

Team Name:

Roll No. :

Time:



- Panem. The far, far future. The games have been re-established, much to the horror of the people. The year is 2022 which, coincidentally also marks the 4th Quarter Quell, i.e. the 100th Hunger Games. This year, the twist is that the Gamemakers are mathematicians and so the Games will not only test your survival skills but also your logical reasoning skills.

1. Since it is the Quarter Quell the number of tributes this year is not the traditional 24. You watch the customary announcement by the President on the television. However, the President decides to have some fun. He gives the following equation:  $f(x^3 + y^3) = xf(x^2) + yf(y^2)$ . Given that  $f'(0) = 5$ , and  $f$  is differentiable, then  $(f'(5))^2 + (f'(0))^2$  gives the number of tributes.

**Solution format:** An integer which represents the number of tributes

2. It's Sorting Day, you are waiting for the names to be called out. Every citizen between the ages of 12-18 has been assigned an identity number. The slips in the giant glass bowls contain the numbers. Effie rummages around the bowl and pulls out a slip. You watch with bated breath, waiting for her to announce the number. However, she says the following:

Let  $A$  and  $B$  be two sets of real numbers with  $x = \sup A = 2431$  and  $y = \sup B = 1329$ . Let  $C$  be the set of all real numbers of the form  $a + b$  where  $a$  is in  $A$  and  $b$  is in  $B$ .  $z = \sup C$  and  $z$  is the identity number of the chosen tribute!

**Solution format:** The real number  $z$ .

**Hint:** (Find a relation between  $x, y, z$  and then find  $z$ )

3. You find that the answer matches your identity number and you reach the stage. Peacekeepers hastily take you to the train and then the Capitol. In the training arena, you are drawn to the puzzle's stall. The trainer comes to you with three boxes A, B, C with 0, 0 and 27 balls respectively. He wants you to sort the balls such that all boxes have the same number of balls. But, he says, at every  $n$ th move, you should move exactly  $n$  balls from one box to another. How many moves would you need to sort the balls.

**Solution format:** Integer representing number of moves.

4. You have your individual session with the Gamemakers today and you are determined to show your best. You confidently ask them to pose any puzzle. They ask you the following question: Let  $A$  and  $B$  are two non singular matrices such that  $3ABA^{-1} + A = 2A^{-1}BA$ , then find the determinant of matrix  $A+B$

**Solution format:** Integer

5. It is the day of the interviews!! You are anxiously waiting for your turn, and go nervously to the stage when Caesar, the interviewer, calls out your name. "We heard about your amazing feat in the training arena", he says. "A solid 12! I am sure the audience would like to see you work that brain." He gives you the quadratic polynomial  $P(x) = ax^2 + bx + c$  and defines some legal operations on  $P(x)$  -

$O_1$ : Switch  $a$  and  $c$ ,

$O_2$ : Replace  $x$  by  $x+t$ ,  $t \in \mathbf{R}$

He then asks you to transform  $x^2 - x - 2$  into  $x^2 - x - 1$  using these operations?

Your reputation is at stake, you must answer quickly before your time is up.

**Solution Format:** Write 0 if the transformation is not possible and 1 if it is possible.

6. The time has come and all of today's tributes are being dropped into the arena. You rush to the cornucopia to grab some supplies. At the cornucopia, there are a large number of boxes labelled with numbers 105 to 210. You see the other tributes approaching and you have to pick fast. A set of boxes is called 'good' if it contains atleast two boxes whose numbers are non-relatively prime. You can carry only a small number of boxes so find out the least number of boxes you need to pick such that the set is 'good'

**Solution Format:** The least number of boxes needed such that out of the boxes picked, two boxes (one pair) have numbers which are non-relatively prime(i.e. have a common factor other than 1)

7. A day of harsh battle later, it is finally nighttime. But how many exactly died, you wonder? This time, the Gamemakers decided to keep the names of those deceased a secret. Instead of the traditional cannons they posed the following puzzle:

Suppose that  $a, b, c, d$  are positive real numbers satisfying  $(a+c)(b+d) = ac + bd$ . Find the smallest possible value of  $S = \frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}$

Where  $S$  is the number of contestants that died the first day?

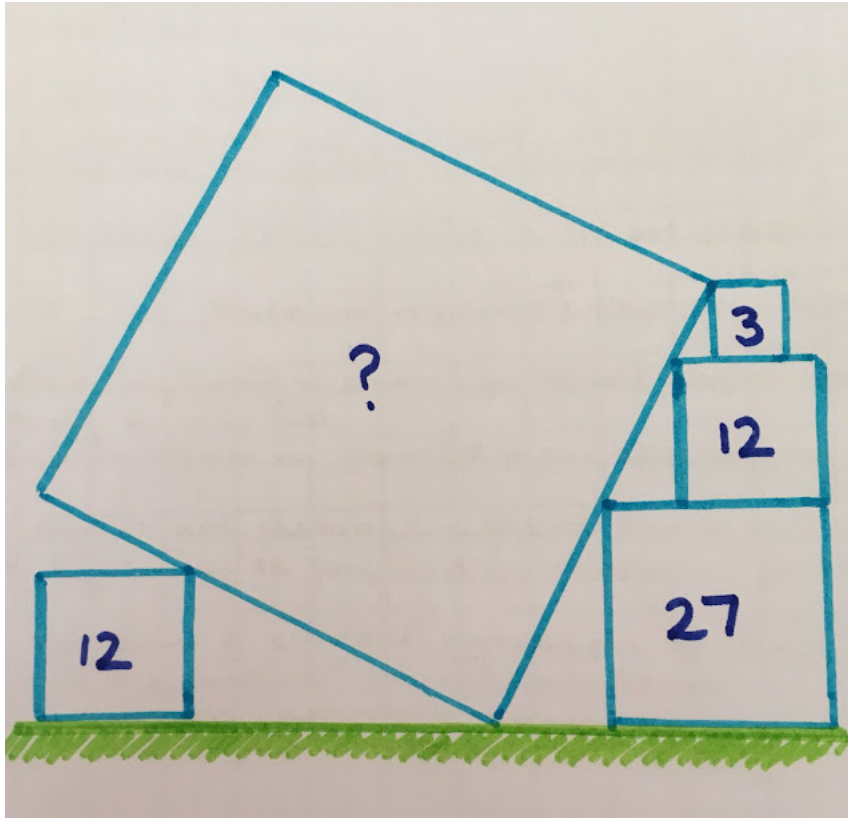
**Solution Format:** Correct integer for  $S$

8. The next morning, you are ambushed by the Careers' pack. You manage to get out of there alive but are left heavily injured. Thankfully, you had enough sponsors and you notice a silver parachute gliding towards you. It carries a small box, the first-aid you need, but your mentor has very smartly encoded the box to avoid others from using it. Find the answer to the lock.

**Solution Format:** Integer

9. Now that you have received the first aid kit, you slowly, and painfully make your way to a nearby cave. The cave has an opening in the middle with weird square shaped boulders on either side of the opening with their areas as given in the figure. As you notice the sun slowly setting, you decide to make camp for the day and decide to cover the opening with a boulder. Unfortunately you only have enough energy to drag only one boulder across the opening. Calculate

the area of the boulder that would be enough to provide you shelter for the night. Use the rough figure given below. The gaps left out will be too small for anyone else to enter, so don't worry about those.



**Solution Format:** Integer

10. The next day, you wake up to find the tribute from District 12, Posy staring at you. Alarmed, you jump up with your knife. You're just about to kill her when you realise that she has been keeping a watch on you. She wants to strike a deal with you. She has received food from her district but her parachute also has a lock. She is willing to share her food with you if you help her with the lock. Realising you haven't eaten for two days, you readily agree. This is what is written:

"Congratulations! That sure was not a cakewalk

$1! + 2! + 3! \dots 2022!$  Is the hint for this lock!

Enter the units digit and the lock will open,

But wait too long and death you have chosen

Find the answer to the riddle to open the box!

**Solution Format:** Correct integer for the units place digit.

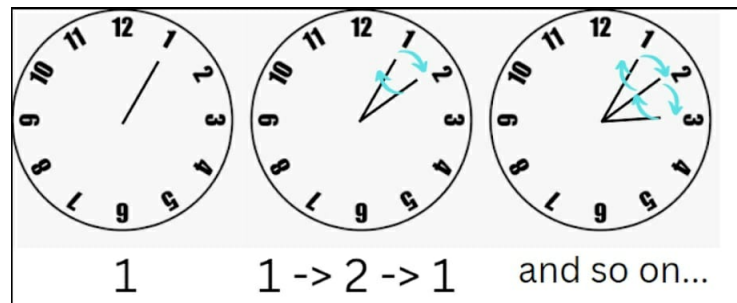
11. After a hasty meal, you and Posy go in search of some water. After a while, you come across a strange hole which is emitting blue light. Posy bends over to see what is in there causing both of you to slip and fall into the hole. To your utter shock, you find yourself in a mirror world, except the sky is red. A deep voice echoes "Turn me blue with 2022 or the redness will take over you". It was a hint! You are supposed to somehow turn the sky to colour blue for this task. You see sticks nearby marked 0,1,2 and powers of 3 written on the ground in the order of 0,3,9,27 and so on. A stone was lying down and it was embossed with "sum the blue divide by 3, the red sky will then flee". You guys connect the dots. The task seems to be writing the sum of first 2022 integers which when written in base 3 are divisible by 3. Turn the sky blue again or get trapped in utter dark redness!

**Solution Format:** Correct integer

12. Whew! You made your way back from that hellhole. But the moment you try to move you realise that you are stuck in a square region. You notice a weird shaped tree in the shape of a 9 in front of you. It was written on the tree "how many ways of stones to get me?" You see another guy and both of your squares were connected by a rope. It was as if both of your lives were entangled for that time. The other guy gets frustrated and picks up three stones in his region and throws it outside and it disappears. Where did it go? Suddenly six stones come out of your region and hit you. It seemed that there was an equation between the squares of two numbers and related with 9 somehow. BAM! You figured it out! You guys are supposed to find out the number of integral solutions to the equation.

**Solution Format:**

13. A hike is supposed to be completed to find food. The hike is very long and you need water for it. You find a pole standing straight up. There were roots coming out of the pole's bottom. The sunlight was falling on the pole and its shadow somehow was imitating a clock. But not just any clock. A clock with multiple hands. You observe their motion. The first second all of them point to 1 and the next second all of them point to 1 and 2. And then they change their position and point to 1. Then they point to 1, 2 and 3 and then again they change their position and point to 2 and then 1. You see it keeps happening for a long time. And after sometime it points to 8 and then 1. The roots and the order of the numbers it was showing seemed interesting. Maybe it was the square root of those numbers. The 81 that it showed at last also signified something. It seemed as if they wanted us to write down 81 times the pattern we were observing. A merchant appears with water and asks "Is it divisible by 81?". Figure out what he is talking about and answer his question for water.



**Solution Format:**

14. A nearby cry jolts you awake and Posy who had been keeping watch whispers to you, "Hey, I guess that's someone out there". You both creep out of your cave and notice that someone had been caught in a trap that you had not noticed before. The poor victim had tripped and fallen into a rapidly filling ditch with a digital gate. The gate is square shaped with 100 square divisions. A inscription inside the ditch reads

"Here are 100 squares for you to play with. Let each square be denoted by  $(i,j)$ . To escape use the digital palette available, colour the grid such that squares centred at  $(i,j)$ ,  $(j,k)$ ,  $(j,l)$  do not have the same colours"

Since the victim is unconscious you both decide to help him out. How many minimum number of colours would you require for this task?

NOTE:  $1 \leq i < j \leq n$  and  $1 \leq k < l \leq n$ .

**Solution Format:** Correct integer for the minimum number of colours.

Right as you help the poor tribute out of the death trap, you hear a booming voice echo throughout the arena.

"All the remaining tributes are kindly requested to make their way, without killing each other, to the main arena in the arena centre! "