

NCERT CBSE Solutions for Class 10 Mathematics

Areas Related to Circles 12.1 Maths

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles. (use $\pi = \frac{22}{7}$)

Solution:

Given radius of first circle be $r_1 = 19$ cm

Radius of second circle be $r_2 = 9$ cm

Let radius of required circle be r

Circumference of first circle = $2\pi r_1 = 2\pi(19) = 38\pi$ cm

Circumference of second circle = $2\pi r_2 = 2\pi(9) = 18\pi$ cm

Circumference of required circle = $2\pi r$

Given that

Circumference of required circle = Circumference of first circle +
Circumference of second circle

$$\Rightarrow 2\pi r = 38\pi + 18\pi = 56\pi \text{ cm}$$

$$\Rightarrow r = \frac{56\pi}{2\pi} = 28$$

Hence, the radius of required circle is 28 cm.

2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles. (use $\pi = \frac{22}{7}$)

Solution:

Given radius of first circle be $r_1 = 8$ cm

Radius of second circle be $r_2 = 6$ cm

Let radius of required circle be r

Area of first circle = $\pi r_1^2 = \pi(8)^2 = 64\pi \text{ cm}^2$

Area of second circle = $\pi r_2^2 = \pi(6)^2 = 36\pi \text{ cm}^2$

Given that

Area of required circle = area of first circle + area of second circle

$$\Rightarrow \pi r^2 = 64\pi + 36\pi$$

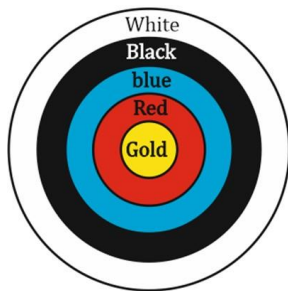
$$\Rightarrow \pi r^2 = 100\pi$$

$$\Rightarrow r^2 = 100 \text{ cm}^2$$

$$\Rightarrow r = \pm 10$$

But radius cannot be negative, so radius of required circle is 10 cm.

3. Given figure depicts an archery target marked with its five-scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions. (use $\pi = \frac{22}{7}$)



Solution:

Given radius (r_1) of gold region = $\frac{21}{2} = 10.5 \text{ cm}$

Given that each circle is 10.5 cm wider than previous circle.

So, radius of second circle $r_2 = 10.5 + 10.5 = 21 \text{ cm}$

Radius of third circle $r_3 = 21 + 10.5 = 31.5 \text{ cm}$

Radius of fourth circle $r_4 = 31.5 + 10.5 = 42 \text{ cm}$

Radius of fifth circle $r_5 = 42 + 10.5 = 52.5 \text{ cm}$

Area of golden region = area of 1st circle = $\pi r_1^2 = \pi(10.5)^2 = 346.5 \text{ cm}^2$

Now, area of Red region = area of second circle - area of first circle

$$= \pi r_2^2 - \pi r_1^2$$

$$= \pi[(21)^2 - (10.5)^2]$$

$$= 441\pi - 110.25\pi = 330.75\pi$$

$$= 1039.5 \text{ cm}^2$$

Area of blue region = area of third circle - area of second circle

$$= \pi r_3^2 - \pi r_2^2$$

$$= \pi[(31.5)^2 - (21)^2]$$

$$= 992.25\pi - 441\pi = 551.25\pi$$

$$= 1732.5 \text{ cm}^2$$

Area of black region = area of fourth circle - area of third circle

$$\begin{aligned}
 &= \pi r_4^2 - \pi r_3^2 \\
 &= \pi[(42)^2 - \pi(31.5)^2] \\
 &= 1764\pi - 992.25\pi \\
 &= 771.75\pi = 2425.5 \text{ cm}^2
 \end{aligned}$$

Area of white region = area of fifth circle – area of fourth circle

$$\begin{aligned}
 &= \pi r_5^2 - \pi r_4^2 \\
 &= \pi[(52.5)^2 - (42)^2] \\
 &= 2756.25\pi - 1764\pi \\
 &= 992.25\pi = 3118.5 \text{ cm}^2
 \end{aligned}$$

Hence, areas of gold, red, blue, black, white regions are 346.5 cm^2 , 1039.5 cm^2 , 1732.5 cm^2 , 2425.5 cm^2 and 3118.5 cm^2 respectively.

4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is traveling at a speed of 66 km per hour? (use $\pi = \frac{22}{7}$)

Solution:

Given diameter of wheel of car = 80 cm

Therefore, radius (r) of wheel of car = $\frac{80}{2} = 40 \text{ cm}$

Now, circumference of wheel = $2\pi r$

$$= 2\pi(40) = 80\pi \text{ cm}$$

Given speed of car = 66 km/hour

$$= \frac{66 \times 100000}{60} \text{ cm/min}$$

$$= 110000 \text{ cm/min}$$

Distance travelled by car in 10 minutes = speed \times time

$$= 110000 \times 10 = 1100000 \text{ cm}$$

Let number of revolutions of wheel of car is n .

$n \times$ distance travelled in 1 revolution (i.e. circumference) = distance traveled in 10 minutes.

$$\Rightarrow n \times 80\pi = 1100000$$

$$\Rightarrow n = \frac{1100000 \times 7}{80 \times 22}$$

$$\Rightarrow \frac{35000}{8} = 4375$$

Therefore, each wheel of car will make 4375 revolutions.

5. Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is (use $\pi = \frac{22}{7}$)

- (A) 2 units
(B) π units
(C) 4 units
(D) 7 units

Solution:

Let radius of circle be r

Circumference of circle = $2\pi r$

Area of circle = πr^2

Given that circumference of circle and area of circle is equal.

$$\Rightarrow 2\pi r = \pi r^2$$

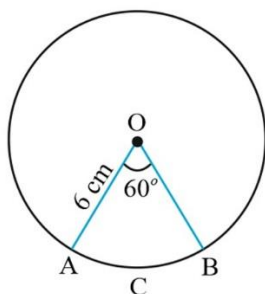
$$\Rightarrow r = 2$$

So radius of circle will be 2 units

Areas Related to Circles 12.2 Maths

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60° .

Solution:



Let OACB be a sector of circle making 60° angle at centre O of circle.

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

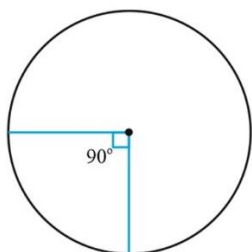
$$\text{So, area of sector OACB} = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2$$

Therefore, area of sector of circle making 60° at centre of circle is 18.85 cm^2

2. Find the area of a quadrant of circle whose circumference is 22 cm.

Solution:



Given circumference of the circle = 22 cm

Let radius of circle be r .

$$\Rightarrow 2\pi r = 22$$

$$\Rightarrow r = \frac{22}{2\pi}$$

$$\Rightarrow r = \frac{7}{2}$$

We know, quadrant of circle will subtend 90° angle at centre of circle.

$$\text{So, area of such quadrant of circle} = \frac{90^\circ}{360^\circ} \times \pi \times r^2$$

$$= \frac{1}{4} \times \pi \times \left(\frac{7}{2}\right)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{49}{4}$$

$$= \frac{77}{8} \text{ cm}^2$$

Therefore, the area of a quadrant of circle whose circumference is 22 cm is $\frac{77}{8} \text{ cm}^2$

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution:

We know that in one hour, minute hand rotates 360° .

So, in 5 minutes, minute hand will rotate $= \frac{360^\circ}{60} \times 5 = 30^\circ$

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

$$\text{Area of sector of } 30^\circ = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{22}{12} \times 2 \times 14$$

$$= \frac{11 \times 14}{3}$$

$$= \frac{154}{3} \text{ cm}^2$$

So, area swept by minute hand in 5 minutes is 51.3 cm^2 .

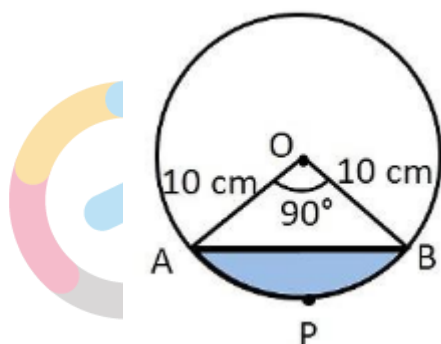
4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector

(Use $\pi = 3.14$)

Solution:



Given a chord of a circle of radius 10 cm subtends a right angle at the centre

Let AB be the cord of circle subtending 90° angle at centre O of circle.

(i) Area of minor sector

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

$$\text{Therefore, area of minor sector (OAPB)} = \frac{90^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 10 \times 10$$

$$= \frac{1100}{14} = 78.5 \text{ cm}^2$$

$$\text{Area of } \triangle OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10$$

$$= 50 \text{ cm}^2$$

$$\text{Area of minor segment APB} = \text{Area of minor sector OAPB} - \text{Area of } \triangle OAB$$

$$= 78.5 - 50 = 28.5 \text{ cm}^2$$

$$(ii) \text{ Area of major sector} = \left(\frac{360^\circ - 90^\circ}{360^\circ} \right) \times \pi r^2 = \left(\frac{270^\circ}{360^\circ} \right) \pi r^2$$

$$= \frac{3}{4} \times \frac{22}{7} \times 10 \times 10$$

$$= \frac{3300}{14} \text{ cm}^2 = 235.7 \text{ cm}^2$$

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre.

Find:

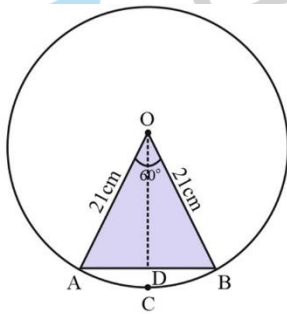
- (i) The length of the arc
- (ii) Area of the sector formed by the arc
- (ii) Area of the segment formed by the corresponding chord

Solution:

Given radius (r) of circle = 21 cm

And angle subtended by given arc = 60°

We know, length of an arc of a sector of angle $\theta = \frac{\theta}{360^\circ} \times 2\pi r$



$$(i) \text{ Length of arc ACB} = \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$$

$$= \frac{1}{6} \times 2 \times 22 \times 3$$

$$= 22 \text{ cm}$$

- (ii) Area of sector formed by the arc

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

$$\Rightarrow \text{Area of sector OACB} = \frac{60^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$$

$$= 231 \text{ cm}^2$$

(iii) Area of the segment formed by the corresponding chord

In ΔOAB

$$\angle OAB = \angle OBA \quad (\text{as } OA = OB)$$

We know, sum of angles in a triangle = 180°

$$\Rightarrow \angle OAB + \angle AOB + \angle OBA = 180^\circ$$

$$\Rightarrow 2\angle OAB + 60^\circ = 180^\circ$$

$$\Rightarrow \angle OAB = 60^\circ$$

So, ΔOAB is an equilateral triangle.

$$\text{Therefore, area of } \Delta OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (21^2) = \frac{441\sqrt{3}}{4} \text{ cm}^2$$

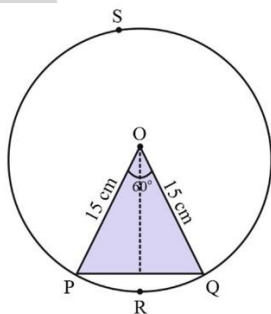
Now, area of segment ACB = Area of sector $OACB$ – Area of ΔOAB

$$= \left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$$

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Solution:



Given radius (r) of circle = 15 cm

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

$$\text{Area of sector } OPRQ = \frac{60^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times (15)^2$$

$$= \frac{11 \times 75}{7} = \frac{825}{7}$$

$$= 117.85 \text{ cm}^2$$

In ΔOPQ

$$\angle OPQ = \angle OQP \quad (\text{as } OP = OQ)$$

$$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$2\angle OPQ = 120^\circ$$

$$\angle OPQ = 60^\circ$$

ΔOPQ is an equilateral triangle

$$\text{Area of equilateral } \Delta OPQ = \frac{\sqrt{3}}{4} (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (15)^2 = \frac{225\sqrt{3}}{4} \text{ cm}^2$$

$$= 56.25\sqrt{3}$$

$$= 97.3125 \text{ cm}^2$$

$$\text{Area of segment PRQ} = \text{Area of sector OPRQ} - \text{Area of } \Delta OPQ$$

$$= 117.85 - 97.3125$$

$$= 20.537 \text{ cm}^2$$

$$\text{Area of major segment PSQ} = \text{Area of circle} - \text{Area of segment PRQ}$$

$$= \pi(15)^2 - 20.537$$

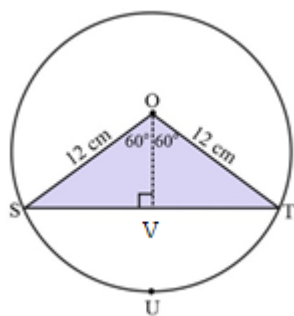
$$= \frac{225 \times 22}{7} - 20.537$$

$$= \frac{4950}{7} - 20.537 = 707.14 - 20.537$$

$$= 686.605 \text{ cm}^2$$

7. A chord of a circle of radius 15 cm subtends an angle of 120° at the centre. Find the areas of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Solution:



Given radius of circle = 15 cm

Draw a perpendicular OV on chord ST. It will bisect the chord ST.

$$SV = VT$$

In ΔOVS

$$\cos 60^\circ = \frac{OV}{OS}$$

$$\Rightarrow \frac{1}{2} = \frac{OV}{12}$$

$$\Rightarrow OV = 6$$

$$\text{And } \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{SV}{SO}$$

$$\Rightarrow \frac{SV}{12} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow SV = 6\sqrt{3}$$

$$\therefore ST = 2SV = 2 \times 6\sqrt{3} = 12\sqrt{3}$$

$$\text{Now, area of } \Delta OST = \frac{1}{2} \times ST \times OV$$

$$= \frac{1}{2} \times 12\sqrt{3} \times 6$$

$$= 36\sqrt{3} = 36 \times 1.73 = 62.28$$

$$\text{We know, area of sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2$$

$$\text{Here } \theta = 120^\circ$$

$$\text{Area of sector OSUT} = \frac{120^\circ}{360^\circ} \times \pi (12)^2$$

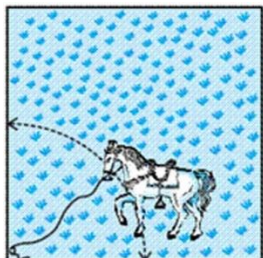
$$= \frac{1}{3} \times \frac{22}{7} \times 144 = 150.72$$

$$\text{Area of segment SUT} = \text{Area of sector OSUT} - \text{Area of } \Delta OST$$

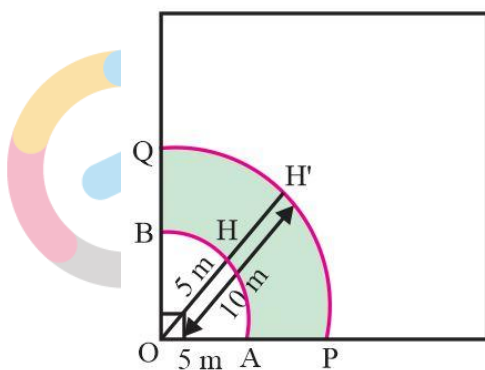
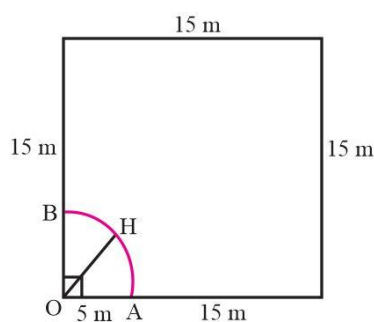
$$= 150.72 - 62.28$$

$$= 88.44 \text{ cm}^2$$

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long as given in figure. Find
- The area of that part of the field in which the horse can graze.
 - The increase in the grazing area if the rope were 10 m long instead of 5 m. (use $\pi = 3.14$)



Solution:



Given horse is tied with a rope of length 5 m.

Horse can graze a sector of 90° in a circle of 5 m radius.

- The area of that part of the field in which the horse can graze

The area that can be grazed by horse = area of sector

$$= \frac{90^\circ}{360^\circ} \pi r^2 = \frac{1}{4} \times \pi (5)^2$$

$$= \frac{25\pi}{4} = 19.62 \text{ m}^2$$

- Area that can be grazed by horse when length of rope is 10 m long

Therefore, radius of sector becomes 10 m

$$\text{Area that can be grazed by horse} = \frac{90^\circ}{360^\circ} \times \pi \times (10)^2$$

$$= \frac{1}{4} \times \pi \times 100$$

$$= 25\pi$$

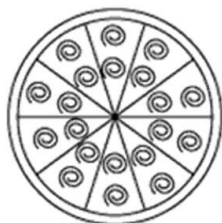
$$\text{Hence increase in grazing area} = 25\pi - \frac{25\pi}{4}$$

$$= 25\pi \left(1 - \frac{1}{4}\right)$$

$$= 25 \times 3.14 \times \frac{3}{4} = 58.87 \text{ cm}^2$$

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find.

- (i) The total length of the silver wire required.
- (ii) The area of each sector of the brooch



Solution:

- (i) The total length of the silver wire required.

Here, total length of wire required will be = length of 5 diameters + circumference of brooch.

Given diameter of circle = 35 mm

$$\Rightarrow \text{Radius of circle} = \frac{35}{2} \text{ mm}$$

We know, circumference of circle = $2\pi r$

$$\text{Therefore, circumference of brooch} = 2\pi \left(\frac{35}{2}\right)$$

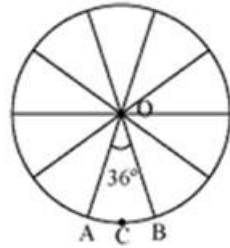
$$= 2 \times \frac{22}{7} \times \left(\frac{35}{2}\right)$$

$$= 110 \text{ mm}$$

$$\text{Total length of silver wire required} = 110 + 5 \times 35$$

$$= 110 + 175 = 285 \text{ mm}$$

- (ii) Each of 10 sectors of circle subtend $\frac{360^\circ}{10} = 36^\circ$ at centre of circle.



We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

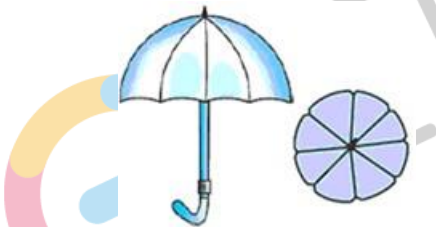
So, area of each sector $= \frac{36^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$$

$$= \frac{385}{4} \text{ mm}^2$$

$$= 96.25 \text{ mm}^2$$

10. An umbrella has 8 ribs which are equally spaced as shown in figure. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Solution:

Given there are 8 ribs in umbrella.

Each of 8 sectors of circle subtend $\frac{360^\circ}{8} = 45^\circ$ at centre of circle.

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

So, area between two consecutive ribs of circle $= \frac{45^\circ}{360^\circ} \times \pi r^2$

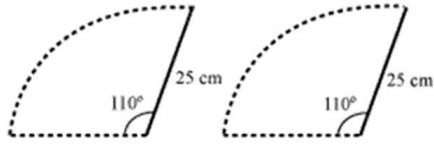
$$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$$

$$= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{ cm}^2$$

$$= 795.8 \text{ cm}^2$$

11. A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.

Solution:



Given that each blade of wiper will sweep an area of a sector of 115° in a circle of 25 cm radius.

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Here $\theta = 115^\circ$ and $r = 25$

Area of such sector $= \frac{115^\circ}{360^\circ} \times \pi \times (25)^2$

$$= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25$$

$$= \frac{158125}{252} \text{ cm}^2$$

Hence area swept by 2 blades $= 2 \times \frac{158125}{252}$

$$= \frac{158125}{126} \text{ cm}^2$$

$$= 1254.96 \text{ cm}^2$$

12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships warned. (Use $\pi = 3.14$)

Solution:

Given lighthouse spreads light over a sector of angle 80° in a circle of 16.5 km radius

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Here $\theta = 80^\circ$ and $r = 16.5$

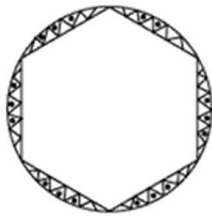
Area of sector OACB $= \frac{80^\circ}{360^\circ} \times \pi r^2$

$$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$$

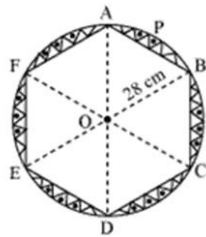
$$= 189.97 \text{ km}^2$$

Therefore, area of the sea over ships warned is 189.97 km^2

13. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2 . (Use $\sqrt{3} = 1.7$)



Solution:



Designs are segments of circle.

Consider segment APB.

Chord AB is a side of hexagon. Each chord will subtend $\frac{360^\circ}{6} = 60^\circ$ at centre of circle.

In $\triangle OAB$

$$\angle OAB = \angle OBA \quad (\text{as } OA = OB)$$

$$\text{And } \angle AOB = 60^\circ$$

We know, sum of interior angles of triangle = 180°

$$\Rightarrow \angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\Rightarrow 2\angle OAB = 180^\circ - 60^\circ = 120^\circ$$

$$\Rightarrow \angle OAB = 60^\circ$$

So, $\triangle OAB$ is an equilateral triangle

$$\text{Hence area of } \triangle OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3} \text{ cm}^2 = 333.2 \text{ cm}^2.$$

$$\text{We know, area of sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2$$

$$\text{Here } \theta = 60^\circ \text{ and } r = 28$$

$$\text{Area of sector OAPB} = \frac{60^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28$$

$$= \frac{1232}{3} = 410.66 \text{ cm}^2$$

Area of segment APB = Area of sector OAPB – Area of ΔOAB

$$= 410.66 - 333.2$$

$$= 77.46 \text{ cm}^2$$

Hence, total area of designs = $6 \times 77.46 = 464.76 \text{ cm}^2$

Cost occurred in making 1 cm^2 designs = Rs. 0.35

Cost occurred in making 464.76 cm^2 designs = $464.76 \times 0.35 = 162.66$

So, cost of making such designs is Rs. 162.66.

14. Tick the correct answer in the following:

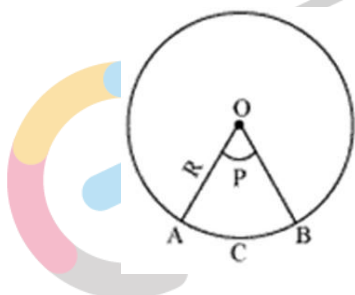
Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$

(B) $\frac{p}{180} \times \pi R^2$

(C) $\frac{p}{360} \times 2\pi R$

(D) $\frac{p}{720} \times 2\pi R^2$



We know that area of sector of angle $\theta = \frac{\theta}{360^\circ} \pi r^2$

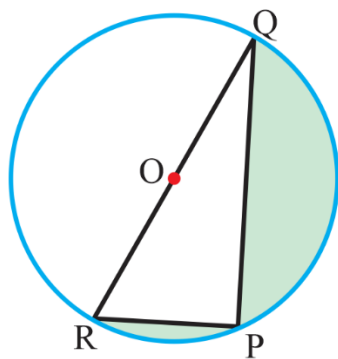
Here $\theta = p$ and $r = R$

Hence, Area of sector of angle $p = \frac{p}{360^\circ} (\pi R^2)$

$$= \left(\frac{p}{720^\circ} \right) (2\pi R^2)$$

Areas Related to Circles 12.3 Maths

1. Find the area of the shaded region in the given figure, if $PQ = 24 \text{ cm}$, $PR = 7 \text{ cm}$ and O is the centre of the circle

**Solution:**

Given $PQ = 24$ cm, and $PR = 7$ cm

RQ is the diameter of circle, so $\angle RPQ$ will be 90° .

Now in ΔPQR by applying Pythagoras theorem

$$RP^2 + PQ^2 = RQ^2$$

$$\Rightarrow (7)^2 + (24)^2 = RQ^2$$

$$\Rightarrow RQ = \sqrt{625} = 25$$

$$\Rightarrow \text{Radius of circle } OR = \frac{RQ}{2} = \frac{25}{2}$$

$$\text{Area of semicircle } RPQOR = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \pi \left(\frac{25}{2} \right)^2$$

$$= \frac{625}{8} \pi \text{ cm}^2$$

$$= 245.53 \text{ cm}^2$$

$$\text{Area of } \Delta PQR = \frac{1}{2} \times PQ \times PR$$

$$= \frac{1}{2} \times 24 \times 7$$

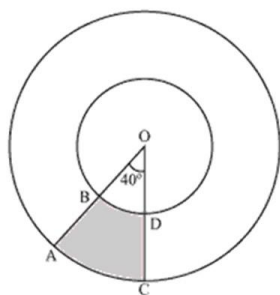
$$= 84 \text{ cm}^2$$

Area of shaded region = area of semicircle $RPQOR$ – area of ΔPQR

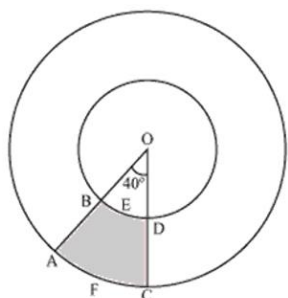
$$= 245.53 - 84$$

$$= 161.53 \text{ cm}^2$$

2. Find the area of the shaded region in the given figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.



Solution:



Given radius of inner circle = 7 cm

And radius of outer circle = 14 cm

Area of shaded region

= area of sector OAFCO – area of sector OBEDO

$$= \frac{40^\circ}{360^\circ} \times \pi(14)^2 - \frac{40^\circ}{360^\circ} \times \pi(7)^2 \quad \left(\text{since area of sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2 \right)$$

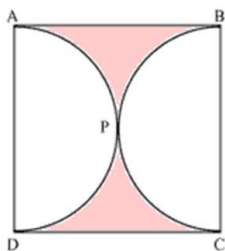
$$= \frac{1}{9} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{9} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9}$$

$$= \frac{154}{3} = 51.33 \text{ cm}^2$$

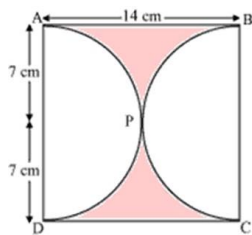
Therefore, area of shaded region is 51.33 cm².

3. Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Solution:

Given length of side of square = 14 cm = length of diameter of semicircle.



\Rightarrow radius of semicircle = 7 cm

We know, area of semicircle = $\frac{1}{2}\pi r^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$

$$= 77 \text{ cm}^2$$

Now, area of square ABCD = $(\text{side})^2 = (14)^2 = 196 \text{ cm}^2$

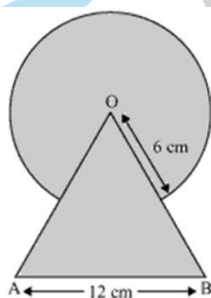
Area of shaded region = area of square ABCD — area of semicircle APD — area of semicircle BPC

$$= 196 - 77 - 77$$

$$= 196 - 154$$

$$= 42 \text{ cm}^2$$

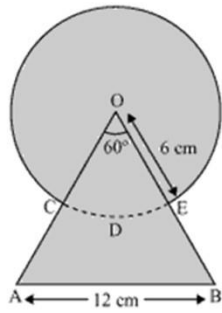
4. Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Solution:

Given length of side of equilateral triangle = 12 cm and radius of circle = 6 cm

$\angle COE = 60^\circ$ (\because interior angle of equilateral triangle)



We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Hence, area of sector OCDE = $\frac{60^\circ}{360^\circ} \pi r^2$

$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} \text{ cm}^2$$

Now, area of $\Delta OAB = \frac{\sqrt{3}}{4} (12)^2$ (\because area of equilateral triangle = $\frac{\sqrt{3}}{4} (\text{side})^2$)

$$= \frac{\sqrt{3} \times 12 \times 12}{4}$$

$$= 36\sqrt{3} \text{ cm}^2$$

And area of circle = πr^2

$$= \frac{22}{7} \times 6 \times 6 = \frac{792}{7} \text{ cm}^2$$

Area of shaded region = area of AOAB + area of circle – area of sector OCDE

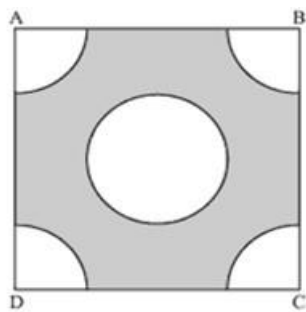
$$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$$

$$= \left(36\sqrt{3} + \frac{660}{7} \right) \text{ cm}^2$$

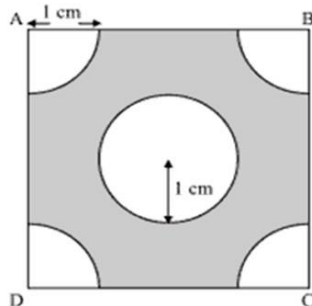
$$= 156.64 \text{ cm}^2$$

Therefore, area of shaded region is 156.64 cm^2

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the given figure. Find the area of the remaining portion of the square.



Solution:



Given length of side of square is 4 cm and radius of quadrant circle = 1 cm.

And diameter of inner circle = 2 cm \Rightarrow radius = 1 cm.

We know, area of quadrant of a circle = $\frac{1}{4}$ (area of a circle)

$$= \frac{1}{4} \times \frac{22}{7} \times (1)^2 = \frac{22}{28} \text{ cm}^2$$

Hence, area of 4 quadrant circles = 4 \times area of one quadrant circle

$$= 4 \times \frac{22}{28} = \frac{22}{7} \text{ cm}^2$$

$$\text{Area of square} = (\text{side})^2 = (4)^2 = 16 \text{ cm}^2$$

$$\text{Area of inner circle} = \pi r^2 = \pi(1)^2$$

$$= \frac{22}{7} \text{ cm}^2$$

Area of shaded region = area of square - area of circle - area of 4 quadrant circles

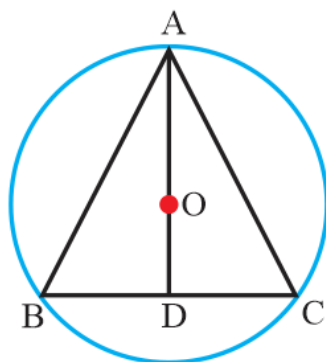
$$= 16 - \frac{22}{7} - \frac{22}{7}$$

$$= 16 - \frac{44}{7}$$

$$= \frac{112 - 44}{7} = \frac{68}{7} \text{ cm}^2$$

$$= 9.71 \text{ cm}^2$$

6. In a circular table cover of radius **32 cm**, a design is formed leaving an equilateral triangle **ABC** in the middle as shown in the given figure. Find the area of the design (Shaded region).



Solution:

Given radius (r) of circle = 32 cm

Let AD be the median of $\triangle ABC$

$$\Rightarrow AO = \frac{2}{3} AD = 32$$

$$\Rightarrow AD = 48 \text{ cm}$$

In $\triangle ABD$, using Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AB^2 = (48)^2 + \left(\frac{AB}{2}\right)^2$$

$$\Rightarrow \frac{3AB^2}{4} = (48)^2$$

$$\Rightarrow AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96}{\sqrt{3}} = 32\sqrt{3} \text{ cm}$$

$$\text{Now, area of equilateral triangle } \triangle ABC = \frac{\sqrt{3}}{4} (32\sqrt{3})^2$$

$$= \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3$$

$$= 96 \times 8 \times \sqrt{3}$$

$$= 768\sqrt{3} \text{ cm}^2$$

$$\text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times (32)^2$$

$$= \frac{22}{7} \times 1024$$

$$= \frac{22528}{7} \text{ cm}^2$$

Area of design = area of circle – area of ΔABC

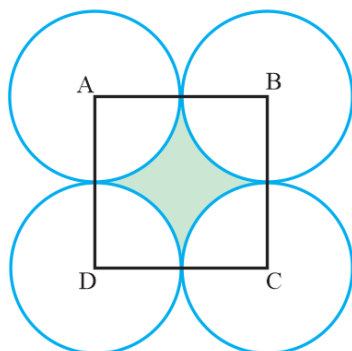
$$= \left(\frac{22528}{7} - 768\sqrt{3} \right)$$

$$= (3218.28 - 1330.18)$$

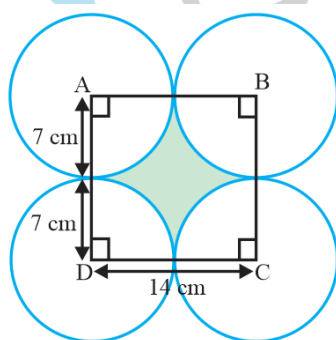
$$= 1888.10 \text{ cm}^2$$

Therefore, area of given design is 1888.10 cm^2

7. In the given figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



Solution:



Given length of side of square = 14 cm

From the figure, radius of circle = 7 cm

Area of quadrant circle = $\frac{1}{4}$ (area of circle)

Area of each quadrant circle = $\frac{1}{4} \times \pi(7)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{77}{2} \text{ cm}^2$$

$$\text{Now, area of square ABCD} = (\text{side})^2 = (14)^2 = 196 \text{ cm}^2$$

Therefore, area of shaded portion = area of square ABCD – 4 × area of each quadrant circle

$$= 196 - 4 \times \frac{77}{2} = 196 - 154$$

$$= 42 \text{ cm}^2$$

Hence, area of shaded region is 42 cm^2 .

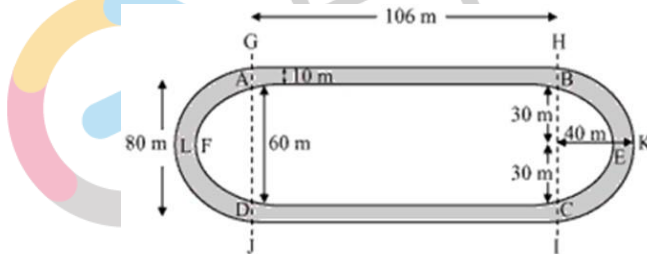
8. The given figure depicts a racing track whose left and right ends are semicircular.



The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

- The distance around the track along its inner edge
- The area of the track

Solution:



Given distance between inner parallel lines = 60 m and length of line segment = 106 m

- Distance around the track along its inner edge = length of AB + length of arc BEC + length of CD + length of arc DFA

$$= 106 + \frac{1}{2} \times 2\pi r + 106 + \frac{1}{2} \times 2\pi r$$

$$= 212 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30$$

$$= 212 + 2 \times \frac{22}{7} \times 30$$

$$\begin{aligned}
 &= 212 + \frac{1320}{7} \\
 &= \frac{1484 + 1320}{7} = \frac{2804}{7} \\
 &= 400.57 \text{ m}
 \end{aligned}$$

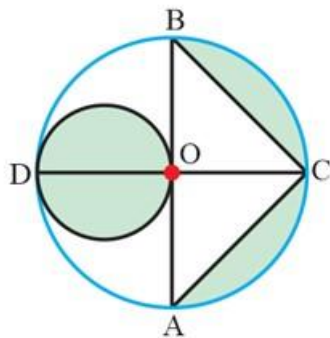
Therefore, distance around the track along its inner edge is = 400.57 m

- (ii) Area of track = area of GHIJ – area of ABCD + area of semicircle HKI
– Area of semicircle BEC + area of Semicircle GLJ – area of
semicircle AFD

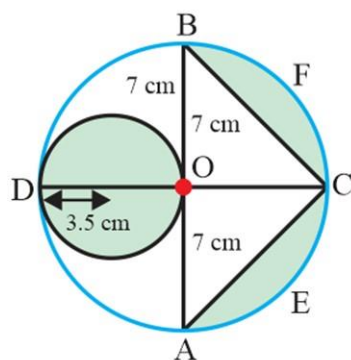
$$\begin{aligned}
 &= 106 \times 80 - 106 \times 60 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 \\
 &\quad + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 \\
 &= 106(80 - 60) + \frac{22}{7} \times (40)^2 - \frac{22}{7} \times (30)^2 \\
 &= 106(20) + \frac{22}{7} [(40)^2 - (30)^2] \\
 &= 2120 + \frac{22}{7} (40 - 30)(40 + 30) \\
 &= 2120 + \left(\frac{22}{7}\right)(10)(70) \\
 &= 2120 + 2200 \\
 &= 4320 \text{ m}^2
 \end{aligned}$$

Hence, area of track is 4320 m²

9. In the given figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



Solution:



Given radius (r_1) of larger circle = 7 cm

Hence, radius (r_2) of smaller circle = $\frac{7}{2}$ cm

Area of smaller circle = πr_2^2

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{77}{2} \text{ cm}^2$$

Area of semicircle AECFB of larger circle = $\frac{1}{2} \pi r_1^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$

$$= 77 \text{ cm}^2$$

Area of $\triangle ABC = \frac{1}{2} \times AB \times OC$

$$= \frac{1}{2} \times 14 \times 7 = 49 \text{ cm}^2$$

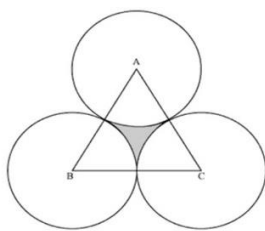
Therefore, area of shaded region = area of smaller circle + area of semicircle AECFB - area of $\triangle ABC$

$$= \frac{77}{2} + 77 - 49$$

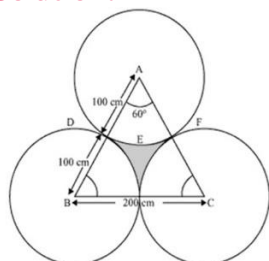
$$= 28 + \frac{77}{2} = 28 + 38.5$$

$$= 66.5 \text{ cm}^2$$

10. The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (See the given figure). Find the area of shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)



Solution:



Given area of equilateral triangle = 17320.5 m^2

Let side of equilateral triangle be a

Area of equilateral triangle = $\frac{\sqrt{3}}{4}(a^2)$

$$\Rightarrow \frac{\sqrt{3}}{4}(a^2) = 17320.5$$

$$\Rightarrow a^2 = 4 \times 10000$$

$$\Rightarrow a = 200 \text{ cm}$$

We know, area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Here $\theta = 60^\circ$ (since equilateral triangle) $r = 100$

So, area of sector ADEF = $\frac{60^\circ}{360^\circ} \times \pi \times r^2$

$$= \frac{1}{6} \times \pi \times (100)^2$$

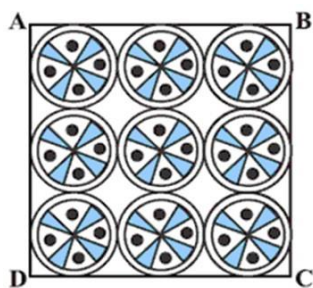
$$= \frac{3.14 \times 10000}{6} = \frac{15700}{3}$$

Area of shaded region = area of equilateral triangle $- 3 \times$ area of each sector

$$= 17320.5 - 3 \times \frac{15700}{3}$$

$$= 17320.5 - 15700 = 1620.5 \text{ cm}^2$$

11. On a square handkerchief, nine circular designs each of radius 7 cm are made as shown in figure. Find the area of the remaining portion of the handkerchief.

**Solution:**

Side of square = $6 \times 7 = 42$ cm,

Area of square = (side)²

$$= (42)^2$$

$$= 1764 \text{ cm}^2$$

Area of each circle = πr^2

$$= \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$$

Therefore, area of 9 circles = 9×154

$$= 1386 \text{ cm}^2$$

Area of remaining portion of handkerchief = $1764 - 1386$

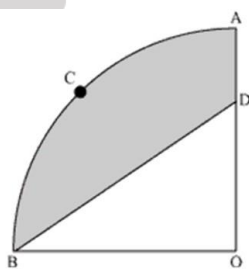
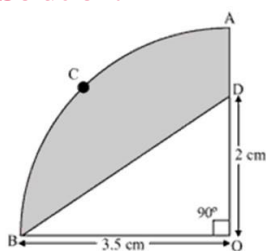
$$= 378 \text{ cm}^2$$

Hence, area of the remaining portion of the handkerchief = 378 cm^2

12. In the given figure, OACB is a quadrant of circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

(i) Quadrant OACB

(ii) Shaded region

**Solution:**

Since OACB is quadrant so it will subtend 90° angle at O.

Area of quadrant OACB = $\frac{1}{4}$ (area of circle)

$$= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{(11 \times 7 \times 7)}{2 \times 7 \times 2 \times 2} = \frac{77}{8} \text{ cm}^2$$

Area of $\triangle OBD = \frac{1}{2} \times OB \times OD$

$$= \frac{1}{2} \times 3.5 \times 2$$

$$= 3.5 \text{ cm}^2$$

Area of shaded region = area of quadrant OACB – area of $\triangle OBD$

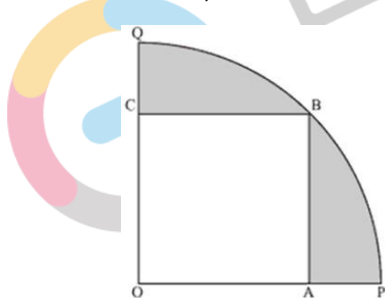
$$= \frac{77}{8} - 3.5$$

$$= \frac{49}{8} \text{ cm}^2$$

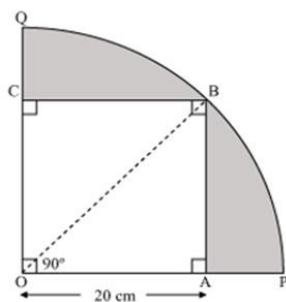
$$= 6.125 \text{ cm}^2$$

Therefore, area of shaded region = 6.125 cm^2

13. In the given figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)



Solution:



Given OA = 20 cm

In $\triangle OAB$, using Pythagoras theorem

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow OB^2 = (20)^2 + (20)^2$$

$$\Rightarrow OB = 20\sqrt{2}$$

Therefore, radius (r) of circle = $20\sqrt{2}$ cm

$$\text{Area of quadrant OPBQ} = \frac{90^\circ}{360^\circ} \times 3.14 \times (20\sqrt{2})^2$$

$$= \frac{1}{4} \times 3.14 \times 800$$

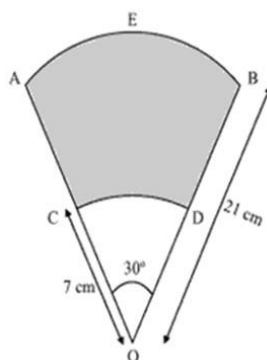
$$= 628 \text{ cm}^2$$

$$\text{Area of square} = (\text{side})^2 = (20)^2 = 400 \text{ cm}^2$$

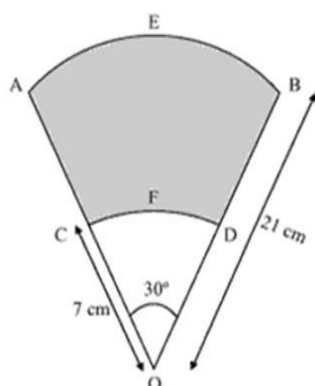
Therefore, area of shaded region = area of quadrant OPBQ – area of square

$$= 628 - 400 = 228 \text{ cm}^2$$

- 14.** AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O as shown in the figure. If $\angle AOB = 30^\circ$, find the area of the shaded region.



Solution:



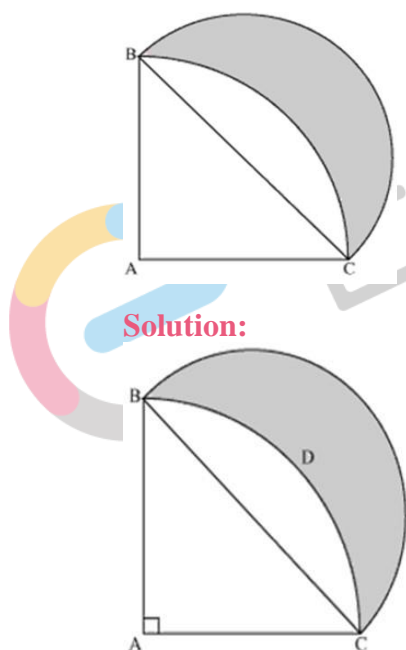
Given radius of larger circle = 21 and radius of smaller circle = 7

Area of shaded region = area of sector OAEF – area of sector OCFD

$$\begin{aligned}
 &= \frac{30^\circ}{360^\circ} \times \pi \times (21)^2 - \frac{30^\circ}{360^\circ} \times \pi \times (7)^2 \\
 &\left(\text{since area of sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2 \right) \\
 &= \frac{1}{12} \times \pi [(21)^2 - (7)^2] \\
 &= \frac{1}{12} \times \frac{22}{7} \times [(21 - 7)(21 + 7)] \\
 &= \frac{22 \times 14 \times 28}{12 \times 7} \\
 &= \frac{308}{3} \text{ cm}^2 \\
 &= 102.6 \text{ cm}^2
 \end{aligned}$$

Therefore, area of shaded portion = 102.6 cm²

15. In the given figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Solution:

Given radius of circle = 14 cm

As ABDC is a quadrant of circle, $\angle BAC$ will be of 90°

In $\triangle ABC$, using Pythagoras theorem,

$$BC^2 = AC^2 + AB^2$$

$$\Rightarrow BC^2 = (14)^2 + (14)^2$$

$$\Rightarrow BC = 14\sqrt{2}$$

From the figure, radius (r_1) of semicircle drawn on BC = $\frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

Now, area of $\triangle ABC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 14 \times 14$$

$$= 98 \text{ cm}^2$$

And area of sector ABDC = $\frac{1}{4} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$

$$= 154 \text{ cm}^2$$

Area of semicircle drawn on BC = $\frac{1}{2} \times \pi \times r_1^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7\sqrt{2})^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 98$$

$$= 154 \text{ cm}^2$$

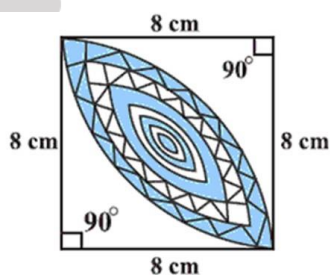
Area of shaded region = area of semicircle – (area of sector ABDC – area of $\triangle ABC$)

$$= 154 - (154 - 98)$$

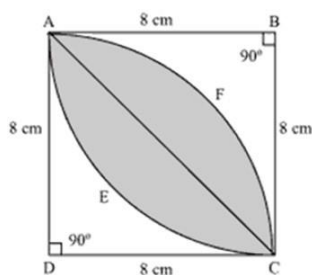
$$= 98 \text{ cm}^2$$

Therefore, area of shade region = 98 cm^2

16. Calculate the area of the designed region in the given figure common between the two quadrants of circles of radius 8 cm each.



Solution:



Given radius of each circle = 8 cm

The designed area is common region between two sectors BAEC and DAFC

$$\begin{aligned}
 \text{Area of sector BAEC} &= \frac{1}{4} \times \frac{22}{7} \times (8)^2 \\
 &= \frac{1}{4} \times \frac{22}{7} \times 64 \\
 &= \frac{22 \times 16}{7} \\
 &= \frac{352}{7} \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of } \triangle BAC &= \frac{1}{2} \times BA \times BC \\
 &= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2
 \end{aligned}$$

So, area of segment AEC = area of sector BAEC – area of $\triangle BAC$

$$= \frac{352}{7} - 32$$

Area of designed portion = 2 × (area of segment AEC)

$$\begin{aligned}
 &= 2 \times \left(\frac{352}{7} - 32 \right) \\
 &= 2 \left(\frac{352 - 224}{7} \right) \\
 &= \frac{2 \times 128}{7} \\
 &= \frac{256}{7} \text{ cm}^2 \\
 &= 36.5 \text{ cm}^2
 \end{aligned}$$

Therefore, area of designed region is 36.5 cm².