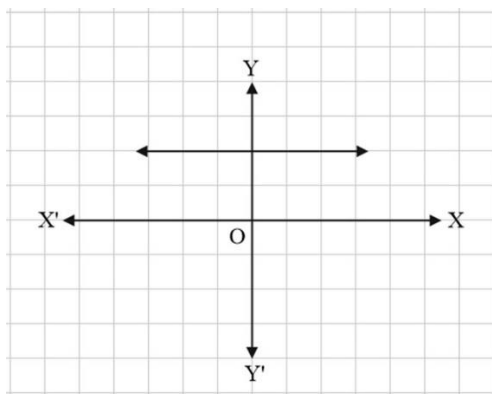


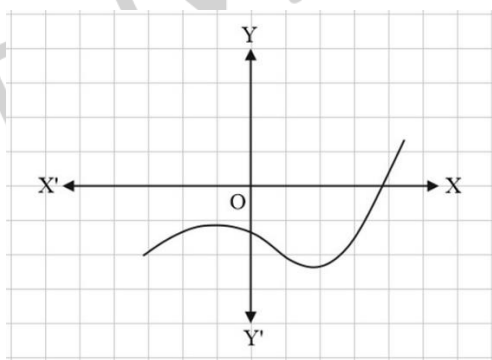
CBSE NCERT Solutions for Class 10 Science Chapter 2 – Ex 2.1

1. The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

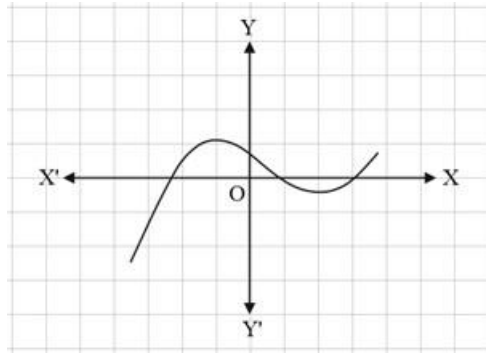
(i)



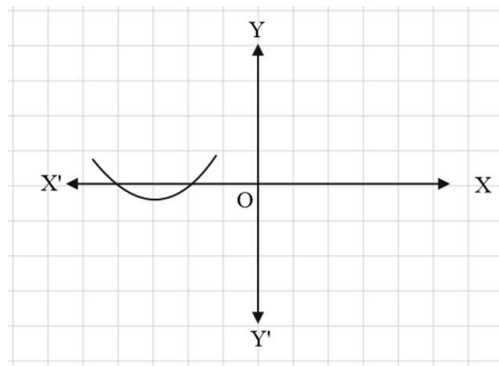
(ii)



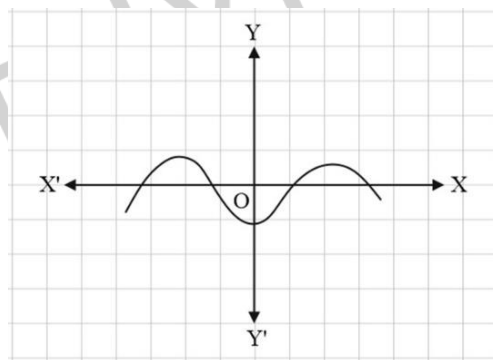
(iii)



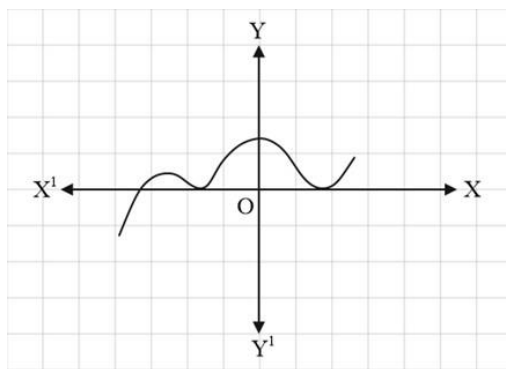
(iv)



(v)



(vi)

**Solution:**

- (i) Since the graph of $p(x)$ does not cut the X-axis at all. Therefore, the number of zeroes is **0**.
- (ii) As the graph of $p(x)$ intersects the X-axis at only **1** point. Therefore, the number of zeroes is **1**.
- (iii) Since the graph of $p(x)$ intersects the X-axis at **3** points. Hence, the number of zeroes is **3**.
- (iv) As the graph of $p(x)$ intersects the X-axis at **2** points. So, the number of zeroes is **2**.
- (v) Since the graph of $p(x)$ intersects the X-axis at **4** points. Therefore, the number of zeroes is **4**.
- (vi) As the graph of $p(x)$ intersects the X-axis at **3** points. So, the number of zeroes is **3**.

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CBSE NCERT Solutions for Class 10 Science Chapter 2 – Ex 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.
 - (i) $x^2 - 2x - 8$
 - (ii) $4s^2 - 4s + 1$
 - (iii) $6x^2 - 3 - 7x$
 - (iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

Solution:

(i) $x^2 - 2x - 8$

$$= x^2 - 4x + 2x - 8 \quad [\text{Factorisation by splitting the middle term}]$$

$$= x(x - 4) + 2(x - 4)$$

$$= (x - 4)(x + 2)$$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$$x^2 - 2x - 8 = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x - 4 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and -2.

$$\text{Sum of zeroes} = 4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

(ii) $4s^2 - 4s + 1 = (2s - 1)^2 \quad [\text{Since, } a^2 - 2ab + b^2 = (a - b)^2]$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$$4s^2 - 4s + 1 = 0$$

$$\Rightarrow (2s - 1)^2 = 0$$

Cancelling square on both the sides,

$$\Rightarrow 2s - 1 = 0$$

$$\Rightarrow s = \frac{1}{2}$$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

$$(iii) \quad 6x^2 - 3 - 7x = 6x^2 - 7x - 3$$

$$= 6x^2 - 9x + 2x - 3 \quad [\text{Factorisation by splitting the middle term}]$$

$$= 3x(2x - 3) + (2x - 3)$$

$$= (3x + 1)(2x - 3)$$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$$6x^2 - 3 - 7x = 0$$

$$\Rightarrow 3x + 1 = 0 \text{ or } 2x - 3 = 0$$

$$\Rightarrow x = \frac{-1}{3} \text{ or } x = \frac{3}{2}$$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

$$\text{Sum of zeroes} = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

$$(iv) \quad 4u^2 + 8u = 4u^2 + 8u + 0 = 4u(u + 2)$$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$$4u^2 + 8u = 0$$

$$\Rightarrow 4u = 0 \text{ or } u + 2 = 0$$

$$\Rightarrow u = 0 \text{ or } u = -2$$

So, the zeroes of $4u^2 + 8u$ are 0 and -2 .

$$\text{Sum of zeroes} = 0 + (-2) = -2 = \frac{-8}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

$$\text{Product of zeroes} = 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

$$(v) \quad t^2 - 15 = t^2 - 0.t - 15 = (t - \sqrt{15})(t + \sqrt{15}) \quad [\text{Since, } a^2 - b^2 = (a + b)(a - b)]$$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$$t^2 - 15 = 0$$

$$\Rightarrow t - \sqrt{15} = 0 \text{ or } t + \sqrt{15} = 0$$

$$\Rightarrow t = \sqrt{15} \text{ or } t = -\sqrt{15}$$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{(\text{Coefficient of } t^2)}$$

$$\text{Product of zeroes} = (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

$$(vi) \quad 3x^2 - x - 4$$

$$= 3x^2 - 4x + 3x - 4 \quad [\text{Factorisation by splitting the middle term}]$$

$$= x(3x - 4) + (3x - 4)$$

$$= (3x - 4)(x + 1)$$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$$3x^2 - x - 4 = 0$$

$$\Rightarrow 3x - 4 = 0 \text{ or } x + 1 = 0$$

$$\Rightarrow x = \frac{4}{3} \text{ or } x = -1$$

Hence, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1 .

$$\text{Sum of zeroes} = \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{4}{3}(-1) = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, the relationship between the zeroes and the coefficients is verified.

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$

(ii) $\sqrt{2}, \frac{1}{3}$

(iii) $0, \sqrt{5}$

(iv) $1, 1$

(v) $-\frac{1}{4}, \frac{1}{4}$

(vi) $4, 1$

Solution:

- (i) We know that if α and β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or,

$p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots $= \alpha + \beta = \frac{1}{4}$ and product of the roots $= \alpha\beta = -1$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x) = a\left\{x^2 - \frac{1}{4}x - 1\right\}$$

$$= a\left\{\frac{4x^2 - x - 4}{4}\right\}$$

By taking $a = 4$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(4x^2 - x - 4)$.

- (ii) We know that if α and β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or,

$p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots $= \alpha + \beta = \sqrt{2}$ and product of the roots $= \alpha\beta = \frac{1}{3}$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x) = a\left\{x^2 - \sqrt{2}x + \frac{1}{3}\right\}$$

$$= a\left\{\frac{3x^2 - 3\sqrt{2}x + 1}{3}\right\}$$

By taking $a = 3$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(3x^2 - 3\sqrt{2}x + 1)$.

- (iii) We know that if α and β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or,

$p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots $= \alpha + \beta = 0$ and product of the roots $= \alpha\beta = \sqrt{5}$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$\begin{aligned} p(x) &= a\{x^2 - 0.x + \sqrt{5}\} \\ &= a\{x^2 + \sqrt{5}\} \end{aligned}$$

By taking $a = 1$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(x^2 + \sqrt{5})$.

- (iv) We know that if α and β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or,

$p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots $= \alpha + \beta = 1$ and product of the roots $= \alpha\beta = 1$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$\begin{aligned} p(x) &= a\{x^2 - 1.x + 1\} \\ &= a\{x^2 - x + 1\} \end{aligned}$$

By taking $a = 1$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(x^2 - x + 1)$.

- (v) We know that if α and β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or,

$p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots $= \alpha + \beta = -\frac{1}{4}$ and product of the roots $= \alpha\beta = \frac{1}{4}$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x) = a\{x^2 + \frac{1}{4}x + \frac{1}{4}\}$$

$$= a \left\{ \frac{4x^2 + x + 1}{4} \right\}$$

By taking $a = 4$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(4x^2 + x + 1)$.

- (vi) We know that if α and β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or,

$p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots $= \alpha + \beta = 4$ and product of the roots $= \alpha\beta = 1$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x) = a\{x^2 - 4x + 1\}$$

By taking $a = 1$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(x^2 - 4x + 1)$.



CBSE NCERT Solutions for Class 10 Science Chapter 2 – Ex 2.3

1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Solution:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

Here, both the polynomials are already arranged in the descending powers of variable.

The polynomial $p(x)$ can be divided by the polynomial $g(x)$ as follows:

$$\begin{array}{r}
 x-3 \\
 x^2-2 \overline{) x^3-3x^2+5x-3} \\
 \underline{x^3 \qquad -2x} \\
 -3x^2+7x-3 \\
 \underline{-3x^2 \qquad +6} \\
 7x-9
 \end{array}$$

$$\text{Quotient} = x - 3$$

$$\text{Remainder} = 7x - 9$$

$$(ii) \quad p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5,$$

Here, the polynomial $p(x)$ is already arranged in the descending powers of variable.

$$g(x) = x^2 + 1 - x$$

Here, the polynomial $g(x)$ is not arranged in the descending powers of variable.

$$\text{Now, } g(x) = x^2 - x + 1$$

The polynomial $p(x)$ can be divided by the polynomial $g(x)$ as follows:

$$\begin{array}{r}
 x^2+x-3 \\
 x^2-x+1 \overline{) x^4+0.x^3-3x^2+4x+5} \\
 \underline{x^4 - x^3 + x^2} \\
 x^3 - 4x^2 + 4x + 5 \\
 \underline{x^3 - x^2 + x} \\
 -3x^2 + 3x + 5 \\
 \underline{-3x^2 + 3x - 3} \\
 8
 \end{array}$$

$$\text{Quotient} = x^2 + x - 3$$

$$\text{Remainder} = 8$$

$$(iii) \quad p(x) = x^4 - 5x + 6 = x^4 + 0 \cdot x^2 - 5x + 6$$

$$g(x) = 2 - x^2$$

Here, the polynomial $g(x)$ is not arranged in the descending powers of variable.

$$\text{Now, } g(x) = -x^2 + 2$$

The polynomial $p(x)$ can be divided by the polynomial $g(x)$ as follows:

$$\begin{array}{r}
 \begin{array}{r} -x^2+2 \end{array} \overline{) \begin{array}{r} x^4+0x^2-5x+6 \\ x^4-2x^2 \\ \hline 2x^2-5x+6 \\ 2x^2 \quad -4 \\ \hline -5x+10 \end{array} }
 \end{array}$$

$$\text{Quotient} = -x^2 - 2$$

$$\text{Remainder} = -5x + 10$$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

$$(i) \quad t^2 - 3, \quad 2t^4 + 3t^3 - 2t^2 - 9t - 12$$

$$(ii) \quad x^2 + 3x + 1, \quad 3x^4 + 5x^3 - 7x^2 + 2x + 2$$

$$(iii) \quad x^3 - 3x + 1, \quad x^5 - 4x^3 + x^2 + 3x + 1$$

Solution:

- (i) The polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$ can be divided by the polynomial $t^2 - 3 = t^2 + 0 \cdot t - 3$ as follows:

$$\begin{array}{r}
 2t^2 + 3t + 4 \\
 t^2 + 0.t - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 \underline{2t^4 + 0.t^3 - 6t^2} \\
 - - + \\
 \underline{3t^3 + 4t^2 - 9t - 12} \\
 3t^3 + 0.t^2 - 9t \\
 \underline{- - + } \\
 4t^2 + 0.t - 12 \\
 4t^2 + 0.t - 12 \\
 \underline{- - + } \\
 0
 \end{array}$$

Since the remainder is 0, hence $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

- (ii) The polynomial $3x^4 + 5x^3 - 7x^2 + 2x + 2$ can be divided by the polynomial $x^2 + 3x + 1$ as follows:

$$\begin{array}{r}
 3x^2 - 4x + 2 \\
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \\
 - - - \\
 \underline{- 4x^3 - 10x^2 + 2x + 2} \\
 - 4x^3 - 12x^2 - 4x \\
 \underline{+ + + + 2} \\
 2x^2 + 6x + 2 \\
 2x^2 + 6x + 2 \\
 \underline{- - - } \\
 0
 \end{array}$$

Since the remainder is 0, hence $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

- (iii) The polynomial $x^5 - 4x^3 + x^2 + 3x + 1$ can be divided by the polynomial $x^3 - 3x + 1$ as follows:

$$\text{Hence, } 3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$$

$$= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$$

$$\text{Now, } x^2 + 2x + 1 = (x + 1)^2$$

Thus, the two zeroes of $x^2 + 2x + 1$ are -1 and -1

Therefore, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1 .

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Solution:

$$\text{Dividend, } p(x) = x^3 - 3x^2 + x + 2$$

$$\text{Quotient} = (x - 2)$$

$$\text{Remainder} = (-2x + 4)$$

$g(x)$ be the divisor.

According to the division algorithm,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$$

Now, $g(x)$ is the quotient when $x^3 - 3x^2 + 3x - 2$ is divided by $x - 2$. (Since, Remainder = 0)

$$\begin{array}{r}
 x^2 - x + 1 \\
 x - 2 \overline{) x^3 - 3x^2 + 3x - 2} \\
 \underline{x^3 - 2x^2} \\
 -x^2 + 3x - 2 \\
 \underline{-x^2 + 2x} \\
 +x - 2 \\
 \underline{+x - 2} \\
 0
 \end{array}$$

$$\therefore g(x) = x^2 - x + 1$$

6. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

- (i) $\deg p(x) = \deg q(x)$
- (ii) $\deg q(x) = \deg r(x)$
- (iii) $\deg r(x) = 0$

Solution:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

- (i) Degree of quotient will be equal to degree of dividend when divisor is constant.

Let us consider the division of $2x^2 + 2x - 16$ by 2.

Here, $p(x) = 2x^2 + 2x - 16$ and $g(x) = 2$

$q(x) = x^2 + x - 8$ and $r(x) = 0$

Clearly, the degree of $p(x)$ and $q(x)$ is the same which is 2.

Verification:

$$p(x) = g(x) \times q(x) + r(x)$$

$$2x^2 + 2x - 16 = 2(x^2 + x - 8) + 0$$

$$= 2x^2 + 2x - 16$$

Thus, the division algorithm is satisfied.

- (ii) Let us consider the division of $4x + 3$ by $x + 2$.

Here, $p(x) = 4x + 3$ and $g(x) = x + 2$

$$q(x) = 4 \text{ and } r(x) = -5$$

Here, degree of $q(x)$ and $r(x)$ is the same which is 0.

Verification:

$$p(x) = g(x) \times q(x) + r(x)$$

$$4x + 3 = (x + 2) \times 4 + (-5)$$

$$4x + 3 = 4x + 3$$

Thus, the division algorithm is satisfied.

- (iii) Degree of remainder will be 0 when remainder obtained on division is a constant.

Let us consider the division of $4x + 3$ by $x + 2$.

Here, $p(x) = 4x + 3$ and $g(x) = x + 2$

$$q(x) = 4 \text{ and } r(x) = -5$$

Here, we get remainder as a constant. Therefore, the degree of $r(x)$ is 0.

Verification:

$$p(x) = g(x) \times q(x) + r(x)$$

$$4x + 3 = (x + 2) \times 4 + (-5)$$

$$4x + 3 = 4x + 3$$

Thus, the division algorithm is satisfied.

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