

CBSE NCERT Solutions for Class 8 Mathematics Chapter 9**Back of Chapter Questions****Exercise 9.1**

1. Identify the terms, their coefficients for each of the following expressions.

(i) $5xyz^2 - 3zy$

(ii) $1 + x + x^2$

(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$

(iv) $3 - pq + qr - rp$

(v) $\frac{x}{2} + \frac{y}{2} - xy$

(vi) $0.3a - 0.6ab + 0.5b$

Solution:

(i) Given expression is $5xyz^2 - 3zy$.

This expression contains two terms $5xyz^2$ and $-3zy$.

Here the coefficient of xyz^2 is 5 and of zy is -3 .

(ii) Given expression is $1 + x + x^2$

This expression contains three terms 1, x and x^2 .

Here the coefficient of x and x^2 is 1.

(iii) Given expression is $4x^2y^2 - 4x^2y^2z^2 + z^2$.

This expression contains three terms $4x^2y^2$, $-4x^2y^2z^2$ and z^2 .

Here the coefficient of x^2y^2 is 4, coefficient of $x^2y^2z^2$ is -4 and coefficient of z^2 is 1.

(iv) Given expression is $3 - pq + qr - rp$.

This expression contains four terms 3, $-pq$, qr and $-rp$

Here the coefficient of pq is -1 , coefficient of qr is 1 and the coefficient of rp is -1 .

(v) Given expression is $\frac{x}{2} + \frac{y}{2} - xy$

This expression contains three terms $\frac{x}{2}$, $\frac{y}{2}$ and $-xy$

Here the coefficient of x is $\frac{1}{2}$, coefficient of y is $\frac{1}{2}$ and the coefficient of xy is -1 .

(vi) Given expression is $0.3a - 0.6ab + 0.5b$

This expression contains three terms $0.3a$, $-0.6ab$ and $0.5b$.

Here the coefficient of a is 0.3 , coefficient of ab is -0.6 and the coefficient of b is 0.5 .

2. Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

$x + y$, 1000 , $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr , $p^2q + pq^2$, $2p + 2q$.

Solution:

Given polynomial is $x + y$

Since $(x + y)$ contains two terms. Therefore, it is binomial.

Given polynomial is 1000

Since 1000 contains only one term. Therefore, it is monomial.

Given polynomial is $x + x^2 + x^3 + x^4$

Since $(x + x^2 + x^3 + x^4)$ contains four terms. Therefore, it is a polynomial and it does not fit in the above three categories.

Given polynomial is $7 + y + 5x$

Since $(7 + y + 5x)$ contains three terms. Therefore, it is trinomial.

Given polynomial is $2y - 3y^2$

Since $(2y - 3y^2)$ contains two terms. Therefore, it is binomial.

Given polynomial is $2y - 3y^2 + 4y^3$

Since $(2y - 3y^2 + 4y^3)$ contains three terms. Therefore, it is trinomial.

Given polynomial is $5x - 4y + 3xy$

Since $(5x - 4y + 3xy)$ contains three terms. Therefore, it is trinomial.

Given polynomial is $4x - 15z^2$

Since $(4x - 15z^2)$ contains two terms. Therefore, it is binomial.

Given polynomial is $ab + bc + cd + da$.

Since $(ab + bc + cd + da)$ contains four terms. Therefore, it is a polynomial and it does not fit in the above three categories.

Given polynomial is pqr .

Since pqr contains only one term. Therefore, it is monomial.

Given polynomial is $p^2q + pq^2$

Since $(p^2q + pq^2)$ contains two terms. Therefore, it is binomial.

Given polynomial is $2p + 2q$

Since $(2p + 2q)$ contains two terms. Therefore, it is binomial.

3. Add the following:

$$ab - bc, bc - ca, ca - ab$$

$$a - b + ab, b - c + bc, c - a + ac$$

$$2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$$

$$l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl$$

Solution:

$$(ab - bc) + (bc - ca) + (ca - ab) = (ab - ab) + (bc - bc) + (ca - ca) = 0$$

$$(a - b + ab) + (b - c + bc) + (c - a + ac) \\ = a - a + b - b + c - c + ab + bc + ac$$

$$= ab + bc + ac$$

$$(2p^2q^2 - 3pq + 4) + (5 + 7pq - 3p^2q^2) \\ = (2 - 3)p^2q^2 + (-3 + 7)pq + (4 + 5)$$

$$= -p^2q^2 + 4pq + 9$$

$$(l^2 + m^2) + (m^2 + n^2) + (n^2 + l^2) + (2lm + 2mn + 2nl) \\ = 2(l^2 + m^2 + n^2 + lm + mn + nl)$$

4. (a) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$
- (b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$
- (c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$.

Solution:

- (a) Given polynomials are $4a - 7ab + 3b + 12$ and $12a - 9ab + 5b - 3$

$$\text{Now, } (12a - 9ab + 5b - 3) - (4a - 7ab + 3b + 12) = (12a - 4a) + \\ (-9ab - (-7ab)) + (5b - 3b) + (-3 - 12) \\ = 8a - 2ab + 2b - 15$$

- (b) Given polynomials are $3xy + 5yz - 7zx$ and $5xy - 2yz - 2zx + 10xyz$

$$\text{Now, } (5xy - 2yz - 2zx + 10xyz) - (3xy + 5yz - 7zx) = (5xy - 3xy) + (-2yz - 5yz) + (-2zx + 7zx) + 10xyz$$

$$= 2xy - 7yz + 5zx + 10xyz$$

- (c) Given polynomials are $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

$$\text{Now, } (18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q) - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$$

$$= (18 - 10) + (-3 - (-8))p + (-11 - 7)q + (5 - (-3))pq + (-2 - 5)pq^2 + (5 - 4)p^2q$$

$$= 28 + 5p - 18q + 8pq - 7pq^2 + p^2q$$

Exercise 9.2

1. Find the product of the following pairs of monomials.

(i) $4, 7p$

(ii) $-4p, 7p$

(iii) $-4p, 7pq$

(iv) $4p^3, -3p$

(v) $4p, 0$.

Solution:

$$4 \times 7p = 28p$$

$$(-4p) \times 7p = (-4 \times 7)(p \times p) = -28p^2$$

$$(-4p) \times 7pq = (-4 \times 7)(p \times p \times q) = -28p^2q$$

$$4p^3 \times (-3p) = (4 \times (-3))(p^3 \times p) = -12p^4$$

$$4p \times 0 = 0$$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

$$(p, q); (10m, 5n); (20x^2, 5y^2); (4x, 3x^2); (3mn, 4np)$$

Solution:

(i) Area of rectangle = length \times breadth = $p \times q = pq$ sq. units.

(ii) Area of rectangle = length \times breadth = $10m \times 5n = 50mn$ sq. units.

(iii) Area of rectangle = length \times breadth = $20x^2 \times 5y^2 = 100x^2y^2$ sq. units

(iv) Area of rectangle = length \times breadth = $4x \times 3x^2 = 12x^3$ sq. units.

(v) Area of rectangle = length \times breadth = $3mn \times 4np = 12mn^2p$ sq. units

3. Complete the table of products:

First Monomial \rightarrow Second monomial \downarrow	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$					
$-5y$			$-15x^2y$			
$3x^2$						
$-4xy$						
$7x^2y$						
$-9x^2y^2$						

Solution:

First Monomial \rightarrow Second monomial \downarrow	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) $5a, 3a^2, 7a^4$

(ii) $2p, 4q, 8r$

(iii) $xy, 2x^2y, 2xy^2$

(iv) $a, 2b, 3c$.

Solution:

Volume of rectangular box = length \times breadth \times height

$$= 5a \times 3a^2 \times 7a^4$$

$$= 105a^7 \text{ cubic units}$$

Volume of rectangular box = length \times breadth \times height

$$= 2p \times 4q \times 8r$$

$$= 64pqr \text{ cubic units.}$$

Volume of rectangular box = length \times breadth \times height

$$= xy \times 2x^2y \times 2xy^2$$

$$= 4x^4 y^4 \text{ cubic units.}$$

Volume of rectangular box = length \times breadth \times height

$$= a \times 2b \times 3c$$

$$= 6abc \text{ cubic units.}$$

5. Obtain the product of

(i) xy, yz, zx

(ii) $a, -a^2, a^3$

(iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$

(v) $m, -mn, mnp$.

Solution:

$$xy \times yz \times zx = x^2y^2z^2$$

$$a \times (-a^2) \times a^3 = -a^6$$

$$2 \times 4y \times 8y^2 \times 16y^3 = 1024y^6$$

$$a \times 2b \times 3c \times 6abc = 36a^2b^2c^2$$

$$m \times (-mn) \times mnp = -m^3n^2p.$$

Exercise 9.3

1. Carry out the multiplication of the expression in each of the following pairs:

$$4p, q + r$$

$$ab, a - b$$

$$a + b, 7a^2b^2$$

$$a^2 - 9, 4a$$

$$pq + qr + rp, 0$$

Solution:

$$\begin{aligned} \text{(i)} \quad 4p \times (q + r) &= 4p \times q + 4p \times r \\ &= 4pq + 4pr \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad ab \times (a - b) &= (ab \times a) - (ab \times b) \\ &= a^2b - ab^2 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (a + b) \times (7a^2b^2) &= (7a^2b^2 \times a) + (7a^2b^2 \times b) \\ &= 7a^3b^2 + 7a^2b^3 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (a^2 - 9) \times 4a &= (a^2 \times 4a) - (9 \times 4a) \\ &= 4a^3 - 36a \end{aligned}$$

$$\text{(v)} \quad (pq + qr + rp) \times 0 = 0$$

Complete the following table:

	First expression	Second expression	Product
(i)	a	b + c + d	...
(ii)	x + y - 5	5xy	...
(iii)	p	6p ² - 7p + 5	...
(iv)	4p ² q ²	p ² - q ²	...
(v)	a + b + c	abc	...

Solution:

	First expression	Second expression	Product
(i)	a	b + c + d	$a(b + c + d)$ $= (a \times b) + (a \times c) + (a \times d)$ $= ab + ac + ad$
(ii)	x + y - 5	5xy	$(x + y - 5)5xy$ $= x(5xy) + y(5xy) - 5(5xy)$ $= 5x^2y + 5xy^2 - 25xy$
(iii)	p	6p ² - 7p + 5	$p(6p^2 - 7p + 5)$ $= (p \times 6p^2) - (p \times 7p) + (p \times 5)$ $= 6p^3 - 7p^2 + 5p$
(iv)	4p ² q ²	p ² - q ²	4p ² q ² (p ² - q ²)

			$= (4p^2q^2 \times p^2) - (4p^2q^2 \times q^2)$ $= 4p^4q^2 - 4p^2q^4$
(v)	$a + b + c$	abc	$abc(a + b + c)$ $= (abc \times a) + (abc \times b) + (abc \times c)$ $= a^2bc + ab^2c + abc^2$

2. Find the product:

(a) $(a^2) \times (2a^{22}) \times (4a^{26})$

(b) $\left(\frac{2}{3}xy\right) \times \left(-\frac{9}{10}x^2y^2\right)$

(c) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$

(d) $x \times x^2 \times x^3 \times x^4$

Solution:

(a) $(a^2) \times (2a^{22}) \times (4a^{26}) = 2 \times 4(a^2 \times a^{22} \times a^{26})$
 $= 8a^{2+22+26}$
 $= 8a^{50}$

(b) $\left(\frac{2}{3}xy\right) \times \left(-\frac{9}{10}x^2y^2\right) = \left(\frac{2}{3} \times -\frac{9}{10}\right)(xy \times x^2y^2)$
 $= -\frac{3}{5}x^3y^3$

(c) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right) = \left(-\frac{10}{3} \times \frac{6}{5}\right) \times (pq^3 \times p^3q)$
 $= -4p^4q^4$

(d) $x \times x^2 \times x^3 \times x^4 = x^{1+2+3+4} = x^{10}$

3. (a) Simplify: $3x(4x - 5) + 3$ and find the value for (i) $x = 3$ (ii) $x = \frac{1}{2}$

(b) Simplify: $a(a^2 + a + 1) + 5$ and find its value for (i) $a = 0$ (ii) $a = 1$ (iii) $a = -1$

Solution:

(a) $3x(4x - 5) + 3 = (3x \times 4x) - (3x \times 5) + 3 = 12x^2 - 15x + 3$
For $x = 3$, $12x^2 - 15x + 3 = 12(3)^2 - 15(3) + 3$
 $= (12 \times 9) - 45 + 3$
 $= 108 - 45 + 3$

$$= 66$$

$$\text{For } x = \frac{1}{2}, 12x^2 - 15x + 3 = 12\left(\frac{1}{2}\right)^2 - 15\left(\frac{1}{2}\right) + 3$$

$$= \left(12 \times \frac{1}{4}\right) - \frac{15}{2} + 3$$

$$= 3 - \frac{15}{2} + 3$$

$$= 6 - \frac{15}{2}$$

$$= \frac{-3}{2}$$

$$(b) \quad a(a^2 + a + 1) + 5 = (a \times a^2) + (a \times a) + a + 5 = a^3 + a^2 + a + 5$$

$$\text{For } a = 0, a^3 + a^2 + a + 5 = (0)^3 + (0)^2 + 0 + 5$$

$$= 0 + 0 + 0 + 5$$

$$= 5$$

$$\text{For } a = 1, a^3 + a^2 + a + 5 = (1)^3 + (1)^2 + 1 + 5$$

$$= 1 + 1 + 1 + 5$$

$$= 8$$

$$\text{For } a = -1, a^3 + a^2 + a + 5 = (-1)^3 + (-1)^2 - 1 + 5$$

$$= -1 + 1 - 1 + 5$$

$$= 4$$

4. (a) Add: $p(p - q)$, $q(q - r)$ and $r(r - p)$
 (b) Add: $2x(z - x - y)$ and $2y(z - y - x)$
 (c) Subtract: $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$
 (d) Subtract: $3a(a + b + c) - 2b(a - b + c)$ from $4c(-a + b + c)$

Solution:

$$(a) \quad p(p - q) + q(q - r) + r(r - p) = (p \times p) - (p \times q) + (q \times q) - (q \times r) + (r \times r) - (r \times p)$$

$$= p^2 - pq + q^2 - qr + r^2 - rp$$

$$= p^2 + q^2 + r^2 - pq - qr - rp$$

$$(b) \quad 2x(z - x - y) + 2y(z - y - x) = 2zx - 2x^2 - 2xy + 2yz - 2y^2 - 2xy$$

$$= -2x^2 - 2y^2 - 4xy + 2yz + 2zx$$

$$(c) \quad 4l(10n - 3m + 2l) - 3l(l - 4m + 5n) = 40ln - 12lm + 8l^2 - 3l^2 + 12lm - 15ln$$

$$= 5l^2 + 25ln$$

$$(d) \quad 4c(-a + b + c) - 3a(a + b + c) + 2b(a - b + c) = -4ac + 4bc + 4c^2 - 3a^2 - 3ab - 3ac + 2ab - 2b^2 + 2bc$$

$$= -3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$$

Exercise 9.4

1. Multiply the binomials.

$$(a) \quad (2x + 5) \text{ and } (4x - 3)$$

$$(b) \quad (y - 8) \text{ and } (3y - 4)$$

$$(c) \quad (2.5l - 0.5m) \text{ and } (2.5l + 0.5m)$$

$$(d) \quad (a + 3b) \text{ and } (x + 5)$$

$$(e) \quad (2pq + 3q^2) \text{ and } (3pq - 2q^2)$$

$$(f) \quad \left(\frac{3}{4}a^2 + 3b^2\right) \text{ and } 4\left(a^2 - \frac{2}{3}b^2\right)$$

Solution:

$$(a) \quad (2x + 5) \times (4x - 3) = (2x \times 4x) + (5 \times 4x) + (2x \times -3) + (5 \times -3)$$

$$= 8x^2 + 20x - 6x - 15$$

$$= 8x^2 + 14x - 15$$

$$(b) \quad (y - 8) \times (3y - 4) = (y \times 3y) + (-8 \times 3y) + (y \times -4) + (-8 \times -4)$$

$$= 3y^2 - 24y - 4y + 32$$

$$= 3y^2 - 28y + 32$$

$$(c) \quad (2.5l - 0.5m) \times (2.5l + 0.5m) = (2.5l \times 2.5l) + (-0.5m \times 2.5l) + (2.5l \times 0.5m) + (-0.5m \times 0.5m)$$

$$= 6.25l^2 - 1.25ml + 1.25ml - 0.25m^2$$

$$= 6.25l^2 - 0.25m^2$$

$$(d) \quad (a + 3b) \times (x + 5) = (a \times x) + (3b \times x) + (a \times 5) + (3b \times 5)$$

$$= ax + 3bx + 5a + 15b$$

$$\begin{aligned} \text{(e)} \quad (2pq + 3q^2) \times (3pq - 2q^2) &= (2pq \times 3pq) + (3q^2 \times 3pq) + (2pq \times -2q^2) + (3q^2 \times -2q^2) \\ &= 6p^2q^2 + 9pq^3 - 4pq^3 - 6q^4 \\ &= 6p^2q^2 + 5pq^3 - 6q^4 \\ \text{(f)} \quad \left(\frac{3}{4}a^2 + 3b^2\right) \times 4\left(a^2 - \frac{2}{3}b^2\right) &= \left(\frac{3}{4}a^2 \times 4a^2\right) + (3b^2 \times 4a^2) + \left(\frac{3}{4}a^2 \times -\frac{8}{3}b^2\right) + (3b^2 \times -\frac{8}{3}b^2) \\ &= 3a^4 + 12a^2b^2 - 2a^2b^2 - 8b^4 \\ &= 3a^4 + 10a^2b^2 - 8b^4 \end{aligned}$$

2. Find the product.

- (a) $(5 - 2x)(3 + x)$
- (b) $(x + 7y)(7x - y)$
- (c) $(a^2 + b)(a + b^2)$
- (d) $(p^2 - q^2)(2p + q)$

Solution:

$$\begin{aligned} \text{(a)} \quad (5 - 2x)(3 + x) &= (5 \times 3) + (-2x \times 3) + (5 \times x) + (-2x \times x) \\ &= 15 - 6x + 5x - 2x^2 \\ &= 15 - x - 2x^2 \\ \text{(b)} \quad (x + 7y)(7x - y) &= (x \times 7x) + (7y \times 7x) + (x \times -y) + (7y \times -y) \\ &= 7x^2 + 49xy - xy - 7y^2 \\ &= 7x^2 + 48xy - 7y^2 \\ \text{(c)} \quad (a^2 + b)(a + b^2) &= (a^2 \times a) + (b \times a) + (a^2 \times b^2) + (b \times b^2) \\ &= a^3 + ab + a^2b^2 + b^3 \\ \text{(d)} \quad (p^2 - q^2)(2p + q) &= (p^2 \times 2p) + (-q^2 \times 2p) + (p^2 \times q) + (-q^2 \times q) \\ &= 2p^3 - 2pq^2 + p^2q - q^3 \end{aligned}$$

3. Simplify.

- (a) $(x^2 - 5)(x + 5) + 25$
- (b) $(a^2 + 5)(b^3 + 3) + 5$
- (c) $(t + s^2)(t^2 - s)$
- (d) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

- (e) $(x + y)(2x + y) + (x + 2y)(x - y)$
 (f) $(x + y)(x^2 - xy + y^2)$
 (g) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$
 (h) $(a + b + c)(a + b - c)$

Solution:

- (a) $(x^2 - 5)(x + 5) + 25 = (x^2 \times x) + (-5 \times x) + (x^2 \times 5) + (-5 \times 5) + 25$
 $= x^3 - 5x + 5x^2 - 25 + 25$
 $= x^3 + 5x^2 - 5x$
- (b) $(a^2 + 5)(b^3 + 3) + 5 = (a^2 \times b^3) + (5 \times b^3) + (a^2 \times 3) + (5 \times 3) + 5$
 $= a^2b^3 + 5b^3 + 3a^2 + 15 + 5$
 $= a^2b^3 + 3a^2 + 5b^3 + 20$
- (c) $(t + s^2)(t^2 - s) = (t \times t^2) + (s^2 \times t^2) + (t \times -s) + (s^2 \times -s)$
 $= t^3 + s^2t^2 - st - s^3$
- (d) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd) = (a \times c) + (a \times -d) + (b \times c) + (b \times -d) + (a \times c) + (a \times d) + (-b \times c) + (-b \times d) + 2ac + 2bd$
 $= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$
 $= 4ac$
- (e) $(x + y)(2x + y) + (x + 2y)(x - y) = (x \times 2x) + (y \times 2x) + (x \times y) + (y \times y) + (x \times x) + (2y \times x) + (x \times -y) + (2y \times -y)$
 $= 2x^2 + 2xy + xy + y^2 + x^2 + 2xy - xy - 2y^2$
 $= 3x^2 + 4xy - y^2$
- (f) $(x + y)(x^2 - xy + y^2) = (x \times x^2) + (y \times x^2) + (x \times -xy) + (y \times -xy) + (x \times y^2) + (y \times y^2)$
 $= x^3 + x^2y - x^2y - xy^2 + xy^2 + y^3$
 $= x^3 + y^3$
- (g) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y = (1.5x \times 1.5x) + (1.5x \times 4y) + (1.5x \times 3) + (-4y \times 1.5x) + (-4y \times 4y) + (-4y \times 3) - 4.5x + 12y$
 $= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$
 $= 2.25x^2 - 16y^2$

$$\begin{aligned}
 \text{(h)} \quad & (a + b + c)(a + b - c) = (a \times a) + (a \times b) + (a \times -c) + (b \times a) + (b \times b) \\
 & + (b \times -c) + (c \times a) + (c \times b) + (c \times -c) \\
 & = a^2 + ab - ca + ab + b^2 - bc + ca + bc - c^2 \\
 & = a^2 + b^2 - c^2 + 2ab
 \end{aligned}$$

Exercise 9.5

1. Use a suitable identity to get each of the following products.

- (i) $(x + 3)(x + 3)$
- (ii) $(2y + 5)(2y + 5)$
- (iii) $(2a - 7)(2a - 7)$
- (iv) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right)$
- (v) $(1.1m - 0.4)(1.1m + 0.4)$
- (vi) $(a^2 + b^2)(-a^2 + b^2)$
- (vii) $(6x - 7)(6x + 7)$
- (viii) $(-a + c)(-a + c)$
- (ix) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$
- (x) $(7a - 9b)(7a - 9b)$

Solution:

$$\text{(i)} \quad (x + 3)(x + 3) = (x + 3)^2$$

We know that $(a + b)^2 = a^2 + 2ab + b^2$

Here $a = x, b = 3$

Hence, $(x + 3)^2 = x^2 + 2x(3) + 3^2$

$$= x^2 + 6x + 9$$

$$\text{(ii)} \quad (2y + 5)(2y + 5) = (2y + 5)^2$$

We know that $(a + b)^2 = a^2 + 2ab + b^2$

Here $a = 2y, b = 5$

Hence, $(2y + 5)^2 = (2y)^2 + 2(2y)(5) + 5^2$

$$= 4y^2 + 20y + 25$$

$$\text{(iii)} \quad (2a - 7)(2a - 7) = (2a - 7)^2$$

We know that $(a - b)^2 = a^2 - 2ab + b^2$

Here $a = 2a, b = 7$

$$\begin{aligned}\text{Hence, } (2a - 7)^2 &= (2a)^2 - 2(2a)(7) + 7^2 \\ &= 4a^2 - 28a + 49\end{aligned}$$

(iv) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right) = \left(3a - \frac{1}{2}\right)^2$

We know that $(a - b)^2 = a^2 - 2ab + b^2$

Here $a = 3a, b = \frac{1}{2}$

$$\begin{aligned}\text{Hence, } \left(3a - \frac{1}{2}\right)^2 &= (3a)^2 - 2(3a)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 \\ &= 9a^2 - 3a + \frac{1}{4}\end{aligned}$$

(v) $(1.1m - 0.4)(1.1m + 0.4)$

We know that $(a + b)(a - b) = a^2 - b^2$

Here $a = 1.1m, b = 0.4$

$$\begin{aligned}\text{Hence, } (1.1m - 0.4)(1.1m + 0.4) &= (1.1m)^2 - 0.4^2 \\ &= 1.21m^2 - 0.16\end{aligned}$$

(vi) $(a^2 + b^2)(-a^2 + b^2) = (b^2 + a^2)(b^2 - a^2)$

We know that $(a + b)(a - b) = a^2 - b^2$

Here $a = b^2, b = a^2$

$$\begin{aligned}\text{Hence, } (b^2 + a^2)(b^2 - a^2) &= (b^2)^2 - (a^2)^2 \\ &= b^4 - a^4\end{aligned}$$

(vii) We know that $(a + b)(a - b) = a^2 - b^2$

Here $a = 6x, b = 7$

$$\begin{aligned}\text{Hence, } (6x - 7)(6x + 7) &= (6x)^2 - 7^2 \\ &= 36x^2 - 49\end{aligned}$$

(viii) $(-a + c)(-a + c) = (c - a)^2$

We know that $(a - b)^2 = a^2 - 2ab + b^2$

Here $a = c, b = a$

$$\text{Hence, } (c - a)^2 = (c)^2 - 2(c)(a) + a^2$$

$$= c^2 - 2ca + a^2$$

$$(ix) \quad \left(\frac{x}{2} + \frac{3y}{4}\right) \left(\frac{x}{2} + \frac{3y}{4}\right) = \left(\frac{x}{2} + \frac{3y}{4}\right)^2$$

$$\text{We know that } (a + b)^2 = a^2 + 2ab + b^2$$

$$\text{Here } a = \frac{x}{2}, b = \frac{3y}{4}$$

$$\text{Hence, } \left(\frac{x}{2} + \frac{3y}{4}\right)^2 = \left(\frac{x}{2}\right)^2 + 2\left(\frac{x}{2}\right)\left(\frac{3y}{4}\right) + \left(\frac{3y}{4}\right)^2$$

$$= \frac{x^2}{4} + \frac{3xy}{4} + \frac{9y^2}{16}$$

$$(x) \quad (7a - 9b)(7a - 9b) = (7a - 9b)^2$$

$$\text{We know that } (a - b)^2 = a^2 - 2ab + b^2$$

$$\text{Here } a = 7a, b = 9b$$

$$\text{Hence, } (7a - 9b)^2 = (7a)^2 - 2(7a)(9b) + (9b)^2$$

$$= 49a^2 - 126a + 81b^2$$

2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.

$$(i) \quad (x + 3)(x + 7)$$

$$(ii) \quad (4x + 5)(4x + 1)$$

$$(iii) \quad (4x - 5)(4x - 1)$$

$$(iv) \quad (4x + 5)(4x - 1)$$

$$(v) \quad (2x + 5y)(2x + 3y)$$

$$(vi) \quad (2a^2 + 9)(2a^2 + 5)$$

$$(vii) \quad (xyz - 4)(xyz - 2)$$

Solution:

$$(i) \quad \text{We know that } (x + a)(x + b) = x^2 + (a + b)x + ab$$

$$\text{Put } a = 3, b = 7$$

$$\text{Hence, } (x + 3)(x + 7) = x^2 + (3 + 7)x + 21$$

$$= x^2 + 10x + 21$$

$$(ii) \quad \text{We know that } (x + a)(x + b) = x^2 + (a + b)x + ab$$

$$(4x + 5)(4x + 1) = 16\left(x + \frac{5}{4}\right)\left(x + \frac{1}{4}\right)$$

$$= 16\left(x^2 + \left(\frac{5}{4} + \frac{1}{4}\right)x + \frac{5}{16}\right)$$

$$= 16x^2 + 24x + 5$$

(iii) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(4x - 5)(4x - 1) = 16\left(x - \frac{5}{4}\right)\left(x - \frac{1}{4}\right)$$

$$= 16\left(x^2 - \frac{6}{4}x + \frac{5}{16}\right)$$

$$= 16x^2 - 24x + 5$$

(iv) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(4x + 5)(4x - 1) = 16\left(x + \frac{5}{4}\right)\left(x - \frac{1}{4}\right)$$

$$= 16\left(x^2 + x - \frac{5}{16}\right)$$

$$= 16x^2 + 16x - 5$$

(v) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(2x + 5y)(2x + 3y) = 4\left(x + \frac{5}{2}y\right)\left(x + \frac{3}{2}y\right)$$

$$= 4\left(x^2 + 4xy + \frac{15}{4}\right)$$

$$= 4x^2 + 16xy + 15$$

(vi) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$\text{Here } x = 2a^2, a = 9, b = 5$$

$$(2a^2 + 9)(2a^2 + 5) = (2a^2)^2 + (9 + 5)2a^2 + 45$$

$$= 4a^4 + 28a^2 + 45$$

(vii) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$\text{Here } x = xyz, a = -4, b = -2$$

$$(xyz - 4)(xyz - 2) = (xyz)^2 + (-4 - 2)xyz + 8$$

$$= x^2y^2z^2 - 6xyz + 8$$

3. Find the following squares by using the identities.

(i) $(b - 7)^2$

(ii) $(xy + 3z)^2$

(iii) $(6x^2 - 5y)^2$

(iv) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2$

(v) $(0.4p - 0.5q)^2$

(vi) $(2xy + 5y)^2$

Solution:

(i) We know that $(a - b)^2 = a^2 - 2ab + b^2$

Here $a = b, b = 7$

$$(b - 7)^2 = b^2 - 2(b)(7) + 49$$

$$= b^2 - 14b + 49$$

(ii) We know that $(a + b)^2 = a^2 + 2ab + b^2$

Here $a = xy, b = 3z$

$$(xy + 3z)^2 = (xy)^2 + 2(xy)(3z) + (3z)^2$$

$$= x^2y^2 + 6xyz + 9z^2$$

(iii) We know that $(a - b)^2 = a^2 - 2ab + b^2$

Here $a = 6x^2, b = 5y$

$$(6x^2 - 5y)^2 = (6x^2)^2 - 2(6x^2)(5y) + (5y)^2$$

$$= 36x^4 - 60x^2y + 25y^2$$

(iv) We know that $(a + b)^2 = a^2 + 2ab + b^2$

Here $a = \frac{2}{3}m, b = \frac{3}{2}n$

$$\left(\frac{2}{3}m + \frac{3}{2}n\right)^2 = \frac{4}{9}m^2 + 2\left(\frac{2}{3}m\right)\left(\frac{3}{2}n\right) + \frac{9}{4}n^2$$

$$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$$

(v) We know that $(a - b)^2 = a^2 - 2ab + b^2$

Here $a = 0.4p, b = 0.5q$

$$(0.4p - 0.5q)^2 = 0.16p^2 - 2(0.4p)(0.5q) + 0.25q^2$$

$$= 0.16p^2 - 0.4pq + 0.25q^2$$

(vi) We know that $(a + b)^2 = a^2 + 2ab + b^2$

Here $a = 2xy$, $b = 5y$

$$\begin{aligned}(2xy + 5y)^2 &= 4x^2y^2 + 2(2xy)(5y) + 25y^2 \\ &= 4x^2y^2 + 20xy^2 + 25y^2\end{aligned}$$

4. Simplify.

- (i) $(a^2 - b^2)^2$
- (ii) $(2x + 5)^2 - (2x - 5)^2$
- (iii) $(7m - 8n)^2 + (7m + 8n)^2$
- (iv) $(4m + 5n)^2 + (5m + 4n)^2$
- (v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$
- (vi) $(ab + bc)^2 - 2ab^2c$
- (vii) $(m^2 - n^2m)^2 + 2m^3n^2$

Solution:

- (i) We know that $(a - b)^2 = a^2 - 2ab + b^2$

Here $a = a^2$, $b = b^2$

$$\begin{aligned}(a^2 - b^2)^2 &= (a^2)^2 - 2(a^2)(b^2) + (b^2)^2 \\ &= a^4 - 2a^2b^2 + b^4\end{aligned}$$

- (ii) We know that $(a + b)^2 - (a - b)^2 = 4ab$

Here $a = 2x$, $b = 5$

$$\begin{aligned}(2x + 5)^2 - (2x - 5)^2 &= 4(2x)5 \\ &= 40x\end{aligned}$$

- (iii) We know that $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

Here $a = 7m$, $b = 8n$

$$\begin{aligned}(7m - 8n)^2 + (7m + 8n)^2 &= 2((7m)^2 + (8n)^2) \\ &= 98m^2 + 128n^2\end{aligned}$$

- (iv) We know that $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned}(4m + 5n)^2 + (5m + 4n)^2 &= 16m^2 + 25n^2 + 2(4m)(5n) + 25m^2 + 16n^2 + 2(5m)(4n) \\ &= 41m^2 + 41n^2 + 80mn\end{aligned}$$

- (v) We know that $(a - b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned}
 & (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2 \\
 &= 6.25p^2 + 2.25q^2 - 7.5pq - (2.25p^2 + 6.25q^2 - 7.5pq) \\
 &= 4p^2 - 4q^2
 \end{aligned}$$

(vi) We know that $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned}
 (ab + bc)^2 - 2ab^2c &= (a^2b^2 + b^2c^2 + 2ab^2c) - 2ab^2c \\
 &= a^2b^2 + b^2c^2
 \end{aligned}$$

(vii) We know that $(a - b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned}
 (m^2 - n^2m)^2 + 2m^3n^2 &= m^4 + n^4m^2 - 2m^3n^2 + 2m^3n^2 \\
 &= m^4 + n^4m^2
 \end{aligned}$$

5. Show that

- (i) $(3x + 7)^2 - 84x = (3x - 7)^2$
- (ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$
- (iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$
- (iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$
- (v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

Solution:

(i) We know that $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned}
 \Rightarrow (a + b)^2 - 4ab &= a^2 + 2ab + b^2 - 4ab \\
 \Rightarrow (a + b)^2 - 4ab &= a^2 - 2ab + b^2 \dots\dots (i)
 \end{aligned}$$

And $(a - b)^2 = a^2 - 2ab + b^2 \dots\dots(ii)$

From (i) and (ii)

$$(a + b)^2 - 4ab = (a - b)^2$$

Put $a = 3x, b = 7$

$$\begin{aligned}
 (3x + 7)^2 - 4(3x)7 &= (3x - 7)^2 \\
 \Rightarrow (3x + 7)^2 - 84x &= (3x - 7)^2
 \end{aligned}$$

(ii) We know that $(a - b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned}
 \Rightarrow (a - b)^2 + 4ab &= a^2 - 2ab + b^2 + 4ab \\
 \Rightarrow (a - b)^2 + 4ab &= a^2 + 2ab + b^2 \dots\dots(i)
 \end{aligned}$$

$$\text{And } (a + b)^2 = a^2 + 2ab + b^2 \dots\dots(ii)$$

From (i) and (ii)

$$(a - b)^2 + 4ab = (a + b)^2$$

$$\text{Put } a = 9p, b = 5q$$

$$(9p - 5q)^2 + 4(9p)(5q) = (9p + 5q)^2$$

$$\Rightarrow (9p - 5q)^2 + 180pq = (9p + 5q)^2$$

$$(iii) \quad \text{We know that } (a - b)^2 = a^2 - 2ab + b^2$$

$$\Rightarrow (a - b)^2 + 2ab = a^2 - 2ab + b^2 + 2ab$$

$$= a^2 + b^2$$

$$\text{Put } a = \frac{4}{3}m, b = \frac{3}{4}n$$

$$\Rightarrow \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2\left(\frac{4}{3}m\right)\left(\frac{3}{4}n\right) = \left(\frac{4}{3}m\right)^2 + \left(\frac{3}{4}n\right)^2$$

$$\Rightarrow \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$$

$$(iv) \quad \text{We know that } (a + b)^2 = a^2 + 2ab + b^2 \dots (i)$$

$$(a - b)^2 = a^2 - 2ab + b^2 \dots (ii)$$

Adding (i) and (ii)

$$(a + b)^2 - (a - b)^2 = (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$$

$$= 4ab$$

$$\text{Put } a = 4pq, b = 3q$$

$$(4pq + 3q)^2 - (4pq - 3q)^2 = 4(4pq)(3q)$$

$$= 48q^2p$$

$$(v) \quad \text{We know that } (a - b)(a + b) = a^2 - b^2$$

$$(b - c)(b + c) = b^2 - c^2$$

$$(c - a)(c + a) = c^2 - a^2$$

$$\text{Hence, } (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = a^2 - b^2 + b^2 - c^2 + c^2 - a^2$$

$$= 0$$

6. Using identities, evaluate

$$(i) \quad 71^2$$

- (ii) 99^2
- (iii) 102^2
- (iv) 998^2
- (v) 5.2^2
- (vi) 297×303
- (vii) 78×82
- (viii) 8.9^2
- (ix) 1.05×9.5

Solution:

- (i) We know that $(a + b)^2 = a^2 + 2ab + b^2$
Put $a = 70, b = 1$
 $(70 + 1)^2 = (70)^2 + 2(70)(1) + 1$
 $= 4900 + 140 + 1 = 5041$
- (ii) We know that $(a - b)^2 = a^2 - 2ab + b^2$
Put $a = 100, b = 1$
 $(100 - 1)^2 = (100)^2 - 2(100)(1) + 1$
 $= 10000 - 200 + 1 = 9801$
- (iii) We know that $(a + b)^2 = a^2 + 2ab + b^2$
Put $a = 100, b = 2$
 $(102)^2 = 100^2 + 2(100)(2) + 4$
 $= 10000 + 400 + 4 = 10404$
- (iv) We know that $(a - b)^2 = a^2 - 2ab + b^2$
Put $a = 1000, b = 2$
 $(1000 - 2)^2 = (1000)^2 - 2(1000)(2) + 2^2$
 $= 1000000 - 4000 + 4$
 $= 996004$
- (v) We know that $(a + b)^2 = a^2 + 2ab + b^2$
Put $a = 5, b = 0.2$
 $(5 + 0.2)^2 = 5^2 + 2(5)(0.2) + (0.2)^2$

$$= 25 + 2 + 0.04 = 27.04$$

(vi) We know that $(a + b)(a - b) = a^2 - b^2$

Put $a = 300, b = 3$

$$(300 + 3)(300 - 3) = (300)^2 - (3)^2$$

$$= 90000 - 9 = 89991$$

(vii) We know that $(a + b)(a - b) = a^2 - b^2$

Put $a = 80, b = 2$

$$(80 - 2)(80 + 2) = (80)^2 - 4$$

$$= 6396$$

(viii) We know that $(a - b)^2 = a^2 - 2ab + b^2$

Put $a = 9, b = 0.1$

$$(9 - 0.1)^2 = 9^2 - 2(9)(0.1) + (0.1)^2$$

$$= 81 - 1.8 + 0.01 = 79.21$$

(ix) $1.05 \times 9.5 = 1.05 \times 0.95 \times 10$

We know that $(a + b)(a - b) = a^2 - b^2$

$$1.05 \times 0.95 \times 10 = (1 + 0.05)(1 - 0.05)(10)$$

$$= (1 - 0.0025)10$$

$$= 9.975$$

7. Using $a^2 - b^2 = (a - b)(a + b)$, find

(i) $51^2 - 49^2$

(ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

Solution:

(i) We know that $a^2 - b^2 = (a - b)(a + b)$

$$51^2 - 49^2 = (51 - 49)(51 + 49)$$

$$= 2(100) = 200$$

(ii) We know that $a^2 - b^2 = (a - b)(a + b)$

$$(1.02)^2 - (0.98)^2 = (1.02 - 0.98)(1.02 + 0.98)$$

$$= (0.04)^2 = 0.08$$

(iii) We know that $a^2 - b^2 = (a - b)(a + b)$

$$153^2 - 147^2 = (153 - 147)(153 + 147)$$

$$= 6(300) = 1800$$

(iv) We know that $a^2 - b^2 = (a - b)(a + b)$

$$12.1^2 - 7.9^2 = (12.1 - 7.9)(12.1 + 7.9)$$

$$= 4.2(20) = 84$$

8. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find

(i) 103×104

(ii) 5.1×5.2

(iii) 103×98

(iv) 9.7×9.8

Solution:

(i) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Put $x = 100, a = 3, b = 4$

$$(100 + 3)(100 + 4) = 100^2 + (3 + 4)100 + 12$$

$$= 10000 + 700 + 12$$

$$= 10712$$

(ii) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Put $x = 5, a = 0.1, b = 0.2$

$$(5 + 0.1)(5 + 0.2) = 25 + 1.5 + 0.02$$

$$= 26.52$$

(iii) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Put $x = 100, a = 3, b = -2$

$$(100 + 3)(100 - 2) = 10000 + 100 - 6$$

$$= 10094$$

(iv) We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Put $x = 10, a = -0.2, b = -0.3$

$$(10 - 0.2)(10 - 0.3) = 100 - 5 + 0.06$$

$$= 95.06$$

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