CBSE NCERT Solutions for Class 10 Mathematics Chapter 6 Back of Chapter Questions

EXERCISE 6.1

1. Fill in the blanks using the correct word given in brackets:

(i) All circles are _________. (congruent, similar)

(ii) All squares are _________. (similar, congruent)

(iii) All _________ triangles are similar. (Isosceles, equilateral)

(iv) Two polygons of the same number of sides re similar, if (a) their corresponding angles are _________ and (b) their corresponding sides are _________. (equal, proportional)

Solution:

(i) similar, since size of circles may be different, but shape will be always same.

(ii) similar, since size of squares may be different, but shape will be always same.

(iii) All equilateral triangles are similar because of their same shape.

(iv) Two polygons of same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.

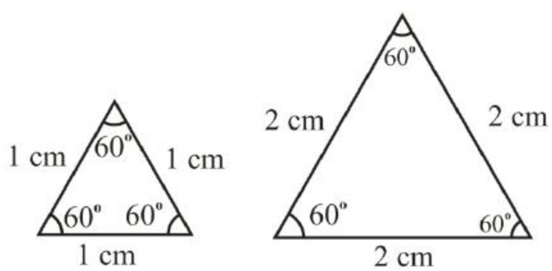
2. Give two different examples of pair of

(i) Similar figures.

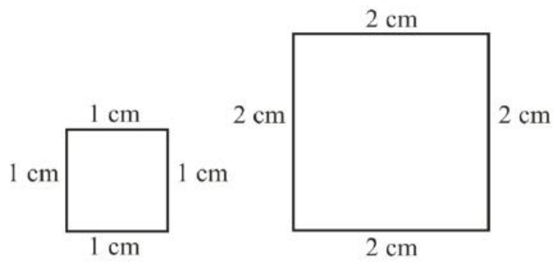
(ii) Non-similar figures.

Solution:

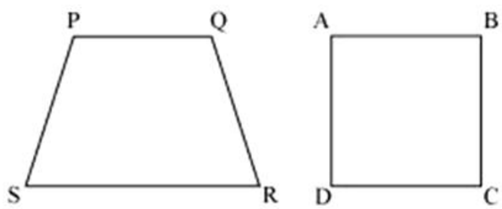
(i) Two equilateral triangles with sides 1cmand 2cm.



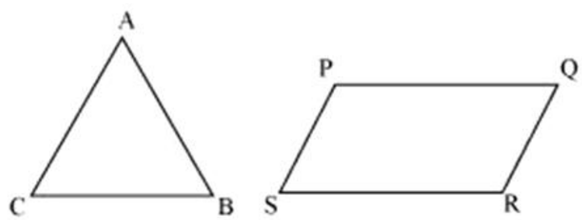
Two squares with sides 1cmand 2cm.



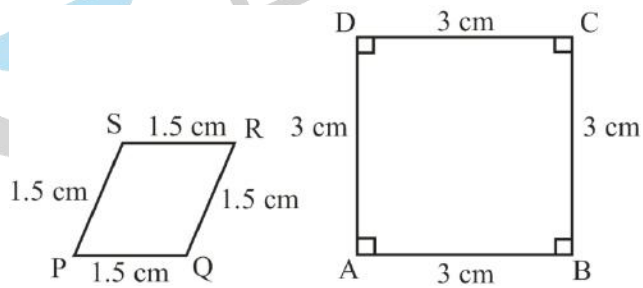
(ii) Trapezium and Square



Triangle and Parallelogram



3. State whether the following quadrilaterals are similar or not:

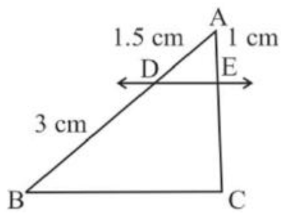


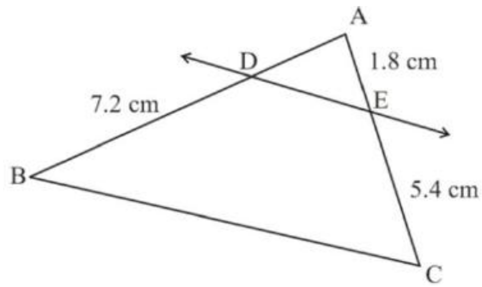
Solution:

Corresponding sides of two quadrilaterals are proportional i. e. $1:2$ but their corresponding angles are not equal. Hence, quadrilateral PQRS and ABCD are not similar.

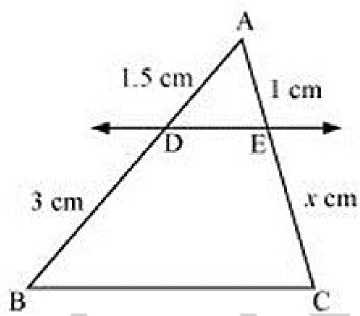
EXERCISE 6.2

1. In Fig. (i) and (ii), DE ∥ BC. Find EC in (i) and AD in (ii).

(i) 

(ii) 

Solution:

(i) 

Let EC $= x$cm

Since DE ∥ BC

Hence, using basic proportionality theorem,

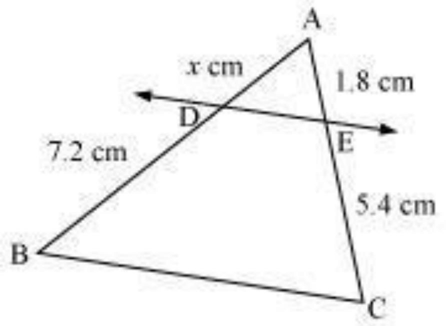
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{1.5}{3} = \frac{1}{x}$$

$$\Rightarrow x = \frac{3 \times 1}{1.5}$$

$$\Rightarrow x = 2$$

Hence, EC $= 2$cm

(ii) 

Let AD $= x$

Since DE ∥ BC,

Hence, using basic proportionality theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x}{7.2} = \frac{1.8}{5.4}$$

$$\Rightarrow x = \frac{1.8 \times 7.2}{5.4}$$

$$\Rightarrow x = 2.4$$

Hence, AD $= 2.4$cm

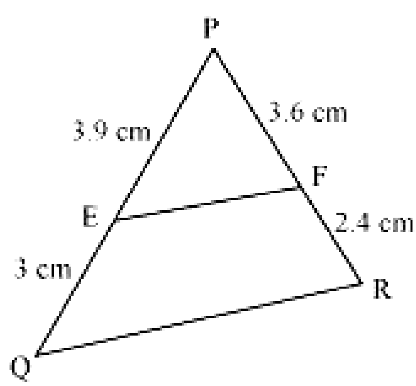
2. Eand F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF ∥ QR:

(i) $PE = 3.9cm, EQ = 3cm, PF = 3.6cm and FR = 2.4cm$

(ii) $PE = 4cm, QE = 4.5cm, PF = 8cm and RF = 9cm$

(ii) $PQ = 1.28cm, PR = 2.56cm, PE = 0.18cm and PF = 0.36cm$

Solution:

(i) 

Given, $PE = 3.9, EQ = 3, PF = 3.6, FR = 2.4$

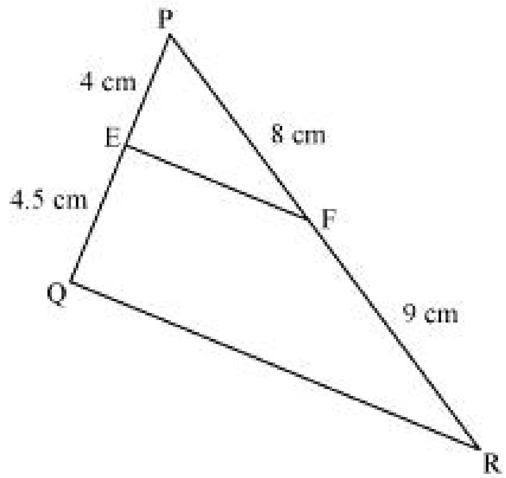
Now,

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Since, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Hence, EF is not parallel to QR.



(ii)

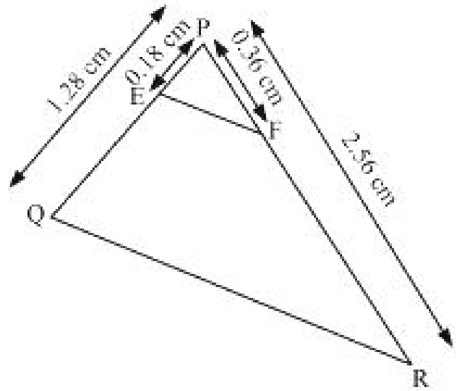
Given, $PE = 4, QE = 4.5, PF = 8, RF = 9$

$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Since $\frac{PE}{EQ} = \frac{PF}{FR}$

Hence, $EF \parallel QR$ (using basic proportionality theorem)



(iii)

Given, $\text{PQ} = 1.28, \text{PR} = 2.56, \text{PE} = 0.18, \text{PF} = 0.36$

$\therefore EQ = PQ - PE = 1.28 - 0.18 = 1.1$ and $FR = PR - PF = 2.56 - 0.36 = 2.2$

$$\frac{\text{PE}}{\text{EQ}} = \frac{0.18}{1.1} = \frac{18}{110} = \frac{9}{55}$$

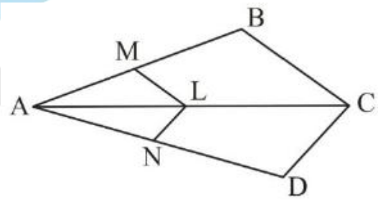
$$\frac{\text{PF}}{\text{FR}} = \frac{0.36}{2.2} = \frac{9}{55}$$

Since $\frac{\text{PE}}{\text{EQ}} = \frac{\text{PF}}{\text{FR}}$

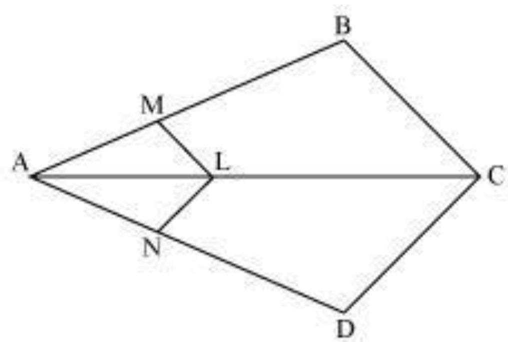
Hence, EF ∥ QR (using basic proportionality theorem)

3. In Fig. if LM ∥ CB and LN ∥ CD, prove that

$$\frac{\text{AM}}{\text{AB}} = \frac{\text{AN}}{\text{AD}}$$



Solution:



In the given figure

Since LM ∥ CB.

Hence, using basic proportionality theorem,

$$\frac{AM}{MB}=\frac{AL}{LC} \qquad \ldots \text{(i)}$$

Again, since LN ∥ CD

Hence, using basic proportionality theorem,

$$\frac{AN}{ND}=\frac{AL}{LC} \qquad \ldots \text{(ii)}$$

From (i) and (ii)

$$\frac{AM}{MB}=\frac{AN}{ND}$$

$$\Rightarrow \frac{MB}{AM}=\frac{ND}{AN}$$

$$\Rightarrow \frac{MB}{AM}+1=\frac{ND}{AN}+$$

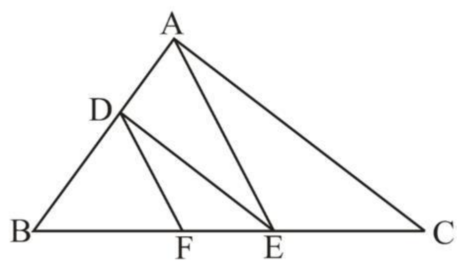
$$\Rightarrow \frac{MB+AM}{AM}=\frac{ND+AN}{AN}$$

$$\Rightarrow \frac{AB}{AM}=\frac{AD}{AN}$$

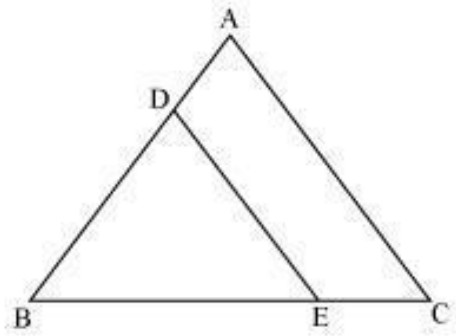
$$\Rightarrow \frac{AM}{AB}=\frac{AN}{AD}$$

4. In Figure, DE ∥ AC andDF ∥ AE. Prove that

$$\frac{BF}{FE}=\frac{BE}{EC}$$



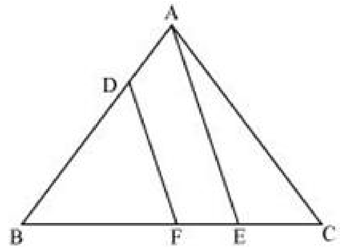
Solution:



In ΔABC

Since $DE \parallel AC$

Hence, $\frac{BD}{DA} = \frac{BE}{EC}$ ... (i) (using basic proportionality theorem)



In ΔBAE,

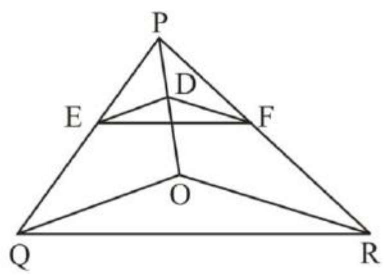
Since $DF \parallel AE$

Hence, $\frac{BD}{DA} = \frac{BF}{FE}$ ... (ii) (using basic proportionality theorem)

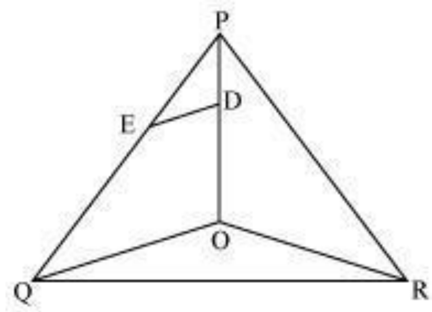
From (i) and(ii)

$$\frac{BE}{EC} = \frac{BF}{FE}$$

5. In Figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.



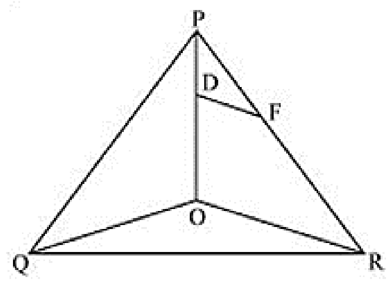
Solution:



In ΔPOQ

Since $DE \parallel OQ$

$\frac{PE}{EQ} = \frac{PD}{DO}$ ... (i)[Using basic proportionality theorem]



In ΔPOR

Since $DF \parallel OR$

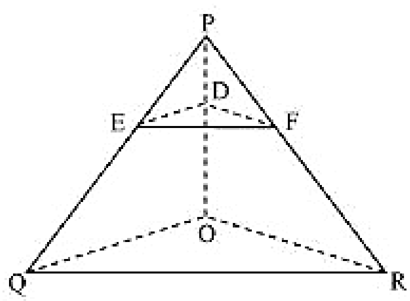
$\frac{PF}{FR} = \frac{PD}{DO}$ ... (ii)[Using basic proportionality theorem]

From (i)and (ii)

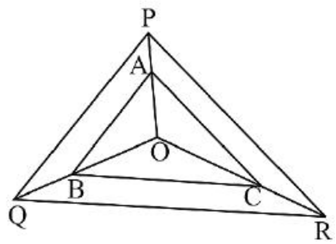
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Using converse of basic proportionality theorem

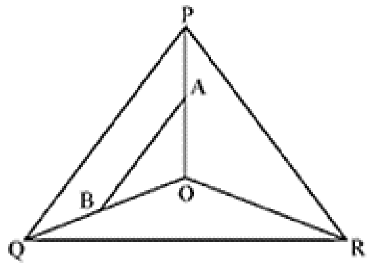
$EF \parallel QR$



6. In Figure A, B andC are points onOP, OQ andOR respectively such that$AB \parallel PQ$ and$AC \parallel PR$. Show that $BC \parallel QR$.



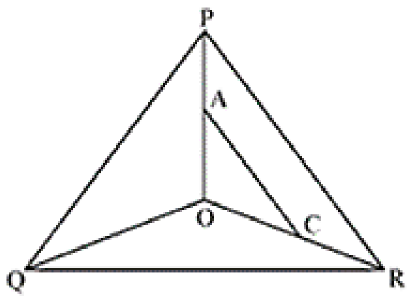
Solution:



In ΔPOQ

Since AB ∥ PQ,

Hence, $\frac{OA}{AP} = \frac{OB}{BQ}$ ... (i)[Using basic proportionality theorem]



In ΔPOR

Since AC ∥ PR

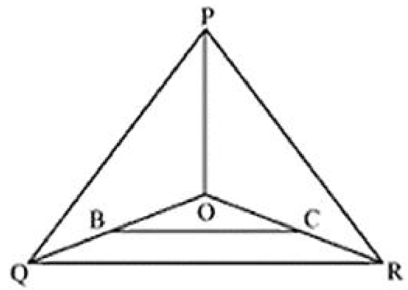
Hence, $\frac{OA}{AP} = \frac{OC}{CR}$ ... (ii)[Using basic proportionality theorem]

From (i)and (ii)

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

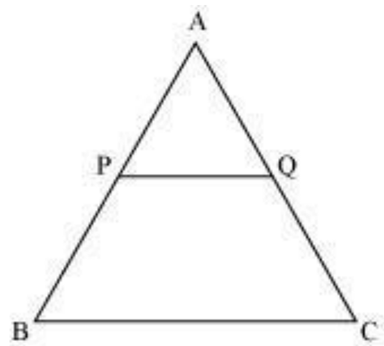
Hence, BC ∥ QR

(Using converse of basic proportionality theorem)



7. Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in ClassIX).

Solution:



Let in the given figure PQ is a line segment drawn through mid-point P of line AB such that PQ ∥ BC

Hence, AP = PB

Now, using basic proportionality theorem

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\Rightarrow \frac{AQ}{QC} = \frac{AP}{AP}$$

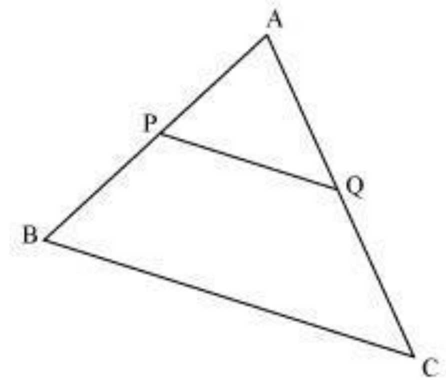
$$\Rightarrow \frac{AQ}{QC} = 1$$

$$\Rightarrow AQ = QC$$

Hence, Q is the mid-point of AC.

8. Prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Solution:



Let in the given figure PQ is a line segment joining mid-points PandQ of line ABand AC respectively.

Hence, $AP = PBandAQ = QC$

Now, since $\frac{AP}{PB} = \frac{AP}{AP} = 1and \frac{AQ}{QC} = \frac{AQ}{AQ} = 1$

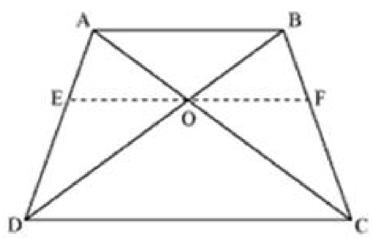
Hence, $\frac{AP}{PB} = \frac{AQ}{QC}$

Now, using converse of basic proportionality theorem $PQ \parallel BC$

9. ABCD is a trapezium in which$AB \parallel DC$ and its diagonals intersect each other at the point O. Show that

$$\frac{AO}{BO} = \frac{CO}{DO}$$

Solution:



Let a line segment EF is drawn through point O such that $EF \parallel CD$

In ΔADC

$EO \parallel CD$

Hence, using basic proportionality theorem

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad \dots(1)$$

Similarly in ΔBDC

FO ∥ CD

Hence, using basic proportionality theorem

$$\frac{BF}{FC} = \frac{BO}{OD} \qquad \ldots (2)$$

Now consider trapezium ABCD

As FE ∥ CD

So, $\frac{AE}{ED} = \frac{BF}{FC} \qquad \ldots (3)$

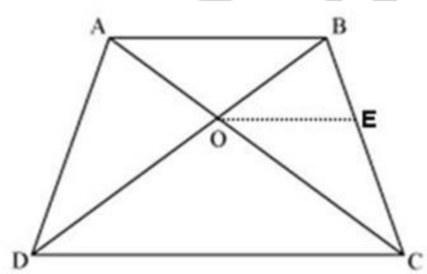
Now from equation (1), (2), (3)

$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

10. The diagonals of a quadrilateralABCD intersect each other at the pointO such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Solution:



Draw a line segment OE ∥ AB

In ΔABC

Since OE ∥ AB

Hence, $\frac{AO}{OC} = \frac{BE}{EC}$

But by the given relation, we have:

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\Rightarrow \frac{AO}{OC} = \frac{OB}{OD}$$

Hence, $\frac{OB}{OD} = \frac{BE}{EC}$

So, using converse of basic proportionality theorem, EO ∥ DC

Therefore AB ∥ OE ∥ DC

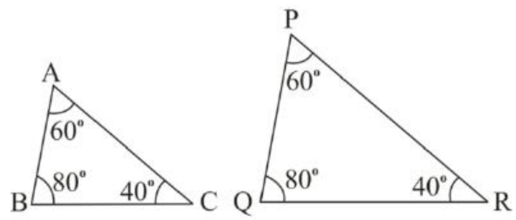
⇒ AB ∥ CD

Therefore, ABCD is a trapezium.

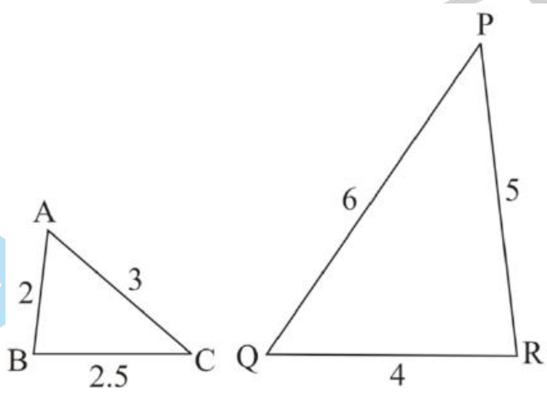
EXERCISE 6.3

1. State which pairs of triangles in Fig. are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

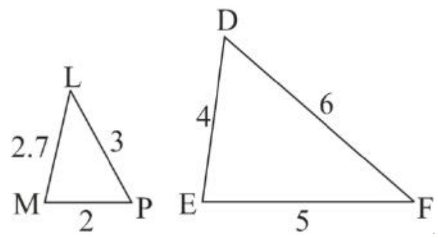
(i)



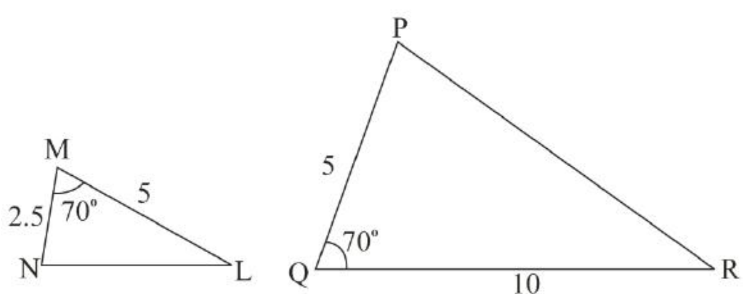
(ii)

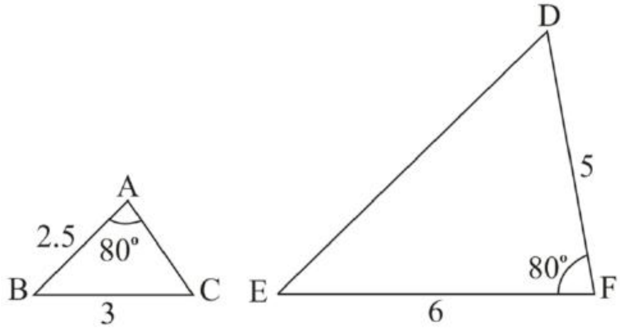


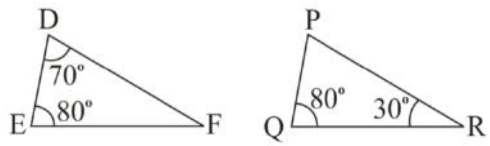
(iii)



(iv)



(v) 

(vi) 

Solution:

(i) $\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Hence by AAA rule $\Delta ABC \sim \Delta PQR$

(ii) $\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$

$$\frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Since, $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$

Hence, by SSS rule

$\Delta ABC \sim \Delta QRP$

(iii) Triangles are not similar as the corresponding sides are not proportional.

(iv) $\frac{MN}{PQ} = \frac{2.5}{5} = \frac{1}{2}$

$$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$$

$\angle M = \angle Q = 70^{o}$

Hence, by SAS rule

$\Delta MNL \sim \Delta PQR$

(v) Triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF

$\angle D + \angle E + \angle F = 180^o$(Sum of angles of a triangle is 180^o)

$\Rightarrow 70^o + 80^o + \angle F = 180^o$

$\Rightarrow \angle F = 30^o$

Similarly in ΔPQR

$\angle P + \angle Q + \angle R = 180^o$ (Sum of angles of a triangle is 180^o)

$\Rightarrow \angle P + 80^o + 30^o = 180^o$

$\Rightarrow \angle P = 70^o$

Now, since

$\angle D = \angle P = 70^o$

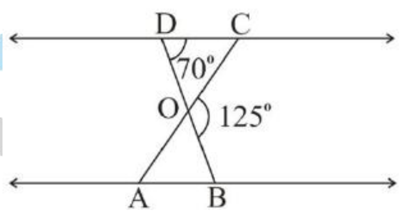
$\angle E = \angle Q = 80^o$

$\angle F = \angle R = 30^o$

Hence, by AAA rule

$\Delta DEF \sim \Delta PQR$

2. In Figure, $\Delta ODC \sim \Delta OBA, \angle BOC = 125^o$and $\angle CDO = 70^o$. Find $\angle DOC, \angle DCO$ and$\angle OAB$.



Solution:

Since DOB is a straight line

Hence, $\angle DOC + \angle COB = 180^o$

$\Rightarrow \angle DOC = 180^o - 125^o = 55^o$

InΔDOC,

$\angle DCO + \angle CDO + \angle DOC = 180^o$

$\Rightarrow \angle DCO + 70^o + 55^o = 180^o$

$\Rightarrow \angle DCO = 55^o$

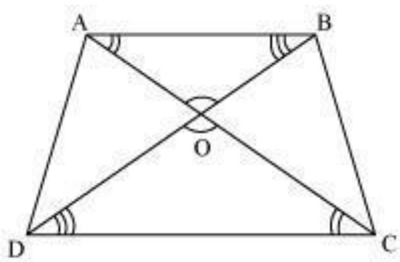
Since, $\Delta ODC \sim \Delta OBA$,

Hence, $\angle OCD = \angle OAB$ [Corresponding angles equal in similar triangles]

Hence, $\angle OAB = 55^o$

3. DiagonalsAC andBD of a trapeziumABCD withAB ∥ DC intersect each other at the pointO. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$

Solution:



In ΔDOCandΔBOA

$AB||CD$

Hence,$\angle CDO = \angle ABO$[Atlernate interior angles]

$\angle DCO = \angle BAO$[Atlernate interior angles]

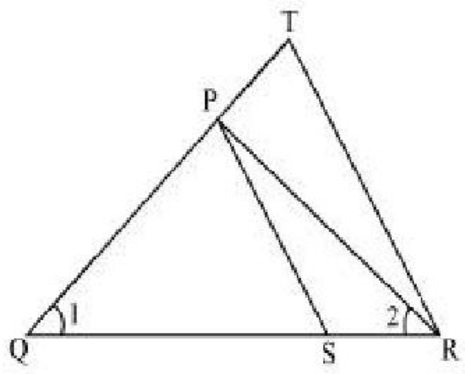
$\angle DOC = \angle BOA$ [Vertically opposite angles]

Hence,$\Delta DOC \sim \Delta BOA$ [AAArule]

$\Rightarrow \frac{DO}{BO} = \frac{OC}{OA}$ [Corresponding sides are proportional]

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

4. In Figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and$\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Solution:

In ΔPQR

$\angle PQR = \angle PRQ$

Hence, $PQ = PR \ldots (i)$

Given

$\frac{QR}{QS} = \frac{QT}{PR}$

Using (i)

$\frac{QR}{QS} = \frac{QT}{PQ} \ldots (ii)$

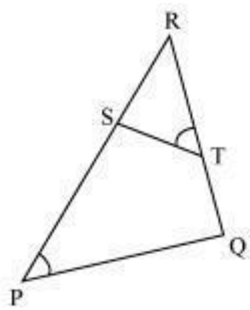
Also, $\angle RQT = \angle PQS = \angle 1$

Hence, by SAS rule

$\Delta PQS \sim \Delta TQR$

5. S andT are points on sidesPR andQR of ΔPQRsuch that$\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Solution:



In ΔRPQ and ΔRTS

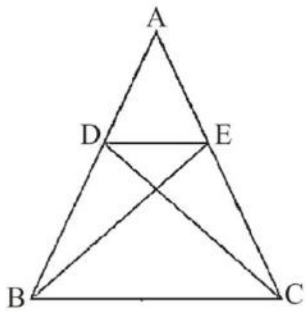
$\angle QPR = \angle RTS$ [Given]

$\angle R = \angle R$ [Common angle]

$\angle RQP = \angle RST$ [Remaining angle]

Hence, $\Delta RPQ \sim \Delta RTS$ [by AAA rule]

6. In Fig., if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



Solution:

Since $\Delta ABE \cong \Delta ACD$

Therefore $AB = AC$ ... (1)

$AE = AD$

$\Rightarrow AD = AE$ ... (2)

Now, in ΔADE and ΔABC,

Dividing equation (2) by (1)

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$\angle A = \angle A$ [Common angle]

Hence, $\Delta ADE \sim \Delta ABC$ [by SAS rule]

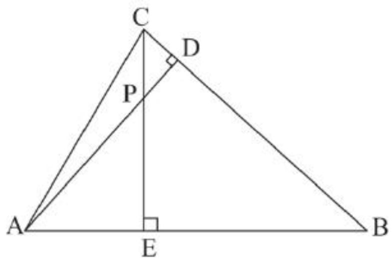
7. In Fig., altitudes AD and CE of ΔABC intersect each other at the point P. Show that:

(i) $\Delta AEP \sim \Delta CDP$

(ii) $\Delta ABD \sim \Delta CBE$

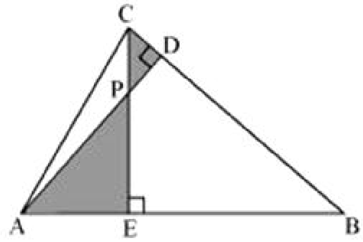
(iii) $\Delta AEP \sim \Delta ADB$

(iii) $\Delta PDC \sim \Delta BEC$



Solution:

(i)



In ΔAEP and ΔCDP

$\angle CDP = \angle AEP = 90^o$

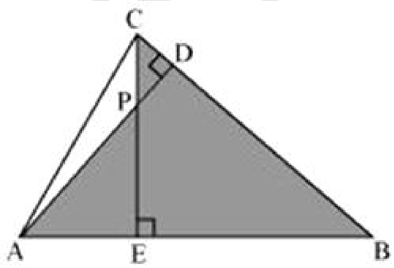
$\angle CPD = \angle APE$(Vertically opposite angles)

$\angle PCD = \angle PAE$(Remaining angle)

Hence, by AArule,

$\Delta AEP \sim \Delta CDP$

(ii)



In ΔABDandΔCBE

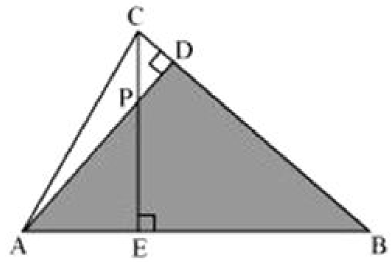
$\angle ADB = \angle CEB = 90^o$

$\angle ABD = \angle CBE$ (Common angle)

$\angle DAB = \angle ECB$(Remaining angle)

Hence, by AArule,

$\Delta ABD \sim \Delta CBE$

(iii) 

In ΔAEP and ΔADB

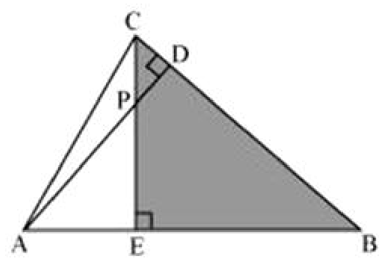
$\angle AEP = \angle ADB = 90^o$

$\angle PAE = \angle DAB$ (Common angle)

$\angle APE = \angle ABD$ (Remaining angle)

Hence, by AAA rule,

$\Delta AEP \sim \Delta ADB$

(iv) 

In ΔPDC and ΔBEC

$\angle PDC = \angle BEC = 90^o$

$\angle PCD = \angle BCE$ (Common angle)

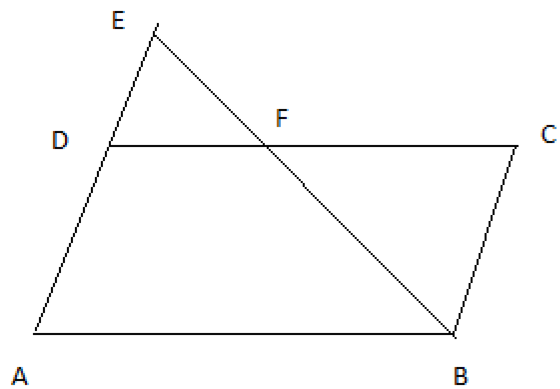
$\angle CPD = \angle CBE$ (Remaining angle)

Hence, by AAArule,

$\Delta PDC \sim \Delta BEC$

8. Eis a point on the sideAD produced of a parallelogram ABCD andBE intersectsCD atF. Show that $\Delta ABE \sim \Delta CFB$.

Solution:



ΔABE and ΔCFB

$\angle A = \angle C$ (Opposite angles of parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles as $AE \parallel BC$)

$\angle ABE = \angle CFB$ (Alternate interior angles as $AB \parallel DC$)

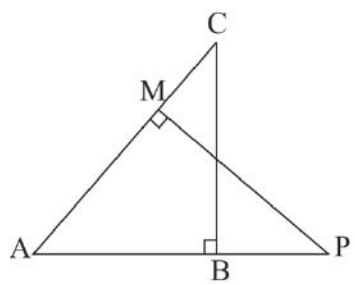
Hence, by AAA rule,

$\Delta ABE \sim \Delta CFB$

9. In Figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



Solution:

(i) In ΔABC and ΔAMP

$\angle ABC = \angle AMP = 90^{o}$

$\angle A = \angle A$(Common angle)

$\angle ACB = \angle APM$(Remaining angle)

Hence, by AArule,

$\Delta ABC \sim \Delta AMP$

(ii) Since, $\Delta ABC \sim \Delta AMP$

Hence, $\frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides are proportional)

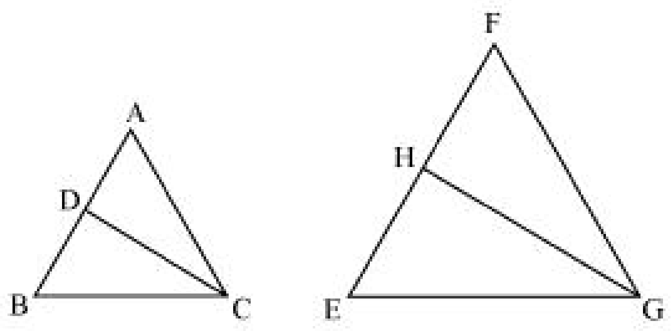
10. CD andGH are respectively the bisectors of$\angle ACB$ and$\angle EGF$ such thatD andH lie on sidesAB andFE ofΔABC andΔEFG respectively. If$\Delta ABC \sim \Delta FEG$, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Solution:



Since $\Delta ABC \sim \Delta FEG$

Hence, $\angle A = \angle F$

$\angle B = \angle E$

$\angle ACB = \angle FGE$

$\Rightarrow \frac{\angle ACB}{2} = \frac{\angle FGE}{2}$

$\Rightarrow \angle ACD = \angle FGH$(Angle bisector)

And $\angle DCB = \angle HGE$ (Angle bisector)

InΔACDandΔFGH

$\angle A = \angle F$

$\angle ACD = \angle FGH$ (Angle bisector)

$\angle ADC = \angle FHG$ (Remaining angle)

Hence, by AAA rule,

$\Delta ACD \sim \Delta FGH$

So, $\frac{CD}{GH} = \frac{AC}{FG}$ (Corresponding sides are proportional)

In ΔDCB and ΔHGE

$\angle B = \angle E$

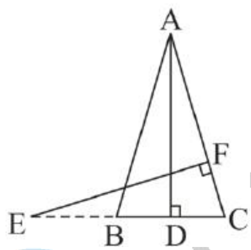
$\angle DCB = \angle HGE$ (Angle bisector)

$\angle BDC = \angle EHG$ (Remaining angle)

Hence, by AAA rule,

$\Delta DCB \sim \Delta HGE$

11. In Fig., E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.



Solution:

In ΔABD and ΔECF,

Since, $AB = AC$ (isosceles triangles)

So, $\angle ABD = \angle ECF$

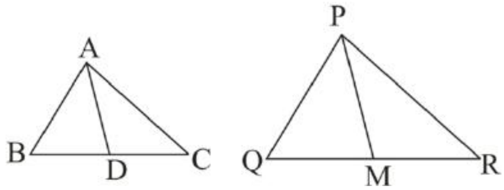
$\angle ADB = \angle EFC = 90^o$

$\angle BAD = \angle CEF$ (Remaining angle)

Hence, by AAA rule,

$\Delta ABD \sim \Delta ECF$

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see Fig.). Show that $\Delta ABC \sim \Delta PQR$.



Solution:

Median divides opposite side.

So, $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

So, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Hence, by SSSrule

$\Delta ABD \sim \Delta PQM$

Hence, $\angle ABD = \angle PQM$ (Corresponding angles of similar triangles)

Hence, $\angle ABC = \angle PQR$

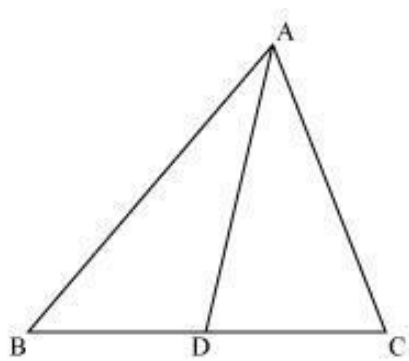
And $\frac{AB}{PQ} = \frac{BC}{QR}$

Hence, by SASrule

$\Delta ABC \sim \Delta PQR$

13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.

Solution:



In ΔACD and ΔBAC

Given that $\angle ADC = \angle BAC$

$\angle ACD = \angle BCA$ (Common angle)

$\angle CAD = \angle CBA$ (Remaining angle)

Hence, by AAA rule,

$\Delta ADC \sim \Delta BAC$

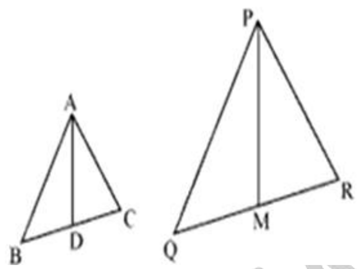
So, corresponding sides of similar triangles will be proportional to each other

$$\frac{CA}{CB} = \frac{CD}{CA}$$

Hence $CA^2 = CB \times CD$

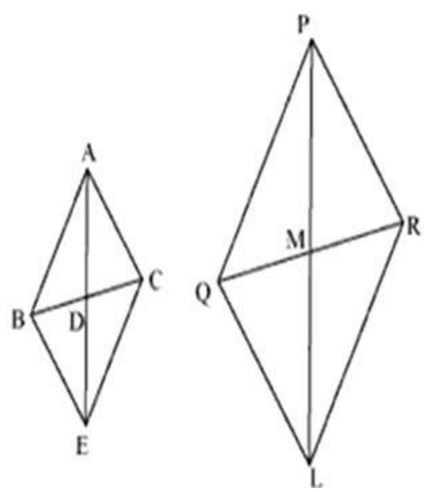
14. SidesAB andAC and medianAD of a triangleABC are respectively proportional to sides PQand PR and medianPM of another trianglePQR. Show that$\Delta ABC \sim \Delta PQR$.

Solution:



Given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$



Let us extend ADandPM up to point EandL respectively such that $AD = DE$ and $PM = ML$. Now join Bto E, CtoE,QtoLandRto L.

We know that medians divide opposite sides.

So, $BD = DC$ and $QM = MR$

Also, $AD = DE$ (by construction)

And $PM = ML$ (By construction)

So, in quadrilateral ABEC, diagonals AE and BC bisects each other at point D.
Also, in quadrilateral PQLR, diagonals PL and QR bisects each other at point M.

So, quadrilaterals ABCD and PQLR are a parallelogram.

$AC = BE$ and $AB = EC$ (Since it is a parallelogram, opposite sides will be equal)

Also $PR = QL$ and $PQ = LR$ (Since it is a parallelogram, opposite sides will be equal)

In ΔABE and ΔPQL,

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$ $\left(\frac{AC}{PR} = \frac{BE}{QL} \, and \, \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PL}\right)$

Hence, by SSS rule,

$\Delta ABE \sim \Delta PQL$

Similarly, $\Delta AEC \sim \Delta PLR$

Hence, $\angle BAE = \angle QPL$ and $\angle EAC = \angle LPR$

Hence, $\angle BAC = \angle QPR$

Now, in ΔABC and ΔPQR,

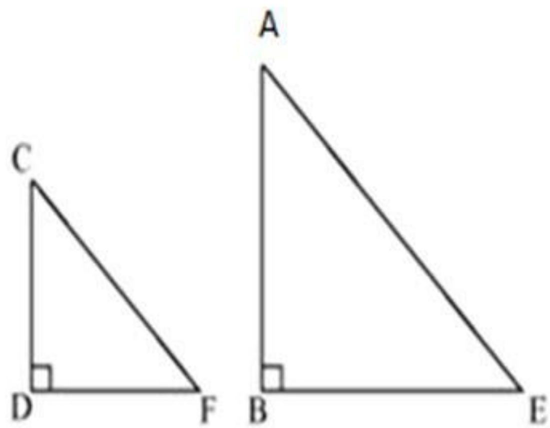
$\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle BAC = \angle QPR$

Hence, by SAS rule,

$\Delta ABC \sim \Delta PQR$

15. A vertical pole of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28m long. Find the height of the tower.

Solution:



Let AB be a tower and CD be a pole

Shadow of ABisBE

Shadow of CDisDF

The sun ray will fall on tower and pole at same angle.

So, $\angle DCF = \angle BAE$ and $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE = 90^o$ (Tower and pole are vertical to ground)

Hence, by AAA rule,

$\Delta ABE \sim \Delta CDF$

Therefore $\frac{AB}{CD} = \frac{BE}{DF}$

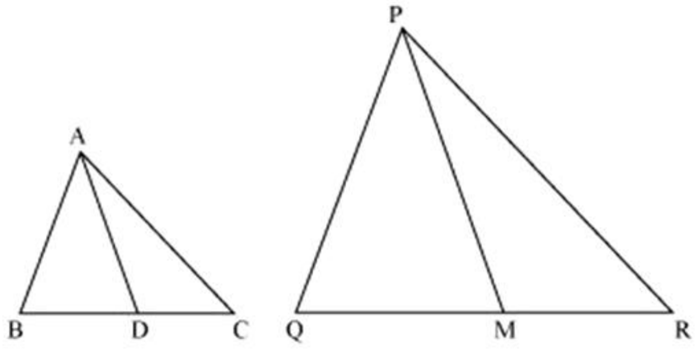
$\Rightarrow \frac{AB}{6} = \frac{28}{4}$

$\Rightarrow AB = 42$

Hence, height of tower $= 42$meters

16. IfAD andPM are medians of trianglesABC andPQR, respectively where$\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Solution:



Since $\Delta ABC \sim \Delta PQR$

So, their respective sides will be in proportion

Or, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ ... (1)

Also, $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ... (2)

Since, ADandPM are medians. So, they will divide their opposite sides equally.

Hence, $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ ... (3)

From equation (1)and(3)

$$\frac{AB}{PQ} = \frac{BD}{QM}$$

$\angle B = \angle Q$ (From equation2)

Hence, by SASrule

$\Delta ABD \sim \Delta PQM$

Hence, $\frac{AB}{PQ} = \frac{AD}{PM}$ (Corresponding sides are proportional)

EXERCISE 6.4

1. Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, $64 cm^2$ and $121 cm^2$. If $EF = 15.4 cm$, find BC.

Solution:

Since, $\Delta ABC \sim \Delta DEF$

Hence, $\frac{area(\Delta ABC)}{area(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$

Since $EF = 15.4, area(\Delta ABC) = 64; area(\Delta DEF) = 121$

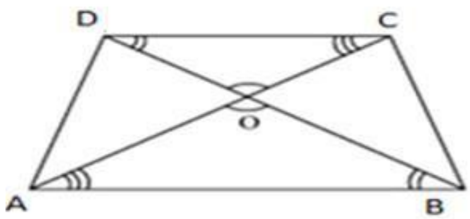
Hence, $\frac{64}{121} = \frac{BC^2}{15.4^2}$

$$\Rightarrow \frac{BC}{15.4} = \frac{8}{11}$$

$$\Rightarrow BC = \frac{8 \times 15.4}{11} = 8 \times 1.4 = 11.2\text{cm}.$$

2. Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O. If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD.

Solution:



Since $AB \parallel CD$

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\angle OBA = \angle ODC$ (Alternate interior angles)

$\angle AOB = \angle COD$ (Vertically opposite angles)

Hence, by AAA rule,

$\Delta AOB \sim \Delta COD$

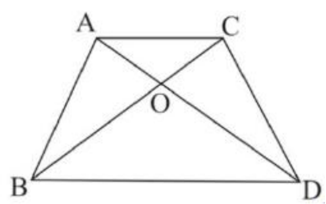
Hence, $\frac{\text{area}(\Delta AOB)}{\text{area}(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$

Since $AB = 2CD$

$$\frac{\text{area}(\Delta AOB)}{\text{area}(\Delta COD)} = \frac{4}{1} = 4:1$$

3. In Figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that

$\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$.



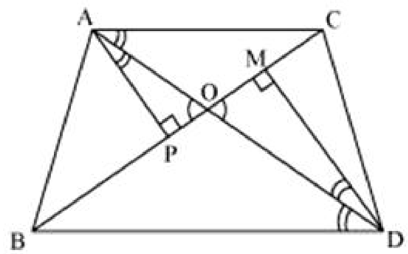
Solution:

We know that area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{height}$

Since ΔABC and ΔDBC are on same base,

Hence, ratio of their areas will be same as ratio of their heights.

Let us draw two perpendiculars AP and DM on BC.



In ΔAPO and ΔDMO

$\angle APO = \angle DMO = 90^o$
$\angle AOP = \angle DOM$ (Vertically opposite angles)

$\angle OAP = \angle ODM$ (Remaining angle)

Hence, by AAArule

$\Delta APO \sim \Delta DMO$

Hence, $\frac{AP}{DM} = \frac{AO}{DO}$

Hence, $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AP}{DM} = \frac{AO}{DO}$

4. If the areas of two similar triangles are equal, prove that they are congruent.

Solution:

Let us assume that $\Delta ABC \sim \Delta PQR$

Now, $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Since area(ΔABC) =area(ΔPQR)

Hence, $AB = PQ$

$BC = QR$

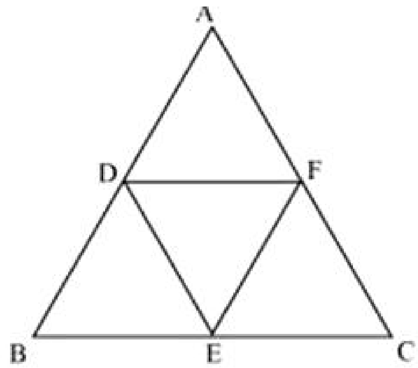
$AC = PR$

Since, corresponding sides of two similar triangles are of same length

Hence, $\Delta ABC \cong \Delta PQR$ (by SSSrule)

5. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Solution:



Since D and E are mid-points of AB and BC of ΔABC

Hence, $DE \parallel AC$ and $DE = \frac{1}{2}AC$ (by mid-point theorem)

Similarly, $EF = \frac{1}{2}AB$ and $DF = \frac{1}{2}BC$

Now in ΔABC and ΔEFD

$$\frac{AB}{EF} = \frac{BC}{FD} = \frac{CA}{DE} = 2$$

Hence, by SSS rule

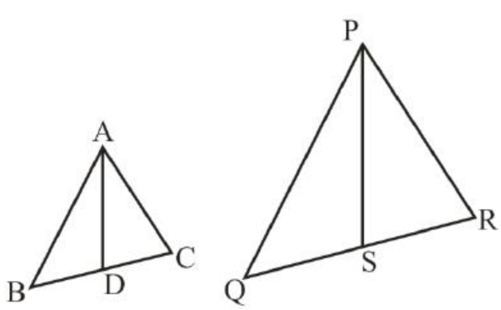
$\Delta ABC \sim \Delta EFD$

Hence, $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \left(\frac{AC}{DE}\right)^2 = 4$

$$\Rightarrow \frac{\text{area}(\Delta DEF)}{\text{area}(\Delta ABC)} = \frac{1}{4} = 1:4$$

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution:



Let us assume that $\Delta ABC \sim \Delta PQR$. Let ADand PSbe the medians of these triangles.

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ... (1)

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$

Since, AD and PS are medians

So, $BD = DC = \frac{BC}{2}$ and $QS = SR = \frac{QR}{2}$

So, equation (1) becomes

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$$

Now in $\Delta ABD$and ΔPQS

$\angle B = \angle Q$ and, $\frac{AB}{PQ} = \frac{BD}{QS}$

Hence, $\Delta ABD \sim \Delta PQS$

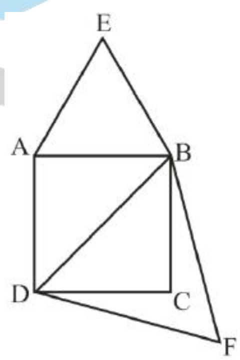
Hence, $\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$ ... (2)

Since, $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$

$\Rightarrow \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$ [from equation (2)]

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Solution:



Let ABCD be a square of side a. Therefore it's diagonal $= \sqrt{2}a$

Let ΔABEandΔDBF are two equilateral triangles.

Hence, $AB = AE = BE = a$ and $DB = DF = BF = \sqrt{2}a$

We know that all angles of equilateral triangles are 60^o.

Hence, all equilateral triangles are similar to each other.

Hence, ratio of areas of these triangles will be equal to the square of the ratio between sides of these triangles.

$$\frac{\text{areaof}\Delta\text{ABE}}{\text{areaof}\Delta\text{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Hence, $\text{areaof}\Delta\text{ABE} = \frac{1}{2}(\text{areaof}\Delta\text{DBF})$

8. ABC andBDE are two equilateral triangles such thatD is the mid-point ofBC. Ratio of the areas of trianglesABC andBDE is

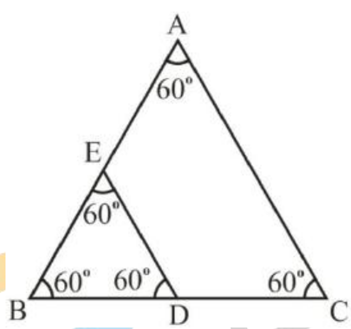
(A) $2:1$

(B) $1:2$

(C) $4:1$

(D) $1:4$

Solution: (C)



Since all angle of equilateral triangles are 60^{o}. Hence, all equilateral triangles are similar to each other. So ratio between areas of these triangles will be equal to the square of the ratio between sides of these triangles.

Let side of $\Delta\text{ABC} = a$

Hence, side of $\Delta\text{BDE} = \frac{a}{2}$

Hence, $\frac{\text{area}(\Delta\text{ABC})}{\text{area}(\Delta\text{BDE})} = \left(\frac{a}{\frac{a}{2}}\right)^2 = \frac{4}{1} = 4:1$

9. Sides of two similar triangles are in the ratio$4:9$. Areas of these triangles are in the ratio

(A) $2:3$

(B) $4:9$

(C) $81:16$

(D) $16:81$

Solution: (D)

If, two triangles are similar to each other, ratio between areas of these triangles will be equal to the square of the ratio between sides of these triangles.
Given that sides are in the ratio$4:9$.

Hence, ratio between areas of these triangles $= \left(\frac{4}{9}\right)^2 = \frac{16}{81} = 16:81$

EXERCISE 6.5

1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) $7\text{cm}, 24\text{cm}, 25\text{cm}$

(ii) $3\text{cm}, 8\text{cm}, 6\text{cm}$

(iii) $50\text{cm}, 80\text{cm}, 100\text{cm}$

(iv) $13\text{cm}, 12\text{cm}, 5\text{cm}$

Solution:

(i) Given that sides are 7cm, 24cm and 25cm.

Squaring the lengths of these sides we get 49,576 and 625.

Clearly, $49 + 576 = 625$or $7^2 + 24^2 = 25^2$.

Since, given triangle satisfies Pythagoras theorem. So, it is a right triangle.

As we know that the longest side in a right triangle is the hypotenuse.

Hence, length of hypotenuse $= 25\text{cm}$.

(ii) Given that sides are $3\text{cm}, 8\text{cm}$ and 6cm.

Squaring the lengths of these sides we may get 9,64 and 36.

Clearly, sum of squares of lengths of two sides is not equal to square of length of third side. Hence, given triangle is not satisfying Pythagoras theorem. So, it is not a right triangle

(iii) Given that sides are $50\text{cm}, 80\text{cm}$and 100cm.

Squaring the lengths of these sides we may get 2500,$6400$and 10000.

Clearly, sum of squares of lengths of two sides is not equal to square of length of third side. Therefore, given triangle is not satisfying Pythagoras theorem. So, it is not a right triangle.

(iv) Given that sides are $13\text{cm}, 12\text{cm}$ and 5cm.

Squaring the lengths of these sides we may get 169,144 and 25.

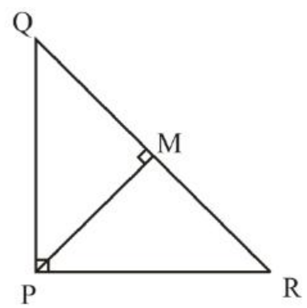
Clearly, $144 + 25 = 169$ or $12^2 + 5^2 = 13^2$.

Since, given triangle is satisfying Pythagoras theorem. So, it is a right triangle. As we know that the longest side in a right triangle is the hypotenuse.

Hence, length of hypotenuse $= 13\text{cm}$.

2. PQR is a triangle right angled atP andM is apoint onQR such thatPM $\perp$ QR. Show that $\text{PM}^2 = \text{QM} \cdot \text{MR}$.

Solution:



Let $\angle\text{MPR} = x$

In ΔMPR

$\angle\text{MRP} = 180^\text{o} - 90^\text{o} - x$

$\Rightarrow \angle\text{MRP} = 90^\text{o} - x$

Similarly in ΔMPQ

$\angle\text{MPQ} = 90^\text{o} - \angle\text{MPR} = 90^\text{o} - x$

$\angle\text{MQP} = 180^\text{o} - 90^\text{o} - (90^\text{o} - x)$

$\Rightarrow \angle\text{MQP} = x$

Now in ΔMPQ and ΔMRP, we may observe that

$\angle\text{MPQ} = \angle\text{MRP}$

$\angle\text{PMQ} = \angle\text{RMP}$

$\angle\text{MQP} = \angle\text{MPR}$

Hence, by AAA rule,

$\Delta\text{MPQ} \sim \Delta\text{MRP}$

Hence, $\frac{\text{QM}}{\text{PM}} = \frac{\text{MP}}{\text{MR}}$

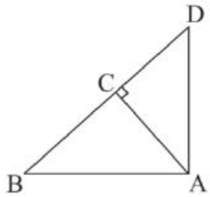
$\Rightarrow \text{PM}^2 = \text{QM} \cdot \text{MR}$

3. In Fig., ABD is a triangle right angled atA andAC $\perp$ BD. Show that

(i) $AB^2 = BC \cdot BD$

(ii) $AC^2 = BC \cdot DC$

(iii) $AD^2 = BD \cdot CD$



Solution:

(i) In $\Delta ABC$and ΔABD

$\angle CBA = \angle DBA$ (common angles)

$\angle BCA = \angle BAD = 90°$

$\angle BAC = \angle BDA$ (remaining angle)

Therefore, $\Delta ABC \sim \Delta ABD$ (by AAA)

$\therefore \frac{AB}{BD} = \frac{BC}{AB}$

$\Rightarrow AB^2 = BC \cdot BD$

(ii) Let $\angle CAB = x$

In ΔCBA

$\angle CBA = 180^o - 90^o - x$

$\angle CBA = 90^o - x$

Similarly in ΔCAD

$\angle CAD = 90^o - \angle CAB = 90^o - x$

$\angle CDA = 180^o - 90^o - (90^o - x)$

$\angle CDA = x$

Now in $\Delta CBA$and ΔCAD,we may observe that

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA = 90^o$

Therefore $\Delta CBA \sim \Delta CAD$(by AAA rule)

Therefore, $\frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = DC \times BC$

(iii) In$\Delta DCA \& \Delta DAB$

$\angle DCA = \angle DAB = 90°$

$\angle CDA = \angle ADB$ (Common angle)

$\angle DAC = \angle DBA$(remaining angle)

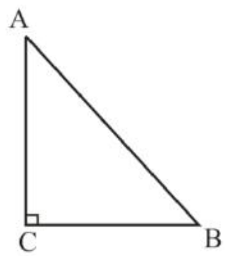
$\Delta DCA \sim \Delta DAB$ (byAAAproperty)

Therefore, $\frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

4. ABC is an isosceles triangle right angled atC. Prove that $AB^2 = 2AC^2$.

Solution:

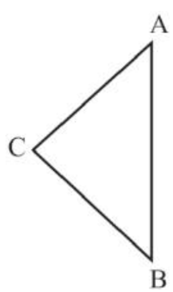


Given that ΔABC is an isosceles triangle.
Therefore, $AC = CB$
Applying Pythagoras theorem in ΔABC (i.e. right angled at point C)
$AC^2 + CB^2 = AB^2$

$2AC^2 = AB^2$ (as $AC = CB$)

5. ABC is an isosceles triangle with$AC = BC$. If$AB^2 = 2AC^2$, prove thatABC is a right triangle.

Solution:



Given that

$AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

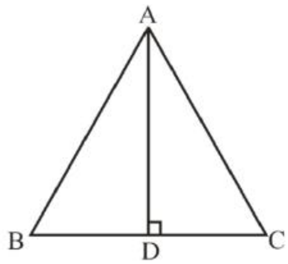
$\Rightarrow AB^2 = AC^2 + BC^2$ (as $AC = BC$)

Since triangle is satisfying the Pythagoras theorem

Therefore, given triangle is a right angled triangle.

6. ABC is an equilateral triangle of side$2a$. Find each of its altitudes.

Solution:



Let AD be the altitude in given equilateral ΔABC.
We know that altitude bisects the opposite side.
So, $BD = DC = a$
in ΔADB

$\angle ADB = 90^o$

Now applying Pythagoras theorem

$AD^2 + BD^2 = AB^2$

$\Rightarrow AD^2 + a^2 = (2a)^2$

$\Rightarrow AD^2 + a^2 = 4a^2$

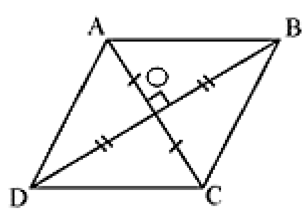
$\Rightarrow AD^2 = 3a^2$

$\Rightarrow AD = a\sqrt{3}$

Since in an equilateral triangle, all the altitudes are equal in length.
So, length of each altitude will be $\sqrt{3}a$

7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Solution:



In $\Delta AOB, \Delta BOC, \Delta COD, \Delta AOD$

Applying Pythagoras theorem

$AB^2 = AO^2 + OB^2$

$BC^2 = BO^2 + OC^2$

$CD^2 = CO^2 + OD^2$

$AD^2 = AO^2 + OD^2$

Adding all these equations,

$AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + OB^2 + OC^2 + OD^2)$

$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$ (diagonals bisect each other.)

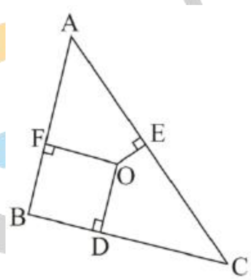
$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$

$= (AC)^2 + (BD)^2$

8. In Fig., O is a point in the interior of a triangle ABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$,

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.



Solution:

(i) In ΔAOF

Applying Pythagoras theorem

$OA^2 = OF^2 + AF^2$

Similarly in ΔBOD

$OB^2 = OD^2 + BD^2$

similarly in ΔCOE

$OC^2 = OE^2 + EC^2$

Adding these equations

$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$

$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$

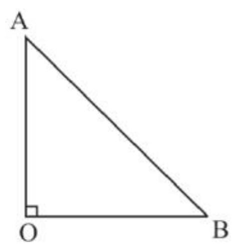
(ii) As from above result

$AF^2 + BD^2 + EC^2 = (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$

Therefore, $AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$

9. A ladder10m long reaches a window8m above the ground. Find the distance of the foot of the ladder from base of the wall.

Solution:



Let OAbe the wall and ABbe the ladder

Therefore by Pythagoras theorem,

$AB^2 = OA^2 + BO^2$

$\Rightarrow 10^2 = 8^2 + OB^2$

$\Rightarrow 100 = 64 + OB^2$

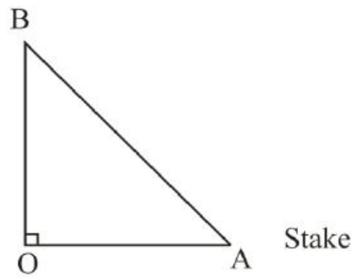
$\Rightarrow OB^2 = 36$

$\Rightarrow OB = 6$

Therefore, distance of foot of ladder from of the wall $= 6\text{m}$

10. A guy wire attached to a vertical pole of height18m is24m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Solution:



Let OB be the pole and AB be the wire.
Therefore, by Pythagoras theorem,

$\mathrm{AB}^2 = \mathrm{OB}^2 + \mathrm{OA}^2$

$\Rightarrow 24^2 = 18^2 + \mathrm{OA}^2$

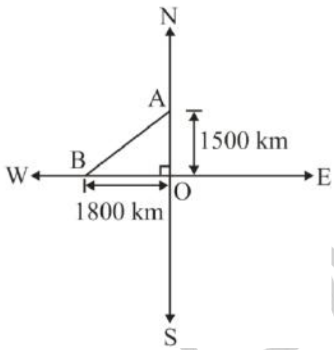
$\Rightarrow \mathrm{OA}^2 = 576 - 324$

$\Rightarrow OA = \sqrt{252} = \sqrt{6 \times 6 \times 7} = 6\sqrt{7}$

Therefore distance from base $= 6\sqrt{7}\mathrm{m}$

11. An aeroplane leaves an airport and flies due north at a speed of 1000km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of1200 km per hour. How far apart will be the two planes after$1\frac{1}{2}$ hours?

Solution:



Distance traveled by the plane flying towards north in $1\frac{1}{2}$hrs

$= 1{,}000 \times 1\frac{1}{2} = 1{,}500\mathrm{km}$

Distance traveled by the plane flying towards west in $1\frac{1}{2}$hrs

$= 1{,}200 \times 1\frac{1}{2} = 1{,}800\mathrm{km}$

Let these distances are represented by OA and OB respectively.
Now applying Pythagoras theorem
Distance between these planes after $1\frac{1}{2}$hrs, $\mathrm{AB} = \sqrt{\mathrm{OA}^2 + \mathrm{OB}^2}$

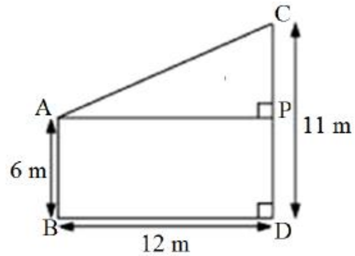
$= \sqrt{(1{,}500)^2 + (1{,}800)^2} = \sqrt{2250000 + 3240000}$

$= \sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$

So, distance between these planes will be $300\sqrt{61}$km. after $1\frac{1}{2}$hrs.

12. Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the poles is 12m, find the distance between their tops.

Solution:



Let CD and AB be the poles of height $11m$ and 6m.
Therefore, $CP = 11 - 6 = 5m$
From the figure we may observe that $AP = 12m$
In ΔAPC, by applying Pythagoras theorem

$AP^2 + PC^2 = AC^2$

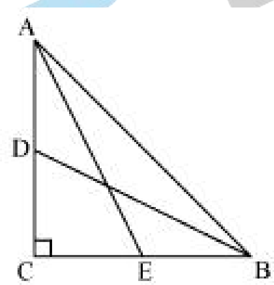
$\Rightarrow 12^2 + 5^2 = AC^2$

$\Rightarrow AC^2 = 144 + 25 = 169$

$\Rightarrow AC = 13$
Therefore, distance between their tops $= 13m$.

13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Solution:



In ΔACE,

$AC^2 + CE^2 = AE^2$ ... (i)

In ΔBCD,

$BC^2 + CD^2 = BD^2$ ... (ii)

Adding (i) and (ii)

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2$ ... (iii)

$\Rightarrow CD^2 + CE^2 + AC^2 + BC^2 = AE^2 + BD^2$

In ΔCDE

$DE^2 = CD^2 + CE^2$

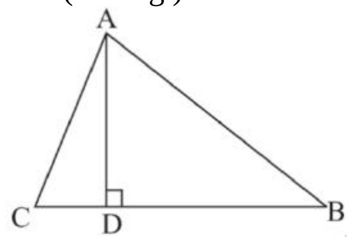
In ΔABC

$AB^2 = AC^2 + CB^2$

Putting the values in equation (iii)

$DE^2 + AB^2 = AE^2 + BD^2$

14. The perpendicular fromA on side BCof aΔABC intersectsBC atD such thatDB $= 3CD$(see Fig.). Prove that $2AB^2 = 2AC^2 + BC^2$.



Solution:

Given that $3DC = DB$

$DC = \frac{BC}{4}$ [DB: DC = 3: 1] ... (1)

and

$DB = \frac{3BC}{4}$ ... (2)

In ΔACD

$AC^2 = AD^2 + DC^2$

$AD^2 = AC^2 - DC^2$ ... (3)

In ΔABD

$AB^2 = AD^2 + DB^2$

$AD^2 = AB^2 - DB^2$ ... (4)

From equation (3) and (4)

$AC^2 - DC^2 = AB^2 - DB^2$

Since, given that $3DC = DB$

$\text{AC}^2 - \left(\frac{\text{BC}}{4}\right)^2 = \text{AB}^2 - \left(\frac{3\text{BC}}{4}\right)^2$ (from(1)and (2))

$\Rightarrow \text{AC}^2 - \dfrac{\text{BC}^2}{16} = \text{AB}^2 - \dfrac{9\text{BC}^2}{16}$

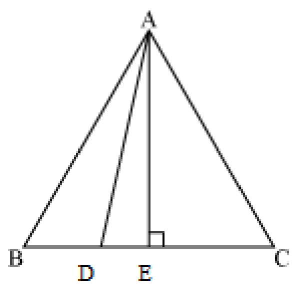
$\Rightarrow 16\text{AC}^2 - \text{BC}^2 = 16\text{AB}^2 - 9\text{BC}^2$

$\Rightarrow 16\text{AB}^2 - 16\text{AC}^2 = 8\text{BC}^2$

$\Rightarrow 2\text{AB}^2 = 2\text{AC}^2 + \text{BC}^2$

15. In an equilateral triangle ABC, D is a point on side BC such that $\text{BD} = \frac{1}{3}\text{BC}$. Prove that $9\text{AD}^2 = 7\text{AB}^2$.

Solution:



Let side of equilateral triangle be a, and AE be the altitude of ΔABC

So, $\text{BE} = \text{EC} = \frac{\text{BC}}{2} = \frac{\text{a}}{2}$

And, $\text{AE} = \frac{\text{a}\sqrt{3}}{2}$

Given that $\text{BD} = \frac{1}{3}\text{BC} = \frac{\text{a}}{3}$

So, $\text{DE} = \text{BE} - \text{BD} = \frac{\text{a}}{2} - \frac{\text{a}}{3} = \frac{\text{a}}{6}$

Now, in ΔADE by applying Pythagoras theorem

$\text{AD}^2 = \text{AE}^2 + \text{DE}^2$

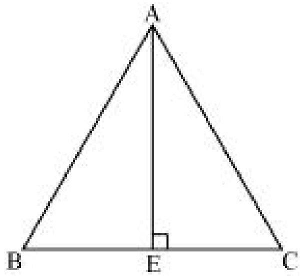
$\Rightarrow \text{AD}^2 = \left(\dfrac{\text{a}\sqrt{3}}{2}\right)^2 + \left(\dfrac{\text{a}}{6}\right)^2$

$= \left(\frac{3\text{a}^2}{4}\right) + \left(\frac{\text{a}^2}{36}\right) = \frac{28\text{a}^2}{36}$

or, $9\text{AD}^2 = 7\text{AB}^2$

16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Solution:



Let side of equilateral triangle be a. And AE be the altitude of ΔABC

So, $BE = EC = \frac{BC}{2} = \frac{a}{2}$

And, $AE = \frac{a\sqrt{3}}{2}$

Now in $\Delta ABE$by applying Pythagoras theorem

$AB^2 = AE^2 + BE^2$

$\Rightarrow a^2 = AE^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow AE^2 = a^2 - \frac{a^2}{4}$

$\Rightarrow AE^2 = \frac{3a^2}{4}$

$\Rightarrow 4AE^2 = 3a^2$

or, $4AE^2 = 3 \times$ square of one side

17. Tick the correct answer and justify: In$\Delta ABC, AB = 6\sqrt{3}cm, AC = 12$cm and$BC =$ 6cm. The angleB is:

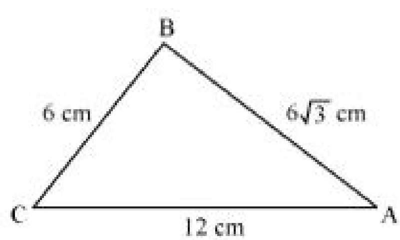
(A) 120^o

(B) 60^o

(C) 90^o

(D) 45^o

Solution: (C)



Given that $AB = 6\sqrt{3}\text{cm}, AC = 12\text{cm}$ and $BC = 6\text{cm}$

We may observe that

$AB^2 = 108AC^2 = 144$

And $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

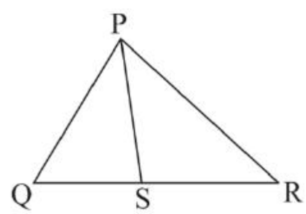
Thus the given ΔABC is satisfying Pythagoras theorem Therefore triangle is a right angled triangle right angled at B

Therefore, $\angle B = 90^o$.

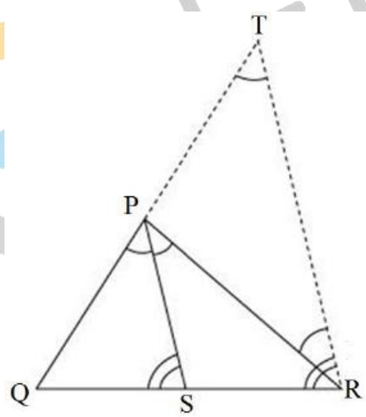
EXERCISE 6.6 (Optional)*

1. In Fig.,PS is the bisector of$\angle QPR$ of ΔPQR. Prove that

$$\frac{QS}{SR} = \frac{PQ}{PR}$$



Solution:



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that PS is angle bisector of $\angle QPR$

$\angle QPS = \angle SPR$ (1)

$\angle SPR = \angle PRT$ (As PS || TR, alternate interior angles) (2)

$\angle QPS = \angle QTR$ (As PS || TR, corresponding angles) (3)

Using these equations we may find

$\angle PRT = \angle QTR$ from (2) and (3)

So, $PT = PR$ (Since ΔPTR is isosceles triangle)

Now in ΔQPS and ΔQTR

$\angle QSP = \angle QRT$ (As $PS \parallel TR$)

$\angle QPS = \angle QTR$ (As $PS \parallel TR$)

$\angle Q$ is common

$\Delta QPS \sim \Delta QTR$ (by AAA property)

So, $\frac{QR}{QS} = \frac{QT}{QP}$

$\Rightarrow \frac{QR}{QS} - 1 = \frac{QT}{QP} - 1$

$\Rightarrow \frac{SR}{QS} = \frac{PT}{QP}$

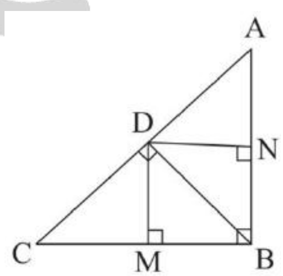
$\Rightarrow \frac{QS}{SR} = \frac{QP}{PT}$

$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR}$

2. In Fig., D is a point on hypotenuse AC of ΔABC, such that $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove that:

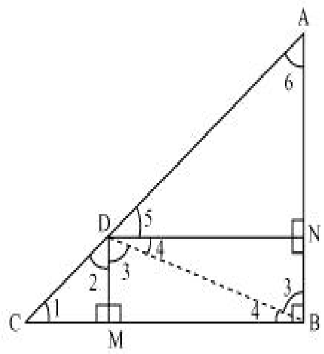
(i) $DM^2 = DN \cdot MC$

(ii) $DN^2 = DM \cdot AN$



Solution:

(i) Let us join DB.



$DN||CB$

$DM||AB$

So, $DN = MB$

$DM = NB$

Then $\angle CDB = \angle ADB = 90°$

$\angle 2 + \angle 3 = 90^o \ldots (1)$

In ΔCDM

$\angle 1 + \angle 2 + \angle DMC = 180°$

$\angle 1 + \angle 2 = 90° \ldots (2)$

In ΔDMB

$\angle 3 + \angle DMB + \angle 4 = 180°$

$\angle 3 + \angle 4 = 90° \ldots (3)$

From equation (1) and (2)

$\angle 1 = \angle 3$

From equation (1) and (3)

$\angle 2 = \angle 4$

$\Delta BDM \sim \Delta DCM$

$$\frac{BM}{DM} = \frac{DM}{MC}$$

$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$$

$\Rightarrow DM^2 = DN \times MC$

(ii) Similarly in ΔDBN

$\angle 4 + \angle 3 = 90^{o}$ …(4)

In ΔDAN

$\angle 5 + \angle 6 = 90^{o}$…(5)

In ΔDAB

$\angle 4 + \angle 5 = 90^{o}$ …(6)

From equation (4) and (6)

$\angle 3 = \angle 5$

From equation (5) and (6)

$\angle 4 = \angle 6$

$\Delta DNA \sim \Delta BND$

$$\frac{AN}{DN} = \frac{DN}{NB}$$

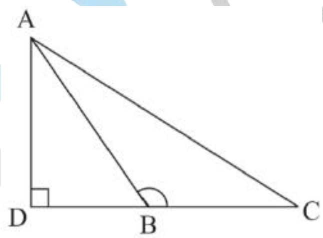
$\Rightarrow DN^2 = AN \times NB$

$\Rightarrow DN^2 = AN \times DM$

(as $NB = DM$)

3. In Fig., ABC is a triangle in which $\angle ABC > 90^{o}$ and $AD \perp CB$ produced. Prove that

$AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.



Solution:

In ΔADB, applying Pythagoras theorem

$AB^2 = AD^2 + DB^2$ ... (1)

In ΔACD, applying Pythagoras theorem

$AC^2 = AD^2 + DC^2$

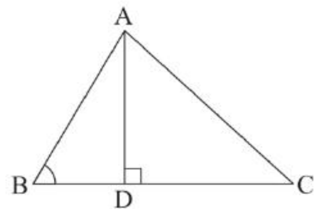
$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$

$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

Now using equation (1)

$AC^2 = AB^2 + BC^2 + 2BC.BD$

4. In Fig., ABC is a triangle in which $\angle ABC < 90^o$ and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Solution:

In ΔADB, applying Pythagoras theorem

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 = AB^2 - DB^2 \ldots (1)$

In ΔADC applying Pythagoras theorem

$D^2 + DC^2 = AC^2$ (2)

Now using equation (1)

$AB^2 - BD^2 + DC^2 = AC^2$

$\Rightarrow AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$\Rightarrow AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC.BD$

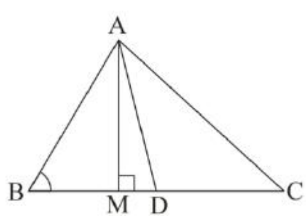
$= AB^2 + BC^2 - 2BC.BD$

5. In Fig., ADis a median of a triangle ABC and $AM \perp BC$. Prove that:

(i) $AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$

(iv) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Solution:

(i) In ΔAMD

$AM^2 + MD^2 = AD^2$ ... (1)

In ΔAMC

$AM^2 + MC^2 = AC^2$ ... (2)

$\Rightarrow AM^2 + (MD + DC)^2 = AC^2$

$\Rightarrow (AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

Using equation (1) we may get

$AD^2 + DC^2 + 2MD.DC = AC^2$

Now using the result, $DC = \frac{BC}{2}$

$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$

$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$

(ii) In ΔABM, applying Pythagoras theorem

$AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD.MD$

$= AD^2 + BD^2 - 2BD.MD$

$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$

$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$

(iii) In ΔAMB

$AM^2 + MB^2 = AB^2$ ... (1)

In ΔAMC

$AM^2 + MC^2 = AC^2$ ... (2)

Adding equation (1) and (2)

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$\Rightarrow 2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$$\Rightarrow 2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$$

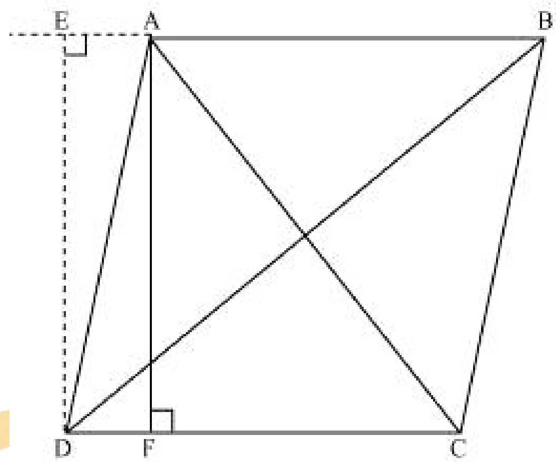
$$\Rightarrow 2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$$

$$\Rightarrow 2(AM^2 + MD^2) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$\Rightarrow 2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Solution:



Let ABCD be a parallelogram

Let us draw perpendicular DE on extended side BA and AF on side DC.
In ΔDEA

$DE^2 + EA^2 = DA^2$ ... (i)

In ΔDEB

$DE^2 + EB^2 = DB^2$

$\Rightarrow DE^2 + (EA + AB)^2 = DB^2$

$\Rightarrow (DE^2 + EA^2) + AB^2 + 2EA.AB = DB^2$

$\Rightarrow DA^2 + AB^2 + 2EA.AB = DB^2$ ... (ii)

In ΔADF

$AD^2 = AF^2 + FD^2$

In ΔAFC

$AC^2 = AF^2 + FC^2$

$= AF^2 + (DC - FD)^2$

$= AF^2 + DC^2 + FD^2 - 2DC \cdot FD$

$= (AF^2 + FD^2) + DC^2 - 2DC. FD$

$\Rightarrow AC^2 = AD^2 + DC^2 - 2DC \cdot FD \ldots$ (iii)

Since ABCD is a parallelogram

$AB = CD$ (iii)

And $BC = AD$ (iv)

In ΔDEA and ΔADF

$\angle DEA = \angle AFD$

$\angle EAD = \angle FDA(EA \parallel DF)$

$\angle EDA = \angle FAD(AF \parallel ED)$

AD is common in both triangles.

Since respective angles are same and respective sides are same
$\Delta DEA \cong \Delta AFD$

$SOEA = DF$

Adding equation (ii) and (iii)

$\Rightarrow DA^2 + AB^2 + 2EA. AB + AD^2 + DC^2 - 2DC. FD = DB^2 + AC^2$

$\Rightarrow DA^2 + AB^2 + AD^2 + DC^2 + 2EA. AB - 2DC. FD = DB^2 + AC^2$

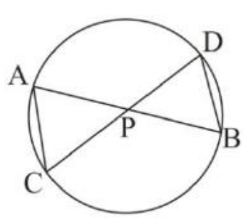
$\Rightarrow BC^2 + AB^2 + AD^2 + DC^2 + 2EA. AB - 2AB. EA = DB^2 + AC^2$

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

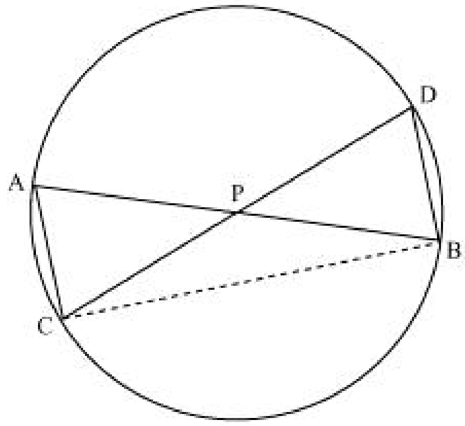
7. In Fig., two chordsAB andCD intersect each other at the point P. Prove that:

(i) $\Delta APC \sim \Delta DPB$

(ii) $AP \cdot PB = CP \cdot DP$



Solution:



Let us join CB

(i) In ΔAPC and ΔDPB

$\angle APC = \angle DPB$ {Vertically opposite angles}

$\angle CAP = \angle BDP$ {Angles in same segment for chord CB}

$\Delta APC \sim \Delta DPB$ {ByAA similarly criterion}

(ii) We know that corresponding sides of similar triangles are proportional

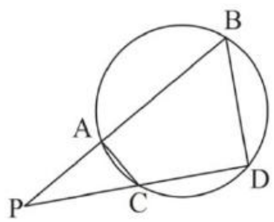
$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$

$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$

$\therefore AP.PB = PC \cdot DP$

8. In Fig., two chordsABandCD of a circle intersect each other at the pointP (when produced) outside the circle. Prove that

(i) $\Delta PAC \sim \Delta PDB$ (ii) $PA \cdot PB = PC \cdot PD$



Solution:

(i) In ΔPAC and ΔPDB

$\angle P = \angle P$(Common)

$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is equal to opposite interior angle)

$\angle PCA = \angle PBD$ (remaining angles)

$\Delta PAC \sim \Delta PDB$ (by AAA)

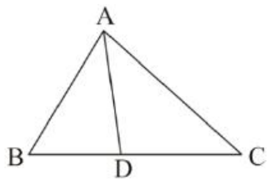
(ii) We know that corresponding sides of similar triangles are proportional.

$$\frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$$

$$\Rightarrow \frac{PA}{PD} = \frac{PC}{PB}$$

$\therefore PA.PB = PC.PD$

9. In Fig.,D is a point on sideBC of ΔABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove thatAD is the bisector of $\angle BAC$.



Solution:

In ΔDBA and ΔDCA

$\frac{BD}{CD} = \frac{AB}{AC}$ (Given)

$AD = AD$(Common)

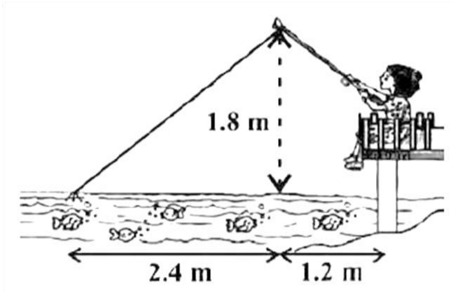
So, $\Delta DBA \sim \Delta DCA$(By SSS)

Now, corresponding angles of similar triangle will be equal.

$\angle BAD = \angle CAD$

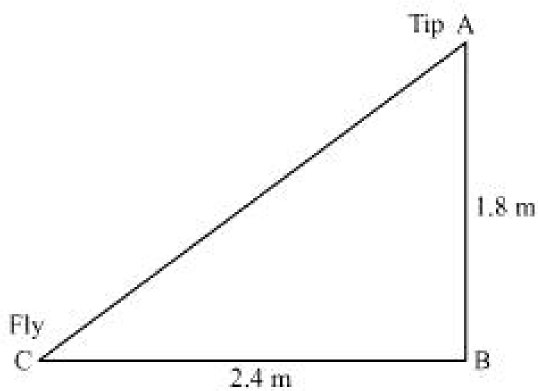
AD is angle bisector of $\angle BAC$

10. Nazima is fly fishing in a stream. The tip of her fishing rod is1.8m above the surface of the water and the fly at the end of the string rests on the water3.6m away and2.4mfrom a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig.)? If she pulls in the string at the rate of5cm per second, what will be the horizontal distance of the fly from her after12 seconds?



A

Solution:



Let AB be the height of tip of fishing rod from water surface. Let BC Be the horizontal distance of fly from the tip of fishing rod.

Then, AC is the length of string.

AC Can be found by applying Pythagoras theorem in ΔABC

$AC^2 = AB^2 + BC^2$

$AC^2 = (1.8)^2 + (2.4)^2$

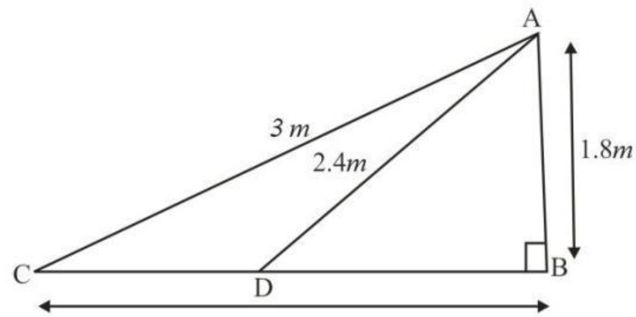
$AC^2 = 3.24 + 5.76$

$AC^2 = 9.00$

Thus, length of string out is3m.

Now, she pulls string at rate of 5cm per second.

So, String Pulled in 12Second $= 12 \times 5 = 60\text{cm} = 0.6\text{m}$



Let after 12second, fly be at point D.

Length of string out after12second is AD

$AD = AC -$ String pulled by Nazima in 12second

$= 3.00 - 0.6$

$= 2.4$

In ΔADB

$AB^2 + BD^2 = AD^2$

$\Rightarrow (1.8)^2 + BD^2 = (2.4)^2$

$\Rightarrow BD^2 = 5.76 - 3.24 = 2.52$

$\Rightarrow BD = 1.587$

Horizontal distance of fly $= BD + 1.2$

$= 1.587 + 1.2$

$= 2.787$

$= 2.79$m