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1 Introduction

To prove:

$$\frac{n!}{n^n} \leq \frac{1}{n}$$

This will be equal to:

$$\frac{n!}{n^n} = \frac{1 \cdot 2 \cdot 3.... \cdot n}{n \cdot n... \cdot n} \tag{1}$$

Cancelling the one n from the denominator and another form the numerator and taking the $1/n$ separately, and separating the fractions we have:

$$\frac{1}{n} \frac{2 \cdot 3 \cdot .... \cdot (n)}{n^{(n)}}$$

By definition, we have the same number of integers to be multiplied in the numertor and the denominator. This also means that except the last term every integer is less than n, creating a number less than 1 (Unless $n = 1$). Multipliying any number by another which is less than one produces a lesser absolute value (here we are dealing with +ve integers anyway.), and multiphying with one brings the same value.
$\therefore$ Proved.

2 Question 92

a.
Solution; It is given that a_{2n+1} approaches L as n becomes arbiterarily large. This means that all terms mapped to an even value of n approach L. What remains is odd numbers, for which simoilar data has been given. This only proves that the values of a sequence as a whole approaches L, a number. Thus the sequence is convergent.
b.
Solution:

$$\lim_{n \to \infty} a_{n+1} = 1 + \frac{1}{1 + a_n}$$

Here, we have to find out the value that the sequence is approaching, and for that we can substitute $n = \infty$.

$$\begin{gathered}\lim_{n\to\infty} a_{n+1} = 1 + \frac{1}{1 + a_n} = a_n \\ \therefore (a_n + 1)(a_n - 1) = 1 \\ \therefore (a_n)^2 - 1 = 1 \\ \therefore (a_n)^2 = 2 \\ \therefore a_n = \pm\sqrt{2}\end{gathered}$$

But seeing that the sequence is strictly increasing after calculating the values of $n = 1, 2, 3, 4, 5, 6, 7, 8$ we can conclude that the value cannot be negative, and so the limit of the sequence is the positive square root of 2.