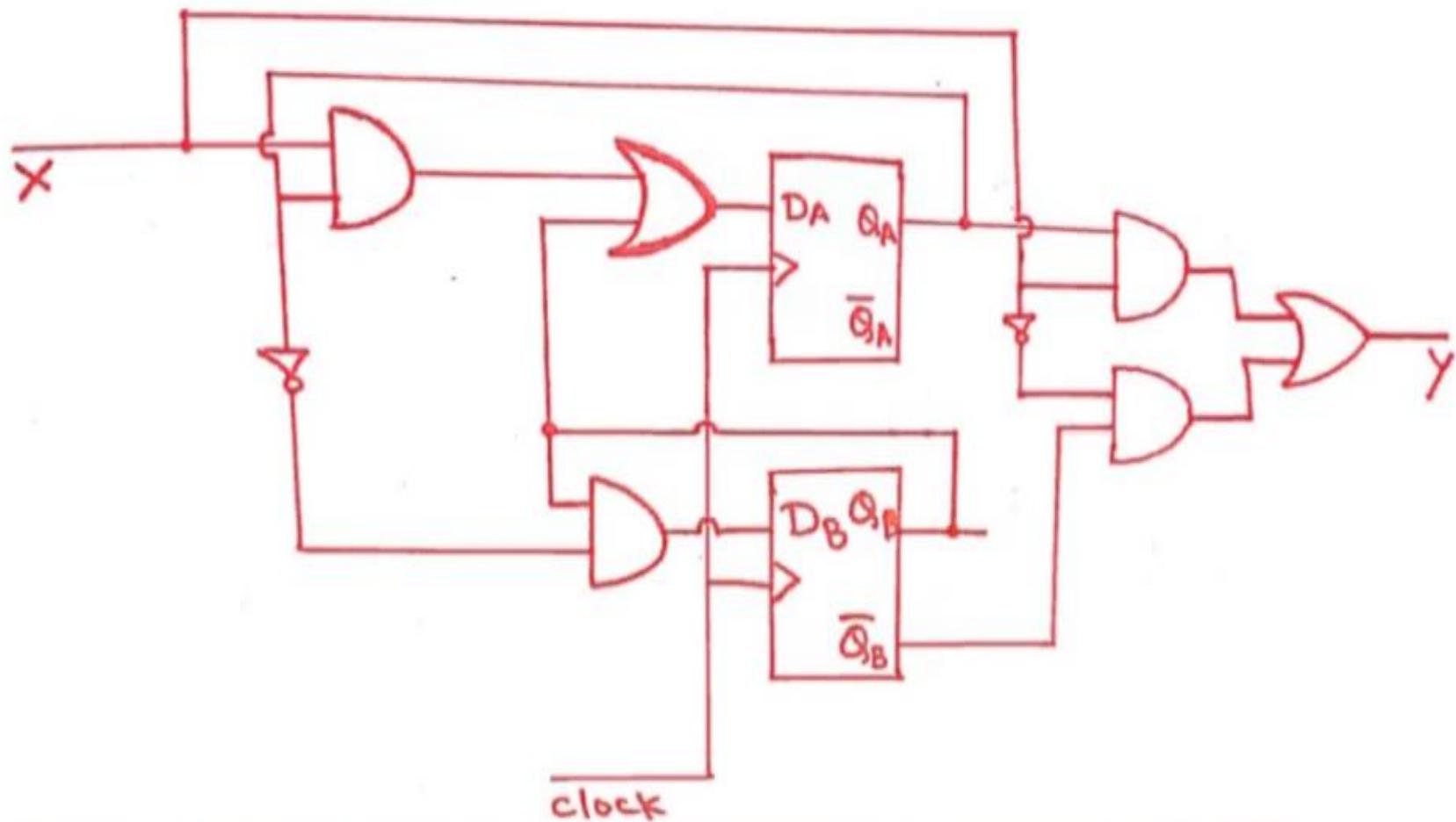
|       |               |   | State Table |           |        |
|-------|---------------|---|-------------|-----------|--------|
| input | Present state |   | Next state  |           | Output |
| X     | A             | B | $A_{n+1}$   | $B_{n+1}$ | Y      |
| 0     | 0             | 0 | 0           | 0         | 0      |
| 0     | 0             | 1 | 0           | 0         | 1      |
| 0     | 1             | 0 | 0           | 0         | 1      |
| 0     | 1             | 1 | 0           | 0         | 1      |
| 1     | 0             | 0 | 0           | 1         | 0      |
| 1     | 0             | 1 | 1           | 1         | 0      |
| 1     | 1             | 0 | 1           | 0         | 0      |
| 1     | 1             | 1 | 1           | 0         | 0      |

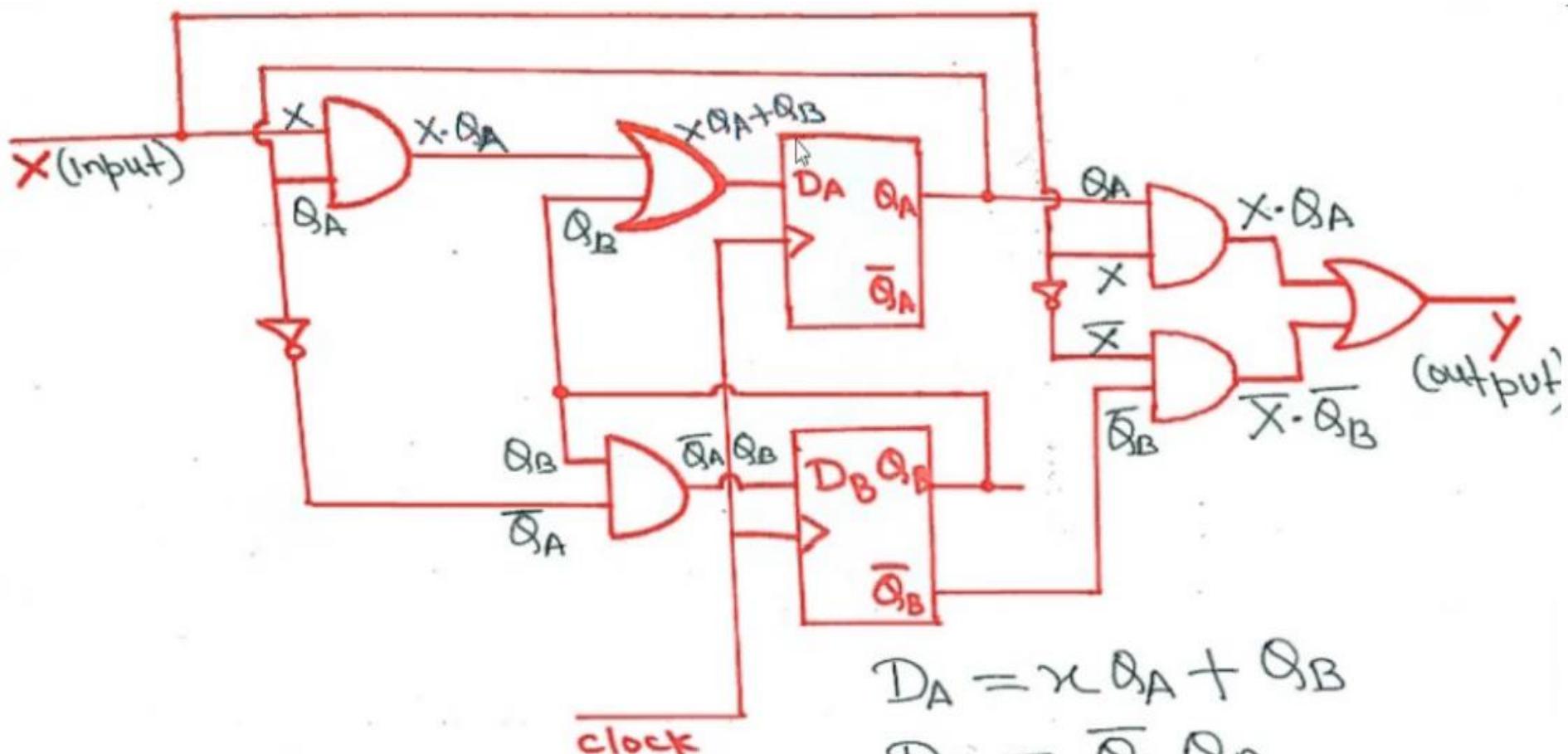
|    | A | B |
|----|---|---|
| S0 | 0 | 0 |
| S1 | 0 | 1 |
| S2 | 1 | 0 |
| S3 | 1 | 1 |



Example - 2

IN THE GIVEN CIRCUIT BELOW, TWO FLIP FLOP ARE USED, BOTH ARE D FLIP FLOP. HERE GATES ARE USED AS A COMBINATIONAL LOGIC(INPUT COMBINATIONAL LOGIC AND OUTPUT COMBINATIONAL LOGIC).



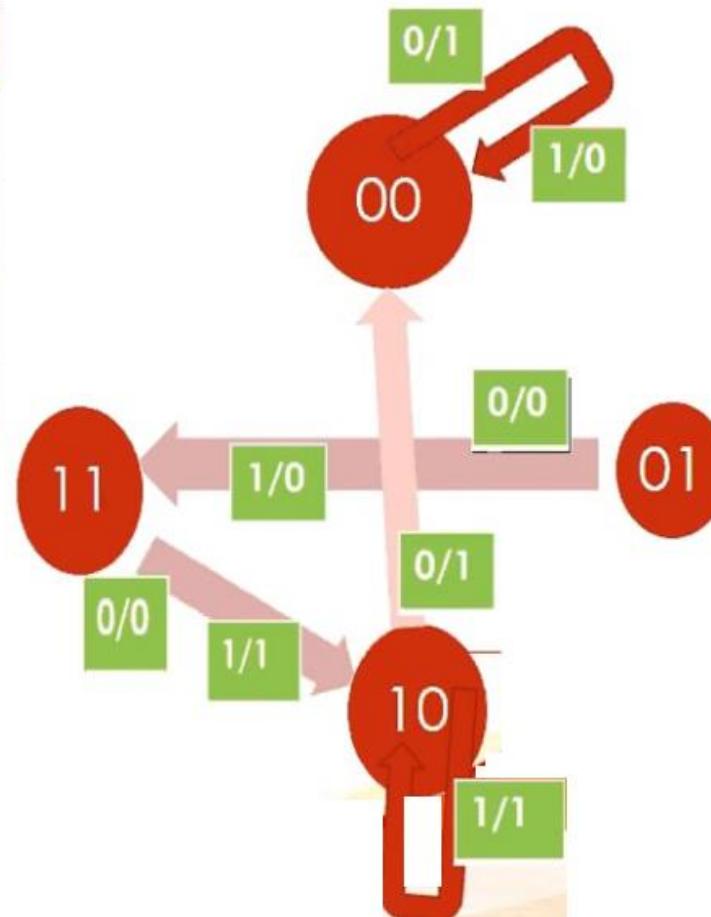
Example - 2

$$D_A = X \cdot Q_B + Q_B$$

$$D_B = \overline{Q_A} \cdot Q_B$$

$$Y = \overline{X} \cdot \overline{Q_B} + X \cdot Q_A$$

|    | QA | QB |
|----|----|----|
| S0 | 0  | 0  |
| S1 | 0  | 1  |
| S2 | 1  | 0  |
| S3 | 1  | 1  |



| Present state  |                |   | Next state      |                 |   |
|----------------|----------------|---|-----------------|-----------------|---|
| Q <sub>A</sub> | Q <sub>B</sub> | X | Q <sub>A+</sub> | Q <sub>B+</sub> | Y |
| 0              | 0              | 0 | 0               | 0               | 1 |
| 0              | 0              | 1 | 0               | 0               | 0 |
| 0              | 1              | 0 | 1               | 1               | 0 |
| 0              | 1              | 1 | 1               | 1               | 0 |
| 1              | 0              | 0 | 0               | 0               | 1 |
| 1              | 0              | 1 | 1               | 0               | 1 |
| 1              | 1              | 0 | 1               | 0               | 0 |
| 1              | 1              | 1 | 1               | 0               | 1 |

$$Q_A+ = D_A = \bar{X} Q_A + Q_B$$

$$Q_B+ = D_B = \bar{Q}_A Q_B$$

$$Y = \bar{X} \bar{Q}_B + X Q_A$$

# State Equations

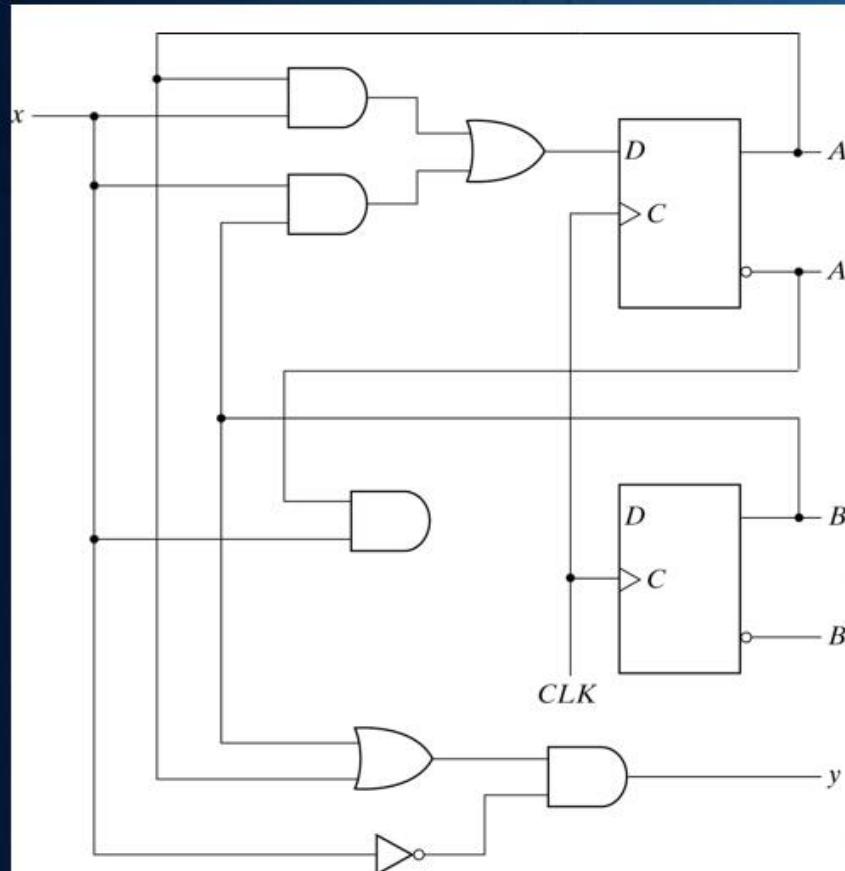


Fig. 5-15 Example of Sequential Circuit

**A state equation shows the next state as a function of the current state and inputs**

$$A(t+1) = A(t)x(t) + B(t)x(t)$$

$$B(t+1) = A'(t)x(t)$$

$$y(t) = [A(t) + B(t)]x'(t)$$

$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

$$y = (A + B)x'$$

# State Table

| <b>Present State</b><br><b>AB</b> | <b>Next State</b> |            | <b>Output</b> |            |
|-----------------------------------|-------------------|------------|---------------|------------|
|                                   | <b>x=0</b>        | <b>x=1</b> | <b>x=0</b>    | <b>x=1</b> |
| 00                                | 00                | 01         | 0             | 0          |
| 01                                | 00                | 11         | 1             | 0          |
| 10                                | 00                | 10         | 1             | 0          |
| 11                                | 00                | 10         | 1             | 0          |

$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

$$y = (A + B)x'$$

# State Diagram

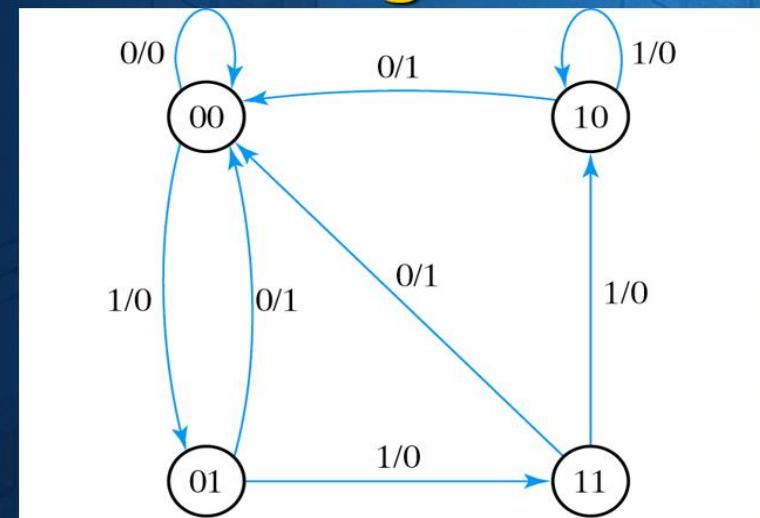
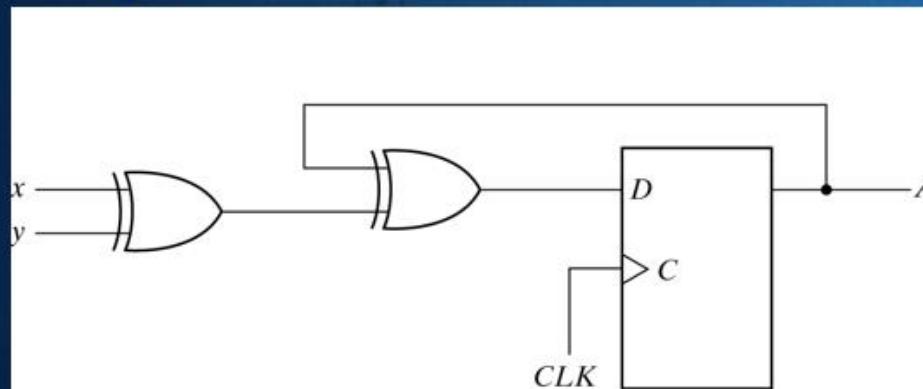


Fig. 5-16 State Diagram of the Circuit of Fig. 5-15

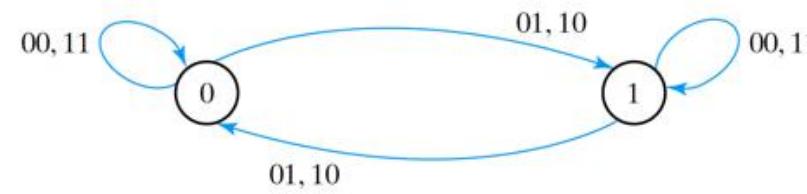
# Analysis with D Flip-Flops



(a) Circuit diagram

| Present state | Inputs  | Nex state |
|---------------|---------|-----------|
| $A$           | $x$ $y$ | $A(t+1)$  |
| 0             | 0 0     | 0         |
| 0             | 0 1     | 1         |
| 0             | 1 0     | 1         |
| 0             | 1 1     | 0         |
| 1             | 0 0     | 1         |
| 1             | 0 1     | 0         |
| 1             | 1 0     | 0         |
| 1             | 1 1     | 1         |

(b) State table



(c) State diagram

Fig. 5-17 Sequential Circuit with D Flip-Flop

$$D_A = A \oplus x \oplus y$$

$$A(t+1) = A \oplus x \oplus y$$

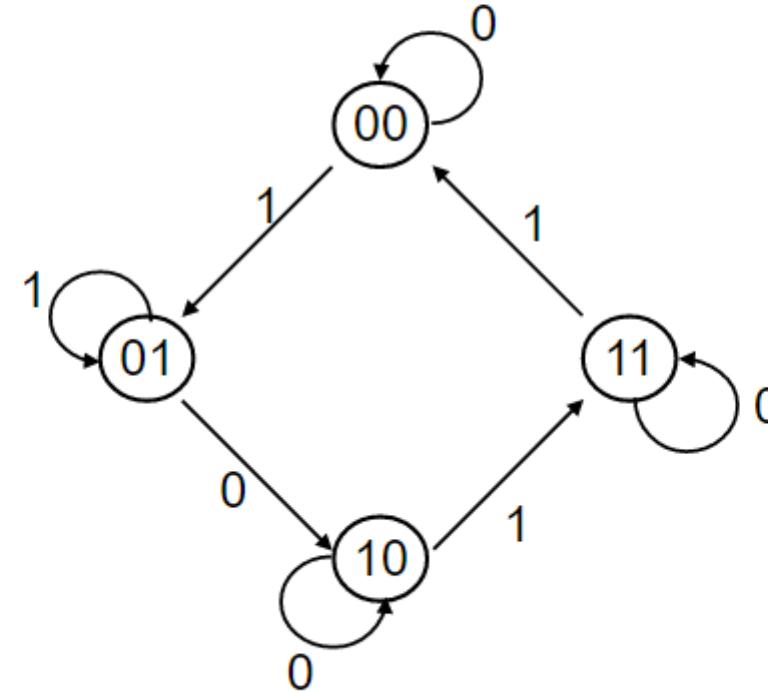
# Sequential Circuit Design

---

- Design procedure:
  - Start with circuit specifications – description of circuit behavior.
  - Derive the state table.
  - Perform state reduction if necessary.
  - Perform state assignment.
  - Determine number of flip-flops and label them.
  - Choose the type of flip-flop to be used.
  - Derive circuit excitation and output tables from the state table.
  - Derive circuit output functions and flip-flop input functions.
  - Draw the logic diagram.

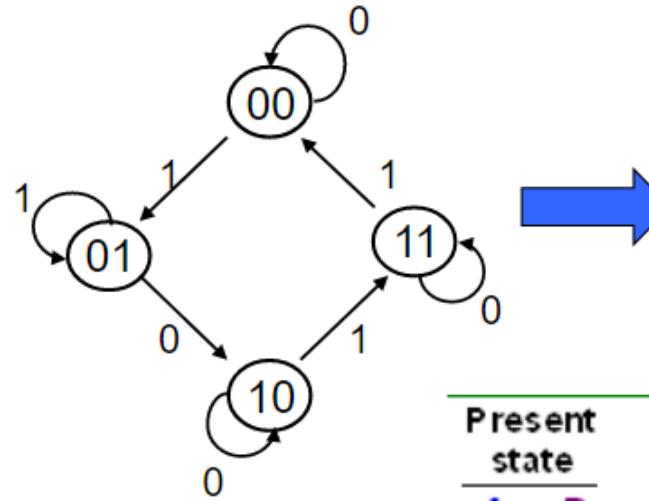
# Design: Example #1

- Given the following state diagram, design the sequential circuit using JK flip-flops.



# Design: Example #1

- Circuit state/excitation table, using JK flip-flops.



| Present State | Next State |           |
|---------------|------------|-----------|
|               | $x=0$      | $x=1$     |
| $A B$         | $A^+ B^+$  | $A^+ B^+$ |
| 00            | 00         | 01        |
| 01            | 10         | 01        |
| 10            | 10         | 11        |
| 11            | 11         | 00        |

| $Q$ | $Q^+$ | $J$ | $K$ |
|-----|-------|-----|-----|
| 0   | 0     | 0   | X   |
| 0   | 1     | 1   | X   |
| 1   | 0     | X   | 1   |
| 1   | 1     | X   | 0   |

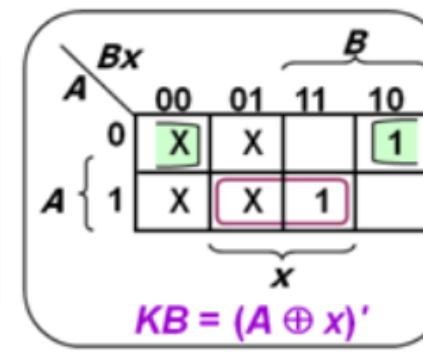
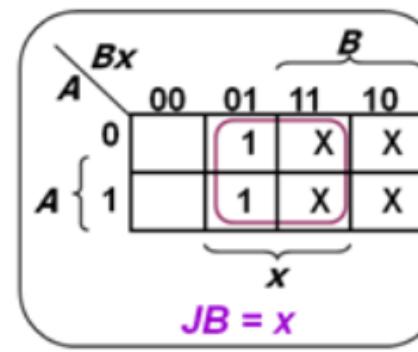
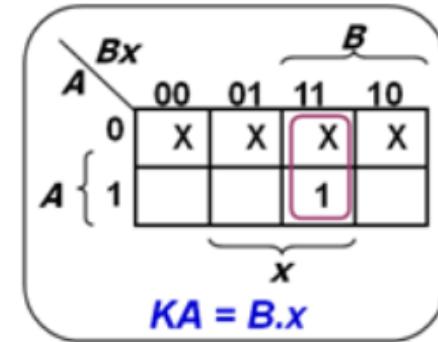
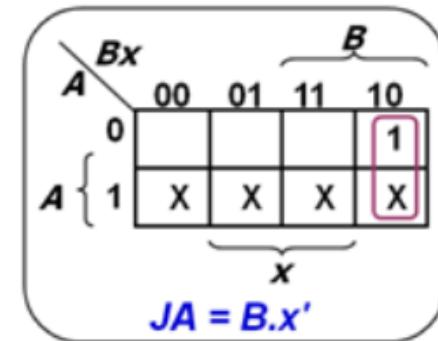
JK Flip-flop's  
excitation table.

| Present state | Input | Next state |       | Flip-flop inputs |       |       |       |
|---------------|-------|------------|-------|------------------|-------|-------|-------|
|               |       | $A^+$      | $B^+$ | $J A$            | $K A$ | $J B$ | $K B$ |
| 0 0           | 0     | 0          | 0     | 0                | X     | 0     | X     |
| 0 0           | 1     | 0          | 0     | 0                | X     | 1     | X     |
| 0 1           | 0     | 1          | 0     | 1                | X     | X     | 1     |
| 0 1           | 1     | 0          | 1     | 0                | X     | X     | 0     |
| 1 0           | 0     | 1          | 0     | 0                | X     | 0     | X     |
| 1 0           | 1     | 0          | 1     | 1                | X     | 0     | 1     |
| 1 1           | 0     | 1          | 1     | 1                | X     | 0     | X     |
| 1 1           | 1     | 0          | 0     | 0                | X     | 1     | X     |

# Design: Example #1

- From state table, get flip-flop input functions.

| Present state |     | Input<br>$x$ | Next state |       | Flip-flop inputs |      |      |      |
|---------------|-----|--------------|------------|-------|------------------|------|------|------|
| $A$           | $B$ |              | $A^+$      | $B^+$ | $JA$             | $KA$ | $JB$ | $KB$ |
| 0             | 0   | 0            | 0          | 0     | 0                | X    | 0    | X    |
| 0             | 0   | 1            | 0          | 1     | 0                | X    | 1    | X    |
| 0             | 1   | 0            | 1          | 0     | 1                | X    | X    | 1    |
| 0             | 1   | 1            | 0          | 1     | 0                | X    | X    | 0    |
| 1             | 0   | 0            | 1          | 0     | X                | 0    | 0    | X    |
| 1             | 0   | 1            | 1          | 1     | X                | 0    | 1    | X    |
| 1             | 1   | 0            | 1          | 1     | X                | 0    | X    | 0    |
| 1             | 1   | 1            | 0          | 0     | X                | 1    | X    | 1    |



# Design: Example #1

- Flip-flop input functions.

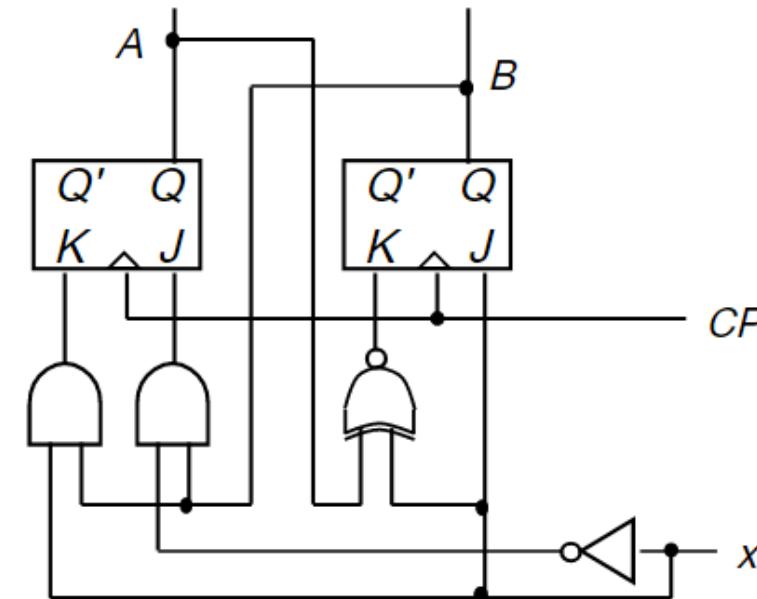
$$JA = B \cdot x'$$

$$KA = B \cdot x$$

$$JB = x$$

$$KB = (A \oplus x)'$$

- Logic diagram:



## Example #2

- Design a circuit (with D flip-flops) that detects three or more consecutive 1's in a string of bits coming through an input line

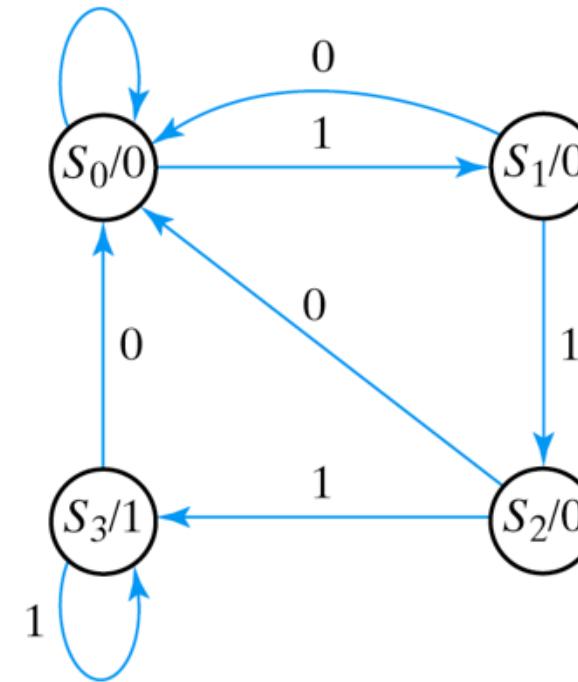
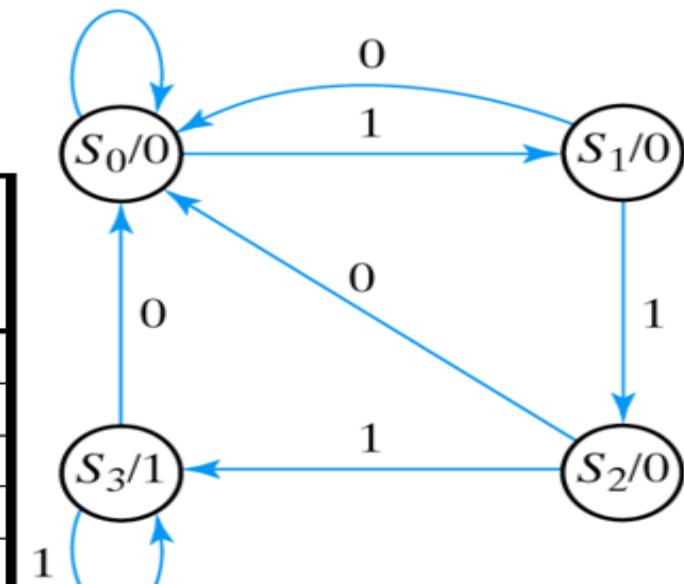


Fig. 5-24 State Diagram for Sequence Detector

## Example (Cont.)

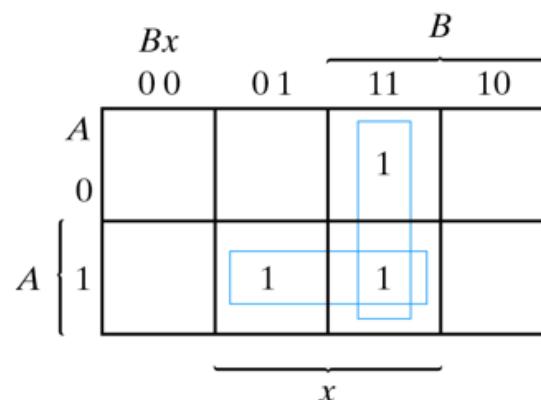
| Present State | Input | Next State | Output |
|---------------|-------|------------|--------|
| A    B        | x     | A    B     | y      |
| 0    0        | 0     | 0    0     | 0      |
| 0    0        | 1     | 0    1     | 0      |
| 0    1        | 0     | 0    0     | 0      |
| 0    1        | 1     | 1    0     | 0      |
| 1    0        | 0     | 0    0     | 0      |
| 1    0        | 1     | 1    1     | 0      |
| 1    1        | 0     | 0    0     | 1      |
| 1    1        | 1     | 1    1     | 1      |



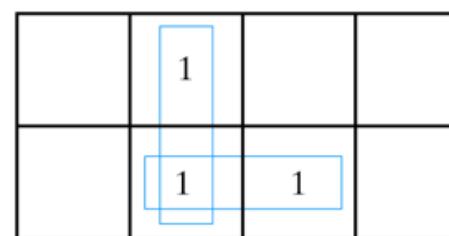
$$A(t+1) = D_A(A, B, x) = \sum(3, 5, 7)$$

$$B(t+1) = D_B(A, B, x) = \sum(1, 5, 7)$$

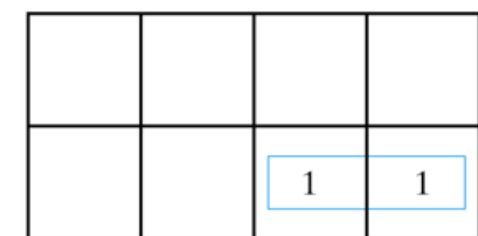
$$y(A, B, x) = \sum(6, 7)$$



$$D_A = Ax + Bx$$



$$D_B = Ax + B'x$$



$$y = AB$$

Fig. 5-25 Maps for Sequence Detector

## Example (Cont.)

$$D_A = Ax + Bx$$

$$D_B = Ax + B'x$$

$$y = AB$$

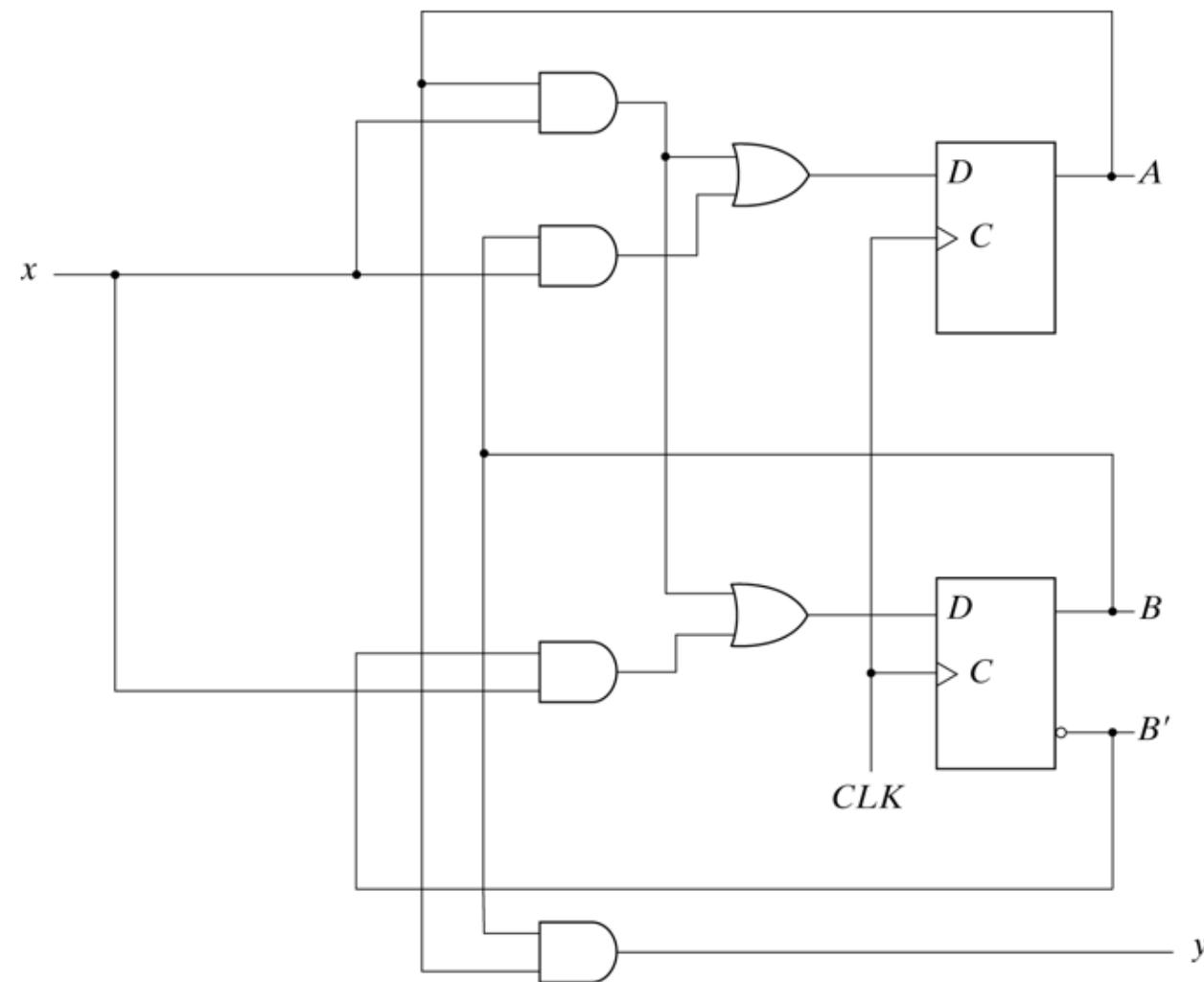


Fig. 5-26 Logic Diagram of Sequence Detector

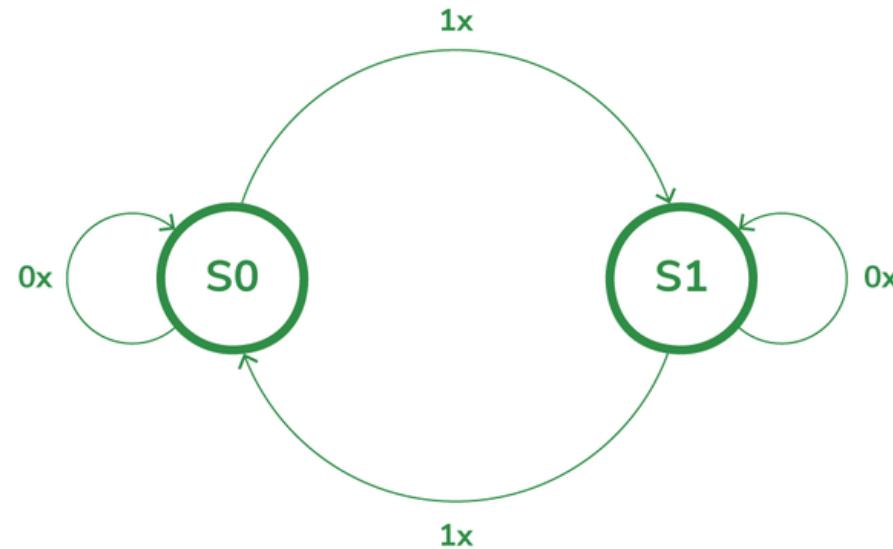
## State Reduction and State Assignment

To illustrate the process of state reduction and state assignment first we have to know the concepts of the state diagram, state table, and state equation. In this article, we are going to learn all the topics related to state reduction and assignment.

**State diagram:** The state graph or state diagram is a pictorial representation of the relationships between the present state, the input state, the next state, and the output state of a sequential circuit i.e. A state diagram is a graphical representation of a sequential circuit's behavior.

Example: Consider an excitation table of J-K flip-flop

| $Q_n$ | $Q_{n+1}$ | J | K |
|-------|-----------|---|---|
| 0     | 0         | 0 | x |
| 0     | 1         | 1 | x |
| 1     | 0         | x | 1 |
| 1     | 1         | x | 0 |



The state diagram of the above table is

*State Diagram of J-K flip-flop*

**State table:** Even though the behavior of a sequential circuit can be conveniently described using a state diagram, for its implementation the information contained in the state diagram is to be translated into a state table. The tabular form of the state diagram is the state table. The present state, the next state, and the output are the three sections of the diagram.

The state table of JK flip-flop is:

| Inputs | Present state | Output |    |          |
|--------|---------------|--------|----|----------|
| J      | K             | Q      | Q+ | (Output) |
| 0      | 0             | 0      | 0  | 0        |
| 0      | 0             | 1      | 1  | 1        |
| 0      | 1             | 0      | 0  | 0        |
| 0      | 1             | 1      | 0  | 0        |
| 1      | 0             | 0      | 1  | 1        |
| 1      | 0             | 1      | 1  | 1        |
| 1      | 1             | 0      | 1  | 1        |
| 1      | 1             | 1      | 0  | 0        |

**State equation:**  $Q_{n+1} = Q_n \bar{J} + Q_n K \bar{bar}$

**State equation:**  $Q_{n+1} = Q_n \bar{J} + Q_n K \bar{bar}$

### State reduction:

The state reduction technique generally prevents the addition of duplicate states. The reduction in redundant states reduces the number of flip-flops and logic gates, reducing the cost of the final circuit. Two states are said to be equivalent if every possible set of inputs generates exactly the same output and the same next state. When two states are equal, one of them can be eliminated without changing the input-output relationship. The state reduction algorithm is applied in the state table to reduce equivalent states.

### State assignment:

State assignment refers to the process of assigning binary values to the states of a sequential machine. The binary values should be given to the states in such a way that flip-flop input functions may be implemented with a minimum number of logic gates.

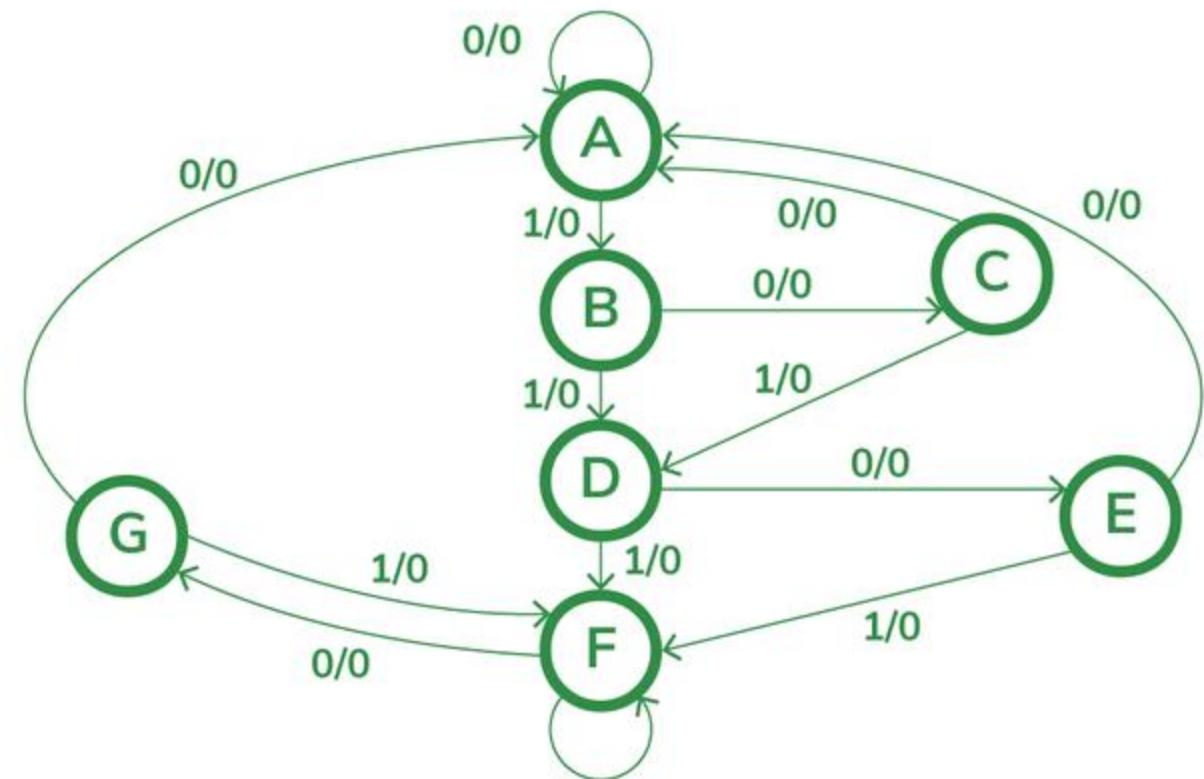
### State assignment rules are as follows:

**Rule 1:** States having the same next state for a given input condition should have assignments that can be grouped into logically adjacent cells in a K-map.

**Rule 2:** States that are the next states of a single state should have assignments that can be grouped into logically adjacent cells in a K-map.

Example 1: To explain the concept of state reduction let us consider the state table as

|   | Present state | Next state | Output |     |
|---|---------------|------------|--------|-----|
|   | X=0           | X=1        | X=0    | X=1 |
| a | a             | b          | 0      | 0   |
| b | c             | d          | 0      | 0   |
| c | a             | d          | 0      | 0   |
| d | e             | f          | 0      | 1   |
| e | a             | f          | 0      | 1   |
| f | g             | f          | 0      | 1   |
| g | a             | f          | 0      | 1   |



The state diagram for the above state table is

State diagram before reduction

**Step1:** First here we are supposed to identify two or more similar states in the next state and output state. In the above table if we observe states of e and g are having the same next state and output values for all combinations of input i.e. X=0 and X=1. So eliminate the g state in the state table and wherever g is present replace it with e. Because e and g both are the same i.e. e=g.

| Present state | Next state |     | Output |     |
|---------------|------------|-----|--------|-----|
|               | X=0        | X=1 | X=0    | X=1 |
| a             | a          | b   | 0      | 0   |
| b             | c          | d   | 0      | 0   |
| c             | a          | d   | 0      | 0   |
| d             | e          | f   | 0      | 1   |
| e             | a          | f   | 0      | 1   |
| f             | e(g=e)     | f   | 0      | 1   |

**Step 2:** Again check if any two states have similar values or not. If any two states have the same next state and output then eliminate one state.

Here d and f are having the same next state value and output. So eliminate f and wherever f is present replace it with d. Because both are the same d=f

| Present state | Next state |        | Output |     |
|---------------|------------|--------|--------|-----|
|               | X=0        | X=1    | X=0    | X=1 |
| a             | a          | b      | 0      | 0   |
| b             | c          | d      | 0      | 0   |
| c             | a          | d      | 0      | 0   |
| d             | e          | d(d=f) | 0      | 1   |
| e             | a          | d(d=f) | 0      | 1   |

**Step 3:** Further observe if any similar states are present or not. The states c and e are having same next states but they are having different outputs. So we can not consider it a reduction state.

**Step 4:** If you observed the state table, the states are represented by using the alphabet. We can not proceed further if we are having alphabets, so, assigning binary numbers to alphabets is called a state assignment.

To assign binary numbers to the state we have to consider the minimum number of bits.

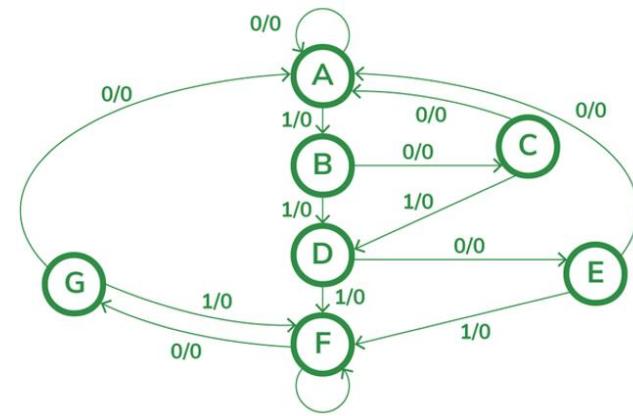
The codes must contain n bits for a circuit with m states, where  $2^n \geq m$ . In this case, each state requires  $2^3 \geq 5 \geq 3$  bits to be represented. With three bits, there are eight possible combinations, of which five can be used to represent the states.

| State  | Assignment 1 |
|--------|--------------|
| Binary |              |
| a      | 000          |
| b      | 001          |
| c      | 010          |
| d      | 011          |
| e      | 100          |

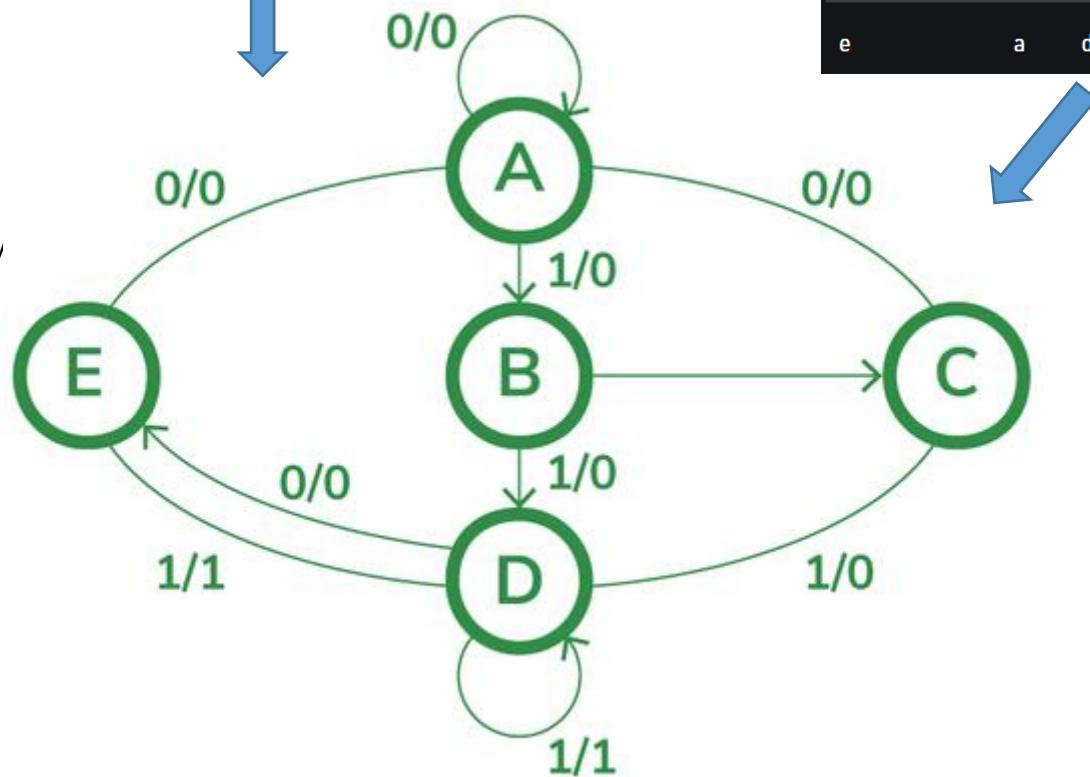
| State | Assignment 1 |
|-------|--------------|
|       | Binary       |
| a     | 000          |
| b     | 001          |
| c     | 010          |
| d     | 011          |
| e     | 100          |

Step 5: Replacing the alphabets with binary numbers.

| Present state | Next state | Output |     |     |
|---------------|------------|--------|-----|-----|
|               | X=0        | X=1    | X=0 | X=1 |
| 000           | 000        | 001    | 0   | 0   |
| 001           | 010        | 011    | 0   | 0   |
| 010           | 000        | 011    | 0   | 0   |
| 011           | 100        | 011    | 0   | 1   |
| 100           | 000        | 011    | 0   | 1   |



| Present state | Next state | Output |     |     |
|---------------|------------|--------|-----|-----|
|               | X=0        | X=1    | X=0 | X=1 |
| a             | a          | b      | 0   | 0   |
| b             | c          | d      | 0   | 0   |
| c             | a          | d      | 0   | 0   |
| d             | e          | d(f=d) | 0   | 1   |
| e             | a          | d(f=d) | 0   | 1   |



# State Reduction Example

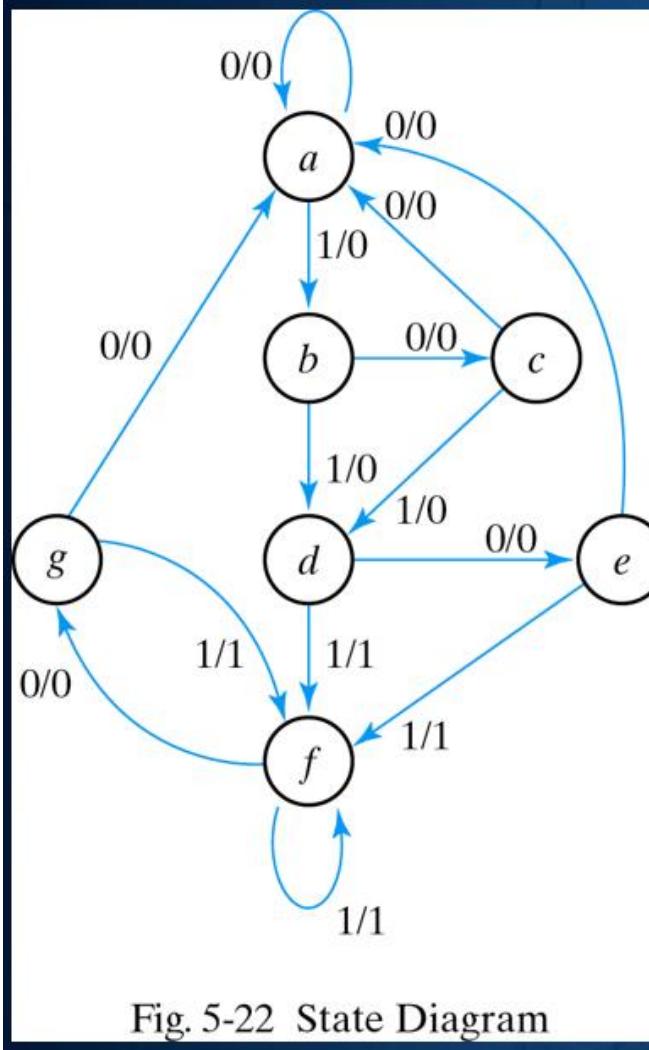


Fig. 5-22 State Diagram

| Present State | Next State<br>$x=0$ $x=1$ | Output<br>$x=0$ $x=1$ |
|---------------|---------------------------|-----------------------|
| a             | a   b                     | 0   0                 |
| b             | c   d                     | 0   0                 |
| c             | a   d                     | 0   0                 |
| d             | e   f                     | 0   1                 |
| e             | a   f                     | 0   1                 |
| f             | g   f                     | 0   1                 |
| g             | a   f                     | 0   1                 |

Find the states for which the next states and outputs are the same

# Example (Cont.)

| Present State | Next State<br>$x=0$ $x=1$ | Output<br>$x=0$ $x=1$ |
|---------------|---------------------------|-----------------------|
| a             | a   b                     | 0   0                 |
| b             | c   d                     | 0   0                 |
| c             | a   d                     | 0   0                 |
| d             | e   f                     | 0   1                 |
| e             | a   f                     | 0   1                 |
| f             | c   f                     | 0   1                 |

In the next state, g is replaced with e

| Present State | Next State<br>$x=0$ $x=1$ | Output<br>$x=0$ $x=1$ |
|---------------|---------------------------|-----------------------|
| a             | a   b                     | 0   0                 |
| b             | c   d                     | 0   0                 |
| c             | a   d                     | 0   0                 |
| d             | e   d                     | 0   1                 |
| e             | a   d                     | 0   1                 |

In the next state, f is replaced with d

# Example (Cont.)

| Present State | Next State<br>$x=0$ $x=1$ | Output<br>$x=0$ $x=1$ |
|---------------|---------------------------|-----------------------|
| a             | a   b                     | 0   0                 |
| b             | c   d                     | 0   0                 |
| c             | a   d                     | 0   0                 |
| d             | e   d                     | 0   1                 |
| e             | a   d                     | 0   1                 |

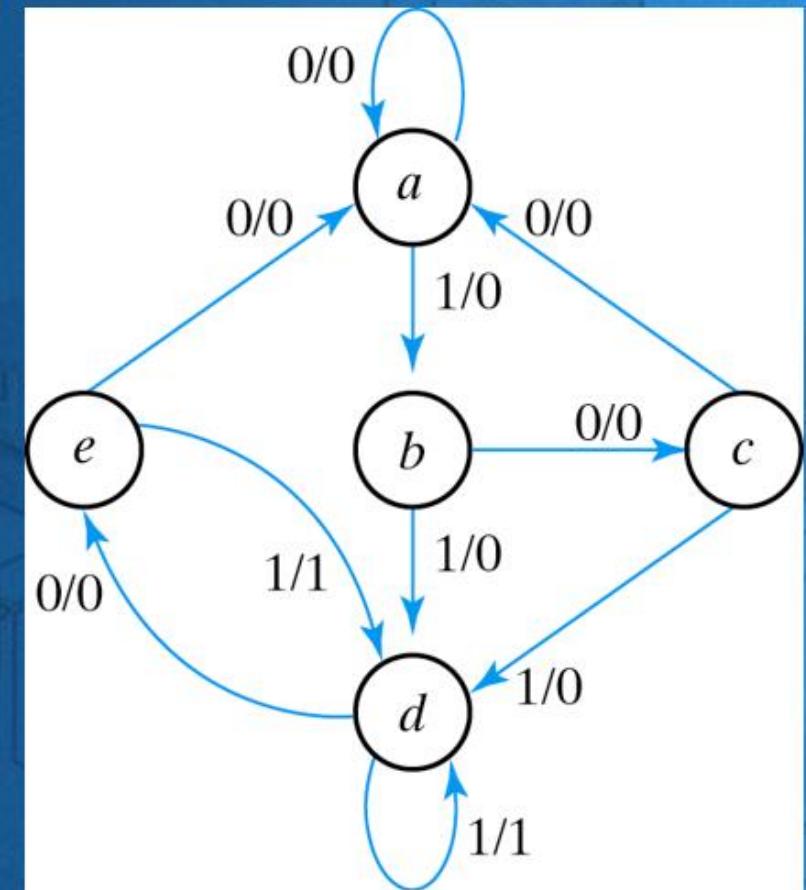


Fig. 5-23 Reduced State Diagram

# State Assignments

- ◆ You need to assign binary values for each state so that they can be implemented
- ◆ You need to use enough number of bits to cover all the states

| Present State | Next State<br>$x=0$ $x=1$ |   | Output<br>$x=0$ $x=1$ |   |
|---------------|---------------------------|---|-----------------------|---|
| a             | a                         | b | 0                     | 0 |
| b             | c                         | d | 0                     | 0 |
| c             | a                         | d | 0                     | 0 |
| d             | e                         | d | 0                     | 1 |
| e             | a                         | d | 0                     | 1 |

| Present State | Next State<br>$x=0$ $x=1$ |     | Output<br>$x=0$ $x=1$ |   |
|---------------|---------------------------|-----|-----------------------|---|
| 000           | 000                       | 001 | 0                     | 0 |
| 001           | 010                       | 011 | 0                     | 0 |
| 010           | 000                       | 011 | 0                     | 0 |
| 011           | 100                       | 011 | 0                     | 1 |
| 100           | 000                       | 011 | 0                     | 1 |



## Department of Electronics and Communication Engineering



# **MARRI LAXMAN REDDY INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

(AN AUTONOMOUS INSTITUTION)

(Approved by AICTE, New Delhi & Affiliated to JNTUH, Hyderabad)

Accredited by NBA and NAAC with 'A' Grade & Recognized Under Section 2(f) & 12(B) of the UGC act, 1956

# DIGITAL LOGIC DESIGN AND COMPUTER ORGANIZATION

- **UNIT-II: Registers, Shift Registers, Ripple counters,  
Synchronous counters**

Presentation By

**Dr. Bapi Debnath**

Assoc. Prof.

Department of ECE



**MARRI LAXMAN REDDY  
INSTITUTE OF TECHNOLOGY AND MANAGEMENT**

(AN AUTONOMOUS INSTITUTION)

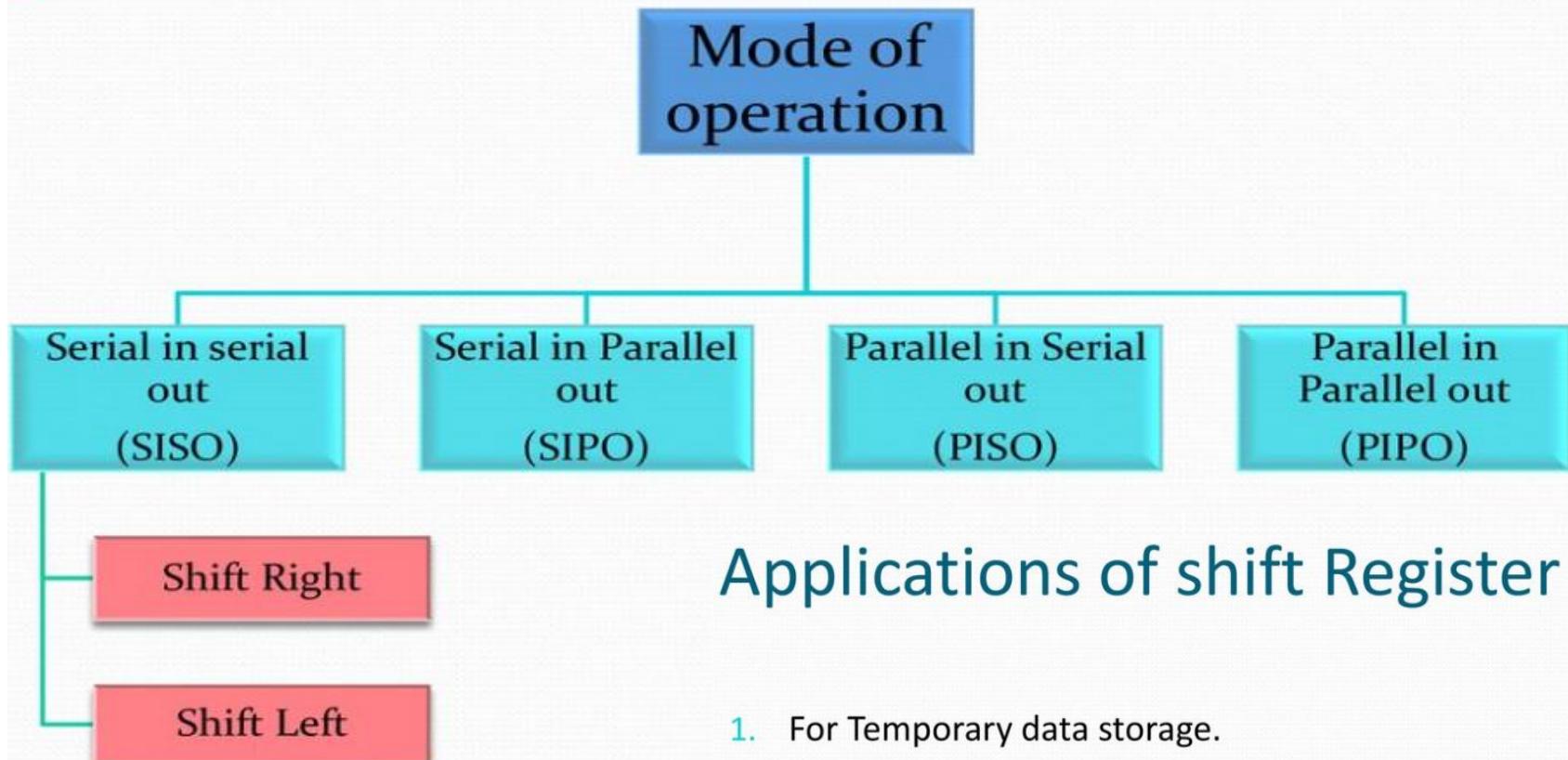
(Approved by AICTE, New Delhi & Affiliated to JNTUH, Hyderabad)

Accredited by NBA and NAAC with 'A' Grade & Recognized Under Section 2(f) & 12(B) of the UGC act, 1956

# INTRODUCTION: REGISTERS

- Register is an important application of Flip-Flop.
- Flip- Flop is a 1 bit memory cell which can be used for storing the digital data.
- To increase the *storage capacity in terms of number of bits*, we have to use a group of flip-flop. *Such a groups of flip-flop is known as a “Register”.*
- Thus register is a group of flip-flops. *The “n-bit” register will consist of “n” number of flip flops.*

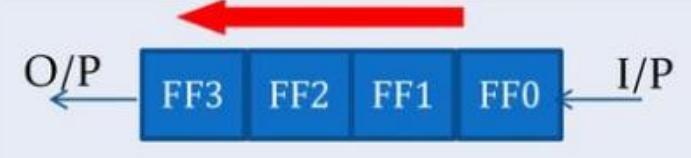
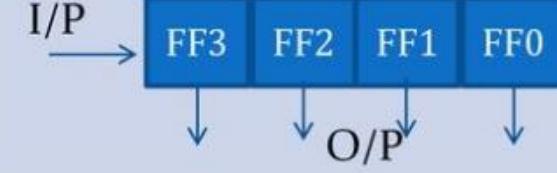
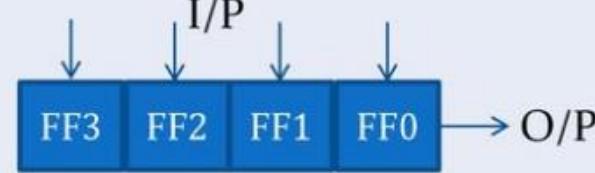
# Classification of Registers :



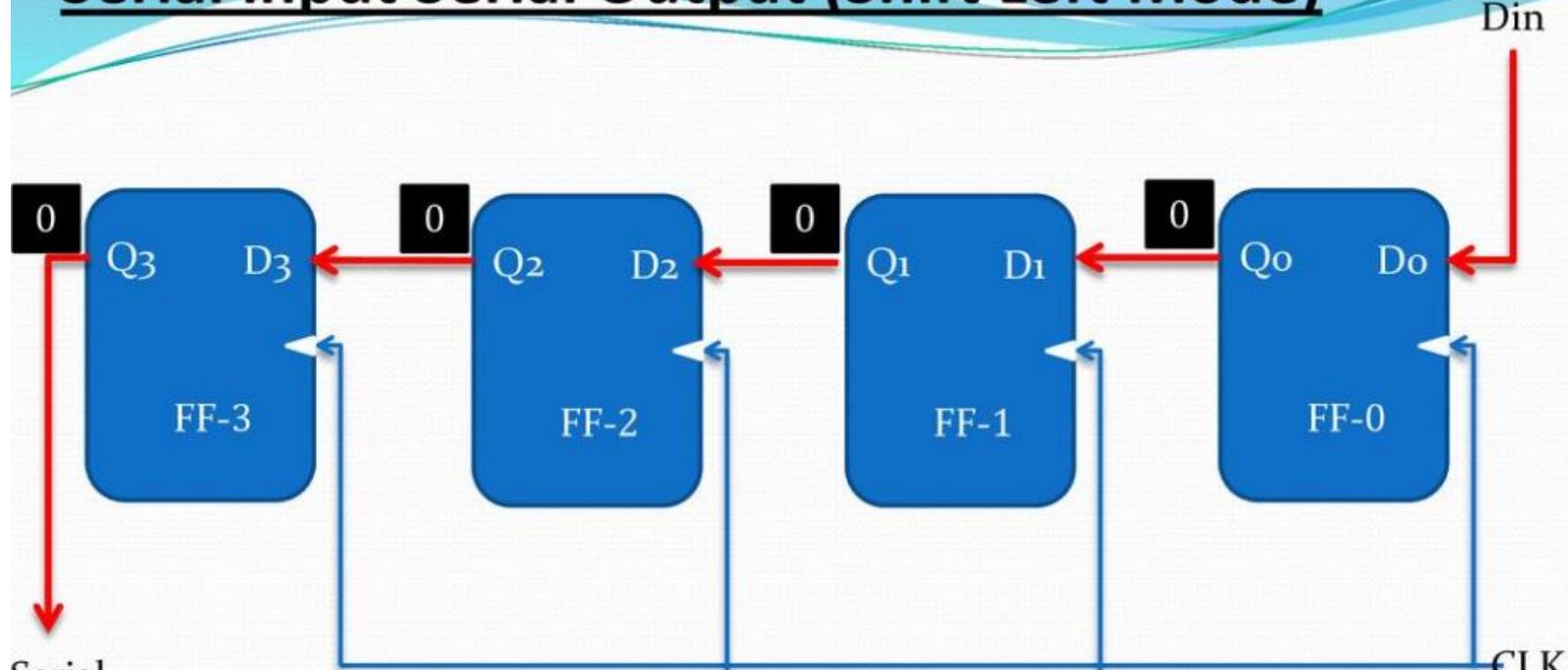
## Applications of shift Register

1. For Temporary data storage.
2. For multiplication and division
3. As a delay line
4. Ring Counter
5. Parallel to serial converter

# Shift Register

| Sr. No. | Mode                  | Illustrative Diagram   | Comments  |
|---------|-----------------------|--|---|
| 1.      | SISO<br>(Shift Right) |    | Data bits shift from Left to Right by 1 position per clock cycle.                                     |
| 2.      | SISO<br>(Shift Left)  |    | Data bits shift from Right to Left by 1 position per clock cycle.                                     |
| 3.      | SIPO                  |   | All o/p bits are made avail. simult. after <b>4-clk pulse</b>   |
| 4.      | PISO                  |  | All i/p bits are applied simult and. After <b>4-clk pulse</b> the required o/p is available serially. |

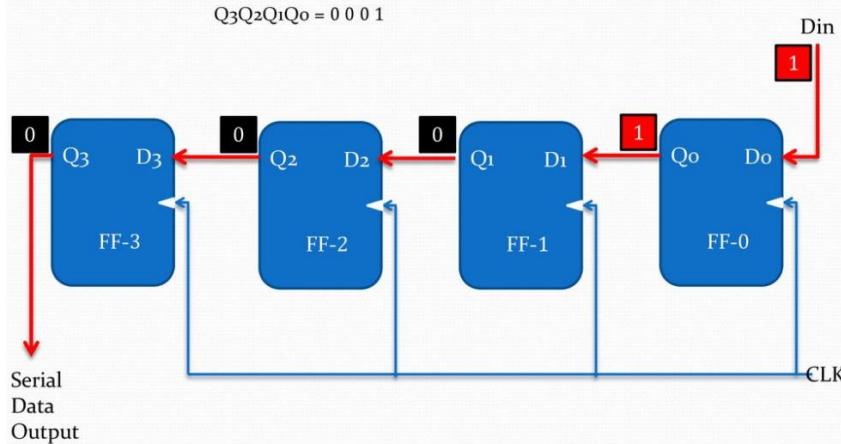
# Serial input Serial Output (Shift Left Mode)



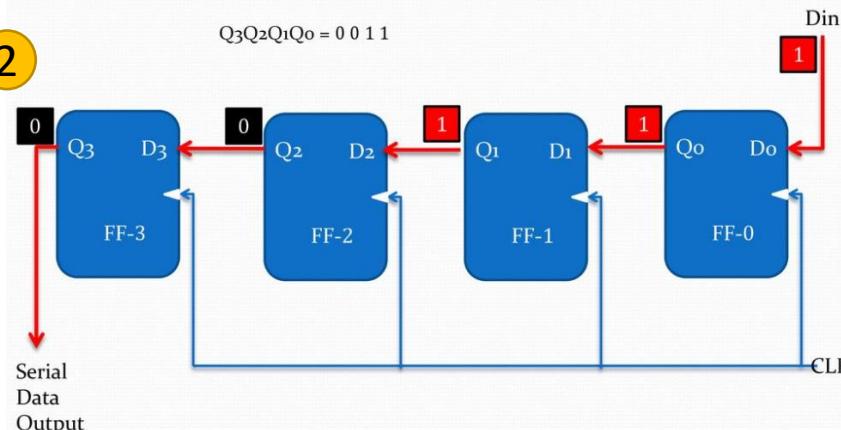
Serial Shift left register

- Before application of clock let assume all outputs are zero and apply MSB bit of the number to entered to Din. So  $Din = Do = 1$ .

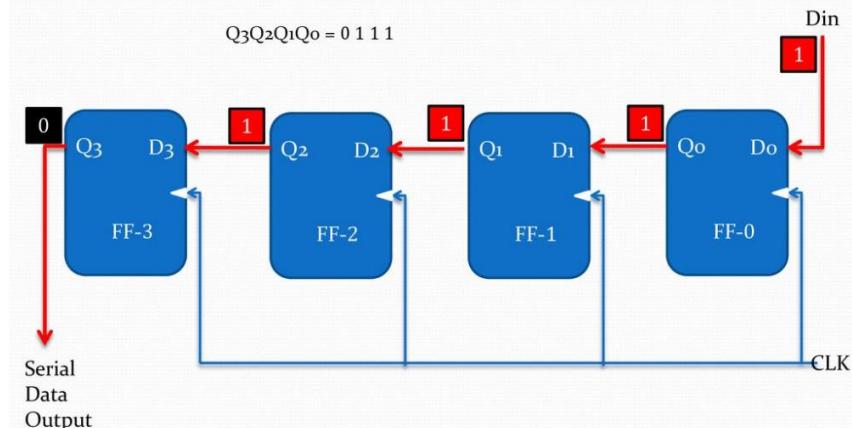
- 1** • Apply the clock . On the first falling edge of clock, the FF-0 is SET and the stored data in the register is



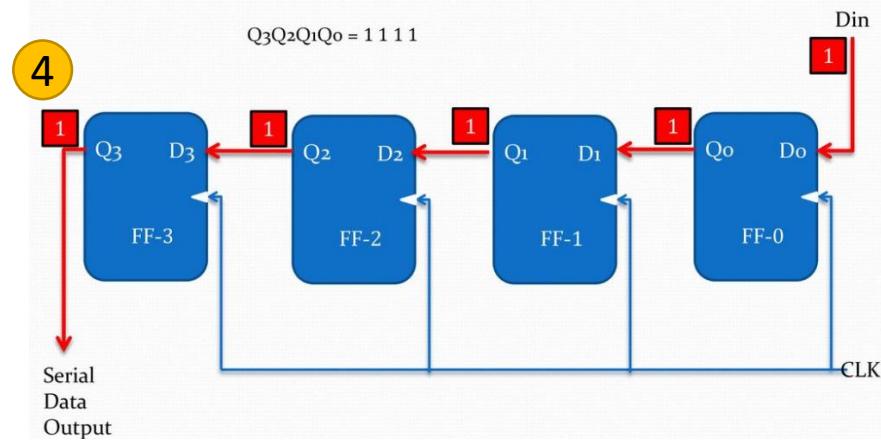
- Apply the NEXT bit to Din . So if Din =1.
- As soon as the next positive edge of the clock hits. FF- 1 will SET and the stored data changes to,



- 3** • Apply the NEXT bit to Din . So if Din =1.  
• As soon as the next positive edge of the clock hits. FF- 2 will SET and the stored data changes to,



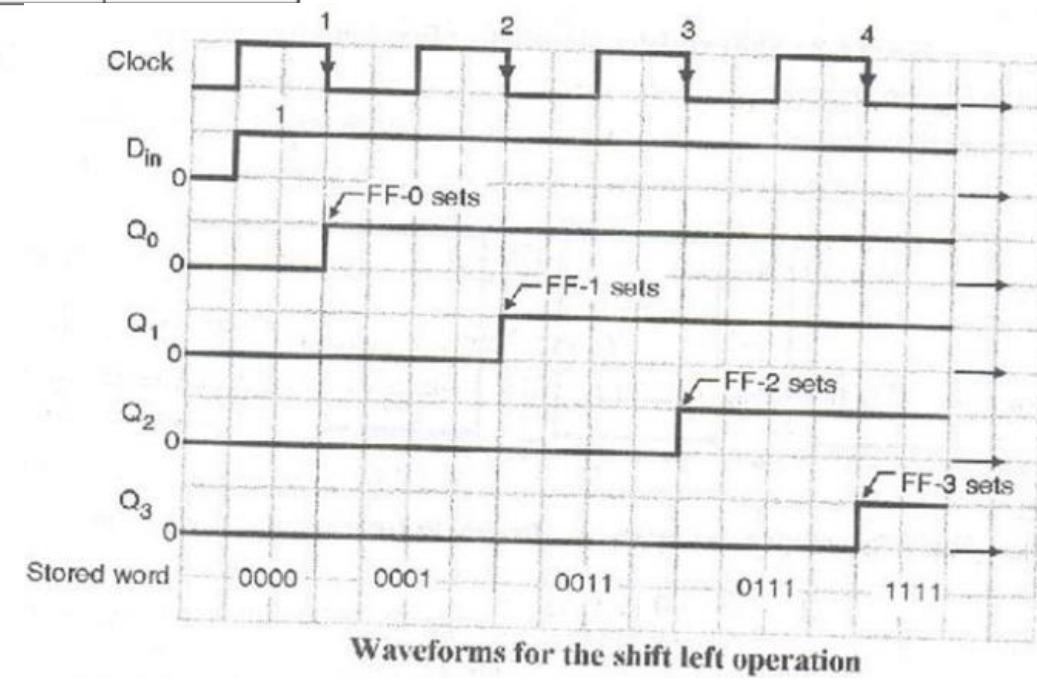
- Apply the NEXT bit to Din . So if Din =1.
- As soon as the next positive edge of the clock hits. FF- 3 will SET and the stored data changes to,



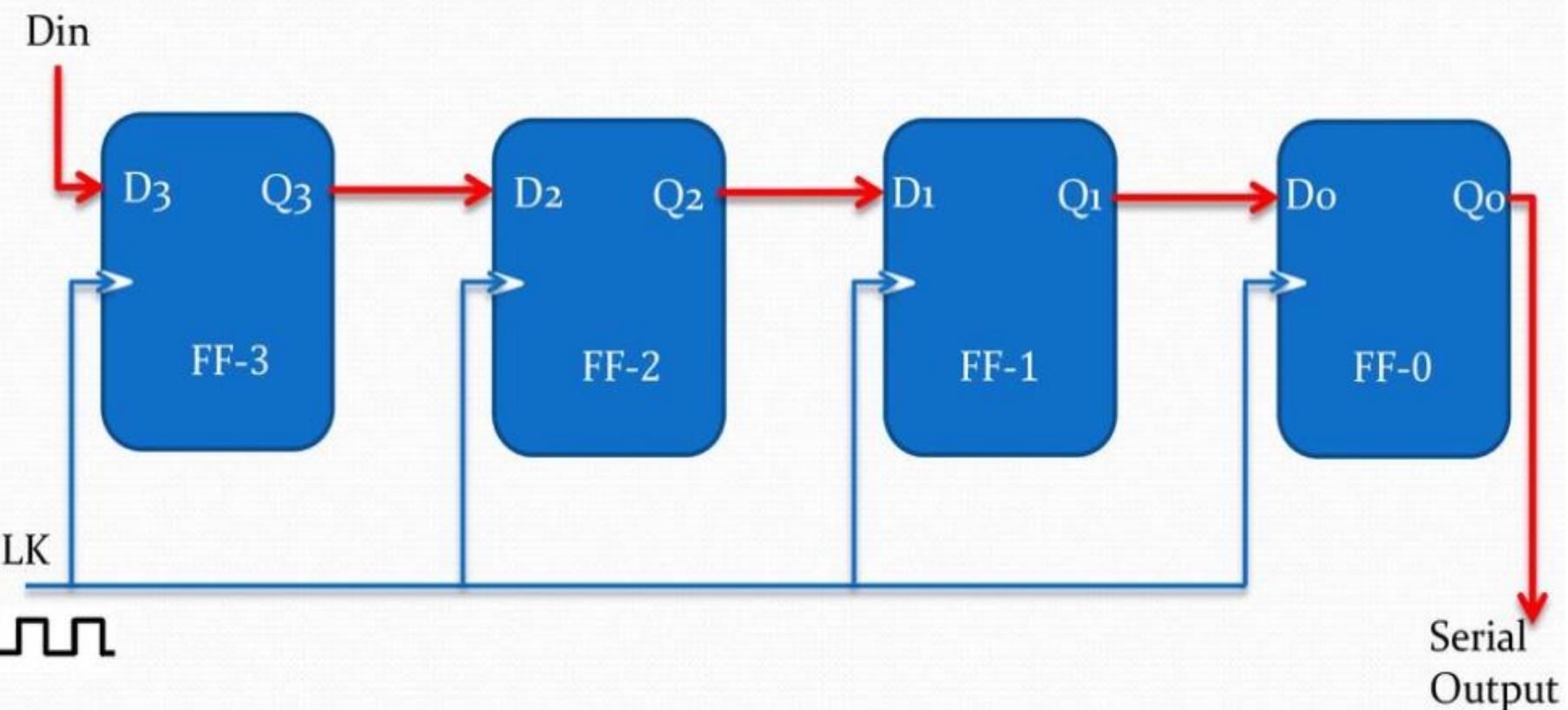
# Summary of shift left operation

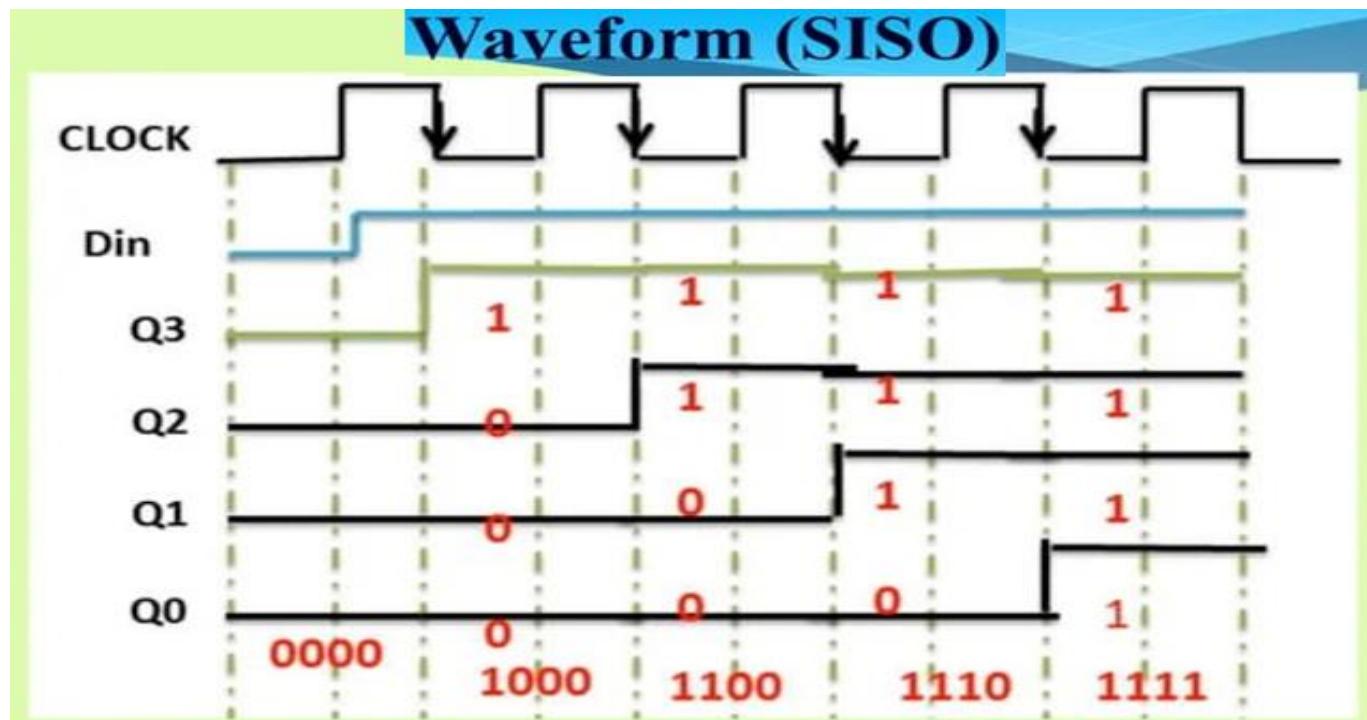
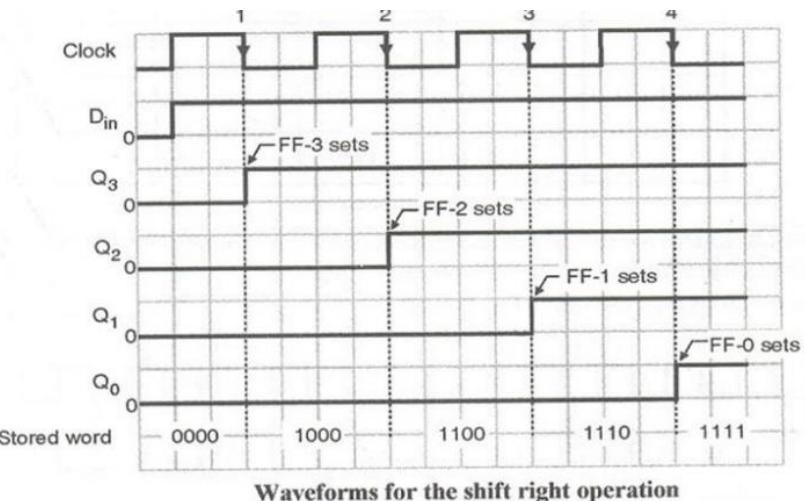
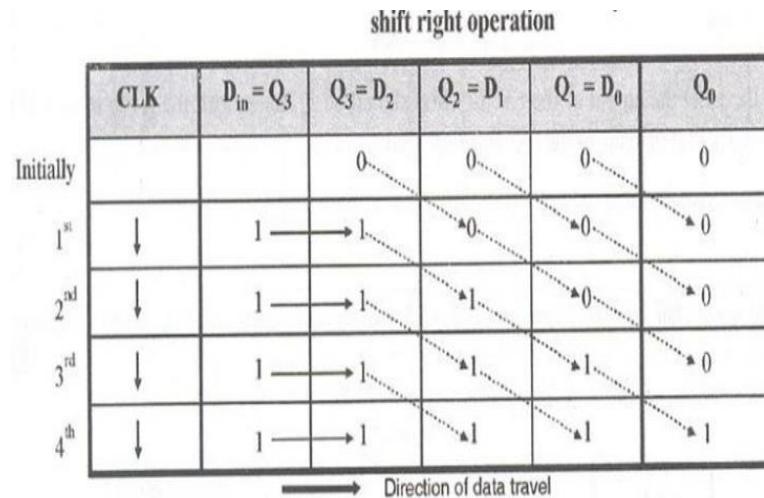
|                 | CLK | Q3 | Q2 | Q1 | Q0 | SERIAL INPUT<br>Din = Do |
|-----------------|-----|----|----|----|----|--------------------------|
| INITIALLY       |     | 0  | 0  | 0  | 0  | -                        |
| 1 <sup>st</sup> | ↓   | 0  | 0  | 0  | 0  | 1 ← 1                    |
| 2 <sup>nd</sup> | ↓   | 0  | 0  | 1  | 1  | 1 ← 1                    |
| 3 <sup>rd</sup> | ↓   | 0  | 1  | 1  | 1  | 1 ← 1                    |
| 4 <sup>th</sup> | ↓   | 1  | 1  | 1  | 1  | 1 ← 1                    |

Direction of data travel ←



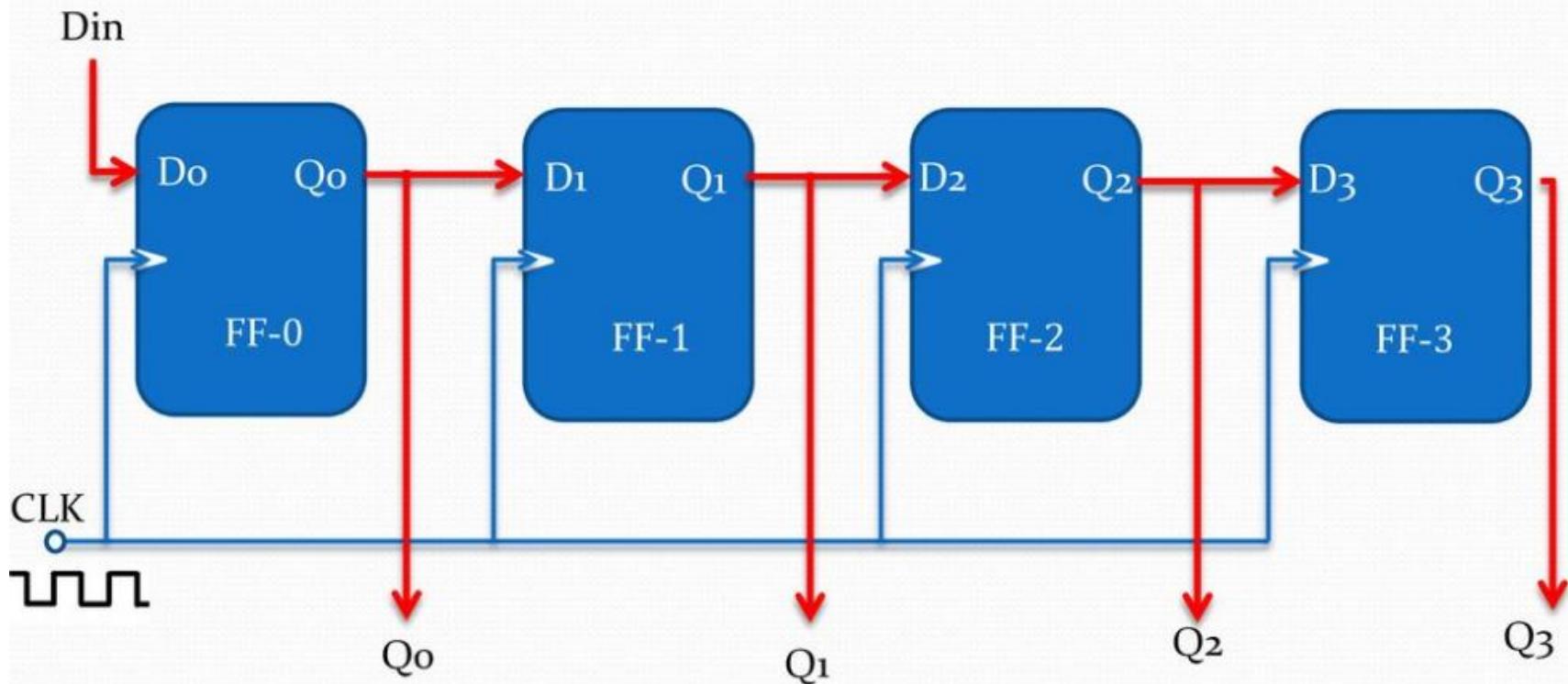
# Serial input Serial Output (Shift Right Mode)





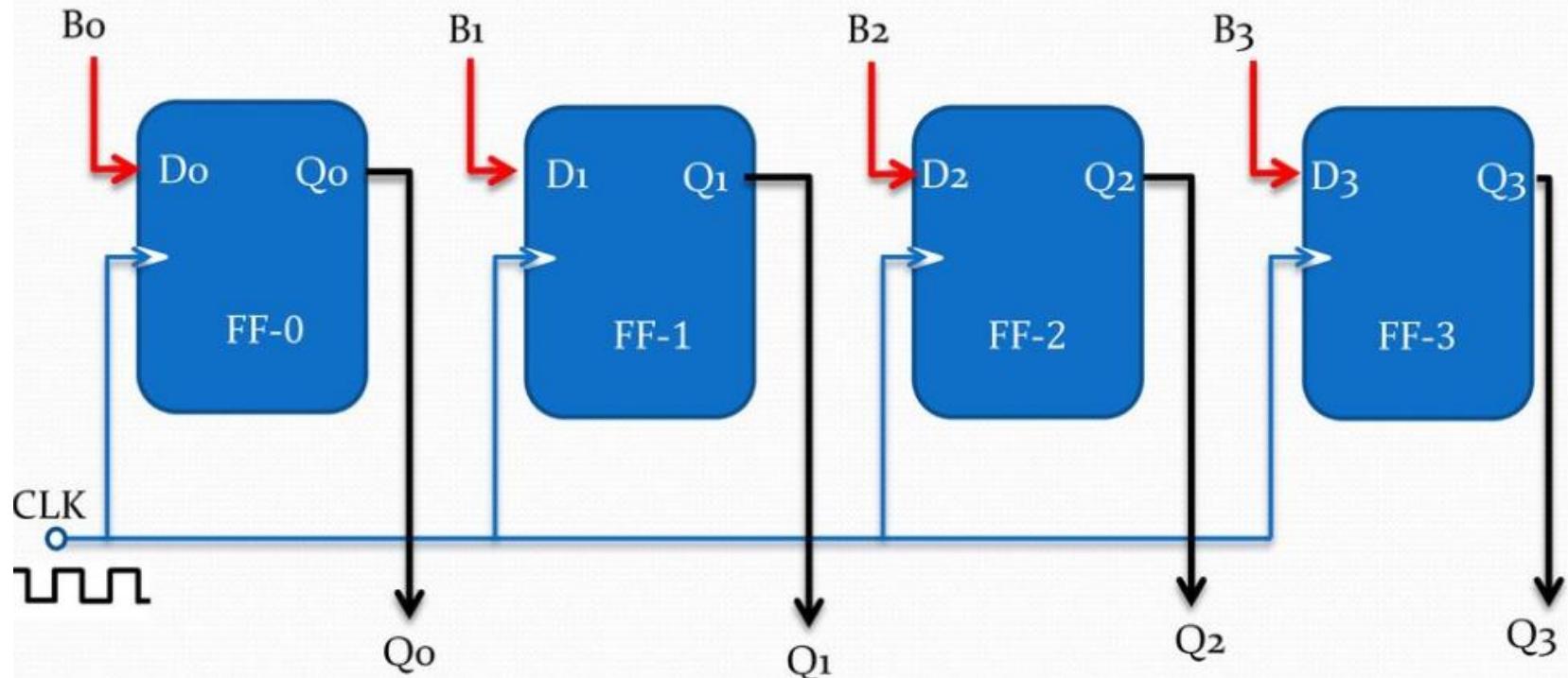
# Serial input Parallel Output (SIPO)

- In this operation the data is entered serially and taken out in parallel.
- That means first the data is loaded bit by bit. The output are disabled as the loading is taking place.
- Number of clock cycles required to load a four bits data is 4. Hence the speed of operation of SIPO mode is same as that of SISO mode.



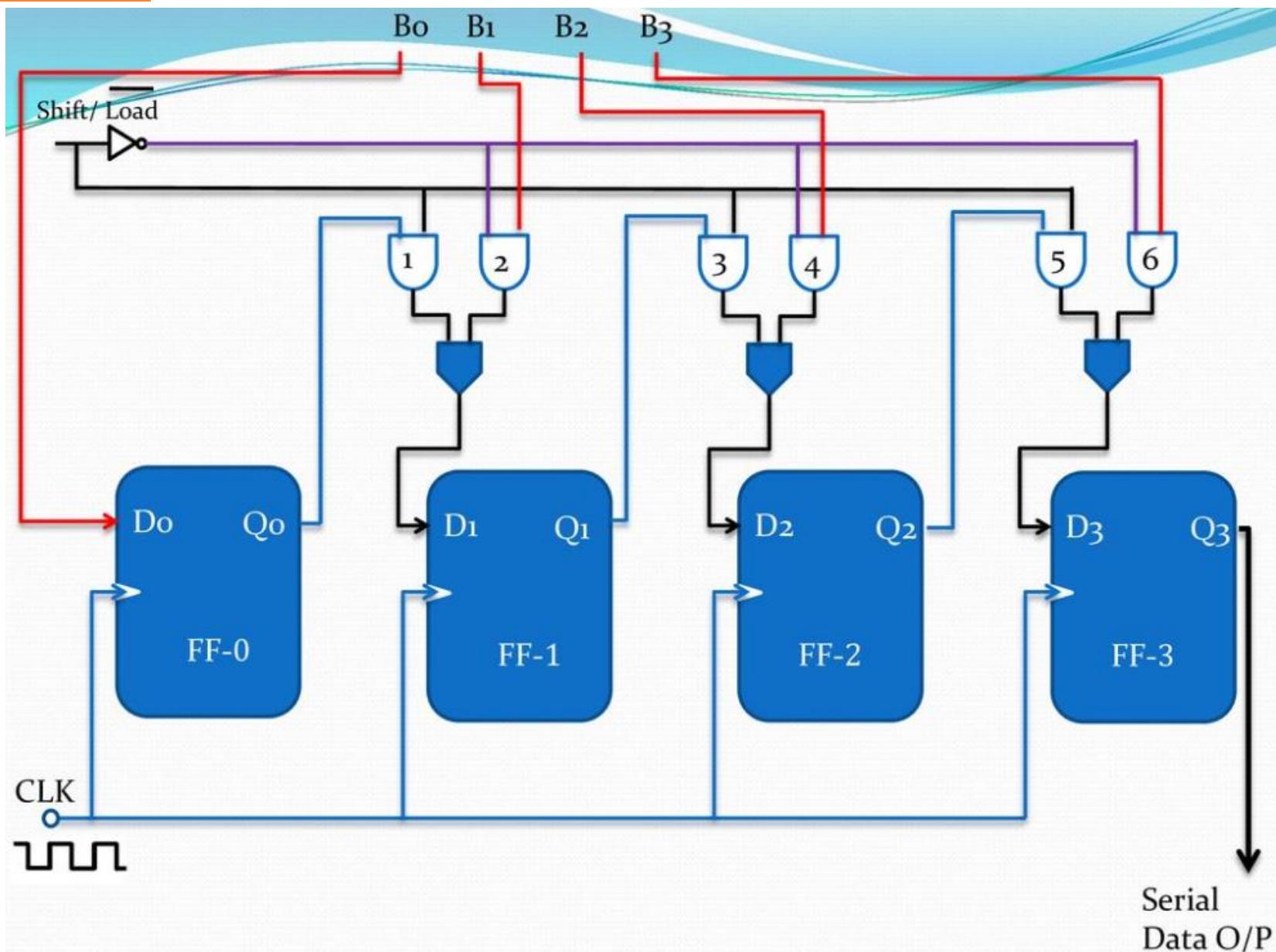
# Parallel input Parallel Output (PIPO)

- In this operation the data are entered parallel.
- The 4-bit binary input  $B_0, B_1, B_2, B_3$  is applied to data inputs  $D_0, D_1, D_2$  and  $D_3$  respectively of the four flip-flops.
- As soon as a positive clock edge is applied, the input binary bits will be loaded into the flip-flops simultaneously.
- The loaded bits will appear simultaneously to the output side. **ONLY ONE CLOCK IS ESSENTIAL TO LOAD ALL THE BITS.**



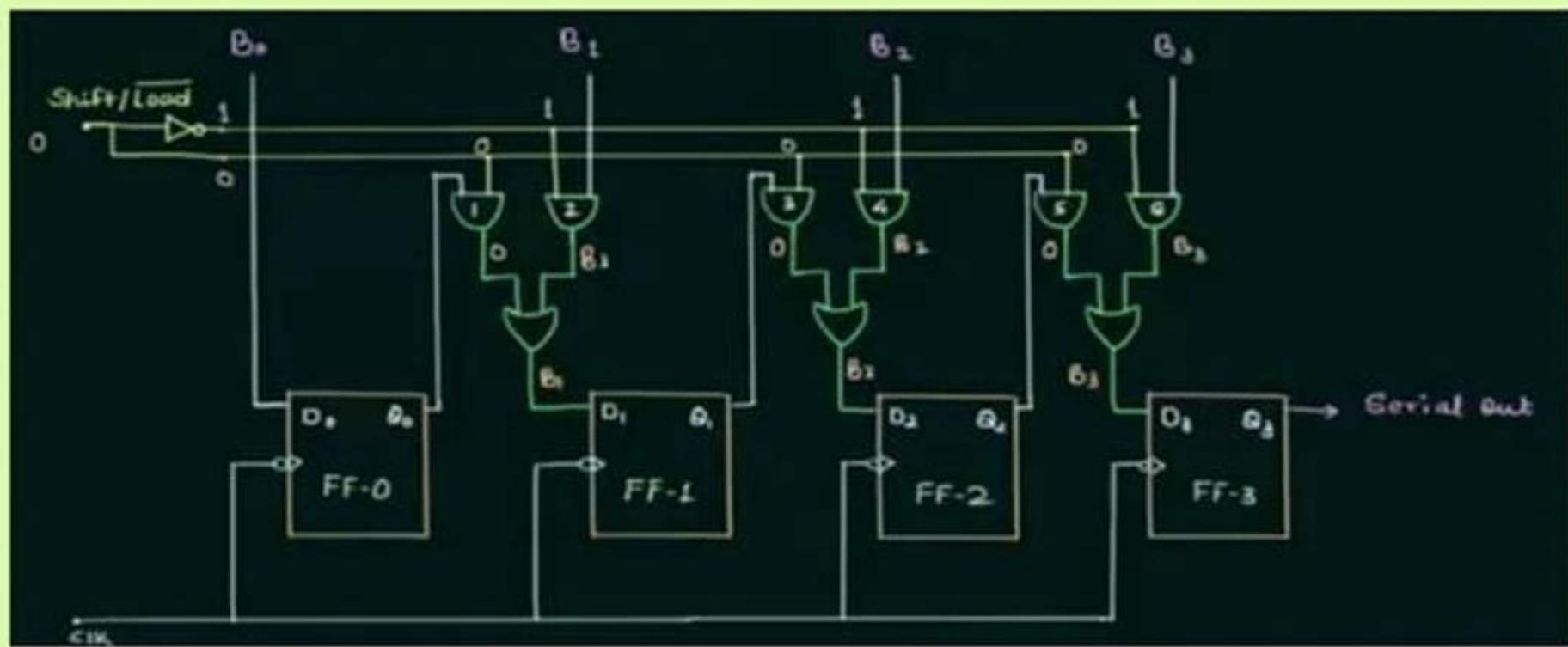
# Parallel input Serial Output (PISO)

- In this operation the data are entered parallel.
- Output of previous FF is connected to the input of the next via a combinational circuit.
- The binary input data  $B_0, B_1, B_2, B_3$  is applied through the same the combinational circuit.
- There are two modes in which this circuit can work namely **shift mode or load mode**.



## Load mode:-

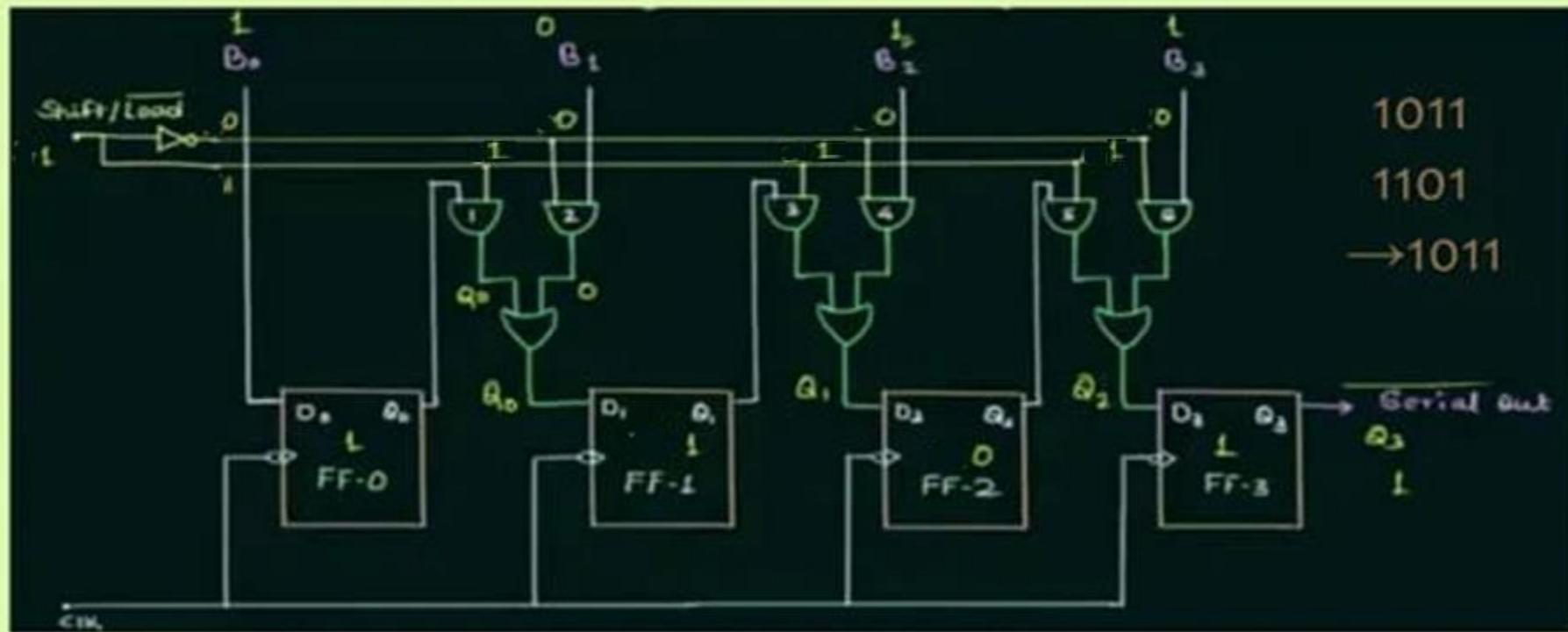
By the load mode we have the parallel input of data in which we have enter or load the data in each of the flip flops simultaneously i.e. in the following example flipflop-0 has bit 0, flipflop- 1 has bit-1, flipflop-2 has bit -2 and flipflop-3 has bit-3. It can be achieve by providing logic 0 at the input.



## Shift mode:-

The second aim of serial output is achieve mode with this operation. It is responsible for serial movement of data i.e. the data will go from flipflop- 0 to flipflop- 1, flipflop- 1 to flipflop- 2, flipflop- 2 to flipflop- 3 and the data store in flipflop -3 is act as serial output.

It can be achieve by providing logic 1 at the input.

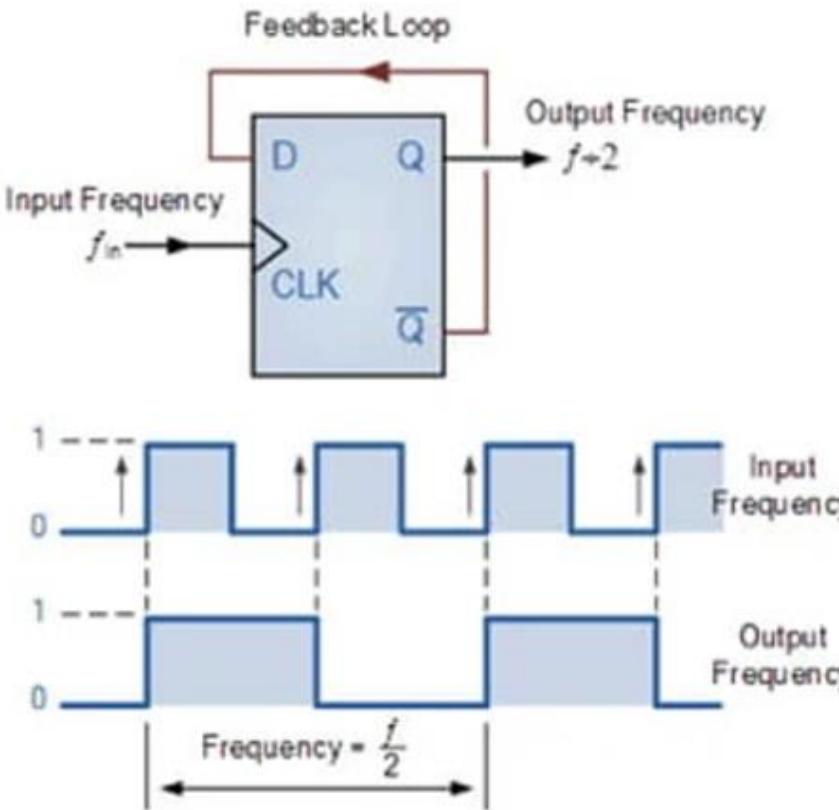


# Introduction –COUNTERS

- A *counter* is a register that goes through a predetermined sequence of states upon the application of clock pulses.
  - Asynchronous counters
  - Synchronous counters
- **Asynchronous Counters** (or *Ripple counters*)
  - the clock signal (CLK) is only used to clock the first FF.
  - Each FF (except the first FF) is clocked by the preceding FF.
- **Synchronous Counters**,
  - the clock signal (CLK) is applied to all FF, which means that all FF shares the same clock signal,
  - **Modulus (MOD)** – the number of states it counts in a complete cycle before it goes back to the initial state.
  - Thus, the number of flip-flops used depends on the MOD of the counter (ie; MOD-4 use 2 FF (2-bit), MOD-8 use 3 FF (3-bit), etc..)
  - Example: MOD-4 Ripple/Asynchronous Up-Counter.

# Frequency Division :

Frequency division is one of the main propose of counters, the division process is used to reduce the frequency of the clock input waveform.



## Divide by 2 counter :

- This type of counters reduces the frequency of the clock input exactly to the half for each flip-flop, this type of counters is called binary ripple counters. For  $n^{\text{th}}$  FFs , the input frequency reduces by the factor  $\frac{f_{in}}{2^n}$  .
- For example, the output frequency :
  - After 4 FFs =  $\frac{f_{in}}{2^4} = \frac{f_{in}}{16}$
  - After 8 FFs =  $\frac{f_{in}}{256}$
  - After 12 FFs =  $\frac{f_{in}}{4096}$

## Asynchronous (Ripple) Counter :

- Definition : Type of counters in which each flip flop output serves as the clock input signal for the next FF in the sequence.
- In Asynchronous counters , the input of some or all FFs are triggered not by the common clock pulse, but rather by the transition that occurs in other FF output.
- The FFs don't change states at exactly the same time as they don't have a common clock pulse.
- For a ripple counter consists of n FFs , the number of its states equals  $2^n$  , and It can count from 0 to  $(2^n - 1)$ .

**Notes :**

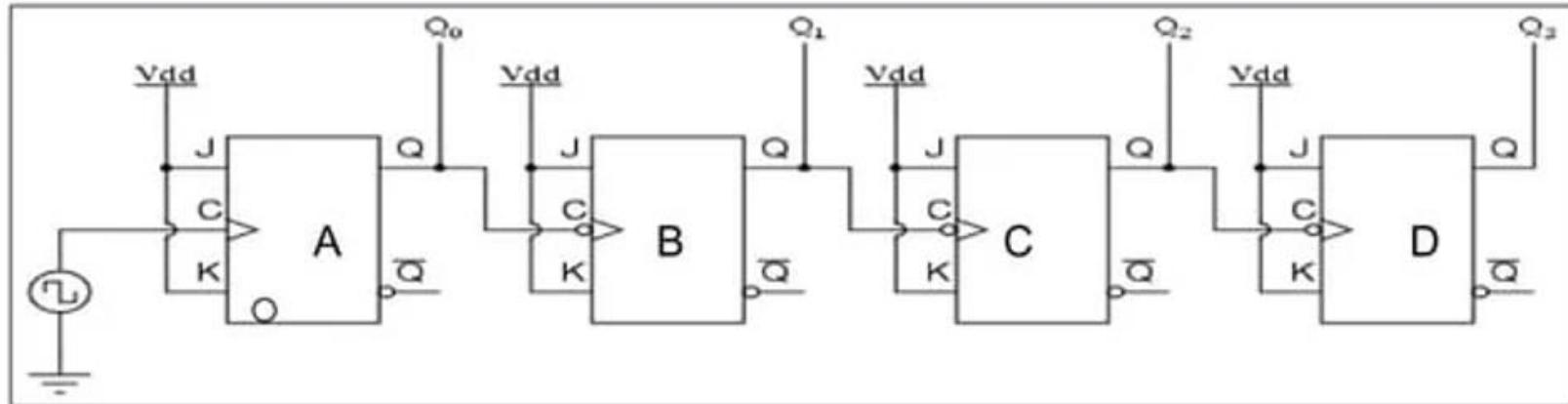
- To operate the toggle mode , the FFs must be connected with  $J=K=1$
- Remember that the FFs are connected in series in Asynchronous systems.

## Binary Ripple

### Counter :

- Binary counter : Group of FFs connected in a special arrangement in which the states of FF represent the binary number equivalent to the number of pulses that have occurred at the input of the counter.
- Binary Ripple counter is called also, a Mod-X counter, where X is the number of counter states and its equal to  $2^n$  , for n FFs .
- The first FF in the counter is called LSB , and the last FF is called MSB.
- The output of MSB divides the input clock frequency by X

- The figure below shows a 4-bit binary counter (Mod-16) :
  - The clock pulse applied only to the Clk input of flip flop A.
  - The input for each of the next FFs is the output of the previous FF. So the output of FF A is the input of FF B , the output of FF B is the input of FF C , and the output of FF C is the input of FF D.
  - FF A will toggle each time the clock pulse make a transition.
  - Since The output of FF A is the input of FF B ,FF B will toggle each time the output of FF A goes from 1 to 0 , and so on.



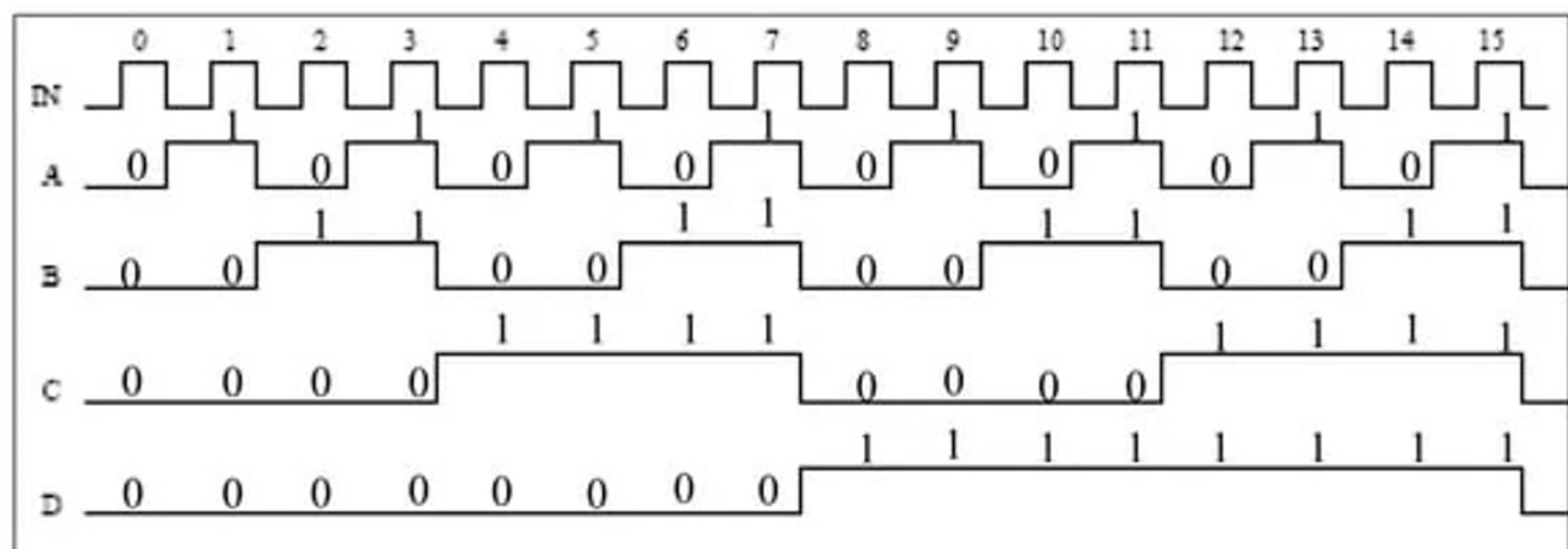
5. This means only FF A responds to the clock pulses. FF B has to wait for FF A to change states before its toggled , and so on.
6. Thus, all FFs don't changes states in exact synchronism with the clock pulse. And because of that it is called asynchronous counters.
7. Because of the phenomenon we talked about in 5 , there is a delay between the response of sequential FFs.
8. So , this type of counters is also commonly called a ripple counter. (simply we can say that the input clock ripples through the counter.)
9. After 15 clock pulse , the counter has finished its first complete cycle (0000 through 1111), and in its 16 clock pulse it will recycle back to (0000) .

Truth table for 4-bit binary counter

| CLK | QA | QB | QC | QD |
|-----|----|----|----|----|
| 0   | 0  | 0  | 0  | 0  |
| 1   | 1  | 0  | 0  | 0  |
| 2   | 0  | 1  | 0  | 0  |
| 3   | 1  | 1  | 0  | 0  |
| 4   | 0  | 0  | 1  | 0  |
| 5   | 1  | 0  | 1  | 0  |
| 6   | 0  | 1  | 1  | 0  |
| 7   | 1  | 1  | 1  | 0  |
| 8   | 0  | 0  | 0  | 1  |
| 9   | 1  | 0  | 0  | 1  |
| 10  | 0  | 1  | 0  | 1  |
| 11  | 1  | 1  | 0  | 1  |
| 12  | 0  | 0  | 1  | 1  |
| 13  | 1  | 0  | 1  | 1  |
| 14  | 0  | 1  | 1  | 1  |
| 15  | 1  | 1  | 1  | 1  |

Note : . A counter may count up or count down or count up and down depending on the input control.

Timing diagram of a 4-bit binary ripple counter



## Asynchronous Down

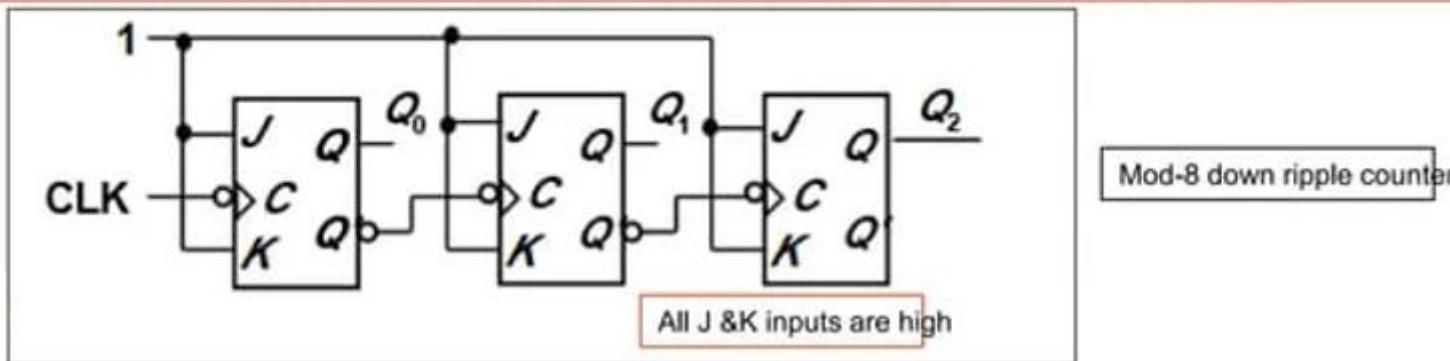
### Counters :

- The ripple down counters, will count downward from maximum count to 0.
- It works in the same method that the ripple up counters works.

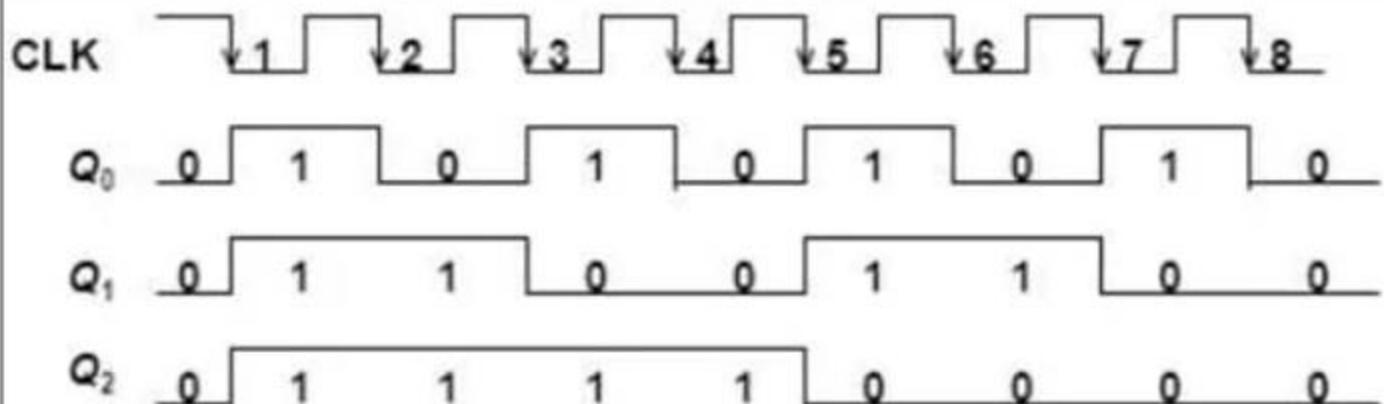
| State | CBA |
|-------|-----|
| 7     | 111 |
| 6     | 110 |
| 5     | 101 |
| 4     | 100 |
| 3     | 011 |
| 2     | 010 |
| 1     | 001 |
| 0     | 000 |

❖ Remark :

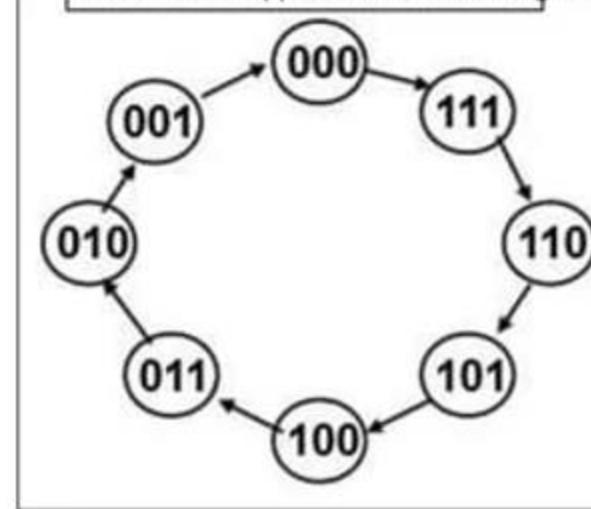
Note that when the clK input of FFs is connected to negative edge , the inverted output (Q) of FF0 will be connected to the input clk input for the FF1 , and so on. While , as you see previously, on the up counters when the clK input of FFs is connected to negative edge , the output (Q) of FF0 will be connected to the input clk input for the FF1



Mod-8 down ripple counter timing diagram



Mod-8 down ripple counter state diagram



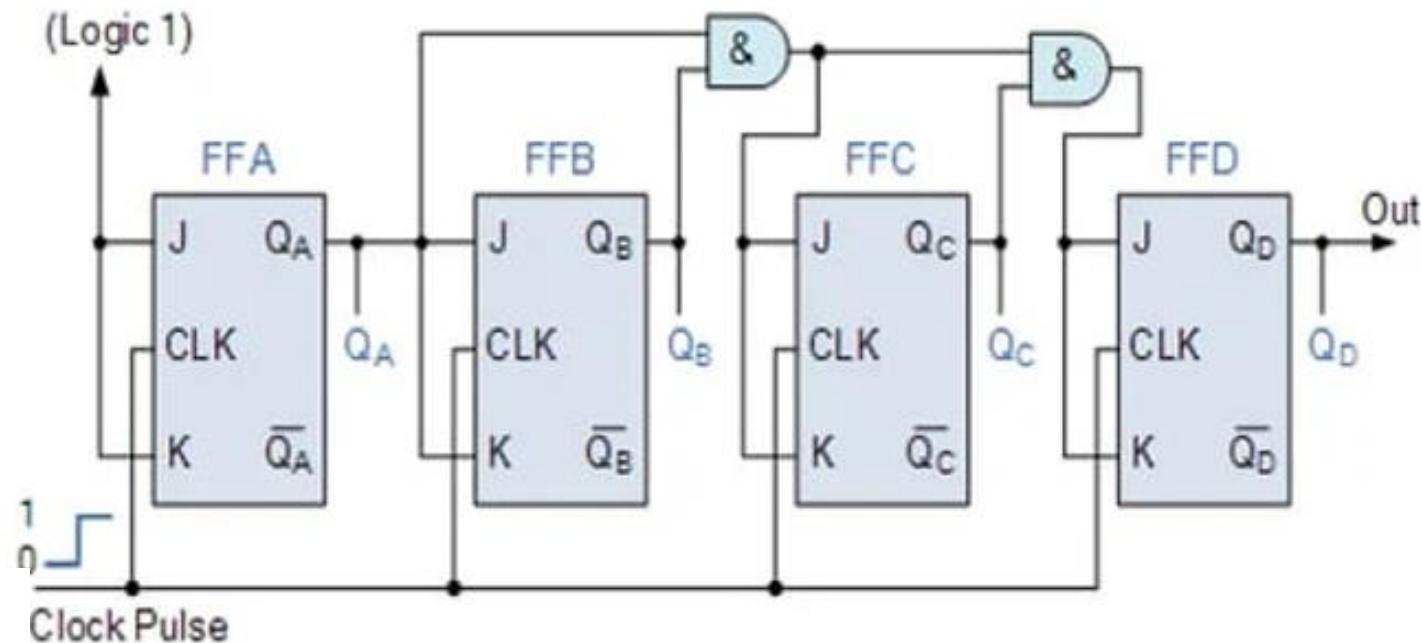
## Asynchronous Counters advantages and disadvantages:

- Advantages :
  1. Asynchronous Counters can easily be made from Toggle or D-type flip-flops.
  2. They are called “Asynchronous Counters” because the clock input of the flip-flops are not all driven by the same clock signal.
  3. Each output in the chain depends on a change in state from the previous flip-flops output.
  4. Asynchronous counters are sometimes called ripple counters because the data appears to “ripple” from the output of one flip-flop to the input of the next.
  5. They can be implemented using “divide-by-n” counter circuits.
  6. Truncated counters can produce any modulus number count.

- Disadvantages :
  1. An extra “re-synchronizing” output flip-flop may be required.
  2. To count a truncated sequence not equal to  $2^n$ , extra feedback logic is required.
  3. Counting a large number of bits, propagation delay by successive stages may become undesirably large.
  4. This delay gives them the nickname of “Propagation Counters”.
  5. Counting errors occur at high clocking frequencies.
  6. Synchronous Counters are faster and more reliable as they use the same clock signal for all flip-flops.

## synchronous binary counters :

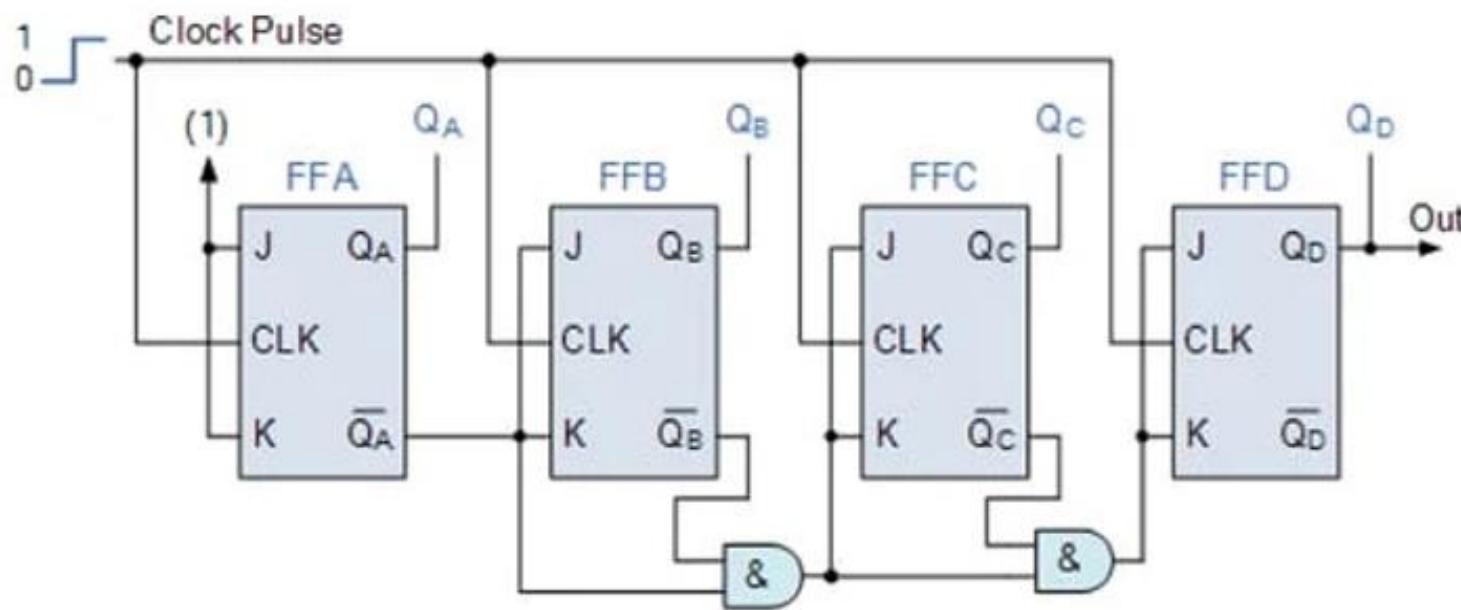
Binary 4-bit Synchronous Up Counter



- the external clock pulses are fed directly to each of the J-K flip flops in the counter chain and that both the J and K inputs are all tied together in toggle mode.
- but only in the first flip-flop, flip-flop FFA (LSB) are they connected **HIGH**, logic “1” allowing the flip-flop to toggle on every clock pulse. Then the synchronous counter follows a predetermined sequence of states in response to the common clock signal, advancing one state for each pulse.
- The J and K inputs of flip-flop FFB are connected directly to the output  $Q_A$  of flip-flop FFA
- The J and K inputs of flip-flops FFC and FFD are driven from separate AND gates which are also supplied with signals from the input and output of the previous stage.
- And the additional AND gates generate the required logic for the JK inputs of the next stage.

- we can easily construct a 4-bit Synchronous Down Counter by connecting the AND gates to the Q output of the flip-flops.

### Binary 4-bit Synchronous Down Counter



## Advantages of synchronous counters over Asynchronous :

- In parallel counter all the FFs will change states simultaneously .Thus ,unlike the asynchronous counters , the propagation delays of the FFs do not add together to produce the overall delay .
- The total response time of a synchronous counter is the time it takes one FF to toggle plus the time for the new logic levels to propagate through a signal AND gate to reach the J,K inputs . that is ,(total delay = FF tpd + AND gate tpd ) .
- This total delay is the same no matter how many FFs are in the counter , and it will generally be much lower than an asynchronous counter with the same number of FFs.
- A synchronous counter can operate at much higher frequency , but the circuitry is more complex than that of the asynchronous counter .

# How To Design Synchronous Counter

- For synchronous counters, all the flip-flops are using the same CLOCK signal. Thus, the output would change synchronously.
- Procedure to design synchronous counter are as follows:-

**STEP 1:** Obtain the State Diagram.

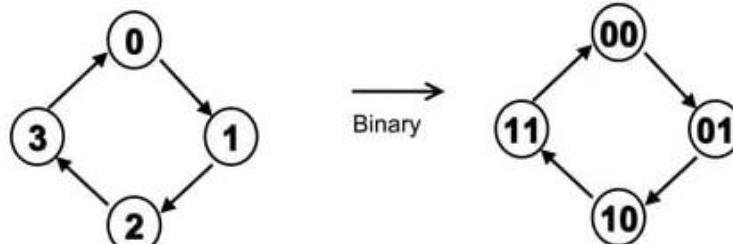
**STEP 2:** Obtain the **Excitation Table** using state transition table for any particular FF (JK or D). Determine number of FF used.

**STEP 3:** Obtain and simplify the function of each FF input using **K-Map**.

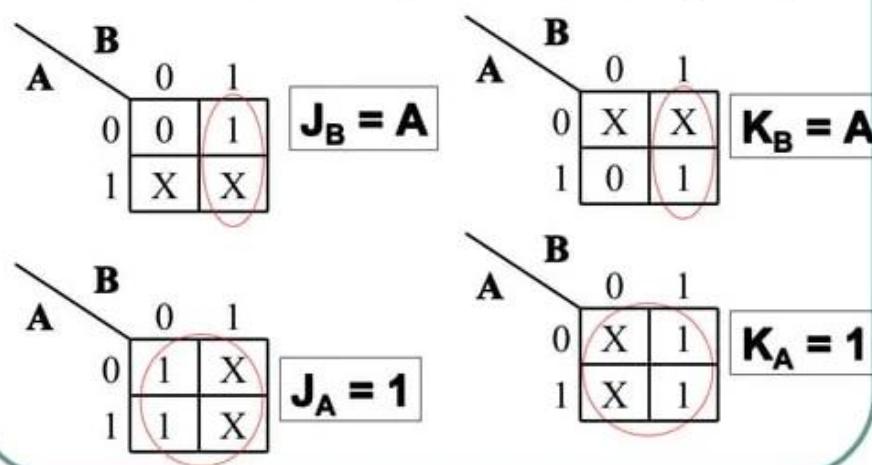
**STEP 4:** Draw the **circuit**.

- Design a MOD-4 synchronous up-counter, using JK FF.

**STEP 1:** Obtain the State transition Diagram



**STEP 3:** Obtain the simplified function using K-Map



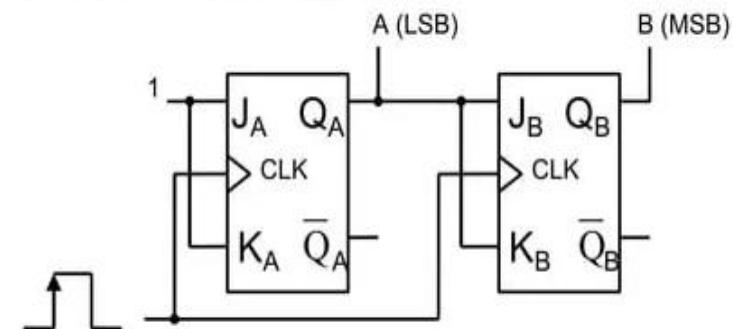
**STEP 2:** Obtain the Excitation table. Two JK FF are used.

| OUTPUT TRANSITION |                  | FF INPUT |   |
|-------------------|------------------|----------|---|
| Q <sub>N</sub>    | Q <sub>N+1</sub> | J        | K |
| 0 → 0             | 0                | 0        | X |
| 0 → 1             | 1                | 1        | X |
| 1 → 0             | 0                | X        | 1 |
| 1 → 1             | 1                | X        | 0 |

Excitation table

| Present State | Next State | Input, J K                    |                               |
|---------------|------------|-------------------------------|-------------------------------|
|               |            | J <sub>B</sub> K <sub>B</sub> | J <sub>A</sub> K <sub>A</sub> |
| 0 0           | 0 1        | 0 X                           | 1 X                           |
| 0 1           | 1 0        | 1 X                           | X 1                           |
| 1 0           | 1 1        | X 0                           | 1 X                           |
| 1 1           | 0 0        | X 1                           | X 1                           |

**STEP 4:** Draw the circuit diagram.



(MOD-4 synchronous up-counter)