#463625

Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

(i) parallel lines

(ii) perpendicular lines

(iii) line segment

(iv) radius of a circle

(v) square

Solution

(i) Parallel line - Two lines are said to be parallel when (a) They never meet or never intersect each other even if they are extended to the infinity. (b) they coplanar.

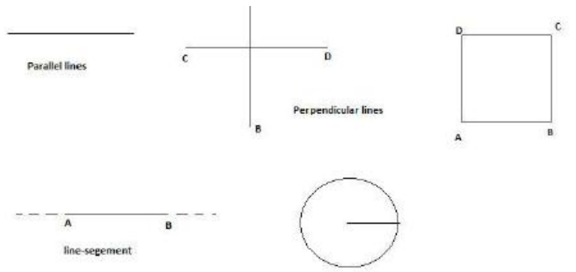
(ii) Perpendicular lines - Two lines AB and CD lying the same plane are said to be perpendicular, if they form a right angle. We write $AB \perp CD$

(iii) Line segment - A line-segment is a part of line. When two distinct points, say A and B on a line are given, then the part of this line with end-points A and B is called the lin

segment.

(iv) Radius of a circle - The distance from the centre to a point on the circle is called the radius of the circle. In the following figure OP is the radius.

(v) Square - A quadrilateral in which all the four angles are right angles and four sides are equal is called a square. $ABCD$ is a square.



#463628

If a point C lies between two points A and B such that $AC = BC$, then prove that $AC = \frac{1}{2}AB$. Explain by drawing the figure.

Solution

As given $AC = BC$......eq (1)

From equation (1)

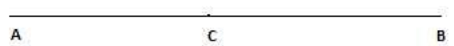
$AC = BC$

Adding AC on both sides,

$AC + AC = BC + AC$

$2AC = AB \ (\because BC + AC = AB)$

$AC = \frac{1}{2}AB$



#463629

Point C is the mid-point of line segment AB, prove that every line segment has one and only one mid-point.

Solution

$AC = BC$ ... (i)

If possible, let D be another mid-point of AB.

$AD = DB$ ... (ii)

Subtracting (ii) from (i)

$AC - AD = BC - DB$

$DC = -DC$ $(\because AC - AD = DC and CB - DB = -DC)$

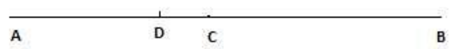
$DC + DC = 0$

$2DC = 0$

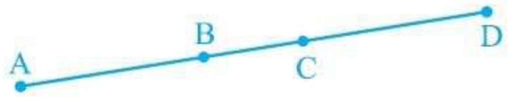
$DC = 0$

So, C and D coincide.

Thus, every line-segment has one and only one mid-point.



#463630



In the figure, if $AC = BD$, then prove that $AB = CD$.

Solution

Given,

$AC = BD$ ... (i)

$AC = AB + BC$ ......(ii) (Point B lies between A and C)

$BD = BC + CD$ ... (iii) (Point C lies between B and D)

Now,

substituting (ii) and (iii) in (i), we get

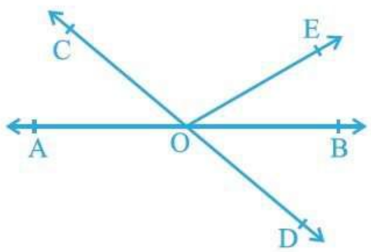
$\Rightarrow AB + BC = BC + CD$

$\Rightarrow AB + BC - BC = CD$

$\Rightarrow AB = CD$

Hence, $AB = CD$.

#463639



In the figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70$ and $\angle BOD = 40$, find $\angle BOE$ and reflex $\angle COE$.

Solution

AB is a straight line and OC and OE meets at point Oo in AB.

$\angle AOC + \angle BOE + \angle COE = 180^\circ$

As given $\angle AOC + \angle BOE = 70^\circ$

$\therefore 70 + \angle COE = 180^\circ$

$\Rightarrow \angle COE = 180 - 70 = 110^\circ$

Reflex $\angle COE = 360^\circ - 110^\circ = 250^\circ$

$\because CD$ is a straight line and OE and OB stand on it

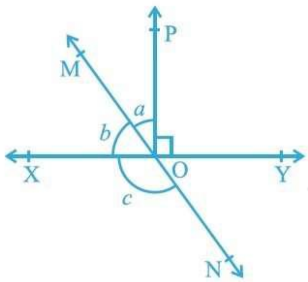
$\angle COE + \angle BOE + \angle BOD = 180^\circ$

$110^\circ + \angle BOE + 40^\circ = 180^\circ$

$\angle BOE = 180 - 110 - 40$

$\angle BOE = 30^\circ$

#463642



In the figure, lines XY and MN intersect at O. If $\angle POY = 90^o$ and $a : b = 2 : 3$ find c.

Solution

Given, $\angle POY = 90^\circ$ and $a : b = 2 : 3$

According to the question,

$\angle POY + a + b = 180^\circ$

$\Rightarrow 90^\circ + a + b = 180^\circ$

$\Rightarrow a + b = 90^\circ$

Let a be $2x$, then will be $3x$

$\Rightarrow 2x + 3x = 90^\circ$

$\Rightarrow 5x = 90^\circ$

$\Rightarrow x = 18^\circ$

$\therefore a = 2 \times 18 = 36^\circ$

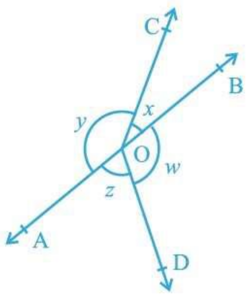
$b = 3 \times 18 = 54^\circ$

$b + c = 180^\circ$ | Linear Pair

$\Rightarrow 54^\circ + c = 180^\circ$

$\Rightarrow c = 126^\circ$

#463645



In the figure, if $x + y = w + z$ then prove that AOB is a line.

Solution

As Given, $x+y=w+z$

To Prove: AOB is a line or $x+y=180^\circ$(linear pair.)

According to the question,

$x+y+w+z=360^\circ$| Angles around a point.

$(x+y)+(w+z)=360^\circ$

$(x+y)+(x+y)=360^\circ$| Given $x+y=w+z$

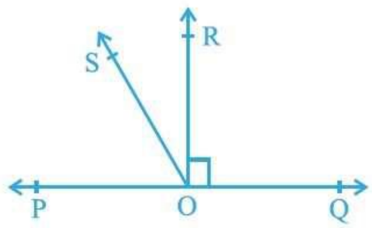
$2(x+y)=360^\circ$

$(x+y)=180^\circ$

Hence, $x+y$ makes a linear pair.

Therefore, AOB is a straight line.

#463647



In the figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS=\dfrac{1}{2}(\angle QOS-\angle POS)$

Solution

Given,

OR is perpendicular to line PQ

To prove: $\angle ROS=\dfrac{1}{2}(\angle QOS–\angle POS)$

According to the question,

$\angle POR=\angle ROQ=90^\circ$| Perpendicular

$\angle QOS=\angle ROQ+\angle ROS=90^\circ+\angle ROQ$-- (i)

$\angle POS=\angle POR-\angle ROS=90^\circ-\angle ROQ$-- (ii)

Subtracting (ii) from (i)

$\angle QOS-\angle POS=90^\circ+\angle ROQ-(90^\circ-\angle ROQ)$

$\Rightarrow \angle QOS-\angle POS=90^\circ+\angle ROQ-90^\circ+\angle ROQ$

$\Rightarrow \angle QOS-\angle POS=2\angle ROQ$

$\Rightarrow \angle ROS=\dfrac{1}{2}(\angle QOS–\angle POS)$

Hence, Proved.

#463649

It is given that $\angle XYZ=64$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Solution

Given,

$\angle XYZ = 64^\circ$

YQ bisects $\angle ZYP$

$\angle XYZ + \angle ZYP = 180^\circ$(Linear Pair)

$\Rightarrow 64^\circ + \angle ZYP = 180^\circ$

$\angle ZYP = 116^\circ$

Also, $\angle ZYP = \angle ZYQ + \angle QYP$

$\angle ZYQ = \angle QYP$ (YQ bisects$\angle ZYP$)

$\Rightarrow \angle ZYP = 2\angle ZYQ$

$\Rightarrow 2\angle ZYQ = 116^\circ$

$\Rightarrow \angle ZYQ = 58^\circ = \angle QYP$

Now, $\angle XYQ = \angle XYZ + \angle ZYQ$

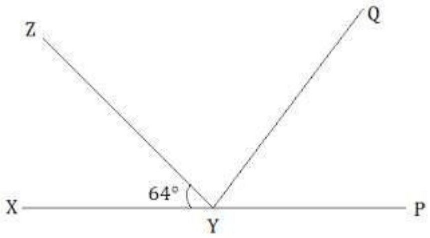
$\Rightarrow \angle XYQ = 64^\circ + 58^\circ$

$\Rightarrow \angle XYQ = 122^\circ$

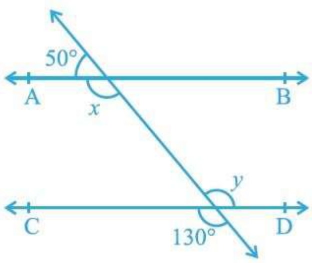
Also, reflex $\angle QYP = 18^{\circ\circ} + \angle XYQ$

$\angle QYP = 180^\circ + 122^\circ$

$\Rightarrow \angle QYP = 302^\circ$



#463653



In the figure, find the values of x and y and then show that $AB \parallel CD$.

Solution

$x + 50^\circ = 180^\circ$(Linear pair)

$\Rightarrow x = 130^\circ$

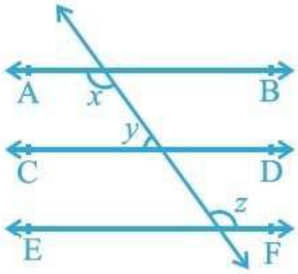
Also, $y = 130^\circ$ (Vertically opposite)

Now, $x = y = 130^\circ$(Alternate interior angles)

Alternate interior angles are equal.

$\therefore AB \parallel CD$.

#463655



In the figure, if $AB \parallel CD, CD \parallel EF$ and $y : z = 3 : 7$ find x.

Solution

Given,

$AB \parallel CD$ and $CD \parallel EF$

$y : z = 3 : 7$

Now, $x + y = 180°$(Angles on the same side of transversal.)

Also, $\angle O = \angle Z$ (Corresponding angles)

and $\angle Y + \angle O = 180°$(Linear pair)

$\Rightarrow y + z = 180°$

According to the question,

$y = 3w$ and $z = 7w$

$3w + 7w = 180°$

$\Rightarrow 10w = 180°$

$\Rightarrow w = 18°$

$\therefore y = 3 \times 18° = 54°$

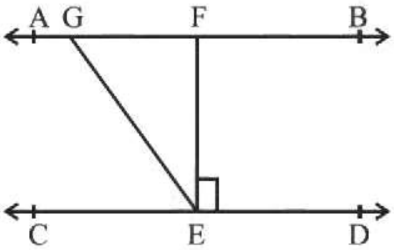
and $z = 7 \times 18° = 126°$

Now, $x + y = 180°$

$\Rightarrow x + 54° = 180°$

$\Rightarrow x = 126°$

#463658



In Fig. if $AB \parallel CD, EF \parallel CD$ and $\angle GED = 126$, find $\angle AGE, \angle GEF$ and $\angle FGE$.

Solution

Given,

$AB \parallel CD, EF \perp CD, \angle GED = 126^\circ$

According to the question,

$\angle FED = 90^\circ (EF \perp CD)$

Now,

$\angle AGE = \angle GED$ (Since, $AB \parallel CD$ and GE is transversal. Alternate interior angles.)

$\therefore \angle AGE = 126^\circ$

Also,

$\angle GEF = \angle GED - \angle FED$

$\Rightarrow \angle GEF = 126^\circ - 90^\circ$

$\Rightarrow \angle GEF = 36^\circ$

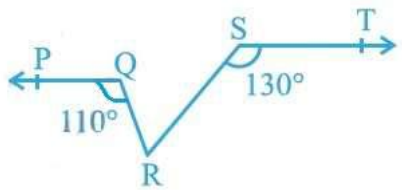
Now,

$\angle FGE + \angle AGE = 180^\circ$(Linear pair)

$\Rightarrow \angle FGE = 180^\circ - 126^\circ$

$\Rightarrow \angle FGE = 54^\circ$

#463659



In the figure, if $PQ \parallel ST$, $\angle PQR = 110$and $\angle RST = 130$, find $\angle QRS$.

Solution

$PQ \parallel ST, \angle PQR = 110^\circ$and $\angle RST = 130^\circ$

Construction: A line XY parallel to PQ and ST is drawn.

$\angle PQR + \angle QRX = 180^\circ$(Angles on the same side of transversal.)

$\Rightarrow 110^\circ + \angle QRX = 180^\circ$

$\Rightarrow \angle QRX = 70^\circ$

Also $\angle RST + \angle SRY = 180^\circ$(Angles on the same side of transversal.)

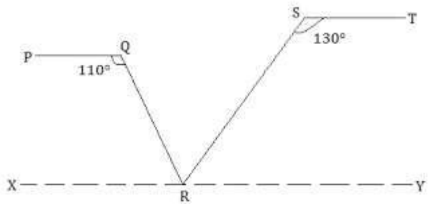
$130^{\circ\circ} + \angle SRY = 180^\circ$

$\angle SRY = 50^\circ$

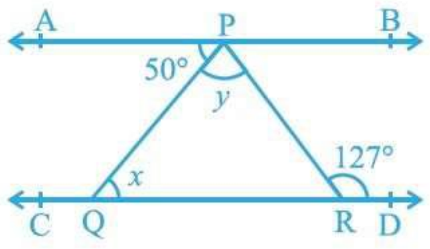
Now, $\angle QRX + \angle SRY + \angle QRS = 180^o$

$\Rightarrow 70^\circ + 50^\circ + \angle QRS = 180^\circ$

$\angle QRS = 60^\circ$



#463661



In the figure, if $AB \parallel CD, \angle APQ = 50$and $\angle PRD = 127$, find x and y.

Solution

Given, $AB \parallel CD, \angle APQ = 50^\circ, \angle PRD = 127^\circ$

According to the question

$x = 50^\circ$ (Alternate interior angles.)

$\angle PRD + \angle RPB = 180^\circ$(Angles on the same side of transversal.)

$\Rightarrow 127^\circ + \angle RPB = 180^\circ$

$\Rightarrow \angle RPB = 53^\circ$

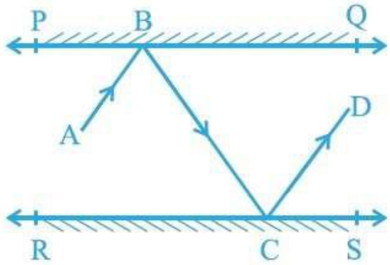
Now, $y + 50^\circ + \angle RPB = 180^\circ$($AB$ is a straight line.)

$\Rightarrow y + 50^\circ + 53^\circ = 180^\circ$

$\Rightarrow y + 103^\circ = 180^\circ$

$\Rightarrow y = 77^\circ$

#463663



In the figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.

Solution

At surface $B, \angle 1 = \angle 2$ (property of reflection)

At surface $C, \ \angle 3 = \angle 4$ (property of reflection))

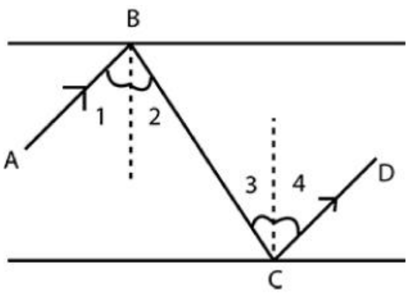
As the sum of the angles are equal AB is parallel to CD,

$\angle 2 = \angle 3$ (Alternate interior angles)

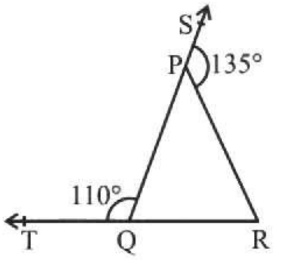
$\angle 1 + \angle 2 = \angle 3 + \angle 4$

Alternate interior angles are equal.

$AB \parallel CD$.



#463672



In Fig. sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135$ and $\angle PQT = 110$, find $\angle PRQ$.

Solution

Given, $\angle SPR = 135^\circ, \angle PQT = 110^\circ$

According to the question

$\angle SPR + \angle QPR = 180^\circ$($SQ$ is a straight line.)

$\Rightarrow 135^\circ + \angle QPR = 180^\circ$

$\Rightarrow \angle QPR = 45^\circ$

Also $\angle PQT + \angle PQR = 180^\circ$($TR$ is a straight line.)

$\Rightarrow 110^\circ + \angle PQR = 180^\circ$

$\Rightarrow \angle PQR = 70^\circ$

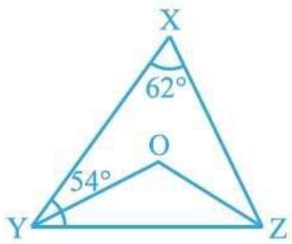
Now, $\angle PQR + \angle QPR + \angle PRQ = 180^\circ$(Sum of the interior angles of the triangle.)

$\angle 70^\circ + 45^\circ + \angle PRQ = 180^\circ$

$\Rightarrow 115^\circ + \angle PRQ = 180^\circ$

$\Rightarrow \angle PRQ = 65^\circ$

#463676



In the figure, $\angle X = 62, \angle XYZ = 54$ If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.

Solution

Given, $\angle X = 62^\circ, \angle XYZ = 54^\circ$

YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively.

According to the question

$\angle X + \angle XYZ + \angle XZY = 180^\circ$(Sum of the interior angles of the triangle.)

$\Rightarrow 62^\circ + 54^\circ + \angle XZY = 180^\circ$

$116^\circ + \angle XZY = 180^\circ$

$\angle XZY = 64^\circ$

Now, $\angle OZY = \frac{1}{2}\angle XZY$(ZO is the bisector.)

$\Rightarrow \angle OZY = 32^\circ$

Also $\angle OYZ = \dfrac{1}{2}\angle XYZ$ (YO is the bisector.)

$\Rightarrow \angle OYZ = 27^\circ$

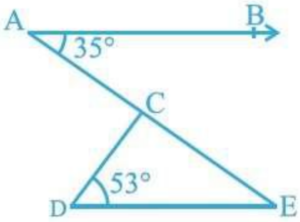
Now, $\angle OZY + \angle OYZ + \angle O = 180^\circ$(Sum of the interior angles of the triangle.)

$\Rightarrow 32^\circ + 27^\circ + \angle O = 180^\circ$

$\Rightarrow 59^\circ + \angle O = 180^\circ$

$\angle O = 121^\circ$

#463680



In the figure, if $AB \parallel DE, \angle BAC = 35$and $\angle CDE = 35$, find $\angle DCE$.

Solution

Given, $AB \parallel DE, \angle BAC = 35^\circ, \angle CDE = 53^\circ$

According to the question,

$\angle BAC = \angle CED$ (Alternate interior angles.)

$\therefore \angle CED = 35^\circ$

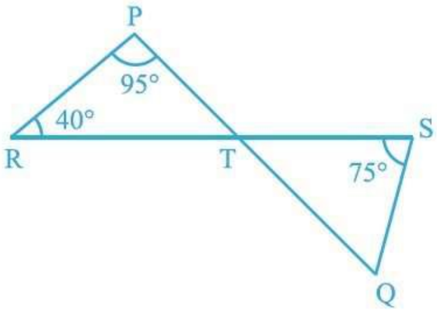
Now, $\angle DCE + \angle CED + \angle CDE = 180^\circ$(Sum of the interior angles of the triangle.)

$\Rightarrow \angle DCE + 35^\circ + 53^\circ = 180^\circ$

$\Rightarrow \angle DCE + 88^\circ = 180^\circ$

$\Rightarrow \angle DCE = 92^\circ$

#463683



In the figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40^o, \angle RPT = 95^o$and $\angle TSQ = 75^o$, find $\angle SQT$.

Solution

Given, $\angle PRT = 40^\circ, \angle RPT = 95^\circ, \angle TSQ = 75^\circ$

According to the question,

$\angle PRT + \angle RPT + \angle PTR = 180^\circ$(Sum of the interior angles of the triangle.)

$\Rightarrow 40^\circ + 95^\circ + \angle PTR = 180^\circ$

$\Rightarrow 40^\circ + 9^\circ + \angle PTR = 180^\circ$

$\Rightarrow 135^\circ + \angle PTR = 180^\circ$

$\Rightarrow \angle PTR = 45^\circ$

$\angle PTR = \angle STQ = 45^\circ$ (Vertically opposite angles.)

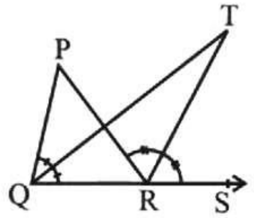
Now, $\angle TSQ + \angle PTR + \angle SQT = 180^\circ$(Sum of the interior angles of the triangle.)

$75^\circ + 45^\circ + \angle SQT = 180^\circ$

$\Rightarrow 120^\circ + \angle SQT = 180^\circ$

$\Rightarrow \angle SQT = 60^\circ$

#463688



In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \dfrac{1}{2}\angle QPR$.

Solution

Given, Bisectors of $\angle PQR$and $\angle PRS$ meet at point T.

To prove: $\angle QTR = \dfrac{1}{2}\angle QPR$.

Proof,

$\angle TRS = \angle TQR + \angle QTR$(Exterior angle of a triangle equals to the sum of the two interior angles.)

$\Rightarrow \angle QTR = \angle TRS - \angle TQR$--- (i)

Also $\angle SRP = \angle QPR + \angle PQR$

$2\angle TRS = \angle QPR + 2\angle TQR$

$\angle QPR = 2\angle TRS - 27\angle TQR$

$\Rightarrow \frac{1}{2}\angle QPR = \angle TRS - \angle TQR$--- (ii)

Equating (i) and (ii),

$\angle QTR - \angle TQR = \dfrac{1}{2}\angle QPR$

Hence proved.