I will change it stfu

r.suhrit2017

March 2021

1 Introduction

Unless specifically mentioned, nth term will always indicate nth term of the sequence b_n.

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a. An alternating series is one where the sign of the term alternates, i.e., when every term of a sequence is multiplied with $(-1)^n$ or $(-1)^{(n+1)}$.
b. If an alternating seies is constantly decreasing, and the terms of the series approach zero as n becomes arbiterarily large, the series is convergent.
c. This shows that the remainder of the partial sums is leser than the next term.

2.

$$\sum_{n=1}^{\infty} \frac{2(-1)^{n-1}}{2n+1}$$

$$n^{th} \text{ term} = \frac{2}{2n+1}$$

The first condition is clearly satisfied. We test for the second condition:

$$\lim_{n\to\infty} \frac{2}{2n+1} = \lim_{n\to\infty} \frac{\frac{2}{n}}{2+\frac{1}{n}} = \frac{0}{2} = 0$$

With both the conditions satisfied, we can conclude that the series is convergent.

3.

$$\sum_{n=1}^{\infty} \frac{2n(-1)^{n-1}}{n+4}$$

We find that the sequence is increasing, for the nth term =

$$\frac{2n}{n+4}$$

Through calculus it is obvious that the rate of increment in the numerator is higher, and thus the sequence starts to increase after the first 4 terms. This proves that it is not strictly increasing, and so the sequrnce is divergent.

4.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$$

The alternating sequence
$$\frac{1}{\sqrt{n}}$$

is clearly convergent what the fuck is wrong with you its clearly decreasing and clearly approaching zero as n approaches infinity.

5.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln n + 4}$$

The series
$$\sum_{n=1}^{\infty} \frac{1}{\ln n + 4}$$

satisfies $b_{n+1} < b_n$ because $\ln n$ is a strictly increasing function. Then, we have

$$\lim_{n\to\infty} \frac{1}{\ln n + 4}$$
$$= \frac{1}{\infty + 4} = \frac{1}{\infty} = 0$$

Thus we have proved that the series is convergent.

8.
$$(-1)^{n-1} \frac{n}{\sqrt{n^3 + 2}}$$

We will divide this but the highest power in the nummerator by the highest in the denominator:

Numerator:

$$\frac{n}{n^{\frac{3}{2}}} = n^{1-\frac{3}{2}} = \frac{1}{\sqrt{n}}$$

Dinominator:

$$\frac{\sqrt{n^3+2}}{n^{\frac{3}{2}}} = \sqrt{\frac{n^3}{n^3} + \frac{2}{n^3}}$$

Now we take the fraction as a whole:t

$$\frac{\frac{1}{\sqrt{n}}}{\sqrt{1+\frac{2}{n^3}}} = \frac{1}{\sqrt{n+\frac{2}{n^2}}}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n+\frac{2}{n^2}}} = 0$$

The second condition asks us to prove that b_n is strictly decreasing. This is true, and one can confirm it by graphing the function. Even though the function b_x is not strictly decreasing, if the domain is restricted to natural numbers then $b_{n+1} \geq b_n$, and thus it can be concluded that the series is convergent.

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Firlty we have to prove that the seires is convergent. It has alreay been seen that the zeta function is convergent with the domain is restricted in this way:

$$\{x : x > 1\}$$

Yet, we will look at a specific proof because bo class is damn boring.

$$\int_1^\infty \frac{1}{n^6}\,dx = \lim_{t\to\infty} int_1^t \frac{1}{n^6}\,dx = \lim_{t\to\infty} -\frac{t^{-5}}{5} + \frac{1}{5} = \lim_{t\to\infty} \frac{-1}{5t^5} + \frac{1}{5} = 0 + \frac{1}{5} = \frac{1}{5}$$

Because 0.2 is a finite number the integral is convergent, and so is the series.
For error less than 0.00005;

$$0.00005 \leq \frac{1}{n^6}$$

$$\therefore 0.00005 \leq n^6$$

$$\therefore \sqrt[6]{\frac{1}{0.00005}}\sqrt[6]{\frac{100000}{5}} = \sqrt[6]{20000} \approx 5.21 \leq n\sqrt[6]{\frac{100000}{5}} = \sqrt[6]{20000} \approx 5.21, \text{ so we can take the value } n = 5$$

$\therefore$ For an approximation with error 0.00005, we can take $n+1=5$

$$\therefore n = 4.$$

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Before proceeding, one must observe that the fraction is multiplied by $(-1)^n$ instead of $(-1)^{n-1}$. Since our formula only works for $(-1)^{n-1}b_n$, we can multiply divide the fraction with -1, resultint in a - sign appearing in front of the faction.
If error less than 0.0001 then b_{n+1} ¿ 0.001.
Solving the inequality:

$$\frac{1}{n5^n} = 0.0001$$
$$\therefore n5^n = 10000$$
$$\therefore n = \frac{2^4 \cdot 5^4}{5^n}$$
$$\therefore n = 16 \cdot 5^{4-n}$$
$$\therefore 10000 = (16 \cdot 5^{4-n})5^n$$

36.
a.
Let us split the sums of s_{2n} and h_{2n} into the sums of their odd and even terms. Since the sums of their odd terms are equal anyway, we can subtract them from both sides of the equation. A new variable, a shall be used. This indicates the nth term but since we have the limits as n and $2n$, a is used for clarity.

$$\sum_{a=1}^{2n} \frac{(-1)^{2a-1}}{a} = \sum_{a=1}^{n} \frac{1}{2a} - \frac{1}{a}$$
$$\therefore \frac{-1}{2a} = \frac{-1}{2a}$$

Both the sides of the equation now mathc, hence proved.
b.

$$\sum_{n=1}^{2n} \frac{(-1)^{n-1}}{n} + \sum_{n \to 1}^{n} \frac{1}{n} - \ln 2n \to \gamma$$

as $n \to \infty$

$$\therefore \lim_{n \to \infty} \sum_{n=1}^{2n} \frac{(-1)^{n-1}}{n} + \sum_{n \to 1}^{n} \frac{1}{n} - \ln 2n = \lim_{n \to \infty} \sum_{n \to 1}^{2n} \frac{1}{n} - \sum_{n=1}^{2n} \frac{(-1)^{n-1}}{n} - \ln n$$

$$\therefore \lim_{n \to \infty} \sum_{n=1}^{2n} \frac{(-1)^{n-1}}{n} + \lim_{n \to \infty} \sum_{n \to 1}^{n} \frac{1}{n} - \lim_{n \to \infty} \ln 2n = \lim_{n \to \infty} \sum_{n \to 1}^{2n} \frac{1}{n} - \lim_{n \to \infty} \sum_{n=1}^{2n} \frac{(-1)^{n-1}}{n} - \lim_{n \to \infty} \ln n$$

$$\therefore \lim_{n \to \infty} \sum_{n=1}^{2n} \frac{(-1)^{n-1}}{n} = \lim_{n \to \infty} \ln 2n - \ln n = \ln(\frac{2x}{x})$$

$$\therefore \lim_{n \to \infty} \sum_{n=1}^{2n} \frac{(-1)^{n-1}}{n} = \ln 2$$

$$\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \ln 2$$

$$\therefore \boldsymbol{QDL}$$