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Applied Physics

Unit-1

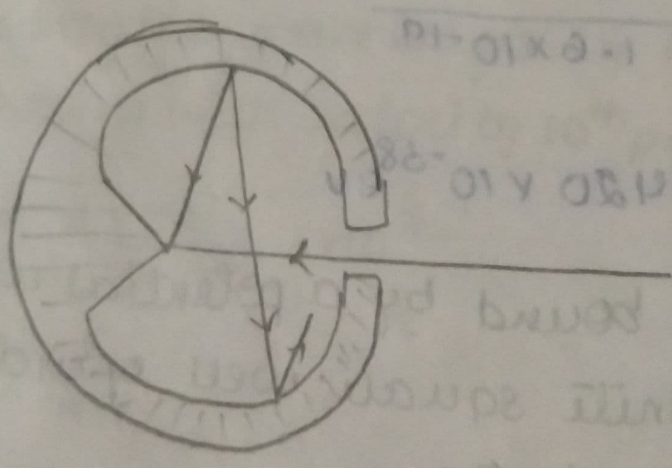
Quantum Mechanics

Black body Radiation:-

A black body which absorbs all the radiation incident on it without reflection is known as black body.

A black body is not only a good absorber of radiation it also a very radiator. It absorbs more when it is cold and radiates more when it is at high temperature.

It is not possible to make a perfect black body in actual practise. A approximate black body can be constructed using hallow object coated from inside with lamp black and having small hole, radiation entering through the hole completely absorbed due to multiple reflection.



In 1859, Kirchhoff stated two laws about black body they are

1. A black body is a perfect radiator at high temperature

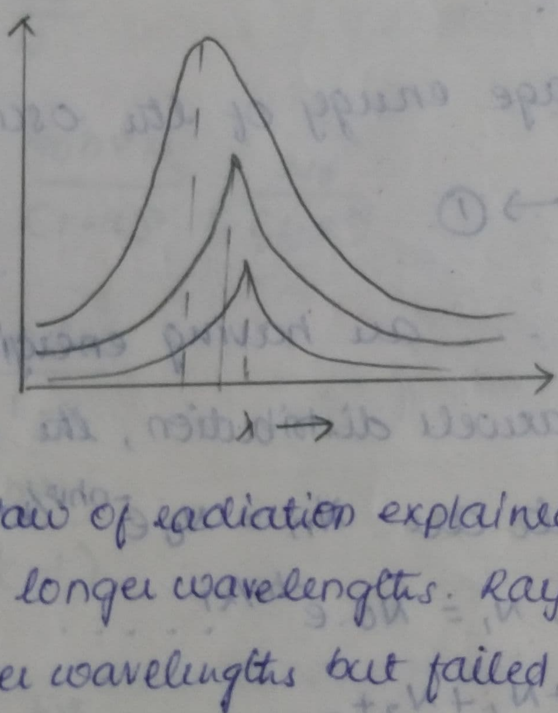
1884, Stephen and Boltzman stated the total energy radiated by black body per unit area, per unit time is proportional to fourth power of absolute temperature

$$E \propto T^4$$

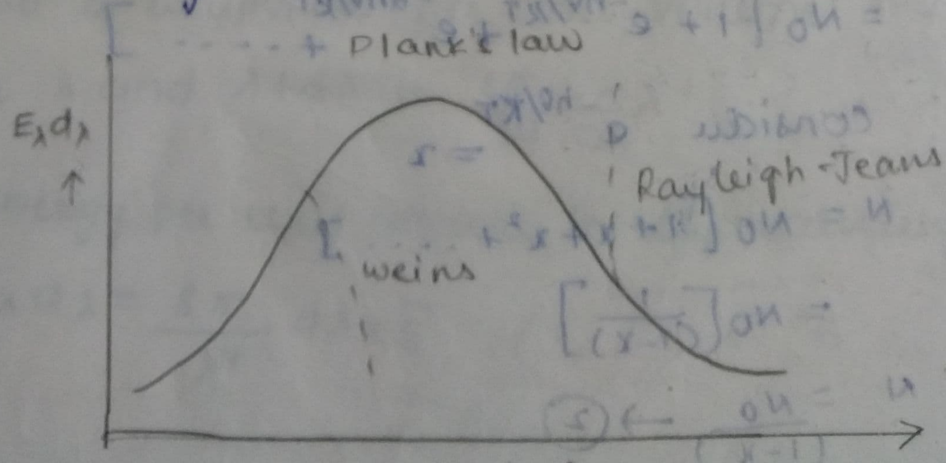
$$E = \sigma T^4$$

where σ is the Stephen constant

- For large frequency means small wavelengths the energy goes on increasing at constant temperature as shown in the below fig.



- The wien's law of radiation explained for lower wavelengths but failed to longer wavelengths. Ray-leigh Jeans formula explained longer wavelengths but failed for shorter wavelengths
- The failure of classical wave theory leads to max planck quantum theory.



Planck's Black body radiation:-

Max planck's theory based on the following assumptions

1. A black body consisting of large no. of oscillating particles, each oscillator, oscillates with a characteristic frequency.

2. The emitted radiation frequency of oscillator is same to oscillator frequency.

3. The radiation emitted by oscillators in the form of quanta or photon instead of continuous. Each photon having the energy $E = h\nu$.

4. Let us consider a body of N oscillators of with total energy E_N.

5. Then the average energy of the oscillator is

$$\bar{E} = \frac{E_N}{N} \rightarrow (1)$$

6. $N_0, N_1, N_2, N_3 \ldots$ are having energies $0, h\nu, 2h\nu, \ldots$

7. According to maxwell distribution, the no. of oscillators in n^{th} energy state are $N_n = N_0 e^{-nh\nu/kT}$

where $N_0 = N_0$, $N_1 = N_0 \cdot e^{-h\nu/kT} \ldots$

Therefore $N = N_0 + N_1 + N_2 + \ldots$

$$= N_0 + N_0 \cdot e^{-h\nu/kT} + N_0 \cdot e^{-2h\nu/kT} + \ldots$$

$$= N_0 \left[1 + e^{-h\nu/kT} + e^{-2h\nu/kT} + \ldots\right]$$

consider $e^{-h\nu/kT} = x$

$$N = N_0 [1 + x + x^2 + \ldots]$$

$$= N_0 \left[\frac{1}{(1-x)}\right]$$

$$N = \frac{N_0}{(1-x)} \rightarrow (2)$$

The total energy will be $E_N = E_0N_0 + E_1N_1 + E_2N_2 + \ldots$

$$= 0 \cdot N_0 + h\nu \cdot N_0 e^{-h\nu/KT} + 2h\nu \cdot N_0 e^{-2h\nu/KT} + \ldots$$

$$= N_0 h\nu e^{-h\nu/KT}\left[1 + 2e^{-h\nu/KT} + 3e^{-2h\nu/KT} + \ldots\right]$$

$$= N_0 h\nu x\left[1 + 2x + 3x^2 + \ldots\right]$$

where $x = e^{-h\nu/KT}$

$$E_N = \frac{N_0 h\nu x}{(1-x)^2} \rightarrow (3)$$

Substitue eqn 2 (4) 3 in one.

$$(1) \Rightarrow \bar{E} = \frac{E_N}{N}$$

$$\bar{E} = \frac{N_0 h\nu x}{(1-x)^2} \Big/ \frac{N_0}{(1-x)}$$

$$= \frac{h\nu x}{1-x}$$

$$\bar{E} = \frac{h\nu}{\left(\frac{1}{x} - 1\right)}$$

$$\bar{E} = \frac{h\nu}{e^{h\nu/KT} - 1}$$

The no. of oscillators per unit volume in the wavelength range λ and $\lambda + d\lambda$ is $\frac{8\pi}{\lambda^4} \times d\lambda$

The energy per unit volume in the wavelength range

$$E_\lambda d\lambda = \frac{8\pi}{\lambda^4} d\lambda \times \bar{E}$$

$$E_\lambda d_\lambda = \frac{8\pi h\nu}{\lambda^4 (e^{h\nu/KT}-1)}$$

$$E_\lambda d_\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{h\nu/KT}-1}$$

This eqn is called planck's law of black body radiation
It is completely explaining black body radiation spectrum. The other laws which are applicable under certain conditions like

case(i)

for shorter wavelengths

$$\exp\left(\frac{h\nu}{KT}\right) \geq 1$$

$$E_\lambda d_\lambda = 8\pi hc\,\lambda^{-5} \cdot \exp\left(\frac{-h\nu}{KT}\right)$$

$$\boxed{E_\lambda d_\lambda = c_1 \lambda^{-5} \exp\left(\frac{-c_2}{KT}\right)}$$

where, $c_1 = 8\pi hc$

$c_2 = hc/K$

this represents weins radiation law

case (ii)

for longer wavelengths, $\frac{h\nu}{KT}$ is small

$$\exp\left(\frac{h\nu}{KT}\right) = \left(1 + \frac{h\nu}{KT}\right)$$

$$E_\lambda = \frac{8\pi hc}{\lambda^5 \left(1 + \frac{h\nu}{KT} - 1\right)}$$

$$E_\lambda = \frac{8\pi hcKT}{\lambda^5 h\nu}$$

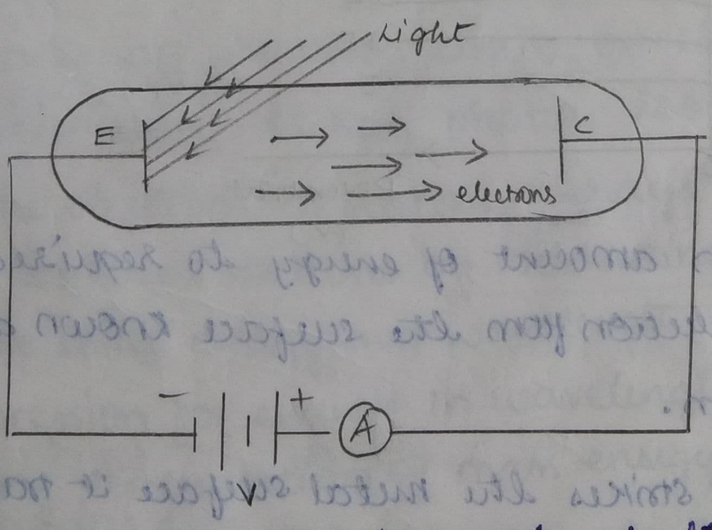
$$\boxed{E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4}}$$ $$\boxed{E_\lambda d_\lambda = \frac{8\pi KT}{\lambda^4}}$$

this represents rayleigh-jeans law

Photoelectric effect:

The emission of electrons from metal plate when illuminated by light or other radiation of same wavelength called photoelectric effect. The emitted electrons are called photo electrons.

As shown in below figure, the equipment consisting of evacuated tube with two electrodes. The electrode E is a metal emits electrons on being illuminated by light. Other electrod C which collects photo electrons to constitute a small current



following are the observations in the photo electric effect

- There is no emission of photo electrons below certain frequency of incident light.

This frequency is known as threshold frequency

• If a retarding potential is applied to the collector the current decreases with increasing retarding potential. This retarding potential minimizes current to zero called stoping potential. Variation of current with potential shown in below figure

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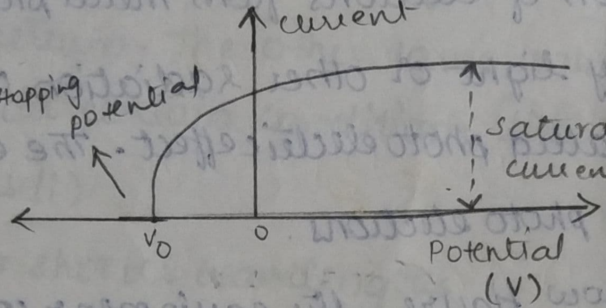
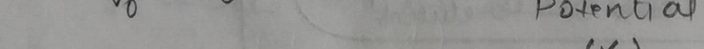


• The effect of ~~incident~~ intensity of same incident radiation as shown in below figure. the stopping potential is independent of intensity of incident radiation.



• The minimum amount of energy to required to release the electron from the surface known as work function.

• When photon strikes the metal surface it transfers its energy to electron. Certain energy used to release electron from surface of metal and remaining energy appears as kinetic energy of electron.

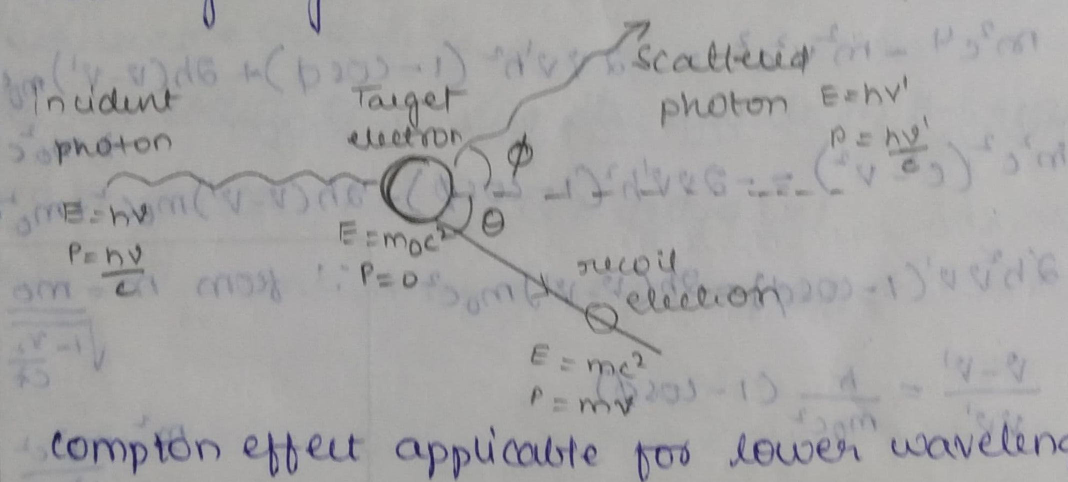
$$h\nu = W + \frac{1}{2}mv^2$$

$$\boxed{E = W + \frac{1}{2}mv^2}$$

$W \rightarrow$ work function

Compton Effect:-

Compton effect studies the scattering of x-rays from a target. the increase in wavelength of x-rays due to scattering is known as compton effect. and change in wavelength known as compton shift. compton shift depends on scattering angle



compton effect applicable for lower wavelength when x-ray photons incident on electrons of target atom. x-ray photon loose some energy. ~~to the electrons to scatter x-rays~~

The scattered electrons by incidenting on electron loose some energy and wavelength increases. The expression for change in wavelength can be derived using conservation of mass energy and momentum equations.

By conservation of mass energy $h\nu + m_0c^2 = h\nu' + mc^2$

$$mc^2 = h(\nu - \nu') + m_0c^2$$

$$m^2c^4 = h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2h(\nu - \nu')m_0c^2 + m_0^2c^4 \quad \text{--- (1)}$$

By conservation of momentum in x-direction.

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + mv\cos\phi$$

By conservation of momentum in y-direction.

$$mvc\cos\theta = h\nu - h\nu'\cos\phi \rightarrow ②$$

$$0 + 0 = \frac{h\nu'}{c}\sin\phi - mv\sin\theta$$

$$mvc\sin\theta = h\nu'\sin\phi \rightarrow ③$$

$$②^2 + ③^2 \Rightarrow m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\phi + \nu'^2) \rightarrow ④$$

subtracting eq ① and ④

$$m^2c^4 - m^2v^2c^2 = -2\nu\nu'h^2(1-\cos\phi) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$m^2c^2(c^2 - v^2) = -2\nu\nu'h^2(1-\cos\phi) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$2h^2\nu\nu'(1-\cos\phi) = 2h(\nu-\nu')m_0c^2$$

$$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\phi)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\phi)$$

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\phi)$$

$$\boxed{\lambda' - \lambda = h/m_0c\,(1-\cos\phi)}$$

$\because$ from $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$$m^2 = \frac{m_0^2c^2}{c^2-v^2}$$

$$m^2c^2(c^2-v^2) = m_0^2c^4$$

In above eqn $\Delta\lambda = \lambda' - \lambda$ is known as compton shift which depends on rest mass and scattering angle of photon.

Waves and particles:-

Particle nature and wave nature both are called as Dual Nature. since radiation has been shown to posses dual nature, wave and particle, matter must also posses dual nature

de-Broglie hypothesis/Matter waves:-

Radiation like electromagnetic waves behave like particles, particles like electrons will behave like waves, matter waves. The matter waves thus conceived by de-Broglie are called de Broglie matter waves.

He derived an expression for the wave length of matter waves

Based on planck's theory of radiation the energy of photon is $E = h\nu = \frac{hc}{\lambda} \rightarrow (1)$

according to einstein mass energy eqn

$$E = mc^2 \rightarrow (2)$$

from eqn (1) & (2)

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc} = \frac{h}{mv} = \frac{h}{p} \rightarrow (3)$$

where $c \rightarrow$ light velocity

$v \rightarrow$ particle velocity

$$\boxed{\lambda = \frac{h}{mv}} \quad - (3)$$

If E is the kinetic energy of the material particle of mass 'm' then

$$E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m v^2 \times \frac{m}{m}$$

$$= \frac{m^2 v^2}{2m} = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}} \rightarrow (4)$$

Hence de-Broglie wave length $\lambda = \frac{h}{mv}$

de-Broglie wave length associated with electron:-

Let us consider an electron of mass 'm' and charge 'e' accelerated by potential V volts.

If v is the velocity attained by the electron due to acceleration

$$\frac{1}{2} m v^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}}$$

By substituting v in eqn (3)

$$(3) \Rightarrow \lambda = \frac{h}{mv} = \frac{h}{m}\sqrt{\frac{m}{2eV}}$$

$$= \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.625 \times 10^{-34}}{(2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V)^{1/2}}$$

$$\boxed{\lambda = \frac{12.26}{\sqrt{V}} A^\circ} \rightarrow (5)$$

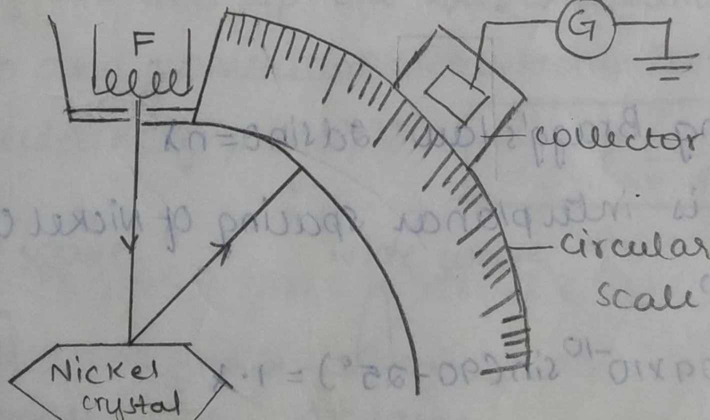
Characteristics of matter waves:-

Since $\lambda = \frac{h}{mv}$

1. Lighter particles are having greater wavelength
2. Less velocity particles having longer wavelength
3. If $v=0$, $\lambda=\infty$ means only moving particles can exhibit wave behaviour
4. Matter waves travel faster than light
5. No single phenomena exhibit both particle and wave nature simultaneously.

Davisson - Germer's Experiment

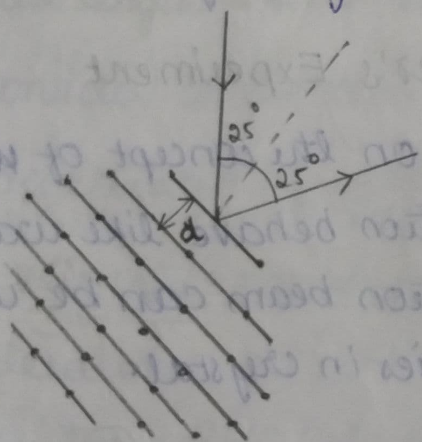
principle:- Based on the concept of matter wave fast moving electron behave like waves. Hence accelerated electron beam can be used for diffraction studies in crystal.



As shown in the figure, electrons are emitted from hot tungsten filament(F) by maintaining potential difference. The electrons from filament incident on surface of nickel crystal normally.

and get scattered in different directions. The intensity of scattered electron is measured in all directions by using collector C. The collector is capable to rotate about the axis. Thus the intensity of a scattered ray can be measured with the help of galvanometer which is connected to the collector.

The surface of nickel crystal with rows of atoms act like diffraction greating (equal parallel slits) or parallel planes, they produce the first order spectrum of 54 volts potential to the filament produces maximum intensity at 50°



Result :-

By applying Bragg's law $2d\sin\theta = n\lambda$

where d is interplanar spacing of Nickel crystal, 0.909×10^{-10} m

$$2 \times 0.909 \times 10^{-10} \sin(90-25°) = 1 \cdot \lambda$$

$$\lambda = 1.648\ A°$$

According to de-Broglie wavelength eqn

$$\lambda = \frac{12.26}{\sqrt{V}} = \frac{12.26}{\sqrt{54}} = 1.668\ A°$$

As the two values are in good agreement, the experiment conforms the de-Broglie concept of matter waves.

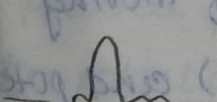
Hysenberg Uncertainity principle:-

Hysenberg Uncertainity principle states " It is impossible to known both the exact position and exact momentum of an object at the same time"

In wave mechanics if we consider a narrow wave group higher will be the accuracy of locating the particle. At the same time one cannot define the wave length of the wave accurately. On the other hand when we consider a wide wave group wavelength λ can be well defined and hence measurement of momentum becomes more accurate, at the same time locating the position of the particle in wave group becomes less accurate. Thus we can admit the uncertainity principle for Narrow and wide wave groups.

If Δx and Δp are the uncertainities in the position and momentum measurements then according to uncertainity principle $\boxed{\Delta x \times \Delta p \geq \frac{h}{4\pi}}$

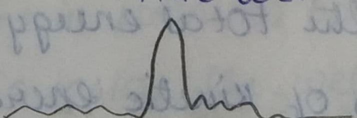
Narrow wave



Δx small

Δp large

Wide wave



Δx large

Δp small

Schrodinger time Independent wave eqⁿ:-

We know that election like particle exhibits wave nature when it is moving with velocity v and having mass m. By considering moving particle as a oscillator in harmonic motion. Let us write a progressive wave eqⁿ for moving particle.

$$\psi(x) = \psi_0 \sin(\omega t - kx) \rightarrow (1)$$

where ψ_0 - amplitude

ω - frequency

$k \rightarrow$ wave number

By differentiating twice both sides

$$\frac{\partial^2 \psi}{\partial x^2} = \underline{-k^2 \psi_0 \sin(\omega t - kx)}$$

from (1)

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0$$

wave number $k = \frac{2\pi}{\lambda}$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2}{\lambda^2}\psi = 0$$

from de-Broglie eq $\lambda = h/mv$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \rightarrow (2)$$

Let us consider the total energy (E) of moving particle is addition of kinetic energy (K) and potential energy (V)

$$\therefore E = K + V$$

$$\Rightarrow K = E - V$$

$$\frac{1}{2} m v^2 \times \frac{m}{m} = (E - V)$$

$$\frac{m^2v^2}{2m} = (E-V)$$

$$m^2v^2 = 2m(E-V) \rightarrow ③$$

By substituting eq^n ③ in eq^n ②

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m(E-V)}{h^2}\psi = 0$$

we know that $\hbar = \frac{h}{2\pi}$

$$\hbar = \frac{h}{2\pi}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2}\psi = 0$$

this is schrodinger time Independent in 1 dimension

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m(E-V)}{\hbar^2}\psi = 0$$

this is schrodinger time Independent in 3-dimension

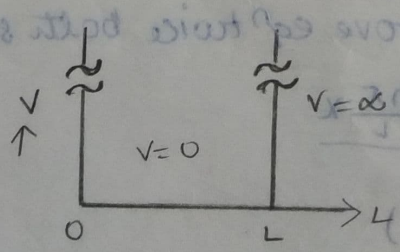
Laplas constant

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2\psi + \frac{2m(E-V)}{\hbar^2}\psi = 0$$

Particle in 1-Dimensional potential box:

Quantum Mechanics examines the behaviour of microlevel units as an ~~exmp~~ example let us consider a particle present in potential box of width 'L'. Let the potential $V=0$ inside the well and $V=\infty$ outside the well



The time independent schrodinger wave eqn in 1-D is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2}\psi = 0 \rightarrow (1)$$

for a particle present inside the well when $V=0$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0 \rightarrow (2)$$

Such a particle called free particle. Let the general solution for above eqn is

$$\psi(x) = A\sin kx + B\cos kx \rightarrow (3)$$

In present case the boundary conditions are

$x = 0 \quad \psi = 0 \rightarrow (a)$

$x = L \quad \psi = 0 \rightarrow (b)$

By substituting boundary condition (a) in eqn (3)

$B = 0$

By substituting boundary condition $B=0$ in eqn (3)

$A\sin kL = 0$

which means $A = 0$ or $\sin kL = 0$

Since $A \neq 0$ as it is constant it cannot be equal to 0

so, $\sin kL = 0$

means $\sin kL = 0$

$kL = n\pi$

$k = \frac{n\pi}{L}$

so eqn (3) becomes

$$3 \Rightarrow \psi(x) = A\sin\frac{n\pi}{L}x + 0$$

By differentiating above eqn twice both sides

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{n^2\pi^2}{L^2}\underline{A\sin\frac{n\pi}{L}x}$$

$$= -\frac{n^2\pi^2}{L^2}\psi(x)$$

$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{n^2\pi^2}{L^2}\psi(x) = 0 \rightarrow (4)$

By comparing (1) & (4)

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

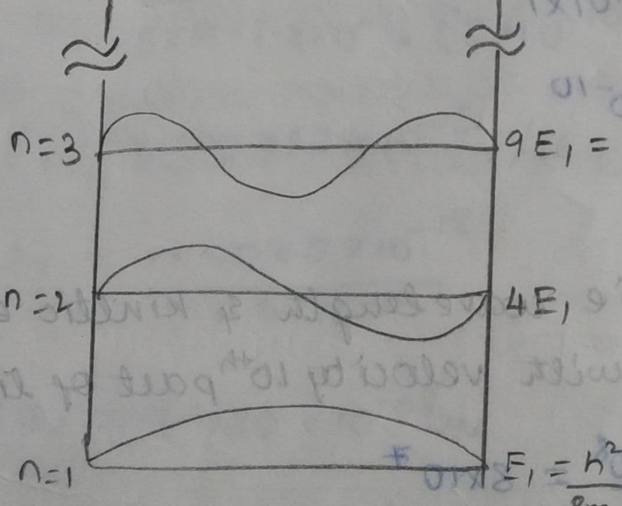
$$E = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \hbar = \frac{h}{2\pi}$$

$$= \frac{n^2\pi^2 h^2}{8m\pi^2 L^2}$$

$$\boxed{E = \frac{n^2h^2}{8mL^2}}$$

where $n \rightarrow$ quantum number

above eqn represent value of energy of discreate set of energy levels. The energy of n^{th} quantum level is $\boxed{E_n = \frac{n^2h^2}{8mL^2}}$



A particle of mass 1.56×10^{-30} kg has free energy 120volts. Find debroglie wave length.

$m = 1.156\times10^{-30}$

$e = 120eV = 120\times1.6\times10^{-19}J$

$\lambda = \frac{h}{\sqrt{2me}} = \frac{6.625\times10^{-34}}{\sqrt{2\times1.156\times10^{-30}\times120\times1.6\times10^{-19}}}$

$1A^\circ = 10^{-10}m$

$= 10^{-8}cm$

$= 9.94\times10^{-11}$

$= 0.995A^\circ$

calculate the wavelength associated with the electron whose speed is 0.01 times of light speed.

$\lambda = \frac{h}{mv}$

$m = 9.1\times10^{-31}$

$v = 0.01\times3\times10^{3}$

$\lambda = \frac{6.625\times10^{-34}}{9.1\times10^{-31}\times0.01\times1}$

$= 2.426\times10^{-10}$

$\lambda = 2.43A^\circ$

calculate debroglie wavelength & kinetic energy of an electron moving with velocity 10^{th} part of light

$V = \frac{3\times10^{8}}{10} = 3\times10^{7}$

$\lambda = \frac{h}{mv}$

$\lambda = \frac{6.625\times10^{-34}}{9.1\times10^{-31}\times\frac{3\times10^{8}}{10}}$

$\lambda = 2.42\times10^{-11}$

$= 0.24A^\circ$

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (3 \times 10^{7})^2$

$= 4.095 \times 10^{-16}$ m/sec

calculate de-Broglie wavelength associated with election accelerated with potential 1600V

$$\lambda = \frac{12.26}{\sqrt{V}}$$

$$= \frac{12.26}{\sqrt{1600}}$$

$$\lambda = 0.3065 A^{\circ}$$

Find out the lowest energy level of an electron trapped or confined in a potential box of width $2A^{\circ}$

$$E_n = \frac{n^2 h^2}{8mL^2} \qquad L = 2 \times 10^{-10}$$

$$E_1 = \frac{(6.625 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$E_1 = 1.5072 \times 10^{-18} \text{ J/sec}$$

$$E_1 = \frac{1.5072 \times 10^{-18}}{1.6 \times 10^{-19}}$$

$$E_1 = 9.420 \times 10^{-38} \text{ ev}$$

An election is bound by a potential which closely approches an infinite square well of width $1A^{\circ}$. calculate 1st 3 energies of election

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$E_1 = \frac{(6.625 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (1 \times 10^{-10})^2}$$

$$= \frac{6.028 \times 10^{-18}}{1.6 \times 10^{-19}}$$

$$= 3.768 \times 10^{-37}$$

$$E_2 = 4 \times 6.028 \times 10^{-18}$$

$$= 2.411 \times 10^{-17}$$

$$E_3 = 9 \times 6.028 \times 10^{-18}$$

$$= 5.426 \times 10^{-17}$$