CBSE NCERT Solutions for Class 10 Mathematics Chapter 7

Back of Chapter Questions

1. Find the distance between the following pairs of points:

(i) $(2,3),(4,1)$

(ii) $(-5,7),(-1,3)$

(iii) $(a,b),(-a,-b)$

Solution:

(i) We know that the distance between two points (x_1,y_1) and (x_2,y_2) is given by,

$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Hence, the distance between $(2,3)$ and $(4,1)$ is given by,

Distance $=\sqrt{(2-4)^2+(3-1)^2}=\sqrt{(-2)^2+(2)^2}=\sqrt{4+4}=\sqrt{8}$

$=2\sqrt{2}$

(ii) We know that the distance between two points (x_1,y_1) and (x_2,y_2) is given by,

$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Hence, the distance between $(-5,7)$ and $(-1,3)$ is given by,

Distance $=\sqrt{\left(-5-(-1)\right)^2+(7-3)^2}=\sqrt{(-4)^2+(4)^2}=\sqrt{16+16}$

$=\sqrt{32}=4\sqrt{2}$

(iii) We know that the distance between two points (x_1,y_1) and (x_2,y_2) is

given by,

$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Hence, the distance between (a,b) and $(-a,-b)$ is given by,

Distance $=\sqrt{\left(a-(-a)\right)^2+\left(b-(-b)\right)^2}$

$=\sqrt{(2a)^2+(2b)^2}=\sqrt{4a^2+4b^2}=2\sqrt{a^2+b^2}$

2. Find the distance between the points $(0,0)$ and $(36,15)$. Can you now find the distance between the two towns A and B discussed in Section 7.2.

Solution:

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is given by,

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Now, the distance between the points $A(0, 0)$ and $B(36, 15)$ is given by,

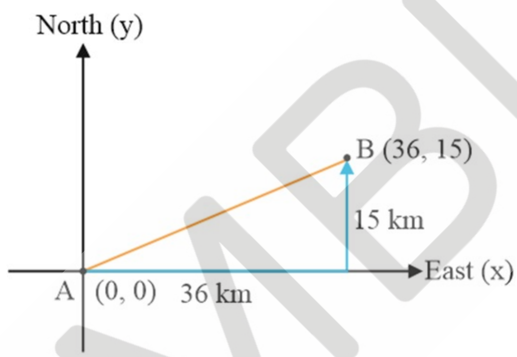
$AB = \sqrt{(36 - 0)^2 + (15 - 0)^2} = \sqrt{36^2 + 15^2}$

$AB = \sqrt{1296 + 225} = \sqrt{1521} = 39$

Now, we are told to find distance between two towns A & B in section 7.2

It is given that B is located 36 km east and 15 km north of town A

Let us take A as origin Therefore, A will be A $(0, 0)$ & B will be B $(36, 15)$



Since, we have already calculated above the distance between the points $A(0, 0)$ and $B(36, 15)$. Hence, we can say that the distance between the two towns A and B discussed in section 7.2 is 39 km.

3. Determine if the points $(1, 5), (2, 3)$ and $(-2, -11)$ are collinear.

Solution:

Three points are collinear if they lie on a same line i.e., one point lies in between the line joining any other two points.

Let $A = (1, 5), B = (2,\ 3), C = (-2, -11)$

We know the distance formula:

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Substituting the values,

So, $AB = \sqrt{(1 - 2)^2 + (5 - 3)^2} = \sqrt{5}$

$BC = \sqrt{(2 - (-2))^2 + (3 - (-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16 + 196} = \sqrt{212}$

$$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9+256} = \sqrt{265}$$

Here, the sum of the distances between any two pairs of points is not equal to the distance between the third pair of points.

i.e., $AB + BC = \sqrt{5} + \sqrt{212} \neq \sqrt{265} = AC$

Similarly, $AB + AC \neq BC$ and $AC + BC \neq AB$.

Hence, the given three points $(1, 5), (2, 3)$ and $(-2, -11)$ are not collinear.

4. Check whether $(5, -2), (6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle.

Solution:

We know that three non-collinear points will represent the vertices of an isosceles triangle if its any of the two sides are of equal length.

Let $A = (5, -2), B = (6, 4), C = (7, -2)$

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

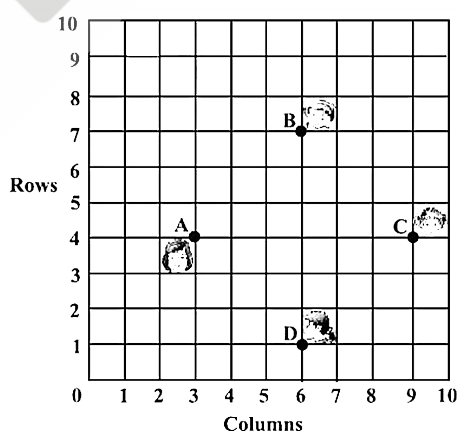
$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

So, $AB = BC$

Since the two sides are equal in length, therefore,ABC is an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees.



Using distance formula, find which of them is correct.

Solution:

From the figure, coordinates of point A, B, C and D are

$A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1)$

$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Diagonal $AC = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$

Diagonal $BD = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$

Here, all four sides of this figure are of equal length and also two diagonals are of equal length.

Therefore, ABCD is a square and thus Champa is correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) $(-1, -2), (1, 0), (-1, 2), (-3, 0)$

(ii) $(-3, 5), (3, 1), (0, 3), (-1, -4)$

(ii) $(4, 5), (7, 6), (4, 3), (1, 2)$

Solution:

(i) Let, $A = (-1, -2), B = (1, 0), C = (-1, 2), D = (-3, 0)$

$AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

$= 2\sqrt{2}$

$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

$= 2\sqrt{2}$

$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$

$= 2\sqrt{2}$

$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

$= 2\sqrt{2}$

Diagonal AC $= \sqrt{\left(-1-(-1)\right)^2+(-2-2)^2} = \sqrt{0^2+(-4)^2} = \sqrt{16} =$ 4

Diagonal BD $= \sqrt{\left(1-(-3)\right)^2+(0-0)^2} = \sqrt{(4)^2+0^2} = \sqrt{16} = 4$

Here, all four sides of this quadrilateral are of equal length and also two diagonals are of equal length.

Therefore, ABCD is a square and the given points are vertices of a square.

(ii) Let $A = (-3, 5), B = (3, 1), C = (0, 3), D = (-1, -4)$

$AB = \sqrt{(-3-3)^2+(5-1)^2} = \sqrt{(-6)^2+(4)^2} = \sqrt{36+16} = \sqrt{52}$

$= 2\sqrt{13}$

$BC = \sqrt{(3-0)^2+(1-3)^2} = \sqrt{(3)^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$

$CD = \sqrt{\left(0-(-1)\right)^2+\left(3-(-4)\right)^2} = \sqrt{(1)^2+(7)^2} = \sqrt{1+49} = \sqrt{50}$

$= 5\sqrt{2}$

$AD = \sqrt{\left(-3-(-1)\right)^2+\left(5-(-4)\right)^2} = \sqrt{(-2)^2+(9)^2} = \sqrt{4+81}$

$= \sqrt{85}$

$AC = \sqrt{(-3-0)^2+(5-3)^2} = \sqrt{(-3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13}$

$BD = \sqrt{\left(3-(-1)\right)^2+\left(1-(-4)\right)^2} = \sqrt{(4)^2+(5)^2} = \sqrt{16+25}$

$= \sqrt{41}$

We can observe very clearly that, $AC + BC = AB$, this means that the point C lies on side AB, which implies that the points A, B and C are collinear.

Hence, no quadrilateral can be formed with the given points.

(iii) Let $A = (4, 5), B = (7, 6), C = (4, 3), D = (1, 2)$

$AB = \sqrt{(4-7)^2+(5-6)^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(7-4)^2+(6-3)^2} = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18}$

$CD = \sqrt{(4-1)^2+(3-2)^2} = \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

$AD = \sqrt{(4-1)^2+(5-2)^2} = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18}$

Diagonal AC $= \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{0^2 + (2)^2} = \sqrt{0+4} = 2$

Diagonal BD $= \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

We can observe that the opposite sides of this quadrilateral are of equal length, but diagonals are of different lengths. Therefore, ABCD is a parallelogram and given points are vertices of a parallelogram.

7. Find the point on the x −axis which is equidistant from $(2,-5)$ and $(-2,9)$.

Solution:

We know that, for a point on the x − axis, its y − coordinate will be 0.

So, let the point on x −axis be $(k, 0)$

Distance between $(k, 0)$ and $(2,-5) = \sqrt{(k-2)^2 + \left(0-(-5)\right)^2} = \sqrt{(k-2)^2 + (5)^2}$

Distance between $(k, 0)$ and $(-2, 9) = \sqrt{(k-(-2))^2 + \left(0-(9)\right)^2} = \sqrt{(k+2)^2 + (9)^2}$

By given condition, these distances are equal in measure.

$\sqrt{(k-2)^2 + (5)^2} = \sqrt{(k+2)^2 + (9)^2}$

$\Rightarrow (k-2)^2 + 25 = (k+2)^2 + 81$

$\Rightarrow k^2 + 4 - 4k + 25 = k^2 + 4 + 4k + 81$

$\Rightarrow 8k = 25 - 81$

$\Rightarrow 8k = -56$

$\Rightarrow k = -7$

Hence, the required point on the x − axis is $(-7, 0)$.

8. Find the values of y for which the distance between the points $P(2,-3)$ and $Q(10, y)$ is 10 units.

Solution:

As per the question, it is given that the distance between $(2,-3)$ and $(10, y)$ is 10.

So, using distance formula:

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Substituting the values,

$\sqrt{(2-10)^2+(-3-y)^2}=10$

$\Rightarrow \sqrt{(-8)^2+(3+y)^2}=10$

Squaring on both the sides,

$64+(y+3)^2=100$

$\Rightarrow (y+3)^2=36$

$\Rightarrow y+3=\pm 6$

$y+3=6$ or $y+3=-6$

Hence, $y=\ -9,3$

9. If $Q(0,1)$ is equidistant from $P(5,-3)$ and $R(x,6)$, find the values of x. Also find the distances QR and PR.

Solution:

According to the question, given $Q(0,1)$ is equidistant from $P(5,-3)$ and $R(x,6)$

So, $PQ=QR$

Using distance formula:

$\Rightarrow \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$\Rightarrow \sqrt{(5-0)^2+(-3-1)^2}=\sqrt{(0-x)^2+(1-6)^2}$

$\Rightarrow \sqrt{(5)^2+(-4)^2}=\sqrt{(-x)^2+(-5)^2}$

$\Rightarrow \sqrt{25+16}=\sqrt{x^2+25}$

Squaring on both sides,

$25+16=x^2+25$

$\Rightarrow 16=x^2$

$\Rightarrow x=\pm 4$

Hence, point R is $(4,6)$ or $(-4,6)$

Case (i):

When point R is $(4,6)$

$PR=\sqrt{(5-4)^2+(-3-6)^2}=\sqrt{1^2+(-9)^2}=\sqrt{1+81}=\sqrt{82}$

$QR=\sqrt{(0-4)^2+(1-6)^2}=\sqrt{(-4)^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$

Case (ii):

When point R is $(-4, 6)$

$$PR = \sqrt{\left(5-(-4)\right)^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81}$$

$$= 9\sqrt{2}$$

$$QR = \sqrt{\left(0-(-4)\right)^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

Hence, $x = \pm 4, QR = \sqrt{41}, PR = \sqrt{82}, 9\sqrt{2}$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Solution:

It is given that point $(x, y)$is equidistant from $(3, 6)$ and $(-3, 4)$

So, $\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{\left(x-(-3)\right)^2 + (y-4)^2}$

$\Rightarrow \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$

Squaring on both sides,

$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$

$\Rightarrow 36 - 16 = 6x + 6x + 12y - 8y$

$\Rightarrow 20 = 12x + 4y$

$\Rightarrow 3x + y = 5$

Hence, the relation between x and y is $3x + y - 5 = 0$.

EXERCISE 7.2

1. Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2 : 3$.

Solution:

Let $P(x, y)$ be the required point.

We know the section formula, which gives coordinates of point P, dividing the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ internally in the ratio m : n

$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$,

Upon substitution of values, we get

$$x = \frac{2 \times 4 + 3 \times (-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

$$y = \frac{2 \times (-3) + 3 \times 7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Hence, the coordinates of the required point are given by $(1, 3)$.

2. Find the coordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

Solution:

Trisection means division into three equal parts.

So, we need to find two points such that they divide the line segment in three equal parts.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are the points of trisection of the line segment joining the given points i.e. $AP = PQ = QB$

Therefore point P divides AB internally in ratio $1 : 2$

$$x_1 = \frac{1 \times (-2) + 2 \times 4}{1+2}, y_1 = \frac{1 \times (-3) + 2 \times (-1)}{1+2}$$

$$x_1 = \frac{-2+8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3-2}{3} = \frac{-5}{3}$$

So, $P(x_1, y_1) = \left(2, -\frac{5}{3}\right)$

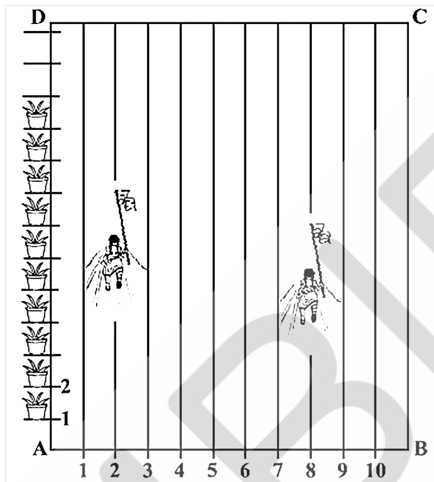
Point Q divides AB internally in ratio $2 : 1$

$$x_2 = \frac{2 \times (-2) + 1 \times 4}{2+1}, y_2 = \frac{2 \times (-3) + 1 \times (-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \quad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

$$Q = (x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

3. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in figure. Niharika runs $\frac{1}{4}th$ the distance AD on the 2^{nd} line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Solution:

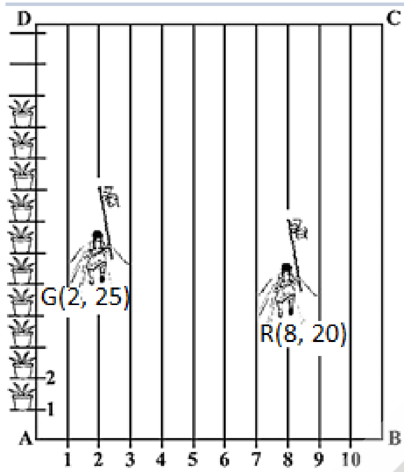
According to the question, it is given that Niharika posted the green flag (G) at $\frac{1}{4}$th the distance AD i. e. $\frac{1}{4} \times 100 = 25$ m from the starting point of 2^{nd} line.

So, coordinates of this point G are $(2, 25)$.

Similarly, Preet posted red flag at $\frac{1}{5}$th the distance AD i. e. $\frac{1}{5} \times 100 = 20$ m from the starting point of 8^{th} line.

So, coordinates of this point R are $(8, 20)$.

Now, below is the figure, which simplifies the given question by considering G as a position of green flag posted by Niharika and similarly R as a position of red flag posted by Preet.



Now distance between these flags by using distance formula = GR

$\mathrm{GR} = \sqrt{(8-2)^2 + (20-25)^2} = \sqrt{36+25} = \sqrt{61}$ m

Since it is given that Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags. Now the point at which Rashmi should post her blue flag is the midpoint of line joining these points.

Let this point be $\mathrm{B}(x, y)$.

$$x = \frac{2+8}{2}, y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5, \quad y = \frac{45}{2} = 22.5$$

So, $\mathrm{B}(x, y) = (5, 22.5)$

So, Rashmi should post her blue flag on 5^{th} line at a distance of 22.5 m.

4. Find the ratio in which the line segment joining the points $(-3, 10)$ and $(6, -8)$ is divided by $(-1, 6)$.

Solution:

Let the ratio in which line segment joining $\mathrm{A}(-3, 10)$ and $\mathrm{B}(6, -8)$ is divided by point $\mathrm{P}(-1, 6)$ is $\mathrm{k} : 1$.

Let $\mathrm{P}(x, y)$ be the required point

We know the section formula, which gives coordinates of point P, dividing the line segment joining $\mathrm{A}(x_1, y_1) \& \mathrm{B}(x_2, y_2)$ internally in the ratio $\mathrm{k} : 1$

$\left(\frac{\mathrm{k}x_2 + x_1}{\mathrm{k}+1}, \frac{\mathrm{k}y_2 + y_1}{\mathrm{k}+1}\right)$,

Upon substitution of values, we get $x = \frac{6\mathrm{k}-3}{\mathrm{k}+1}$

But given, $x = -1$ and $y = 6$

So, $-1 = \frac{6k-3}{k+1}$

$-k - 1 = 6k - 3$

$7k = 2$

$k = \frac{2}{7}$

Similarly, we can get k using y as follows:

$6 = \frac{-8k + 10}{k + 1}$

$6k + 6 = -8k + 10$

$14k = 4$

$k = \frac{2}{7}$

Hence, the required ratio is $2 : 7$.

5. Find the ratio in which the line segment joining $A(1, -5)$ and $B(-4, 5)$ is divided by the x −axis. Also find the coordinates of the point of division.

Solution:

If the ratio in which P divides AB is $k : 1$, then the co-ordinates of the point P will be

$\left(\frac{kx_2 + x_1}{k + 1}, \frac{ky_2 + y_1}{k + 1}\right)$

Let the ratio in which line segment joining $A(1, -5)$ and $B(-4, 5)$ is divided by x axis be $k : 1$.

So, coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$

We know that y coordinate of any point on x −axis is 0.

Thus, $\frac{5k-5}{k+1} = 0$

$\Rightarrow k = 1$

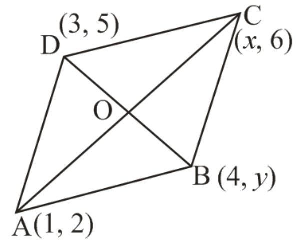
Hence, x −axis divides it in the ratio $1 : 1$.

Coordinates of point of division are given by,

$\left(\frac{-4(1) + 1}{1 + 1}, \frac{5(1) - 5}{1 + 1}\right) = \left(\frac{-4 + 1}{2}, \frac{5 - 5}{2}\right) = \left(\frac{-3}{2}, 0\right)$

6. If $(1, 2), (4, y), (x, 6)$ and $(3, 5)$ are the vertices of a parallelogram taken in order, find x and y.

Solution:



Let $(1, 2), (4, y), (x, 6)$ and $(3, 5)$ are the coordinates of A, B, C and D respectively, which are the vertices of a parallelogram ABCD.

We know that the diagonals of a parallelogram bisect each other. So, O is midpoint of AC and BD.

If O is midpoint of AC, then coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is midpoint of BD, then coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

As both the coordinates are of same point O, we compare both the sides.

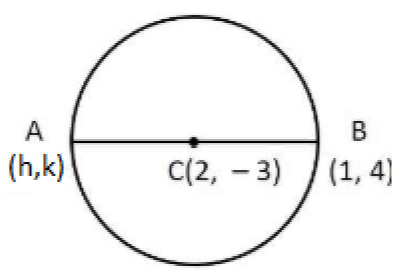
So, $\frac{x+1}{2} = \frac{7}{2}$ and $4 = \frac{5+y}{2}$

$x + 1 = 7$ and $5 + y = 8$

$x = 6$ and $y = 3$

7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is $(2, -3)$ and B is $(1, 4)$.

Solution:



Let Coordinates of point A be (h, k)

We know that the midpoint of diameter AB is the centre of circle.

$(2, -3) = \left(\frac{h+1}{2}, \frac{k+4}{2}\right)$

$\frac{h+1}{2} = 2$ and $\frac{k+4}{2} = -3$

$h + 1 = 4$ and $k + 4 = -6$

$h = 3$ and $k = -10$

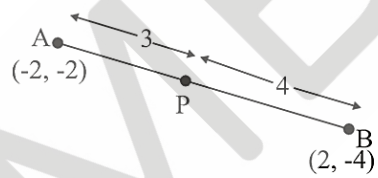
Hence, the coordinates of A are $(h, k) = (3, -10)$.

8. If A and B are$(-2, -2)$ and$(2, -4)$, respectively, find the coordinates of P such that AP $= \frac{3}{7}$AB and P lies on the line segment AB.

Solution:

Coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), internally, in the ratio $m_1 : m_2$ are

$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$



The coordinates of point A and B are $(-2, -2)$ and $(2, -4)$ respectively.

Since given that AP $= \frac{3}{7}$AB ……….(i)

But, AB = AP + PB ……….(ii)

Substituting (ii) in (i), we get,

$7\text{AP} = 3(\text{AP} + \text{PB})$

$4\text{AP} = 3\text{PB}$

So, AP : PB = $3 : 4$

Therefore, point P divides the line segment AB in a ratio $3 : 4$

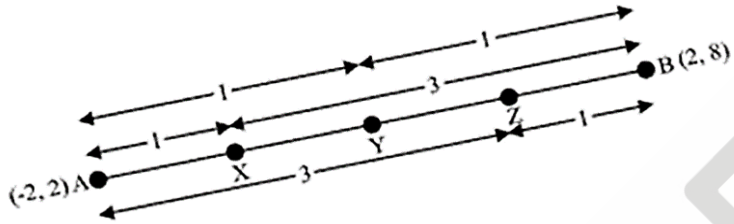
Hence, the coordinates of P $= \left(\frac{3\times2+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right)$

$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$

$= \left(-\frac{2}{7}, -\frac{20}{7}\right)$

9. Find the coordinates of the points which divide the line segment joining $A(-2, 2)$ and $B(2, 8)$ into four equal parts.

Solution:



It is clear from the figure that we have three points X, Y, Z dividing the line segment AB in a ratio $1 : 3, 1 : 1, 3 : 1$ respectively.

Coordinates of $X = \left(\frac{1\times2+3\times(-2)}{1+3}, \frac{1\times8+3\times2}{1+3}\right) = \left(-1, \frac{7}{2}\right)$

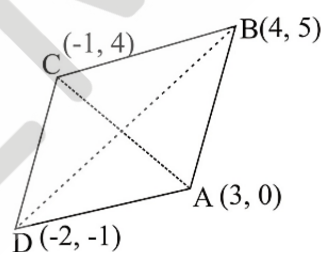
Coordinates of $Y = \left(\frac{2+(-2)}{2}, \frac{2+8}{2}\right) = (0, 5)$

Coordinates of $Z = \left(\frac{3\times2+1\times(-2)}{3+1}, \frac{3\times8+1\times2}{3+1}\right) = \left(1, \frac{13}{2}\right)$

10. Find the area of a rhombus if its vertices are $(3, 0), (4, 5), (-1, 4)$ and $(-2, -1)$ taken in order.

[Hint: Area of a rhombus $= \frac{1}{2}$(product of its diagonals)]

Solution:



Let $(3, 0), (4, 5), (-1, 4)$ and $(-2, -1)$ are the vertices A, B, C and D respectively of a rhombus ABCD.

Using distance formula:

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

The length of diagonal AC of a rhombus $= \sqrt{[3 - (-1)]^2 + (0 - 4)^2} = \sqrt{16 + 16} = 4\sqrt{2}$

The length of diagonal BD of a rhombus $= \sqrt{[4 - (-2)]^2 + [5 - (-1)]^2} = \sqrt{36 + 36} = 6\sqrt{2}$

Hence, the area of rhombus ABCD $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ square units

EXERCISE 7.3

1. Find the area of the triangle whose vertices are:

(i) $(2, 3), (-1, 0), (2, -4)$

(ii) $(-5, -1), (3, -5), (5, 2)$

Solution:

(i) We know that area of a triangle$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Substituting the values,

Area of given triangle $= \frac{1}{2}[2\{0 - (-4)\} + (-1)\{(-4) - (3)\} + 2(3 - 0)]$

$= \frac{1}{2}\{8 + 7 + 6\} = \frac{21}{2}$ square units

(ii) We know that area of a triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Substituting the values,

Area of given triangle $= \frac{1}{2}[(-5)\{(-5) - (2)\} + 3(2 - (-1)) + 5\{-1 - (-5)\}]$

$= \frac{1}{2}\{35 + 9 + 20\} = 32$ square units

2. In each of the following find the value of 'k', for which the points are collinear.

(i) $(7, -2), (5, 1), (3, k)$

(ii) $(8, 1), (k, -4), (2, -5)$

Solution:

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points $(7, -2), (5, 1)$ and $(3, k)$, area of triangle $= 0$

That means, $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$

$\Rightarrow \frac{1}{2}[7\{1 - k\} + 5\{k - (-2)\} + 3\{(-2) - 1\}] = 0$

$\Rightarrow 7 - 7k + 5k + 10 - 9 = 0$

$\Rightarrow -2k + 8 = 0$

$\Rightarrow k = 4$

(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points $(8, 1), (k, -4), (2, -5)$, area of triangle= 0

That means, $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$

$\Rightarrow \frac{1}{2}[8\{-4 - (-5)\} + k\{(-5) - (1)\} + 2\{1 - (-4)\}] = 0$

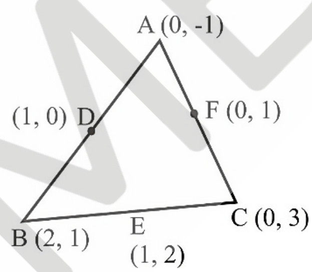
$\Rightarrow 8 - 6k + 10 = 0$

$\Rightarrow 6k = 18$

$\Rightarrow k = 3$

3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are$(0, -1), (2, 1)$ and $(0, 3)$. Find the ratio of this area to the area of the given triangle.

Solution:



Let vertices of the triangle be A $(0, -1)$, B $(2, 1)$, C $(0,3)$

Let D, E, F are midpoints of the sides of this triangle. Coordinates of D, E, and F are given by-

$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

$$E = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$F = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

We know that area of a triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Area of $\Delta DEF = \frac{1}{2}\{1(2-1) + 1(1-0) + 0(0-2)\}$

$= \frac{1}{2}(1+1) = 1$ square units

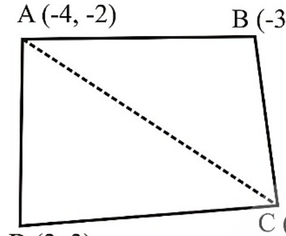
Area of $\Delta ABC = \frac{1}{2}[0(1-3)+2\{3-(-1)\}+0(-1-1)]$

$= \frac{1}{2}\{8\} = 4$ square units

Therefore, the required ratio $= 1:4$

4. Find the area of the quadrilateral whose vertices, taken in order, are $(-4,-2), (-3,-5), (3,-2)$ and $(2,3)$.

Solution:



Let vertices of the quadrilateral be $A(-4,-2), B(-3,-5), C(3,-2)$ and $D(2,3)$.

Also, join AC to form two triangles, ΔABC and ΔACD

We know that area of a triangle $= \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

Substituting the values,

Area of $\Delta ABC = \frac{1}{2}[(-4)\{(-5)-(-2)\}+(-3)\{(-2)-(-2)\}+3\{(-2)-(-5)\}]$

$= \frac{1}{2}(12+0+9) = \frac{21}{2}$ square units

Substituting the values,

Area of $\Delta ACD = \frac{1}{2}[(-4)\{(-2)-(3)\}+3\{(3)-(-2)\}+2\{(-2)-(-2)\}]$

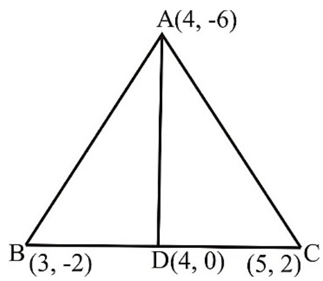
$= \frac{1}{2}\{20+15+0\} = \frac{35}{2}$ square units

Area of quadrilateral ABCD $=$ area of $\Delta ABC +$ area of ΔACD

$= \frac{21}{2}+\frac{35}{2} = 28$ square units

5. You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for Δ ABC whose vertices are $A(4,-6), B(3,-2)$ and $C(5,2)$

Solution:



Let vertices of the triangle be A $(4, -6)$, B $(3, -2)$, C $(5, 2)$

Let D be the midpoint of side BC of ΔABC. Therefore, AD is the median in ΔABC.

Coordinates of point D $= \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4,0)$

We know that area of a triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Area of ΔABD $= \frac{1}{2}[(4)\{(-2) - (0)\} + (3)\{(0) - (-6)\} + (4)\{(-6) - (-2)\}]$

$= \frac{1}{2}(-8 + 18 - 16) = -3$ square units

But, area cannot be negative. Hence, the area of ΔABD is 3 square units.

Area of ΔADC $= \frac{1}{2}[(4)\{0 - (2)\} + (4)\{(2) - (-6)\} + (5)\{(-6) - (0)\}]$

$= \frac{1}{2}(-8 + 32 - 30) = -3$ square units

But, area cannot be negative. Hence, area of ΔADC is 3 square units.

Therefore, clearly median AD has divided ΔABC in two triangles of equal areas.

EXERCISE 7.4(Optional)*

1. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points $A(2, -2)$ and $B(3, 7)$.

Solution:

If the ratio in which P divides AB is $k : 1$, then the coordinates of the point P will be

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$$

Let the given line divides the line segment joining the points $A(2, -2)$ and $B(3, 7)$ in a ratio $k : 1$.

The coordinates of the point of division $= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

This points also lies on $2x + y - 4 = 0$

$$2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow \frac{6k+4+7k-2-4k-4}{k+1}=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

2. Find a relation between x and y if the points $(x, y), (1, 2)$ and $(7, 0)$ are collinear.

Solution:

If the given points are collinear, the area of triangle formed by these points will be 0.

Area of a triangle $=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

Area $=\frac{1}{2}[x(2-0)+1(0-y)+7(y-2)]$

$$\Rightarrow 0=\frac{1}{2}[2x-y+7y-14]$$

$$\Rightarrow 0=\frac{1}{2}[2x+6y-14]$$

$$\Rightarrow 2x+6y-14=0$$

$$\Rightarrow x+3y-7=0$$

This is the required relation between x and y.

3. Find the centre of a circle passing through the points $(6, -6), (3, -7)$ and $(3, 3)$.

Solution:

Let $O(x, y)$ be the centre of circle. And let $(6, -6), (3, -7)$ and $(3, 3)$ are A, B, and C points on the circumference of circle.

$$OA=\sqrt{(x-6)^2+(y+6)^2}$$

$$OB=\sqrt{(x-3)^2+(y+7)^2}$$

$$OC=\sqrt{(x-3)^2+(y-3)^2}$$

$OA=OB$ (Radius of circle)

$$\sqrt{(x-6)^2+(y+6)^2}=\sqrt{(x-3)^2+(y+7)^2}$$

$$\Rightarrow x^2+36-12x+y^2+36+12y=x^2+9-6x+y^2+49+14y$$

$\Rightarrow -6x - 2y + 14 = 0$

$\Rightarrow 3x + y = 7$ ....(i)

$OA = OC$ (Radius of circle)

$\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$

$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$

$\Rightarrow -6x + 18y + 54 = 0$

$\Rightarrow -3x + 9y = -27$ ....(ii)

Adding equation (i)and (ii)

$10y = -20$

$\Rightarrow y = -2$

From equation (i)

$3x - 2 = 7$

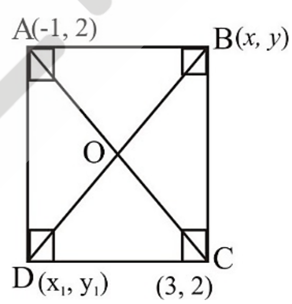
$\Rightarrow 3x = 9$

$\Rightarrow x = 3$

So, the centre of circle is $(3, -2)$

4. The two opposite vertices of a square are$(-1, 2)$ and $(3, 2)$. Find the coordinates of the other two vertices.

Solution:



Let ABCD be a square having $(-1, 2), (3, 2)$ as vertices A and C respectively and $(x, y), (x_1, y_1)$ be the coordinate of vertex B and D respectively.

We know that, sides of a square are equals to each other, hence $AB = BC$

$\sqrt{(x+1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y-2)^2}$

$\Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y$

$\Rightarrow 8x = 8$

$\Rightarrow x = 1$

We know that in a square all interior angles are of 90^o

So, in ΔABC

$AB^2 + BC^2 = AC^2$

$\left(\sqrt{(1+1)^2 + (y-2)^2}\right)^2 + \left(\sqrt{(1-3)^2 + (y-2)^2}\right)^2$

$= \left(\sqrt{(3+1)^2 + (2-2)^2}\right)^2$

$\Rightarrow 4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

$\Rightarrow 2y^2 + 16 - 8y = 16$

$\Rightarrow 2y^2 - 8y = 0$

$\Rightarrow y(y-4) = 0$

$y = 0$ or 4

We know that in a square diagonals are of equal length and bisect each other at 90^o. Let O be the midpoint of AC so it will also be the midpoint of BD.

Coordinate of point $O = \left(\frac{-1+3}{2}, \frac{2+2}{2}\right)$

$\left(\frac{1+x_1}{2}, \frac{y+y_1}{2}\right) = (1, 2)$

$\frac{1+x_1}{2} = 1$

$1 + x_1 = 2$

$\Rightarrow x_1 = 1$

$\frac{y+y_1}{2} = 2$

$\Rightarrow y + y_1 = 4$

If $y = 0$

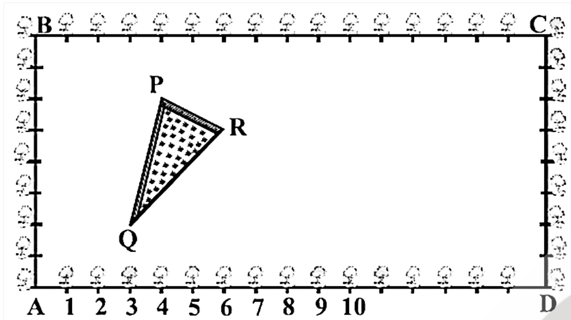
$y_1 = 4$

If $y = 4$

$y_1 = 0$

So, Coordinates of other vertices are $(1, 0)(1, 4)$.

5. The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure the students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

Solution:

(i) Taking A as origin, we will take AD as x axis and AB as y aixs. Now we may observes that coordinates of points P, Q and R are $(4, 6), (3, 2), (6, 5)$

Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \dfrac{1}{2}[4(2-5) + 3(5-6) + 6(6-2)]$

$= \dfrac{1}{2}[-12 - 3 + 24]$

$= \frac{9}{2}$ square units

(ii) Taking C as origin and CB as x −axis and CD as y −axis the coordinates of vertices P, Q, R are $(12, 2), (13, 6), (10,3)$.

Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 + y_2)]$

$= \dfrac{1}{2}[12(6-3) + 13(3-2) + 10(2-6)]$

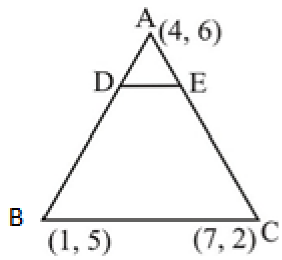
$= \dfrac{1}{2}[36 + 13 - 40]$

$= \frac{9}{2}$ square units

Area of the triangle is same both in the cases.

6. The vertices of a ΔABC are $A(4,6)$, $B(1,5)$ and $C(7,2)$. A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of Δ ABC.

Solution:



Given that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$

$$\frac{AD}{AD+BD}=\frac{AE}{AE+EC}=\frac{1}{4}$$

$$\frac{AD}{DB}=\frac{AE}{EC}=\frac{1}{3}$$

So, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1 : 3.

Coordinates of the point $P(x,y)$ which divides the line segment joining the points points $A(x_1,y_1)$ and $B(x_2y_2)$ internally in the ratio $m_1 : m_2$are

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

Coordinates of Point D $=\left(\frac{1\times1+3\times4}{1+3},\frac{1\times5+3\times6}{1+3}\right)=\left(\frac{13}{4},\frac{23}{4}\right)$

Coordinates of point E $=\left(\frac{1\times7+3\times4}{1+3},\frac{1\times2+3\times6}{1+3}\right)=\left(\frac{19}{4},\frac{20}{4}\right)$

Area of a triangle $=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Area of $\Delta ADE=\frac{1}{2}\left[4\left(\frac{23}{4}-\frac{20}{4}\right)+\frac{13}{4}\left(\frac{20}{4}-6\right)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$

$=\frac{1}{2}\left[3-\frac{13}{4}+\frac{19}{16}\right]=\frac{1}{2}\left[\frac{48-52+19}{16}\right]=\frac{15}{32}$ square units

Area of $\Delta ABC=\frac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$

$=\frac{1}{2}[12-4+7]=\frac{15}{2}$ square units

Clearly the ratio between the areas of ΔADE and of ΔABC is 1 : 16.

7. Let A $(4, 2)$, B$(6, 5)$and C$(1, 4)$ be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

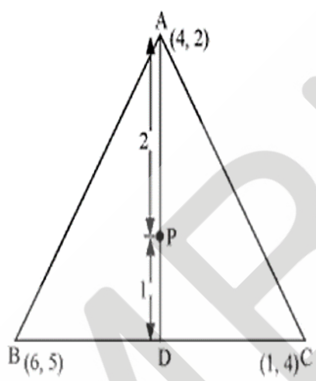
(ii) Find the coordinates of the point P on AD such that AP : PD $= 2 : 1$

(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE $= 2 : 1$ and CR : RF $= 2 : 1$.

(iv) What do you observe?

(v) If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Solution:



(i) Median AD of the triangle will divide the side BC in two equal parts. So D is the midpoint of side BC.

Coordinates of D $= \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Point P divides the side AD in a ratio $2 : 1$.

Coordinates of P $= \left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Median BE of the triangle will divide the side AC in two equal parts, So E is the midpoint of side AC.

Coordinates of E $= \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Point Q divides the side BE in a ratio $2 : 1$.

Coordinates of Q $= \left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Median CF of the triangle will divide the side AB in two equal parts. So F is the midpoint of side AB.

Coordinates of F $= \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Point R divides the side CF in a ratio 2 : 1.

Coordinates of R $= \left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) Now we may observe that coordinates of point P, Q, R are same. So, all these are representing same point on the plane i. e. centroid of the triangle.

(v) Now consider a ΔABC having its vertices as $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$. Median AD of the triangle will divide the side BC in two equal parts. So D is the midpoint of side BC.

Coordinates of D $= \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

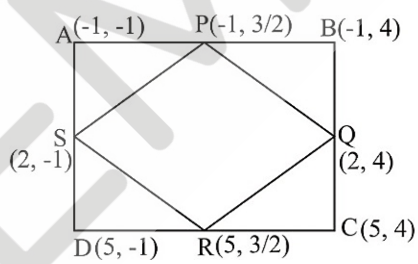
Let centroid of this triangle is O.

Point O divides the side AD in a ratio 2 : 1.

Coordinate of O $= \left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

8. ABCD is a rectangle formed by the points $A(-1, -1)$, $B(-1, 4)$, $C(5, 4)$ and $D(5, -1)$. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Solution:



Length of PQ $= \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of QR $= \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of RS $= \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of SP $= \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of PR $= \sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$

Length of QS $= \sqrt{(2-2)^2 + (4+1)^2} = 5$

Here all sides of given quadrilateral is of same measure but the diagonals are of different lengths. So, quadrilateral PQRS is a rhombus.

♦ ♦ ♦