Introduction to Linear Algebra Excersises

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Chapter 1

Exercise 3

Problem 3

Given that A and B are two vectors defined on the same dimentions, prove that $(A+B)^2 = A^2 + B^2 + 2AB$, and that $(A-B)^2 = A^2 + B^2 - 2AB$

Solution

$$
\begin{gathered}
A = (a_1, a_2, a_3, ...a_n) \\
\therefore A^2 = (a_1^2, a_2^2, a_3^2, ...a_n^2) \\
B = (b_1, b_2, b_3, ...b_n) \\
\therefore B^2 = (b_1^2, b_2^2, b_3^2, ...b_n^2) \\
\therefore 2AB = (2a_1b_1, 2a_2b_2, 2a_nb_n) \\
\therefore (A+B) = (a_1 + b_1, a_2 + b_2 .....a_n + b_n) \\
\therefore (A+B)^2 = ((a_1^2 + b_1^2 + 2a_1b_1), (a_2^2 + b_2^2 + 2a_2b_2), ...(a_n^2 + b_n^2 + 2a_nb_n)) \\
= (a_1^2, a_2^2, a_3^2, ...a_n^2) + (b_1^2, b_2^2, b_3^2, ...b_n^2) + (2a_1b_1, 2a_2b_2, 2a_nb_n) \\
= A^2 + B^2 + 2AB
\end{gathered}
$$

$\therefore$ ***Q.E.D.***

PTO

Now, we move on to the next proof.

$$A = (a_1, a_2, a_3, ...a_n)$$
$$\therefore A^2 = (a_1^2, a_2^2, a_3^2, ...a_n^2)$$
$$B = (b_1, b_2, b_3, ...b_n)$$
$$\therefore B^2 = (b_1^2, b_2^2, b_3^2, ...b_n^2)$$
$$\therefore 2AB = (2a_1b_1, 2a_2b_2, 2a_nb_n)$$
$$\therefore (A - B) = (a_1 - b_1, a_2 - b_2.....a_n - b_n)$$
$$\therefore (A - B)^2 = ((a_1 - b_1)^2, (a_2 - b_2)^2, ...(a_n - b_n)^2)$$
$$= ((a_1^2 + b_1^2 - 2a_1b_1), (a_2^2 + b_2^2 - 2a_2b_2), ...(a_n^2 + b_n^2 - 2a_nb_n))$$
$$= (a_1^2, a_2^2, a_3^2, ...a_n^2) + (b_1^2, b_2^2, b_3^2, ...b_n^2) - (2a_1b_1, 2a_2b_2, 2a_nb_n)$$
$$= A^2 + B^2 - 2AB$$

$\therefore$ ***Q.E.D.***

Problem 5

Given that vector A is perpendicular to any and all vectors, prove that A is a 0 vector. You may take any and all vectors to be X.

Solution

$$A = (a_1, a_2, a_3, ...a_n)$$
$$X = (x_1, x_2, x_3, ...x_n)$$
$$AX = \sum_{i=1}^{n} a_i x_i$$

Because x_i can be any number, the only way the sum of all the terms is 0 is by ensuring that all the terms are equal to 0, and that can only be the case if a_i is 0 in all cases, because 0 is the only number which yieds 0 as the product when multiplied with any number.

$\therefore a_i = 0$ for all i.

$\therefore A$ is a 0 vector.

Exercise 4

Find the norm of the vector A in the following cases.

Part A

A=(2, -1)

Solution

$$||A|| = \sqrt{\sum_{i=1}^{n} a_i^2}$$
$$= \sqrt{2^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

Part E

$$A = (\pi, 3, -1)$$

Solution

$$||A|| = \sqrt{\sum_{i=1}^{n} a_i^2}$$
$$= \sqrt{\pi^2 + 3^2 + (-1^2)}$$
$$= \sqrt{\pi^2 + 10}$$

Problem 3

Find the projection of A along B.

Part B

$$A = (-1, 3);\ B = (0, 4)$$

Solution

$$c = \frac{A \cdot B}{B \cdot B}$$
$$= \frac{(-1 \cdot 0) + (3 \cdot 4)}{(0 \cdot 0) + (4 \cdot 4)} = \frac{12}{16} = \frac{3}{4}$$
$$\text{Projection of A along B } = cB$$
$$= \frac{3}{4}(0, 4) = (0, 3)$$

Problem 5

Find the cosine between the following vectors A and B.

Part A

$$A = (1, -2);\ B = (5, 3)$$

Solution

$$A \cdot B = 1 \cdot 5 + -2 \cdot 3 = -1$$
$$||A|| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$
$$||B|| = \sqrt{5^2 + 3^2} = \sqrt{34}$$
$$\cos\theta = \frac{A \cdot B}{||A||\,||B||}$$
$$= -\frac{1}{\sqrt{5}\sqrt{34}}$$

Problem 6 A

Find the cosine of the angle formed by these vertices:

$$(2, -1, 1),\ (1, -3, 5),\ (3, -4, -4)$$

Solution

I shall consider the angle to be denoted by ϑ instead of θ to phleks the seski math symbol.
Firstly, we shall lable the points.

$$(2, -1, 1) = A$$
$$(1, -3, 5) = B$$
$$(3, -4, -4) = C$$
$$\overrightarrow{AB} = B - A = (-1, -2, 4)$$
$$\overrightarrow{BC} = C - B = (2, -1, -9)$$
$$\therefore \cos\vartheta = \frac{A \cdot B}{||A||\,||B||}$$
$$= \frac{(-1) \cdot 2 + (-2) \cdot (-1) + 4 \cdot 9}{(\sqrt{(-1)^2 + (-2)^2 + 4^2})(\sqrt{2^2 + (-1)^2 + (-9)^2})}$$
$$= \frac{36}{\sqrt{21}\sqrt{41}}$$

$$f'(x) = f'(g(x)) \cdot g'(x)$$

Problem 7

Actually fuck it if you wanna see the solution head to my chat with *yang_neo* so ya.
No wait we chat about a lot of shit which you dont wanna see youre gonna judge us so dont

Exercise 6

Problem 2

Let $y = mx + b$ and $y = m'x + c$ be the equations of two lines in the plane. Write down vectors perpendicular to these lines. Show that these vectors are perpendicular to each other if and only if $mm' = -1$

Solution

$$
\begin{gathered}
b = y - mx \\
c = y - m'x \\
\therefore (1, -m) = \text{Vector } \beta\{\text{seski phlex}\}, \\
(1, -m') = \text{Vector } \gamma\{\text{seski phlex again}\} \\
\beta \cdot \gamma = 0 \\
\therefore 1 \cdot 1 + -m \cdot (-m') = 0 \\
\therefore 1 + mm' = 0 \\
\therefore mm' = (-1)
\end{gathered}
$$

$\therefore$ ***Q.E.D.***