Classification

Rahmad Mahendra, M.Sc

with credit to
Samuel Louvan, M.Sc
Meganingrum Arista Jiwanggi, M.Kom

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Data Mining for Big Data
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Classification

A mapping f from input data x (drawn from instance space **X**)
to a label y from some enumerable output space **Y**

X = set of all fruits
Y = {orange, apple, banana}

f [ x] = banana y

As humans how do you **discriminate** between each of the instance?

x

f

```
if x.color == "orange":
    y = "orange"
elif  x.color == "red":
    y = "apple"
elif x.color == "yellow":
    y =  "banana"
```

y orange

Recognizing a Classification Problem

- Can you formulate your question as a choice among some universe of possible classes?
- Can you create (or find) labeled data that marks that choice for a bunch of examples? Can you make that choice?
- Can you create features that might help in distinguishing those classes?

Problems ?

x

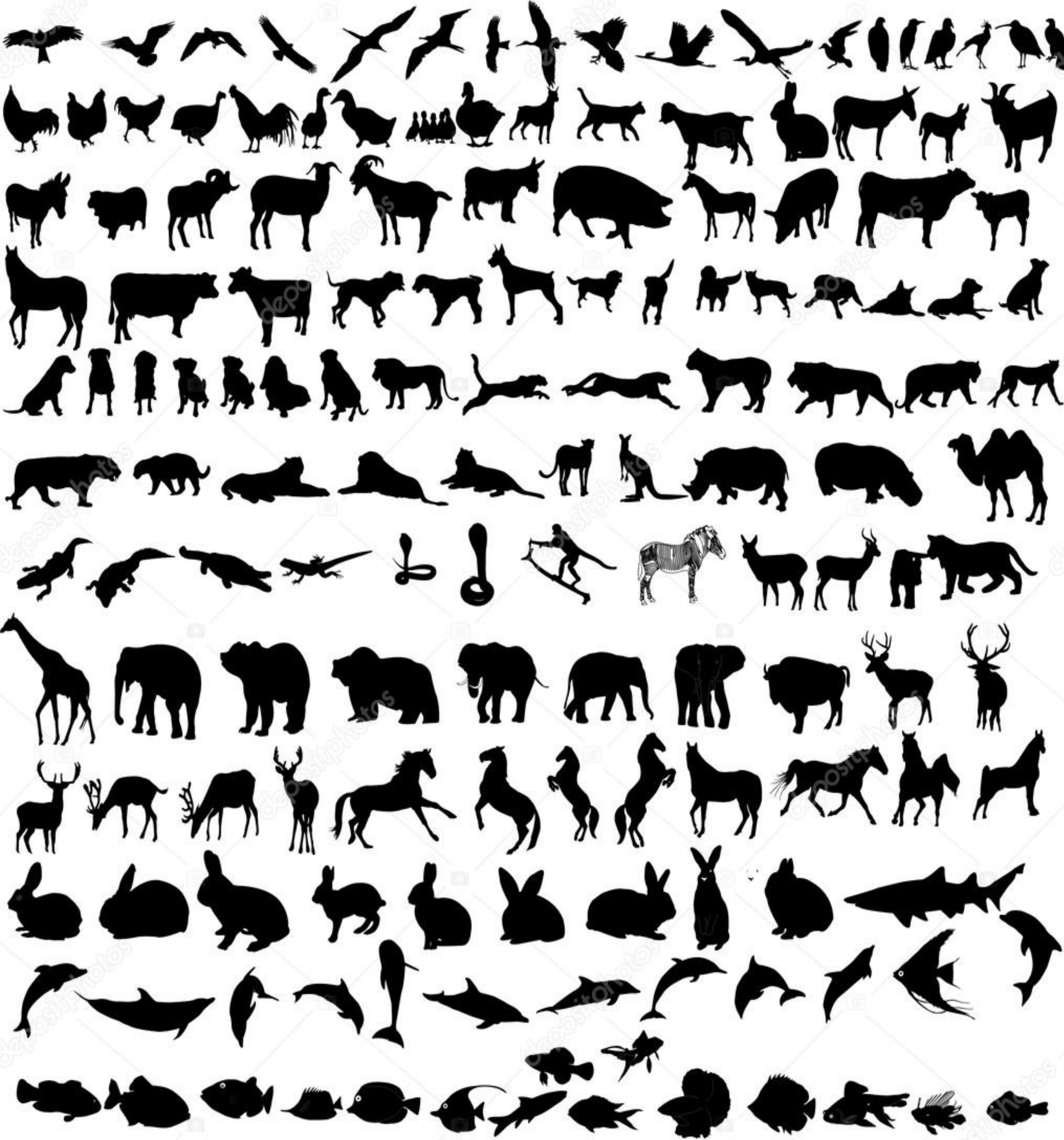
f

```
if x.color == “orange”:
    y = “orange”
elif  x.color == “red”:
    y = “apple”
elif x.color == “yellow”:
    y =  “banana”
```

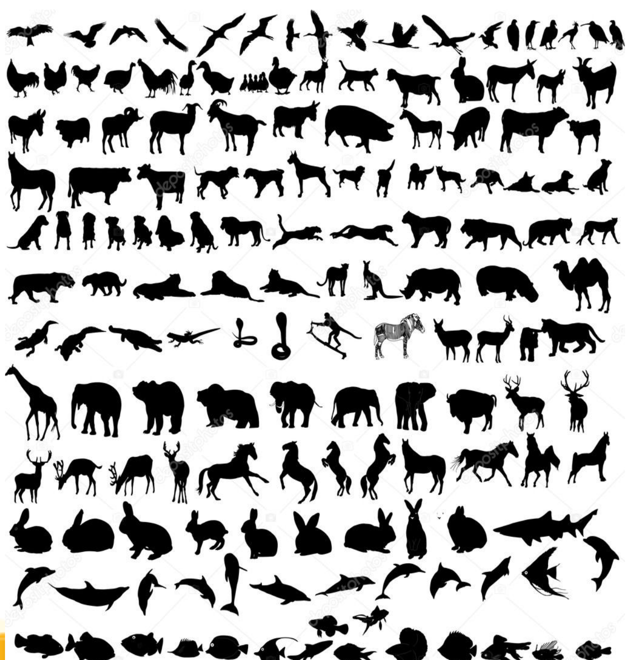
?

Problems ?



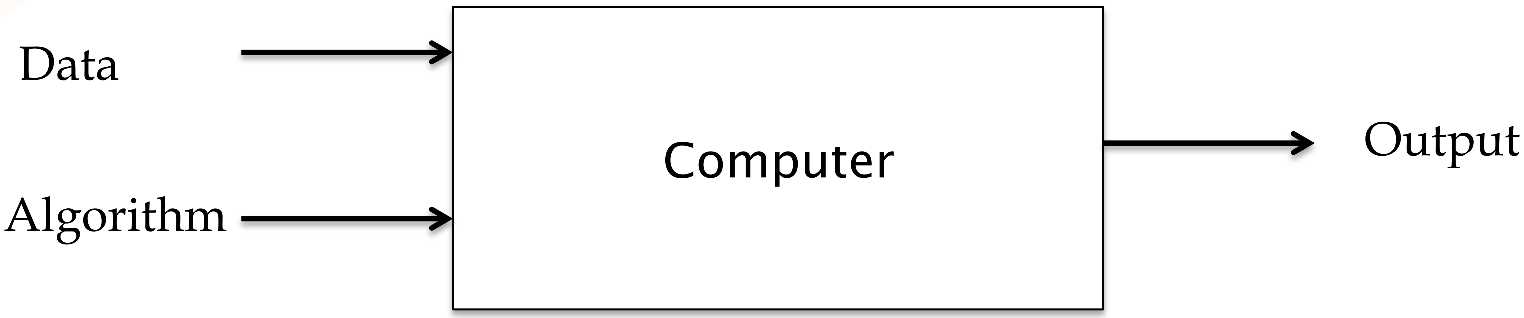
Problems ?



Manually creating rules are not scalable

Instead, can we let computers to *learn* the rules automatically from data?

Traditional Programming



Machine Learning (ML)

[Pedros Domingos]

ML Framework

$$y_{pred} = f(x)$$

output / label/class

prediction function/model

input (features/attributes)

To learn the function **f,** you need to *train* it

Apply **f** to a *never before seen test* example x and output the predicted value $y_{pred} = f(x)$

ML Framework – Training (Supervised Learning)

Training data

x y

x_1  [color = … , shape = …, texture = …] orange y_1

x_2  [color = … , shape = …, texture = …] banana y_2

x_3  [color = … , shape = …, texture = …] apple y_3

x_4  [color = … , shape = …, texture = …] banana y_4

x_5  [color = … , shape = …, texture = …] apple y_5

feature vector representation

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ML Framework - Testing

Unseen data

x

$\rightarrow f(x) \rightarrow$

orange ✔

orange ✖

banana ✔

y_{pred}

Learning Algorithms

Decision Tree

k-NN

Naïve Bayes

Support Vector Machine

Logistic Regression

Neural Network

……..

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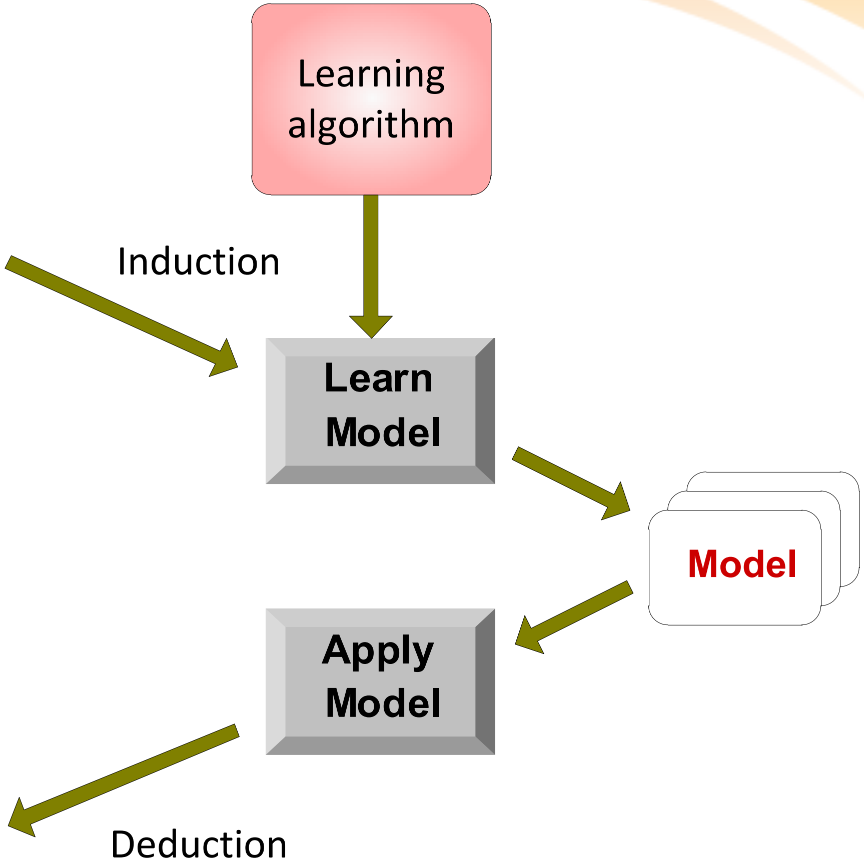
Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



k - NN

Instance-Based Classifiers

- Store the training records
- Use training records to predict the class label of unseen cases

Set of Stored Cases

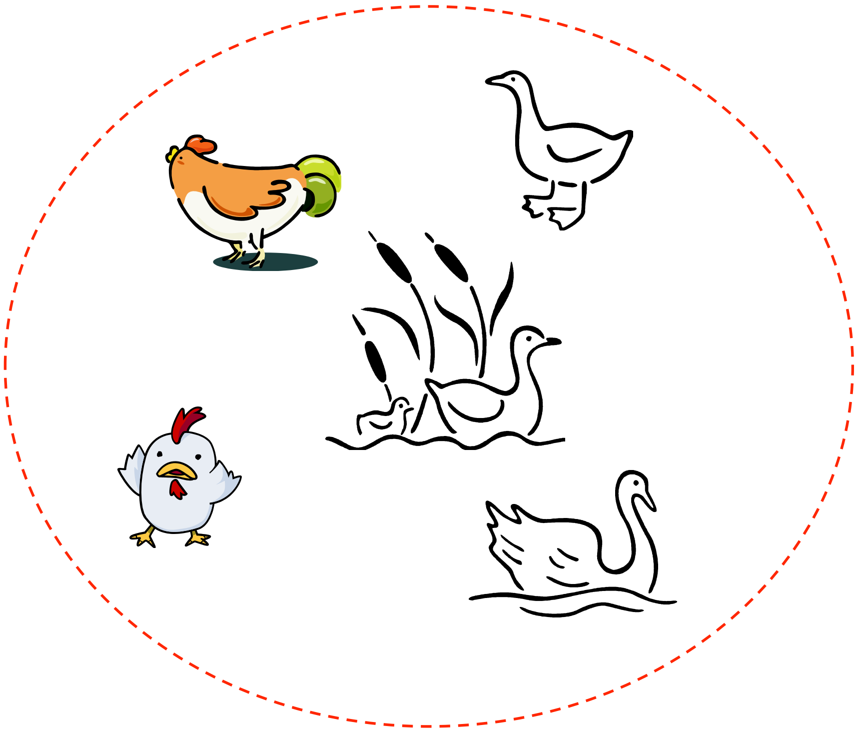
Atr1	………	AtrN	Class
			A
			B
			B
			C
			A
			C
			B

Unseen Case

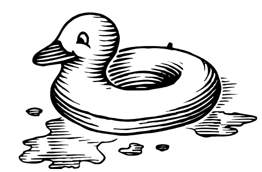
Atr1	………	AtrN

Basic idea:

"If it walks like a duck, quacks like a duck, then it's probably a duck"

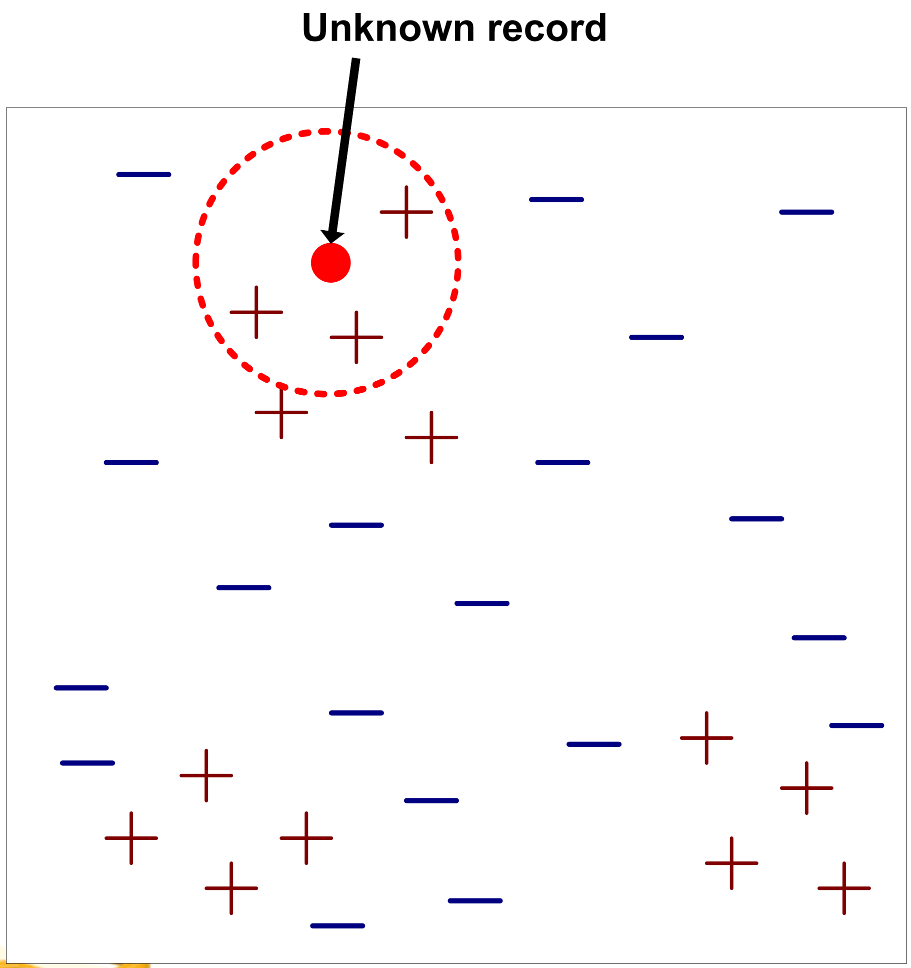


Training Data



Test Data

Nearest-Neighbor Classifiers



- Requires three things
  - The set of stored records
  - Distance Metric to compute distance between records
  - The value of *k*, the number of nearest neighbors to retrieve

- To classify an unknown record:
  1. Compute distance to other training records
  2. Identify *k* nearest neighbors
  3. Use class labels of nearest neighbors to determine the class label of unknown record (e.g., by taking majority vote)

Nearest Neighbor Classification

- Compute distance between points:
  - Euclidean distance

$$d(p,q) = \sqrt{\sum_i (p_i - q_i)^2}$$

- Determine the class from the nearest neighbor list
  - Take the majority class labels among the k-nearest neighbors
  - Weight the vote according to the distance

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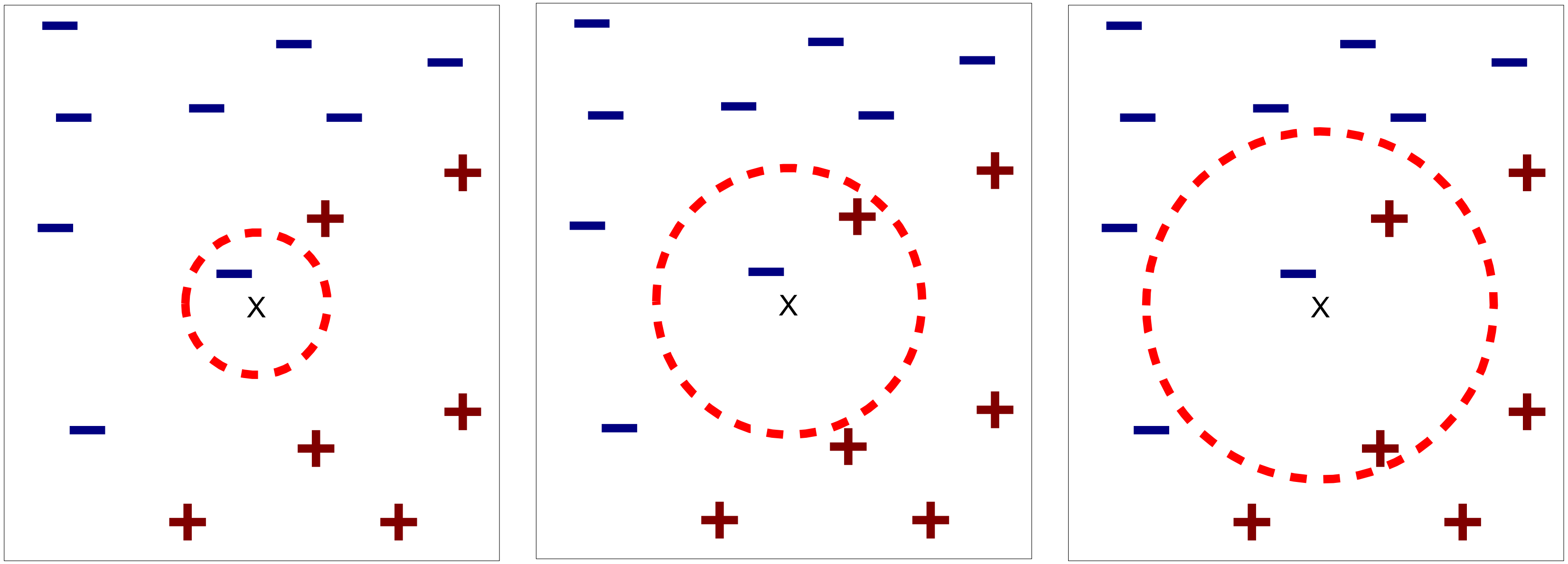
Definition of Nearest Neighbor

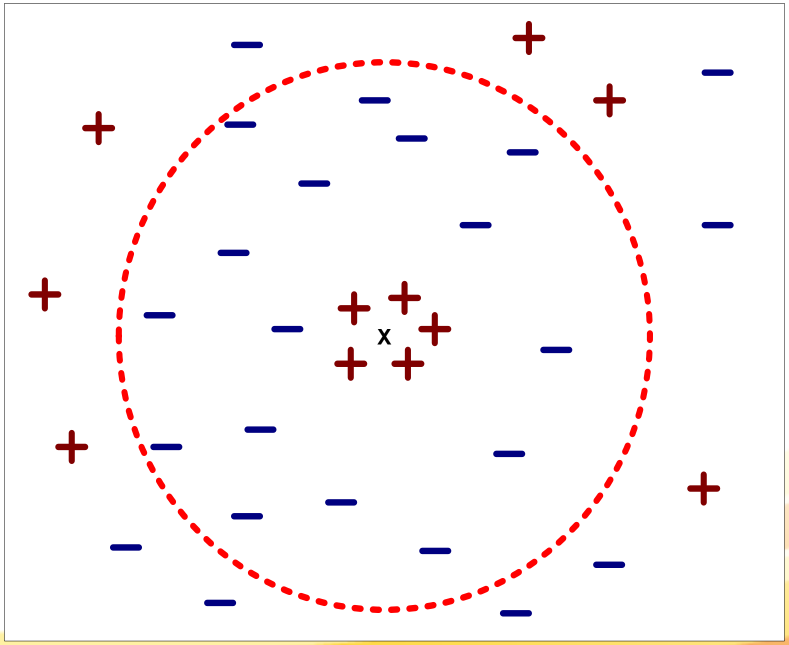


(a) 1-nearest neighbor (b) 2-nearest neighbor (c) 3-nearest neighbor

K-nearest neighbors of a record x are data points that have the k smallest distance to x

Nearest Neighbor Classification…

- Choosing the value of k:
  - If k is too small, sensitive to noise points
  - If k is too large, neighborhood may include points from other classes



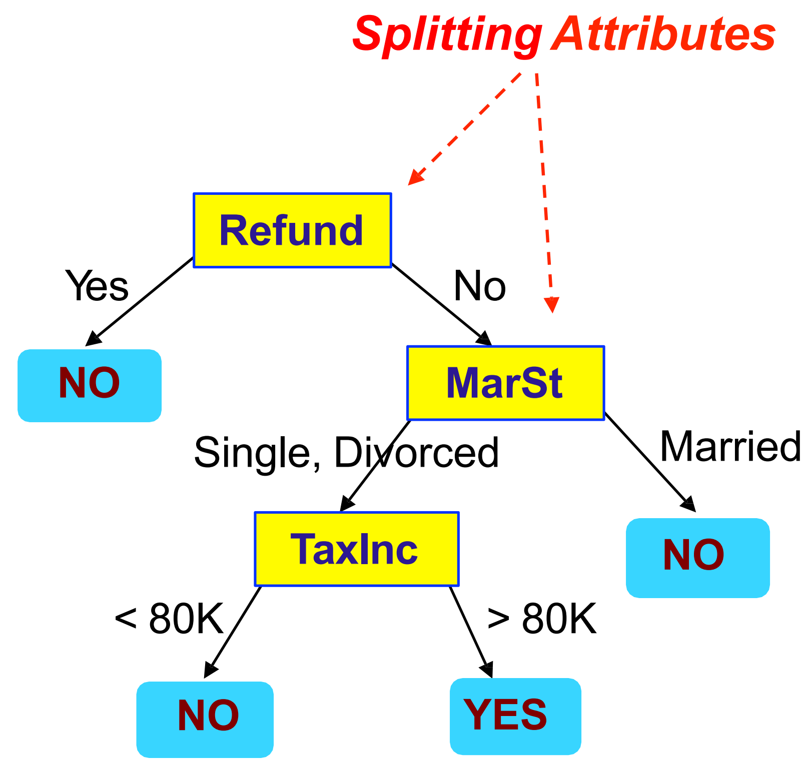
Decision Tree

Example of a Decision Tree

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data



Model: Decision Tree

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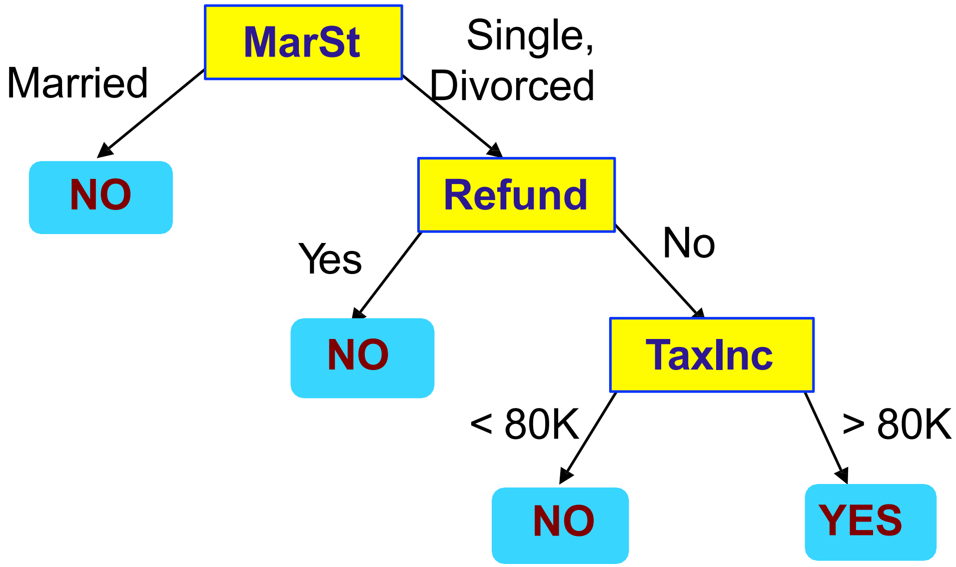
Another Example of Decision Tree

Tid categorical	Refund categorical	Marital Status continuous	Taxable Income class	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

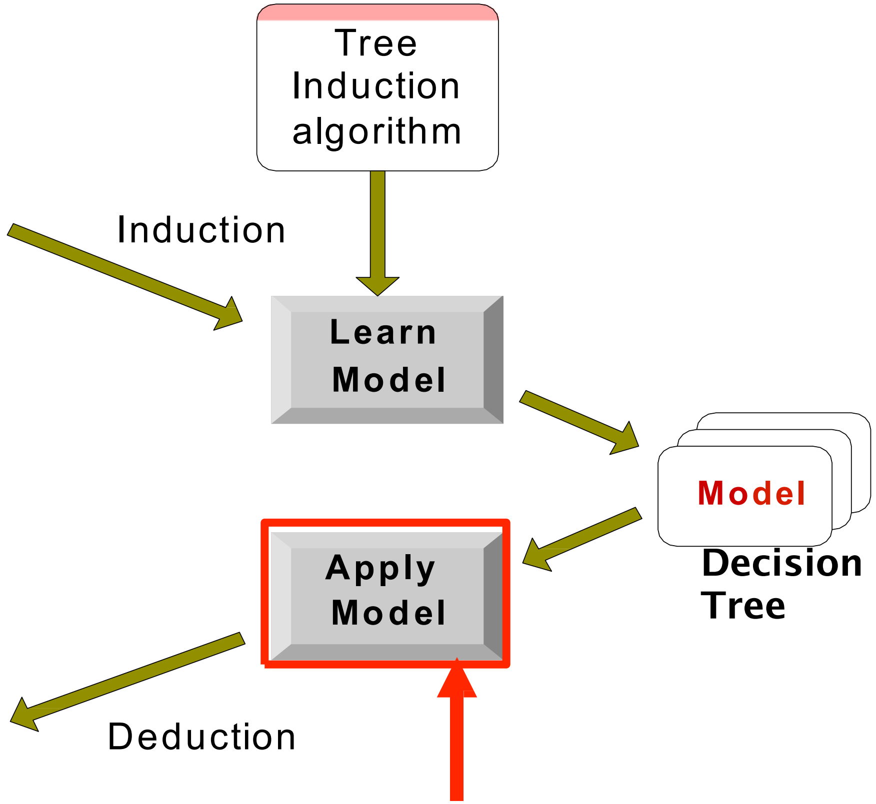


There could be more than one tree that fits the same data!

Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

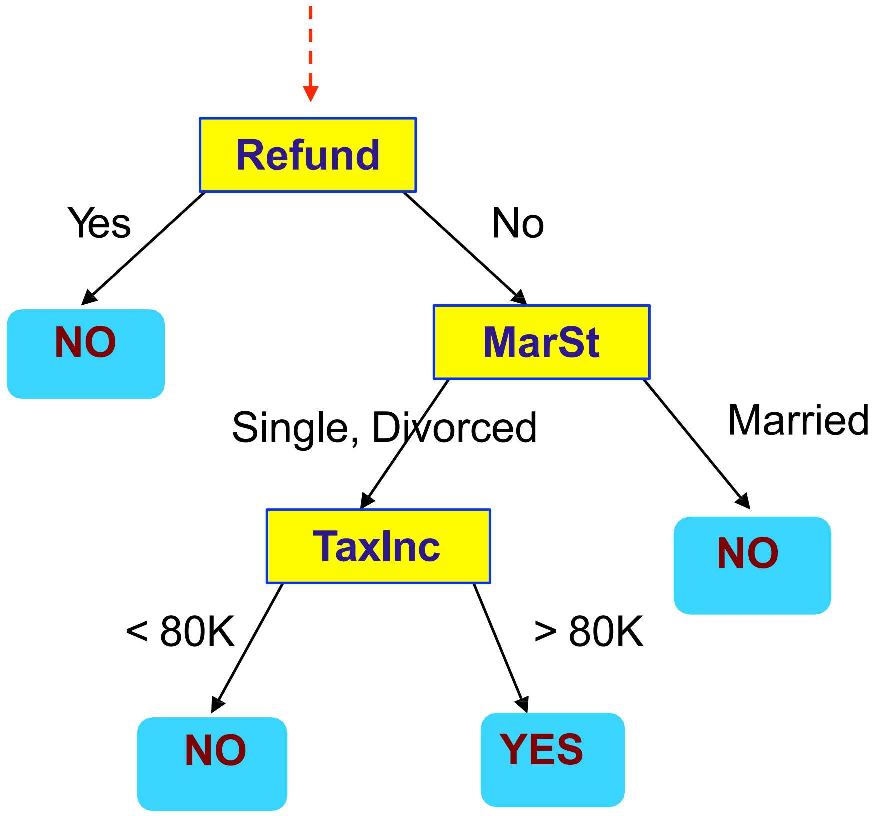


Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

Apply Model to Test Data

Start from the root of tree.



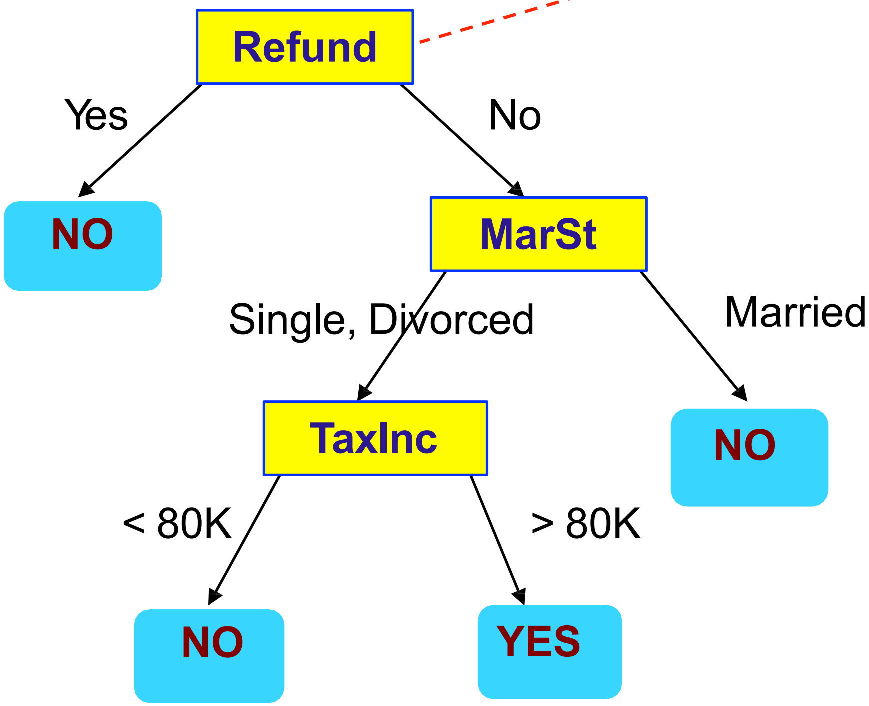
Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Apply Model to Test Data

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



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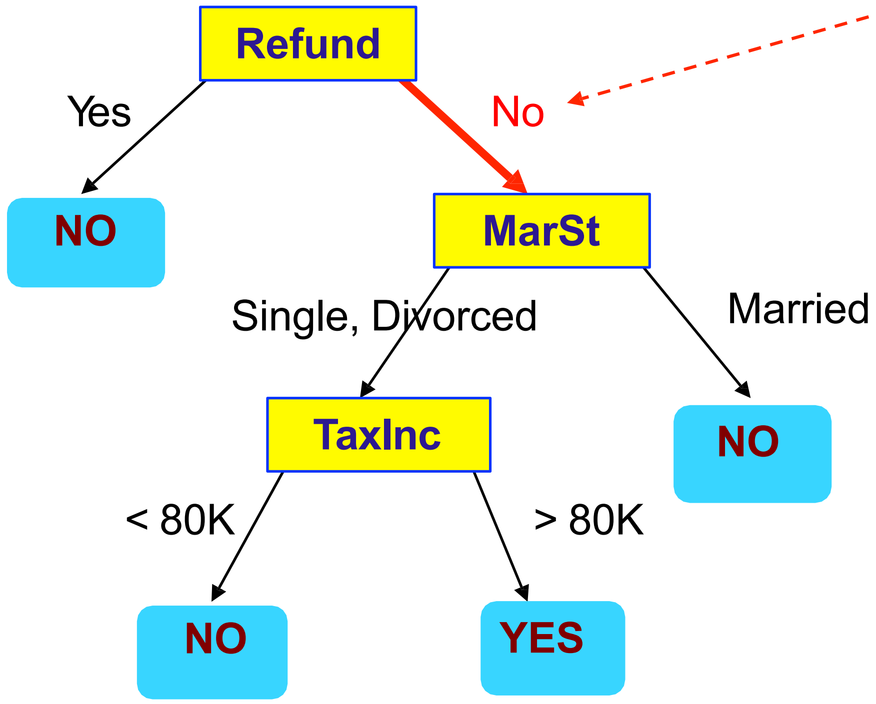
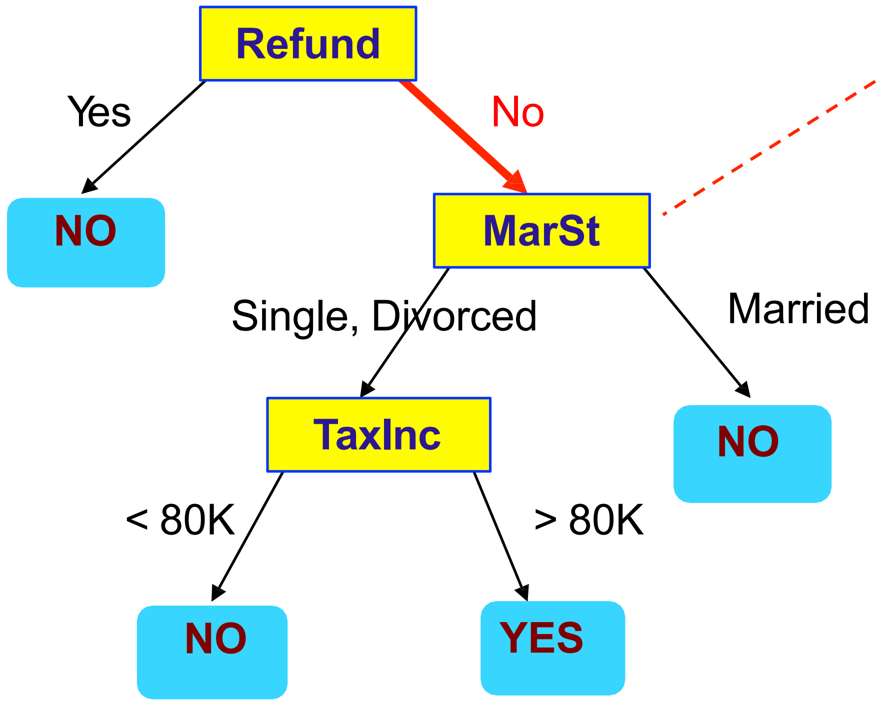
Apply Model to Test Data

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data



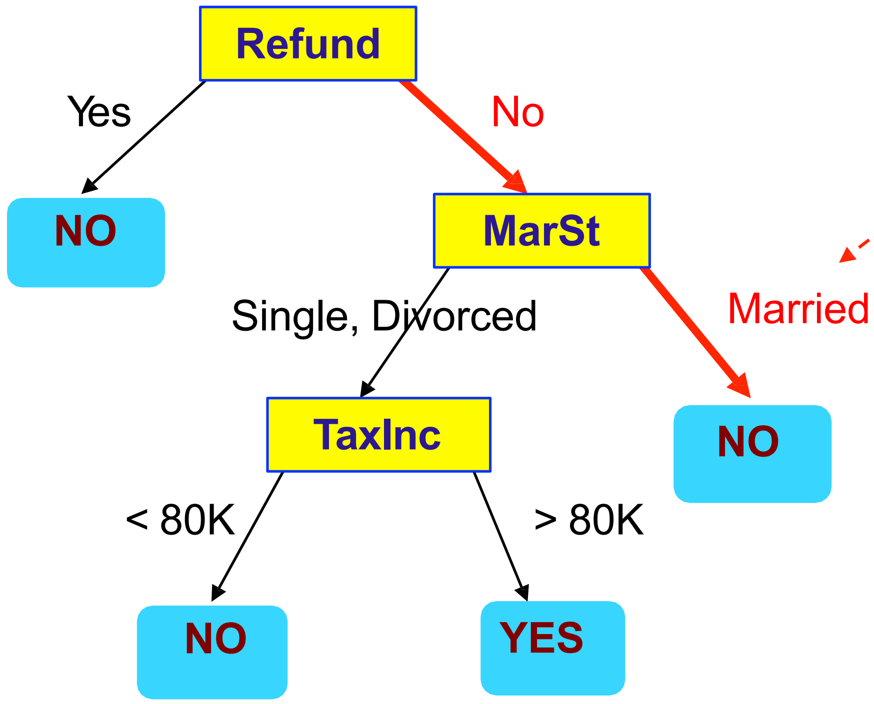
Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Apply Model to Test Data

Test Data

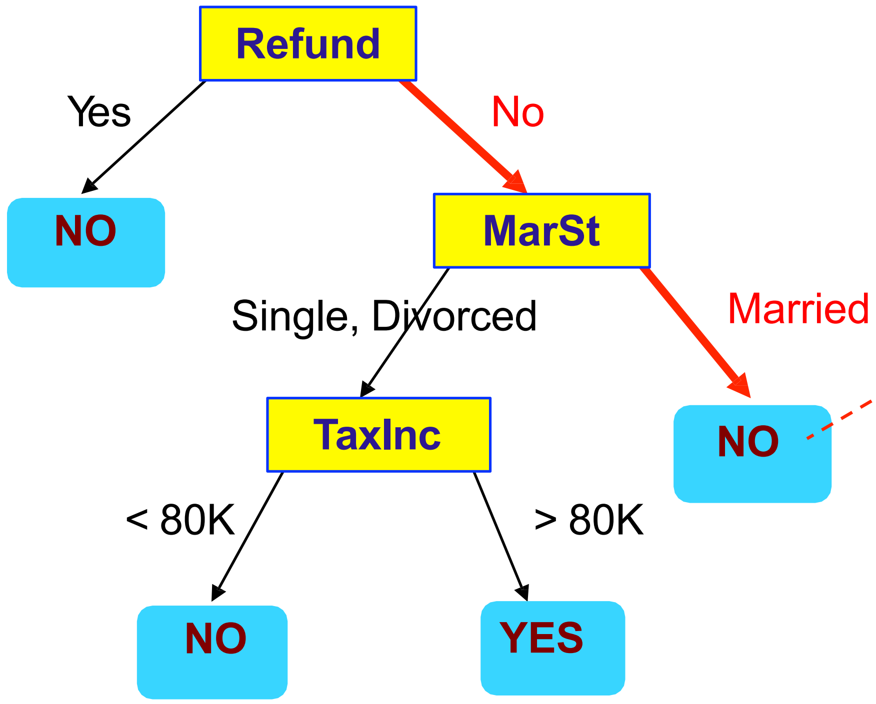
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Assign Cheat to "No"

Training examples: **9 yes / 5 no**

Day	Outlook	Humidity	Wind	Play
D1	Sunny	High	Weak	No
D2	Sunny	High	Strong	No
D3	Overcast	High	Weak	Yes
D4	Rain	High	Weak	Yes
D5	Rain	Normal	Weak	Yes
D6	Rain	Normal	Strong	No
D7	Overcast	Normal	Strong	Yes
D8	Sunny	High	Weak	No
D9	Sunny	Normal	Weak	Yes
D10	Rain	Normal	Weak	Yes
D11	Sunny	Normal	Strong	Yes
D12	Overcast	High	Strong	Yes
D13	Overcast	Normal	Weak	Yes
D14	Rain	High	Strong	No

New data:

D15	Rain	High	Weak	?

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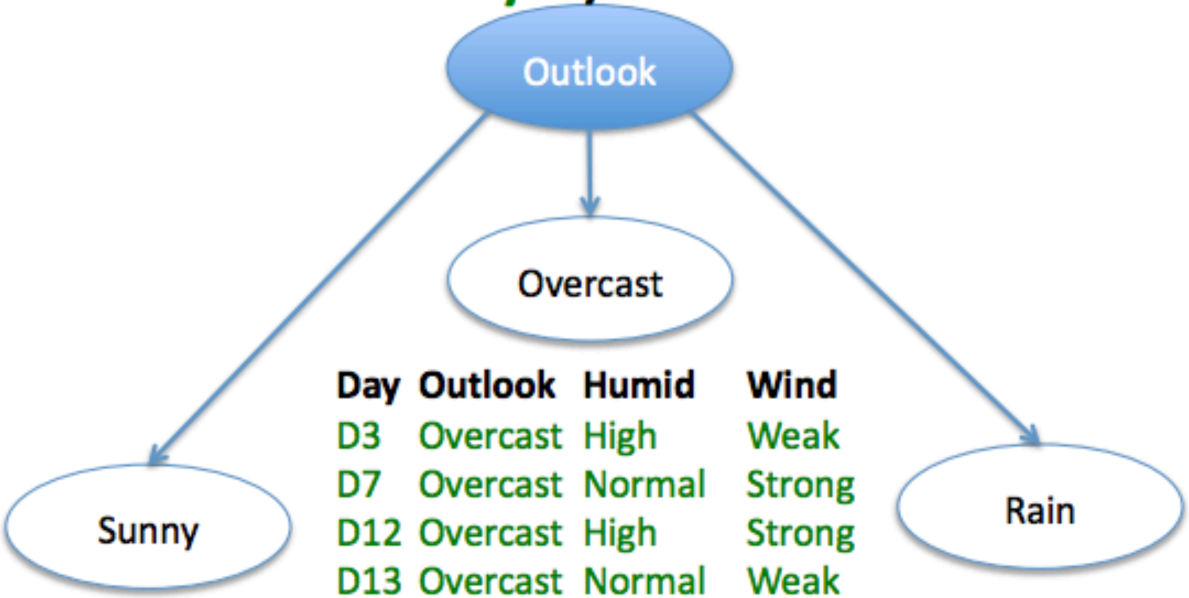
9 yes / 5 no



Day	Outlook	Humid	Wind
D3	Overcast	High	Weak
D7	Overcast	Normal	Strong
D12	Overcast	High	Strong
D13	Overcast	Normal	Weak

4 yes / 0 no

pure subset

Day	Outlook	Humid	Wind
D1	Sunny	High	Weak
D2	Sunny	High	Strong
D8	Sunny	High	Weak
D9	Sunny	Normal	Weak
D11	Sunny	Normal	Strong

2 yes / 3 no

split further

Day	Outlook	Humid	Wind
D4	Rain	High	Weak
D5	Rain	Normal	Weak
D6	Rain	Normal	Strong
D10	Rain	Normal	Weak
D14	Rain	High	Strong

3 yes / 2 no

split further

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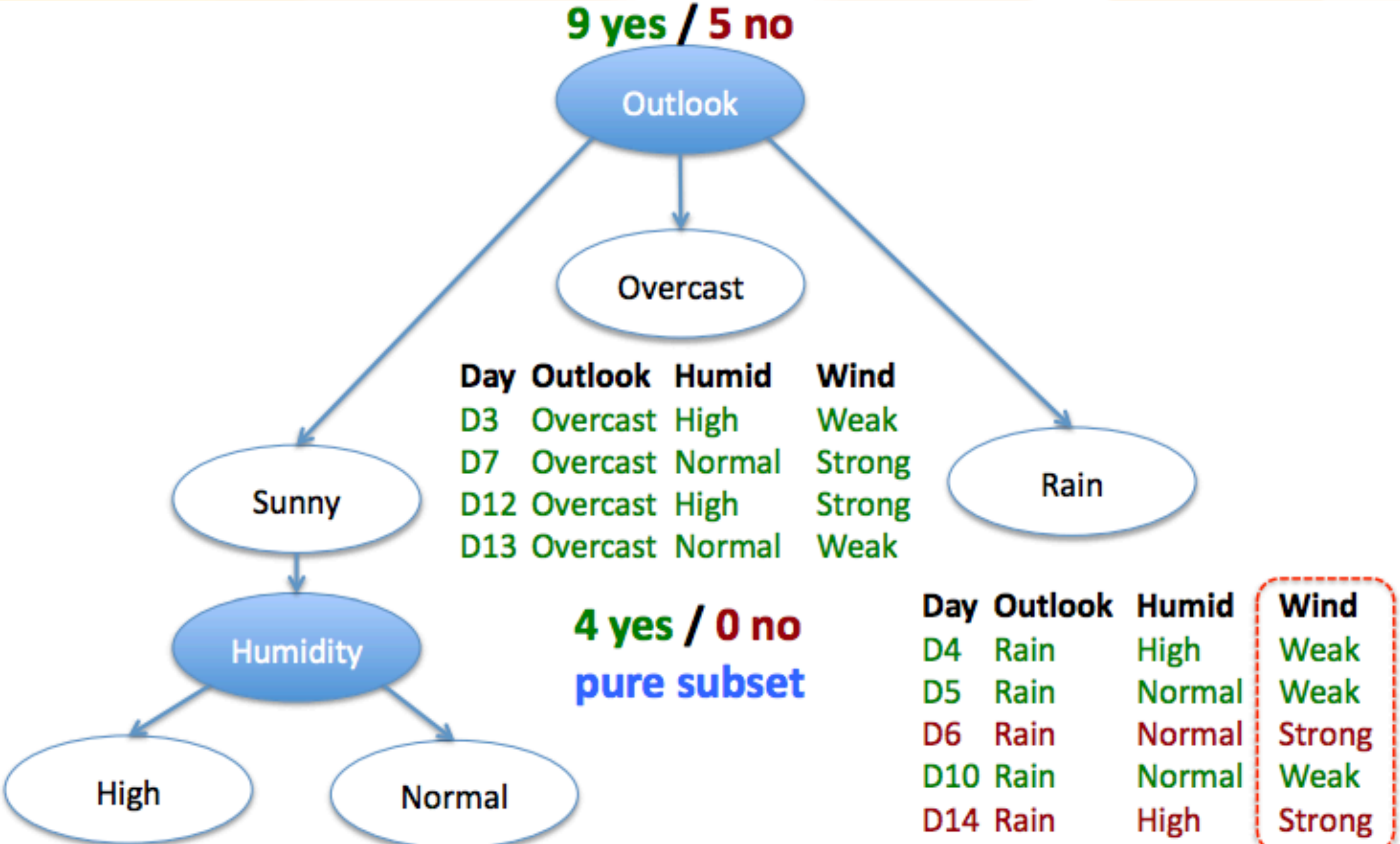


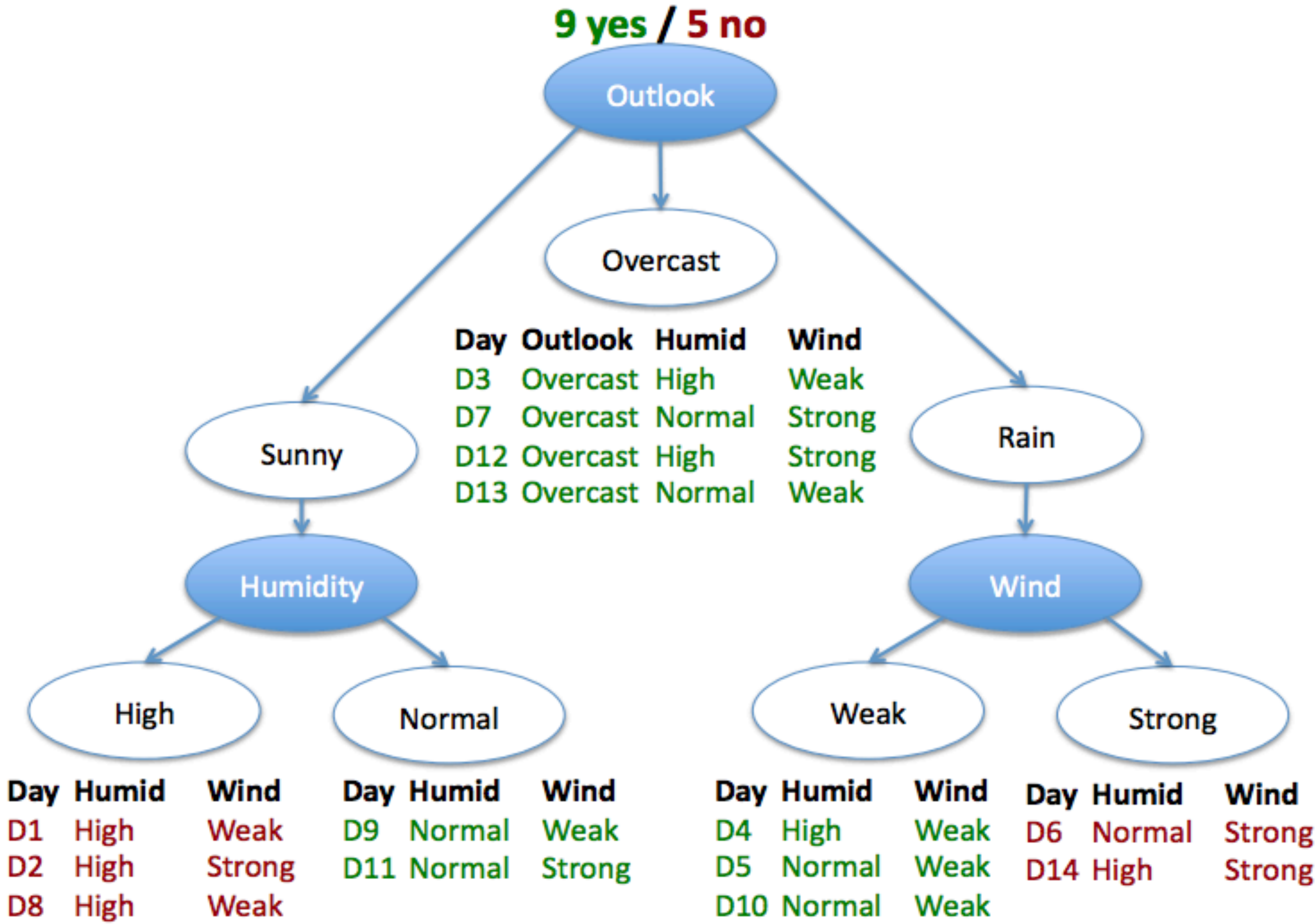
Day	Outlook	Humid	Wind
D3	Overcast	High	Weak
D7	Overcast	Normal	Strong
D12	Overcast	High	Strong
D13	Overcast	Normal	Weak

Day	Outlook	Humid	Wind
D4	Rain	High	Weak
D5	Rain	Normal	Weak
D6	Rain	Normal	Strong
D10	Rain	Normal	Weak
D14	Rain	High	Strong

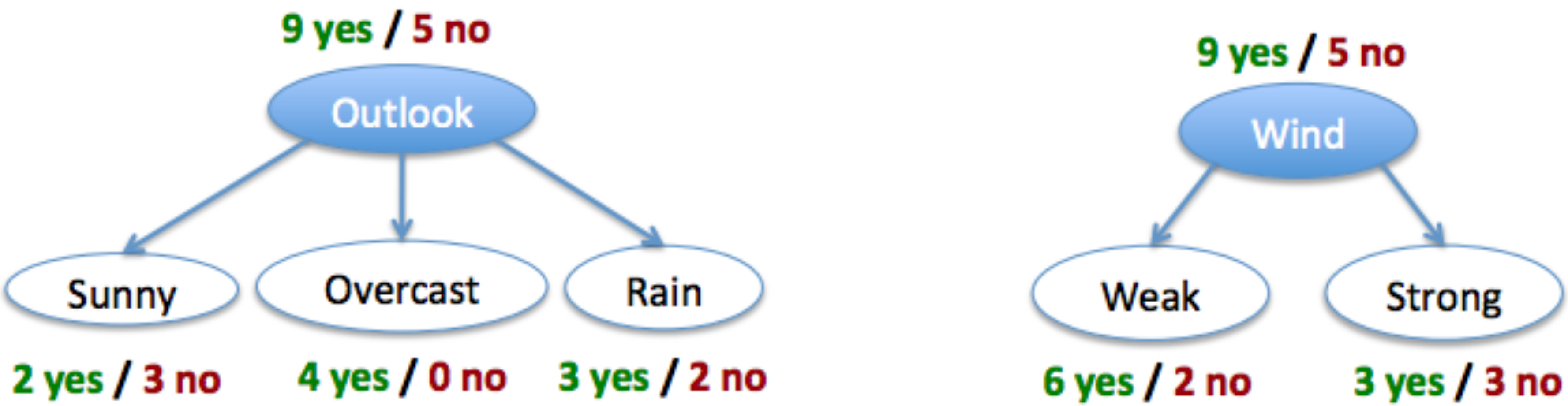
Day	Humid	Wind
D1	High	Weak
D2	High	Strong
D8	High	Weak

Day	Humid	Wind
D9	Normal	Weak
D11	Normal	Strong

New data:	Day	Outlook	Humid	Wind	
	D15	Rain	High	Weak	→ Yes



Which attribute to split on?



- Want to measure "purity" of the split
  - more certain about Yes/No after the split
    - pure set (4 yes / 0 no) => completely certain (100%)
    - impure (3 yes / 3 no) => completely uncertain (50%)

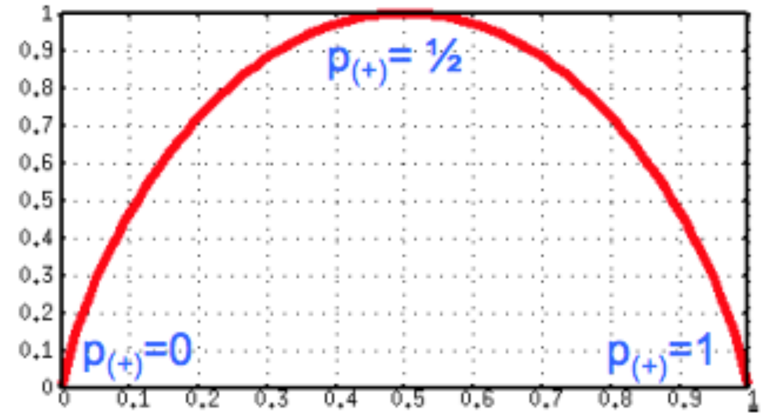
Entropy

- Entropy: $H(S) = - p_{(+)} \log_2 p_{(+)} - p_{(-)} \log_2 p_{(-)}$ bits
  - S … subset of training examples
  - $p_{(+)}$ / $p_{(-)}$ … % of positive / negative examples in S
- Interpretation: assume item X belongs to S
  - how many bits need to tell if X positive or negative
- impure (3 yes / 3 no):

$$H(S) = -\frac{3}{6}\log_2\frac{3}{6} - \frac{3}{6}\log_2\frac{3}{6} = 1 \text{ bits}$$

- pure set (4 yes / 0 no):

$$H(S) = -\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4} = 0 \text{ bits}$$



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Information Gain

- Want many items in pure sets
- Expected drop in entropy after split:

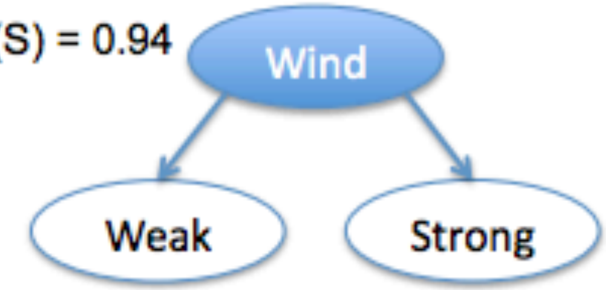
$$Gain(S,A) = H(S) - \sum_{V \in Values(A)} \frac{|S_V|}{|S|} H(S_V)$$

V … possible values of A
S … set of examples {X}
S_v … subset where X_A = V

- Mutual Information
  - between attribute A and class labels of S

$-\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14}$ **9 yes / 5 no**

H(S) = 0.94



6 yes / 2 no

$-\frac{6}{8}\log_2\frac{6}{8} - \frac{2}{8}\log_2\frac{2}{8}$

$H(S_{weak}) = 0.81$

3 yes / 3 no

$-\frac{3}{6}\log_2\frac{3}{6} - \frac{3}{6}\log_2\frac{3}{6}$

$H(S_{strong}) = 1.0$

Gain (S, Wind)
$= H(S) - {}^{8}/_{14}\, H(S_{weak}) - {}^{6}/_{14}\, H(S_{weak})$
$= 0.94 - {}^{8}/_{14} * 0.81 - {}^{6}/_{14} * 1.0$
$= 0.049$

Naïve Bayes

Bayes Classifier

- A probabilistic framework for solving classification problems
- Conditional Probability:

$$P(C|A) = \frac{P(A,C)}{P(A)} \qquad P(A|C) = \frac{P(A,C)}{P(C)}$$

- Bayes theorem: $$P(C|A) = \frac{P(A|C)P(C)}{P(A)}$$

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Example of Bayes Theorem

- Given:
  - A doctor knows that meningitis causes stiff neck 50% of the time
  - Prior probability of any patient having meningitis is 1/50,000
  - Prior probability of any patient having stiff neck is 1/20
- If a patient has stiff neck, what's the probability he/she has meningitis?

$$P(M \mid S) = \frac{P(S \mid M)P(M)}{P(S)} = \frac{0.5 \times 1/50000}{1/20} = 0.0002$$

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Bayesian Classifiers

- Consider each attribute and class label as random variables
- Given a record with attributes (A_1, A_2,…,A_n)
  - Goal is to predict class C
  - Specifically, we want to find the value of C that maximizes $P(C| A_1, A_2,\ldots,A_n)$
- Can we estimate $P(C| A_1, A_2,\ldots,A_n)$ directly from data?

Bayesian Classifiers

- Approach:
  - compute the posterior probability $P(C \mid A_1, A_2, \ldots, A_n)$ for all values of C using the Bayes theorem

$$\boxed{P(C \mid A_1 A_2 \ldots A_n) = \frac{P(A_1 A_2 \ldots A_n \mid C) P(C)}{P(A_1 A_2 \ldots A_n)}}$$

  - Choose value of C that maximizes $P(C \mid A_1, A_2, \ldots, A_n)$
  - Equivalent to choosing value of C that maximizes $P(A_1, A_2, \ldots, A_n \mid C) P(C)$
- How to estimate $P(A_1, A_2, \ldots, A_n \mid C)$?

Naïve Bayes Classifier

- Assume independence among attributes A_i when class is given:
  - $P(A_1, A_2, \ldots, A_n \mid C) = P(A_1 \mid C_j)\, P(A_2 \mid C_j) \ldots P(A_n \mid C_j)$
  - Can estimate $P(A_i \mid C_j)$ for all A_i and C_j.
  - New point is classified to C_j if $P(C_j)\, \Pi\, P(A_i \mid C_j)$ is maximal.

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How to Estimate Probabilities from Data?

Tid	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

- Class: $P(C) = N_c/N$
  - e.g., $P(No) = 7/10$, $P(Yes) = 3/10$

For discrete attributes:

$$P(A_i \mid C_k) = |A_{ik}| / N_{c_k}$$

- where $|A_{ik}|$ is number of instances having attribute A_i and belongs to class C_k
- Examples:

P(Status=Married|No) = 4/7
P(Refund=Yes|Yes)=0

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How to Estimate Probabilities from Data?

- For continuous attributes:
  - Discretize the range into bins
    - one ordinal attribute per bin k
    - violates independence assumption
  - Two-way split: (A < v) or (A > v)
    - choose only one of the two splits as new attribute

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How to Estimate Probabilities from Data?

	Taxable Income	Evade
	125K	No
	100K	No
	70K	No
	120K	No
d	95K	Yes
	60K	No
d	220K	No
	85K	Yes
	75K	No
	90K	Yes

- Normal distribution:
  - One for each (A_i, c_j) pair

$$P(A_i \mid c_j) = \frac{1}{\sqrt{2\pi\sigma_{ij}^2}} e^{-\frac{(A_i-\mu_{ij})^2}{2\sigma_{ij}^2}}$$

- For (Income, Class=No):
  - If Class=No
    - sample mean = 110
    - sample variance = 2975

$$P(Income=120 \mid No) = \frac{1}{\sqrt{2\pi}(54.54)} e^{-\frac{(120-110)^2}{2(2975)}} = 0.0072$$

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Example of Naïve Bayes Classifier

Given a Test Record:

$$X = (\text{Refund} = \text{No}, \text{Married}, \text{Income} = 120\text{K})$$

naive Bayes Classifier:

P(Refund=Yes|No) = 3/7
P(Refund=No|No) = 4/7
P(Refund=Yes|Yes) = 0
P(Refund=No|Yes) = 1
P(Marital Status=Single|No) = 2/7
P(Marital Status=Divorced|No)=1/7
P(Marital Status=Married|No) = 4/7
P(Marital Status=Single|Yes) = 2/7
P(Marital Status=Divorced|Yes)=1/7
P(Marital Status=Married|Yes) = 0

- P(X|Class=No) = P(Refund=No|Class=No)
  $\times$ P(Married| Class=No)
  $\times$ P(Income=120K| Class=No)
  = 4/7 $\times$ 4/7 $\times$ 0.0072 = 0.0024

- P(X|Class=Yes) = P(Refund=No| Class=Yes)
  $\times$ P(Married| Class=Yes)
  $\times$ P(Income=120K| Class=Yes)
  = 1 $\times$ 0 $\times$ 1.2 $\times$ 10^{-9} = 0

Since P(X|No)P(No) > P(X|Yes)P(Yes)

Therefore P(No|X) > P(Yes|X)

=> Class = No

Naïve Bayes Classifier

- If one of the conditional probability is zero, then the entire expression becomes zero
- Probability estimation:

$$\text{Original} : P(A_i | C) = \frac{N_{ic}}{N_c}$$

$$\text{Laplace} : P(A_i | C) = \frac{N_{ic} + 1}{N_c + c}$$

$$\text{m - estimate} : P(A_i | C) = \frac{N_{ic} + mp}{N_c + m}$$

c: number of classes

p: prior probability

m: parameter

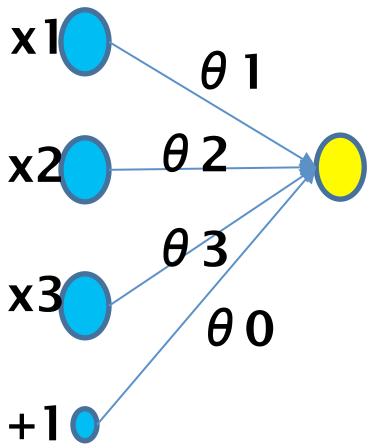
Logistic Regression

- Can be visualized as a "**single neuron**".

$$P(y=1|x;\theta)=\sigma(\theta_0+\theta_1 x_1+\ldots+\theta_n x_n)=\sigma(\theta_0+\sum_{i=1}^{n}\theta_i x_i)$$

$$P(y=0|x;\theta)=1-P(y=1|x;\theta)$$

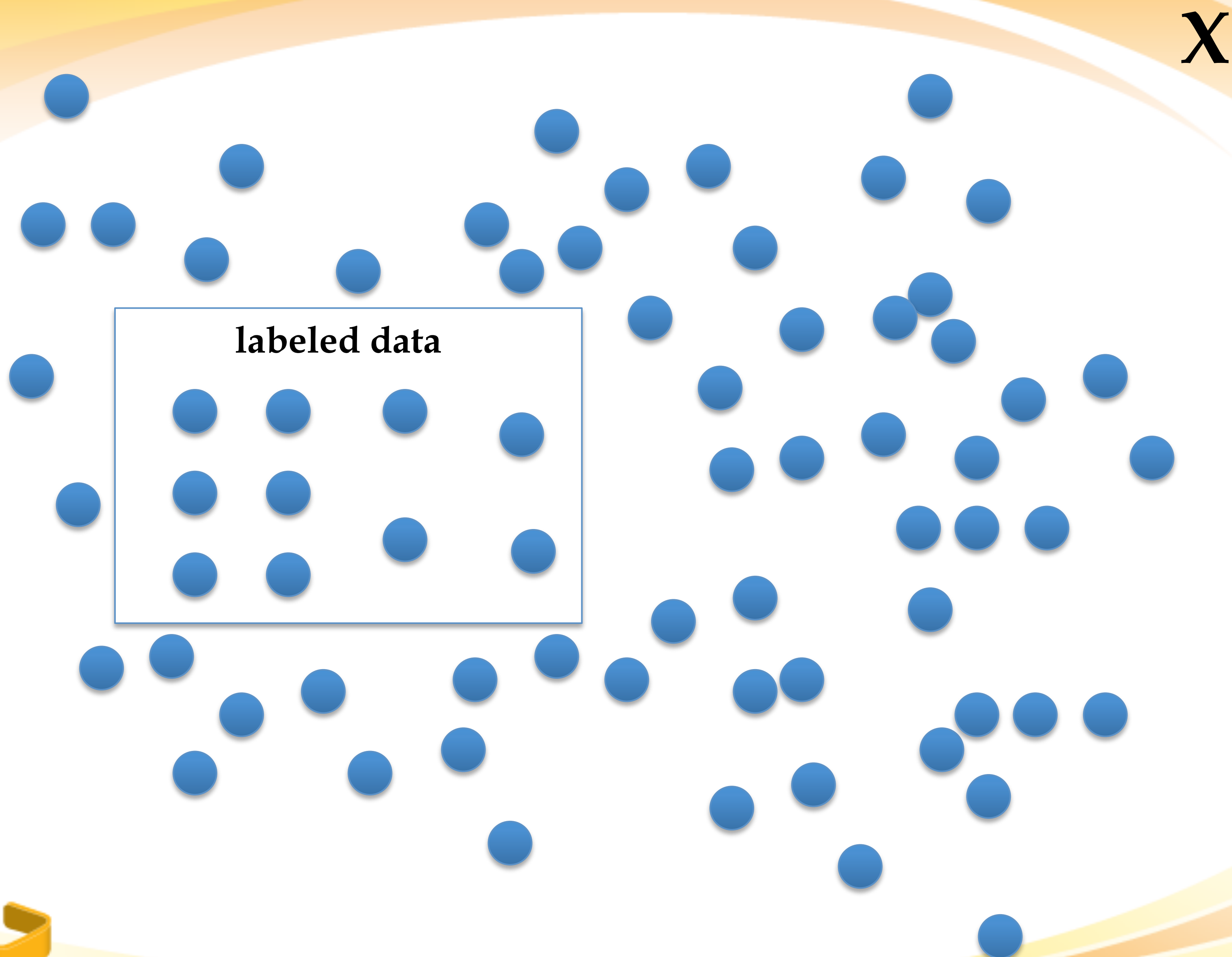
$$\sigma(z)=\frac{1}{1+e^{-z}}$$



Using the **sigmoid function** as the **activation function**

Evaluation

- For all supervised problems, it's important to understand how well your model is performing
- What we try to estimate is how well you ***will*** perform in the future, on new data also drawn from **X**



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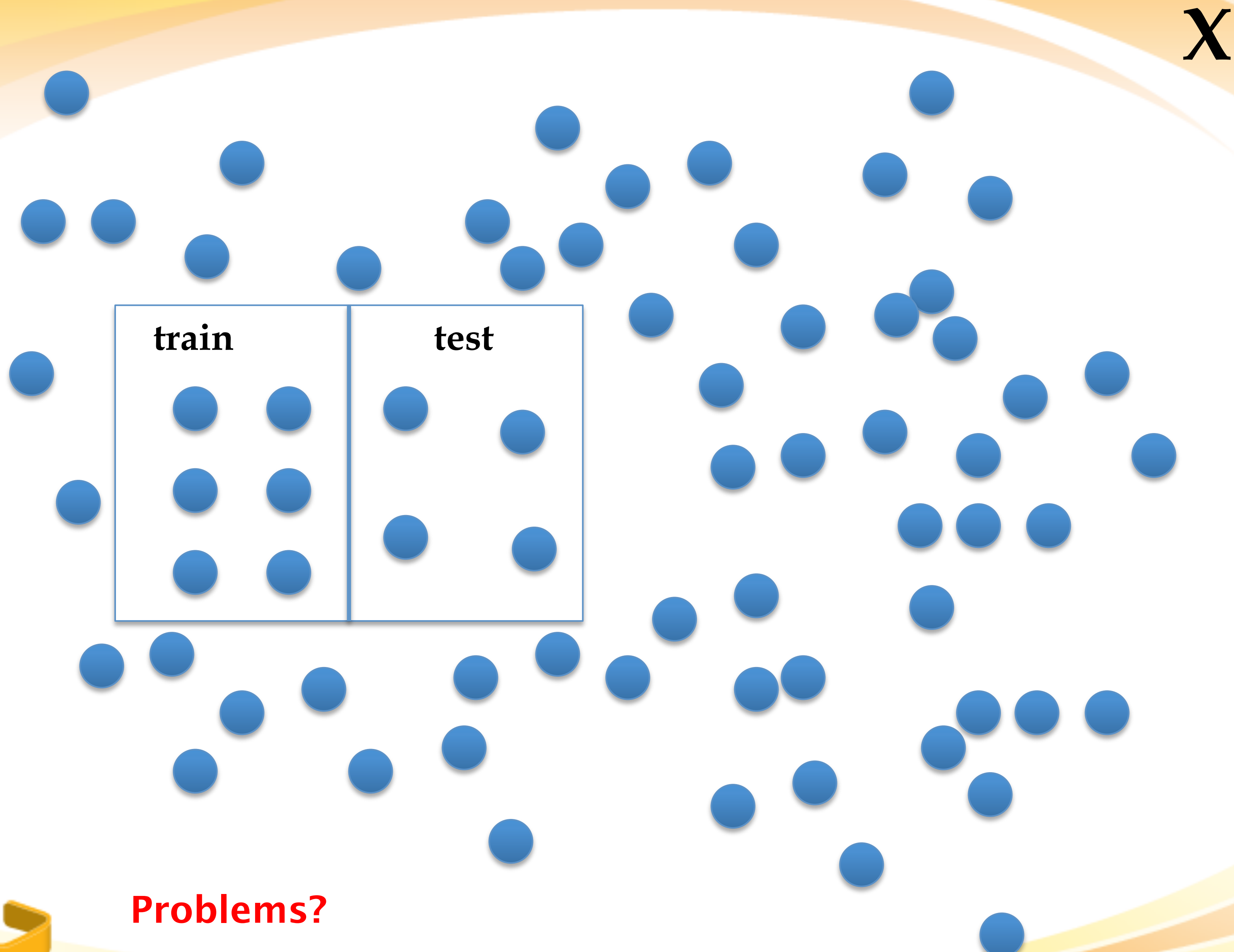
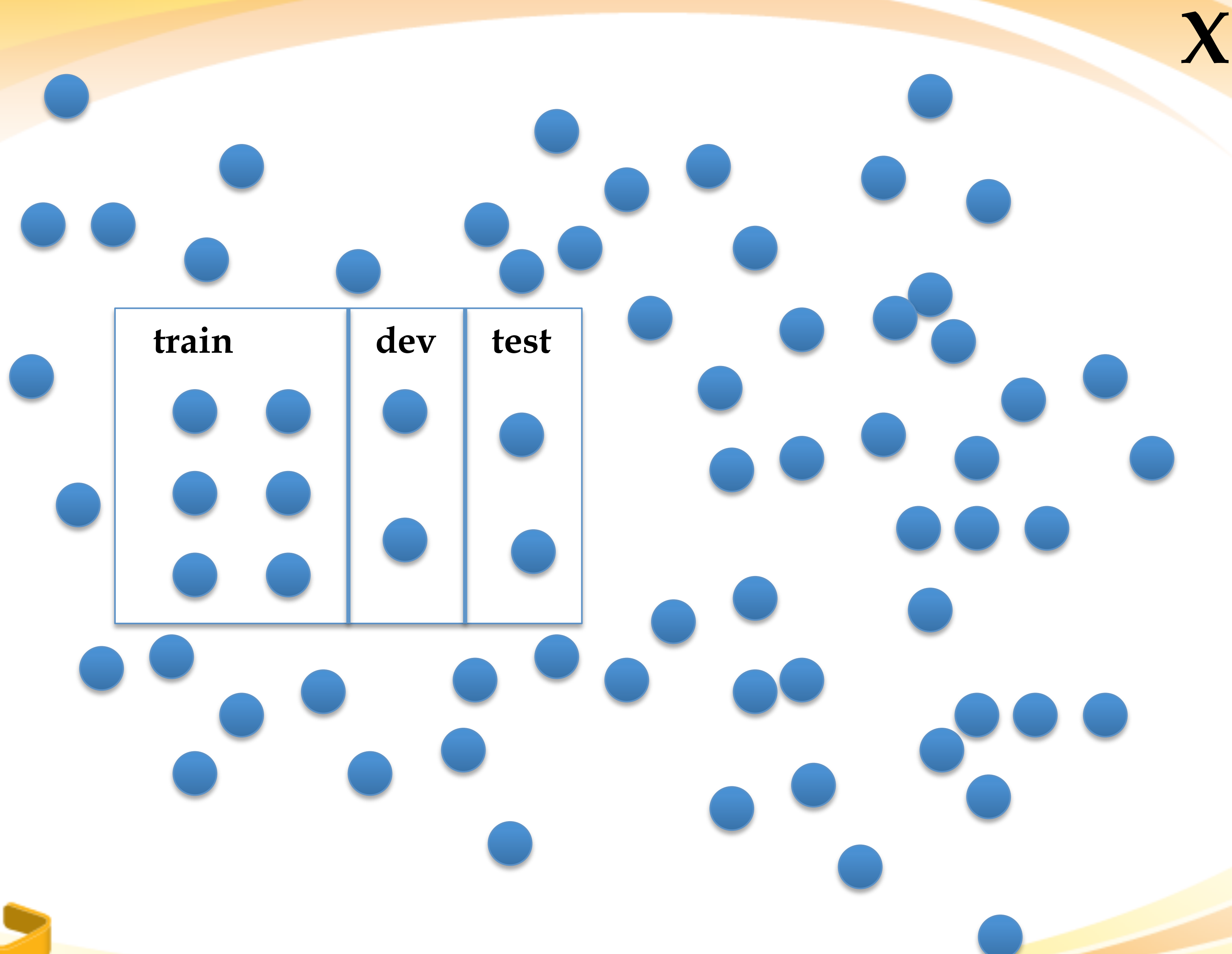
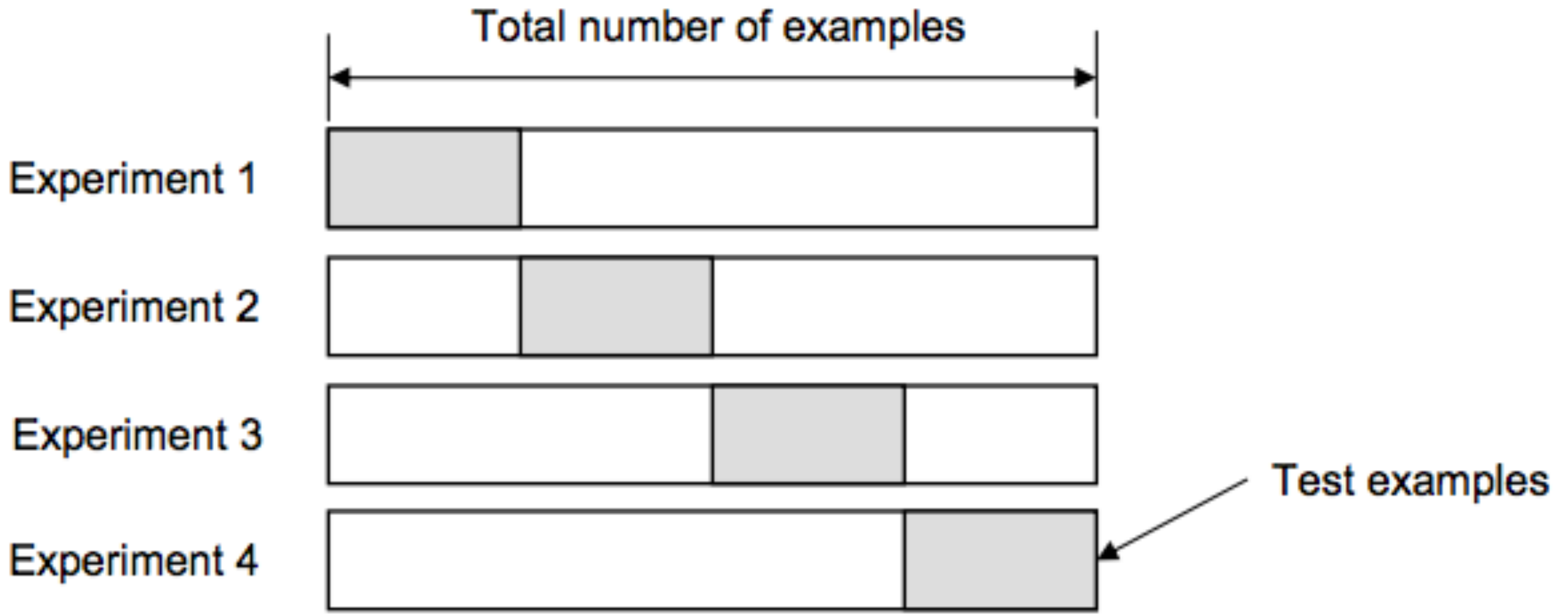


Problems?


X
train
dev
test

K-fold cross validation



K = 4

Evaluation Metric

Binary Classification

Confusion Matrix:

		PREDICTED (y_{pred})	
		Class=Yes	Class=No
ACTUAL CLASS (y_{true})	Class=Yes	**a**	**b**
	Class=No	**c**	**d**

a: TP (true positive)

b: FN (false negative)

c: FP (false positive)

d:TN (true negative)

$$\text{Accuracy} = \frac{a+d}{a+b+c+d} = \frac{TP+TN}{TP+TN+FP+FN}$$

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Precision

Precision: proportion of predicted class that are actually that class. i.e., if a class prediction is made, should you trust it?

$$\text{Precision} = \frac{a}{a+c+} = \frac{TP}{TP+FP}$$

		Predicted (y_{pred})	
Actual class (y_{true})		Class=Yes	Class=No
	Class=Yes	48	70
	Class=No	0	10347

Recall

Recall: proportion of true class actually predicted to be that class

$$\text{Recall} = \frac{a}{a+b} = \frac{TP}{TP+FN}$$

	Predicted (y_{pred})	
	Class=Yes	Class=No
Actual class (y_{true}) Class=Yes	48	70
Actual class (y_{true}) Class=No	0	10347

Precision, Recall, F1

$$\text{Precision (p)} = \frac{a}{a+c} = \frac{TP}{TP+FP}$$

$$\text{Recall (r)} = \frac{a}{a+b} = \frac{TP}{TP+FN}$$

$$\text{F-measure (F)} = \frac{1}{\left(\frac{1/r+1/p}{2}\right)} = \frac{2rp}{r+p} = \frac{2a}{2a+b+c} = \frac{2TP}{2TP+FP+FN}$$

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Multiclass classification

	Apple	Orange	Banana
Precision	?	?	?
Recall	?	?	?

Predicted

True label

100	2	15
0	104	30
30	40	70

Multiclass classification

	Apple	Orange	Banana
Precision	0.769	0.712	0.609
Recall	0.855	0.776	0.500

Predicted

True label

100	2	15
0	104	30
30	40	70

Summary

- Classification task
- Some learning algorithms
- Evaluation

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