1. Big omega Notation: prove that $g(n) = n^3+2n^2+4n$ is $\Omega(n^3)$

Sol $g(n) \geq c \cdot n^3$

$g(n) = n^3+2n^2+4n$

for finding constants c and n_0

$n^3+2n^2+4n \geq c \cdot n^3$

divide both sides with n^3

$$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$$

$$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$$

$$1 + \frac{2}{n} + \frac{4}{n^2} \simeq 1$$

example $c = 1/2$

$1 + \frac{2}{n} + 4/n^2 \geq 1/2$ $(1 \geq 1/2, n \geq 1)$

$1 + 2/n + 4/n^2 \geq 1$ $(n \geq 1, n_0 = 1)$

$1 + 2/n + 4/n^2 \geq 1/2$

Thus, $g(n) = n^3+2n^2+4n$ is indeeded $\Omega(n^3)$

2. Big theta notation. determine whether $h(n) = 4n^2+3n$ is $\theta(n^2)$ or not

$c_1 n^2 \leq h(n) \leq c_2 n^2$

In upper band $h(n)$ is $O(n^2)$

In lower band $h(n)$ is $\Omega(n^2)$

upper bound $(1 O(n^2))$

$h(n) = 4n^2+3n$

$h(n) \leq c_2 n^2$

$4n^2+3n \leq c_2 n^2 \Rightarrow 4n^2+3n \leq cn^2$

let $c_2 = 5$

divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2+3n$ is $O(n^2)$ $[c_2 = 5, n_0 = 1]$

$h(n) = 4n^2 + 3n$ is $\Theta(n^2)$

$$1 + \frac{n}{n\log n} \leq 2$$ (Simplify)

$$1 + \frac{1}{\log n} \leq C_2$$ $C_2 = 2$

$$1 + \frac{1}{\log n} \leq 2$$ $(C_2 = 2, n_0 = 2)$

Then $h(n)$ is $O(n\log n)$

Lower bound.

$$h(n) \geq C_1(n\log n)$$

$$h(n) = n\log n + n$$

$$n\log n + n \geq C_1 \cdot n\log n$$

divide both sides by $n\log n$

$$1 + \frac{n}{n\log n} \geq C_1$$ (Simplify)

$C_1 = 1$

$$1 + \frac{1}{\log n} \geq C_1$$

$$\frac{1}{\log n} \geq 0 \text{ for all } n > 1$$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

3. Solve the following recurrence relations & find the growth of Solution $T(n) = 4T(n/2) + n^2 / T(1) = 1$

$$f(n) \geq C_1 g(n)$$

Substuting $f(n)$ & $g(n)$ into this inequality we get.

$$n^3 - 2n^2 + n \geq C(-n^2)$$

A and C and n_0 holds $n \geq n_0$

$$n^3 - 2n^2 + n \geq Cn^2$$

$$n^3 + ((-2))n^2 + n \geq 0$$

$$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

$\therefore$ The statement $f(n) = \Omega g(n)$ is true

4. Determine whether $h(n) = n\log n + n$ is $O(n\log n)$ prove a vigorous proof for your conclusion.

$$c_1 n\log n \le h(n) \le c_2 n\log n.$$

upper bound :-

$$h(n) \le c_2 \cdot n\log n$$

$$h(n) = n\log n + n$$

$$n\log n + n \le c_2 n\log n$$

divide both sides with $n\log n$

$$1 + \frac{n}{n\log n} \le 2$$

$$1 + \frac{1}{\log n} \le c_2$$

$$1 + \frac{1}{\log n} \le 2$$

Then $h(n)$ is $O(n\log n)$

lower bound :-

$$h(n) \geq c_1 n\log n$$

$$h(n) = n\log n + n$$

$$n\log n + n \geq c_1 n\log n$$

divide bothsides with $n\log n$

$$1 + \frac{n}{n\log n} \geq c_1$$

$$1 + \frac{1}{\log n} \geq c_1$$

$$1 + \frac{1}{\log n} \geq 1$$

$$\frac{1}{\log n} \geq 0$$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

5. Solve the following recurrence relations and find the order of growth of solutions.

$T(n) = 4T(n/2) + n2, T(1) = 1$

Sol. $T(n) = 4T(n/2) + n^2, T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4 \quad b = 2 \quad f(n) = n^2$

Applying masters thoerem

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b a - \epsilon})$

$f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + \epsilon})$, Then $T(n) = f(n)$,

Caluclating $\frac{a}{b}$:

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$ (comparing $f(n)$ with $n^{\log_b a}$)

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

Order of growth:

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$

is $\Theta(n^2 \log n)$