Excs

Sammit

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Problem 41

Prove the root test.

Solution

Firstly, I must clarify that the if the absolute value of a series is convergent then the series is, and this has been proven.

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$$

$$\therefore \sqrt[n]{|a_n|} < r \qquad\qquad \text{when } n \geq N$$

$$\therefore a_N < r^N$$

$$\therefore |a_{N+1}| < r^N \cdot r$$

$$\because r < 1,\ r^N \cdot r < r^N$$

$$\therefore |a_N|r^k > |a_N|r^{k+1}$$

$$\because n \text{ need not be greater then } N \text{ for } r \text{ to be less than } 1$$

$$\therefore \sum_{n=1}^{\infty} |a_N|r^n \text{ is convergent.}$$

$$\therefore \sum_{n=1}^{\infty} |a_N|a_n = |a_N|\sum_{n=1}^{\infty} |a_n| \text{ is also convergent}$$

$$\therefore \frac{|a_N|}{|a_N|}\sum_{n=1}^{\infty} |a_n| \text{ is also convergent}$$

Because the series a_n is absolutely convergent, it is convergent.