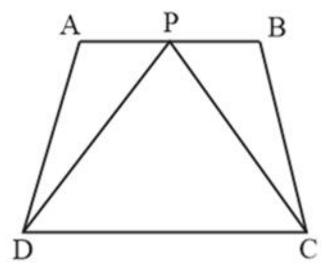
CBSE NCERT Solutions for Class 9 Mathematics Chapter 9

Back of Chapter Questions

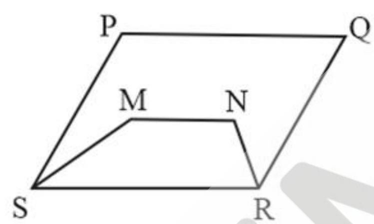
Exercise: 9.1

1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

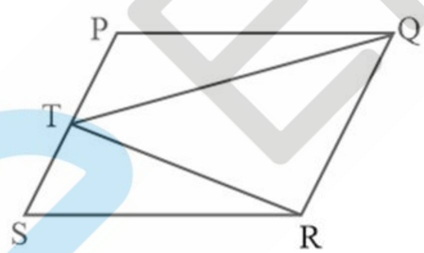
(i)



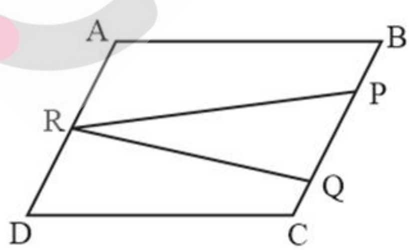
(ii)



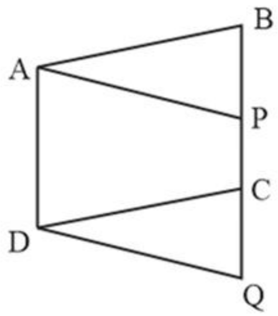
(iii)



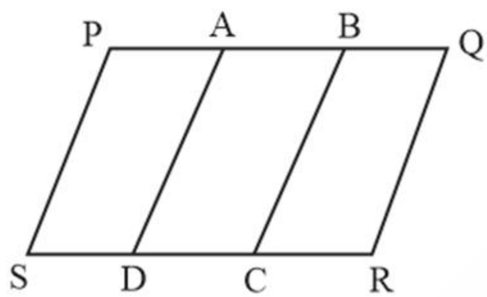
(iv)



(v)

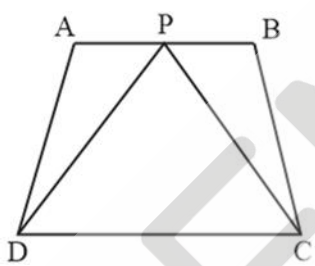


(vi)



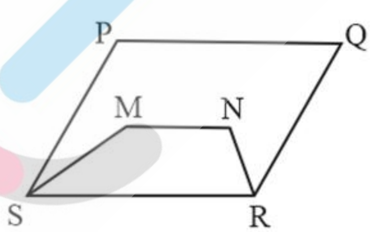
Solution:

(i)



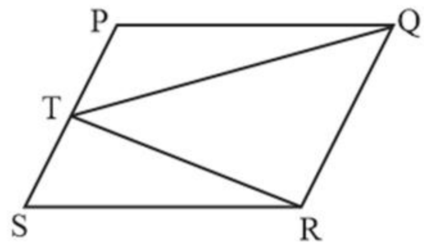
Yes. It is clearly seen that the trapezium ABCD and the triangle PCD lie on a common base CD and between the same parallel lines AB and CD.

(ii)



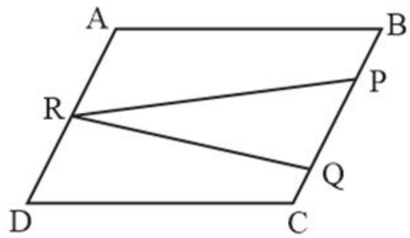
No. We observe that parallelogram PQRS and trapezium MNRS have a common base RS. But, their vertices, that are opposite to the common base RS of parallelogram and of trapezium are not lying on the same line.

(iii)



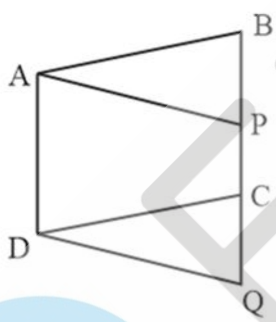
Yes. We see that parallelogram PQRS and triangle TQR have a common base QR and they are lying between the same parallel lines PS and QR.

(iv)



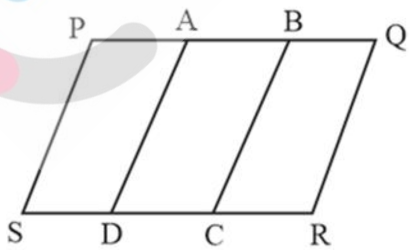
No. We see that parallelogram ABCD and triangle PQR are lying between same parallel lines AD and AC but not on a common base.

(v)



Yes. We see that parallelogram ABCD and parallelogram APQD have a common base AD and lie between the same parallel lines AD and BQ.

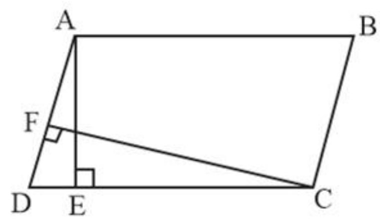
(vi)



No. It is seen that parallelogram ABCD and PQRS are lying between two parallel lines but not on the same.

Exercise: 9.2

1. In Figure, ABCD is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, find AD.



Solution:

Given that length of $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm

We know that in a parallelogram opposite sides are equal.

So, in parallelogram ABCD, $AB = CD = 16$ cm.

Also, we know that

Area of a parallelogram $=$ Base $\times$ Corresponding altitude

Therefore, area of parallelogram $ABCD = CD \times AE = AD \times CF$

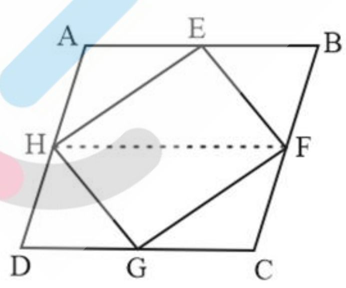
$\Rightarrow 16\text{ cm} \times 8\text{ cm} = AD \times 10\text{ cm}$

$\Rightarrow AD = \frac{(16 \times 8)}{10}$ cm

Hence, the length of AD is 12.8 cm.

2. If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) $= \frac{1}{2}$ar (ABCD).

Solution:



Given E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD.

Now, in parallelogram ABCD, join HF

We know, opposite sides of a parallelogram are equal and parallel

$AD = BC$ and $AD \parallel BC$

And $AB = CD$

$\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$

$\Rightarrow AH = BF$

and $AH \parallel BF$

Therefore, ABFH is a parallelogram.

Since ΔHEF and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,

$\therefore$ ar $(\Delta HEF) = \frac{1}{2}$ ar $(ABFH) \ldots (1)$

Similarly, we can prove that

ar $(\Delta HGF) = \frac{1}{2}$ ar $(HDCF) \ldots (2)$

On adding equations (1) and (2), we obtain

$$\text{ar }(\Delta HEF) + \text{ar }(\Delta HGF) = \frac{1}{2}\text{ar }(ABFH) + \frac{1}{2}\text{ar }(HDCF)$$

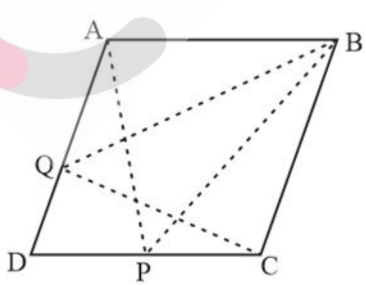
$$= \frac{1}{2}[\text{ar }(ABFH) + \text{ar }(HDCF)]$$

$$\Rightarrow \text{ar }(EFGH) = \frac{1}{2}\text{ar }(ABCD)$$

Hence proved.

3. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that $ar(APB) = ar(BQC)$.

Solution:



It can be observed that ΔBQC and parallelogram ABCD lie on the same base BC and between the same parallel lines AD and BC.

$\therefore \text{ar}(\Delta BQC) = \frac{1}{2}\text{ar}(ABCD) \ldots (1)$

Similarly, ΔAPB and parallelogram ABCD lie on the same base AB and between the same parallel lines AB and DC.

$\therefore \text{ar}(\Delta APB) = \frac{1}{2}\text{ar}(ABCD) \ldots (2)$

From equation (1) and (2), we obtain

$\text{ar}\ (\Delta BQC) = \text{ar}\ (\Delta APB)$

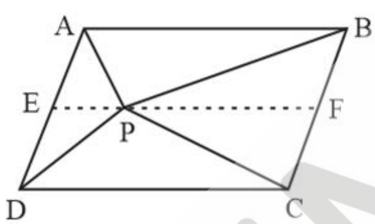
Hence proved.

4. In Figure, P is a point in the interior of a parallelogram ABCD. Show that

(i) $\text{ar}\ (APB) + \text{ar}\ (PCD) = \frac{1}{2}\text{ar}\ (ABCD)$

(ii) $\text{ar}\ (APD) + \text{ar}\ (PBC) = \text{ar}\ (APB) + \text{ar}\ (PCD)$

Solution:



(i) Draw a line segment EF, passing through point P and parallel to line segment AB.

In parallelogram ABCD,

$AB \parallel EF$ (By construction)… (1)

ABCD is a parallelogram.∴ $AD \parallel BC$ (Opposite sides of a parallelogram)

$\Rightarrow AE \parallel BF \ldots (2)$

From equations (1) and (2), we obtain

$AB \parallel EF$ and $AE \parallel BF$

Therefore, quadrilateral ABFE is a parallelogram

Since parallelogram ABFE and ΔAPB are lying between the same parallel lines EF and AB and on the same base AB.

$\therefore \text{ar}\ (\Delta APB) = \frac{1}{2}\text{ar}\ (ABFE) \ldots (3)$

Similarly, for ΔPCD and parallelogram EFCD,

$\text{ar}(\Delta PCD) = \frac{1}{2}\text{ar}(EFCD)\ldots(4)$

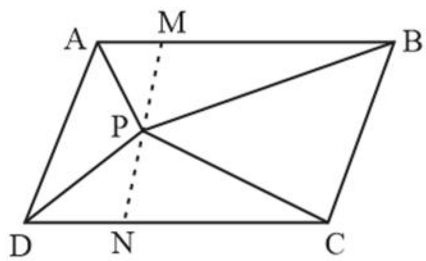
Adding equation (3) and (4), we obtain

$\text{ar}(\Delta APB) + \text{ar}(\Delta PCD) = \frac{1}{2}[\text{ar}(ABFE) + \text{ar}(EFCD)]$

$\Rightarrow \text{ar}(\Delta APB) + \text{ar}(\Delta PCD) = \frac{1}{2}\text{ar}(ABCD)\ldots(5)$

Hence, proved.

(ii)



Draw a line segment MN, passing through point P and parallel to line segment AD.

In parallelogram ABCD,

MN ∥ AD (By construction)... (6)

ABCD is a parallelogram.

∴ AB ∥ DC (Opposite sides of a parallelogram)

⇒ AM ∥ DN… (7)

From equations (6) and (7), we obtain

MN ∥ AD and AM ∥ DN

Therefore, quadrilateral AMND is a parallelogram.

It can be observed that ΔAPD and parallelogram AMND are lying between the same parallel lines AD and MN and on the same base AD.

$\therefore \text{ar}(\Delta APD) = \frac{1}{2}\text{ar}(AMND)\ldots(8)$

Similarly, for ΔPCB and parallelogram MNCB,

$\text{ar}(\Delta PCB) = \frac{1}{2}\text{ar}(MNCB)\ldots(9)$

Adding equations (8) and (9), we obtain

$(\Delta APD) + \text{ar}(\Delta PCB) = \frac{1}{2}[\text{ar}(AMND) + \text{ar}(MNCB)]$

$\Rightarrow (\Delta APD) + \text{ar}(\Delta PCB) = \frac{1}{2}\text{ar}(ABCD)\ldots(10)$

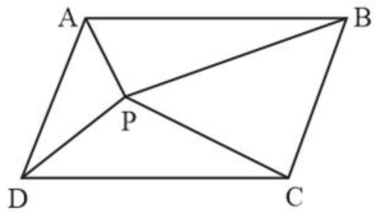
On comparing equations (5) and (10), we obtain

$(\Delta APD) + ar\ (\Delta PBC) = ar\ (\Delta APB) + ar\ (\Delta PCD)$

Hence proved.

Hint:

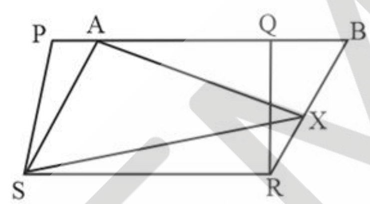
Through P, draw a line parallel to AB.



5. In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i) $ar\ (PQRS) = ar\ (ABRS)$

(ii) $ar\ (AXS) = \frac{1}{2} ar\ (PQRS)$



Solution:

It is seen that parallelogram PQRS and ABRS lie on the same base SR and also, these lie in between the same parallel lines SR and PB.

$\therefore ar\ (PQRS) = ar\ (ABRS)$ ... (1)

(ii) Consider ΔAXS and parallelogram ABRS.

As these lie on the same base and between the same parallel lines AS and BR,

$\therefore ar\ (\Delta AXS) = \frac{1}{2} ar\ (ABRS)$ ... (2)

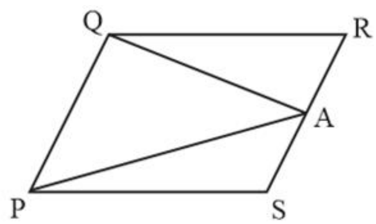
From equations (1) and (2), we obtain

$$ar\ (\Delta AXS) = \frac{1}{2} ar\ (PQRS)$$

6. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided?

What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Solution:



From the figure, it is clear that point A divides the field into three triangles $\Delta PSA, \Delta PAQ$, and ΔQRA

It is clear that,

Area of ΔPSA + Area of ΔPAQ +Area of ΔQRA = Area of parallelogram PQRS ... (1)

We know that if a parallelogram and a triangle are on the same base and between the same parallels, then the area of the triangle is half the area of the parallelogram.

$\therefore$ ar (PAQ) $= \frac{1}{2}$ar (PQRS)... (2)

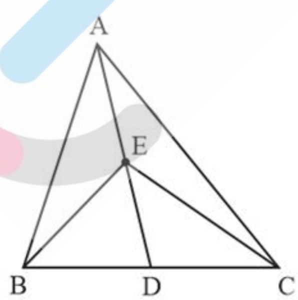
From equations (1) and (2), we obtain

ar (PSA) + ar (QRA) $= \frac{1}{2}$ar (PQRS) ... (3)

Therefore, the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.

Exercise: 9.3

1. In the given figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE)



Solution:

Given AD is the median of ΔABC. Therefore, AD divides ΔABC into two triangles of equal areas.

$\therefore$ ar (ΔABD) = ar (ΔACD)...(1)

ED is the median of ΔEBC.

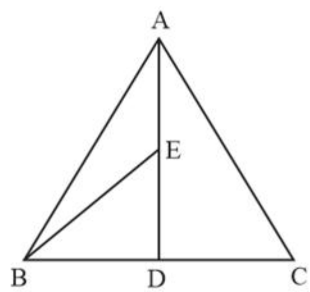
$\therefore$ ar (ΔEBD) = ar (ΔECD) ... (2)

On subtracting equation (2) from equation (1), we obtain

ar (ABD) − ar (EBD) = ar (ACD) − ar (ECD)

$\Rightarrow$ ar (ABE) = ar (ACE).

2. In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC)



Solution:

Given AD is the median of ΔABC. Therefore, it will divide ΔABC into two triangles of equal areas.

$\therefore$ ar (ΔABD) = ar (ΔACD)

$\Rightarrow$ ar (ΔABD) $= \frac{1}{2}$ ar (ΔABC) ... (1)

In ΔABD, E is the mid-point of AD. Therefore, BE is the median.

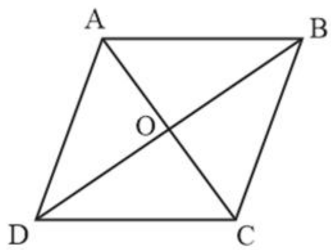
$\therefore$ ar (ΔBED) = ar (ΔABE)

$\Rightarrow$ ar (ΔBED) $= \frac{1}{2}$ ar (ΔABD)

$\Rightarrow$ ar (ΔBED) $= \frac{1}{2} \times \frac{1}{2}$ ar (ΔABC) [From equation (1)]

$\Rightarrow$ ar (ΔBED) $= \frac{1}{4}$ ar (ΔABC).

3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.



Solution:

We know that diagonals of parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in ΔABC. Therefore, it will divide it into two triangles of equal areas.

$\therefore$ ar (ΔAOB) = ar (ΔBOC) ... (1)

In ΔBCD, CO is the median.

$\therefore$ ar (ΔBOC) = ar (ΔCOD) ... (2)

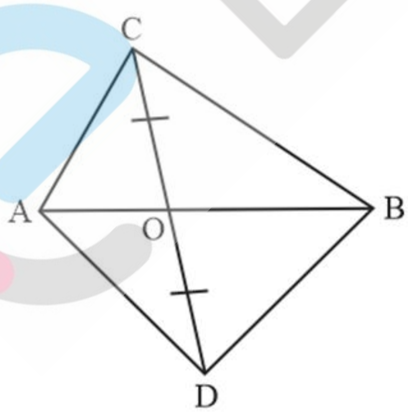
Similarly, ar (ΔCOD) = ar (ΔAOD) ... (3)

From equations (1), (2), and (3), we obtain

ar (ΔAOB) = ar (ΔBOC) = ar (ΔCOD) = ar (ΔAOD)

Therefore, the diagonals of a parallelogram divide it into four triangles of equal area.

4. In the given figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Solution:

Consider ΔACD.

Given that the line-segment CD is bisected by AB at O, AO becomes the median of ΔACD.

$\therefore$ ar (ΔACO) = ar (ΔADO) ... (1)

Considering ΔBCD, BO is the median.

$\therefore$ ar (ΔBCO) = ar (ΔBDO) ... (2)

Adding equations (1) and (2), we obtain

ar (ΔACO) + ar (ΔBCO) = ar (ΔADO) + ar (ΔBDO)

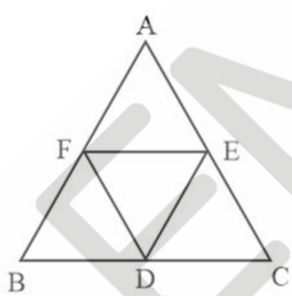
$\Rightarrow$ ar (ΔABC) = ar (ΔABD)

5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that

(i) BDEF is a parallelogram.

(ii) ar (DEF) $= \frac{1}{4}$ar (ABC)

ar (BDEF) $= \dfrac{1}{2}$ar (ABC)



Solution:

In ΔABC,

E and F are the mid-points of side AC and AB respectively.

From mid-point theorem, we have EF $\parallel$ BC and EF $= \frac{1}{2}$BC

Also, BD $= \frac{1}{2}$BC (D is the mid-point of BC)

Therefore, BD = EF and BD $\parallel$ EF

Therefore, BDEF is a parallelogram.

From (i), it can be said that quadrilaterals BDEF, DCEF, AFDE are parallelograms.

Since the diagonal of a parallelogram divides it into two triangles of equal area.

We have, ar (ΔBFD) = ar (ΔDEF) … (For parallelogram BD)

ar (ΔCDE) = ar (ΔDEF) … (For parallelogram DCEF)

ar (ΔAFE) = ar (ΔDEF) … (For parallelogram AFDE)

$\therefore$ ar (ΔAFE) = ar (ΔBFD) = ar (ΔCDE) = ar (ΔDEF)

Also,

ar (ΔAFE) + ar (ΔBDF) + ar (ΔCDE) + ar (ΔDEF) = ar (ΔABC)

$\Rightarrow$ ar (ΔDEF) + ar (ΔDEF) + ar (ΔDEF) + ar (ΔDEF) = ar (ΔABC)

$\Rightarrow$ 4ar (ΔDEF) = ar (ΔABC)

$\Rightarrow$ ar (ΔDEF) $= \frac{1}{4}$ar (ΔABC)

(iii) ar(BDEF) = ar (ΔDEF) + ar (ΔBDF)

$\Rightarrow$ ar (BDEF) = ar (ΔDEF) + ar (ΔDEF)

$\Rightarrow$ ar (BDEF) = 2ar (ΔDEF)

$\Rightarrow$ ar (BDEF) $= 2 \times \frac{1}{4}$ar (ΔABC)

$\Rightarrow$ ar (BDEF) $= \frac{1}{2}$ar (ΔABC)

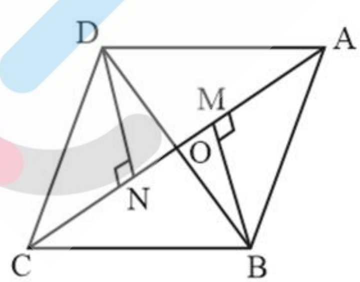
6. In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA $\parallel$ CB or ABCD is a parallelogram.

Solution:



Construct DN $\perp$ AC and BM $\perp$ AC.

(i) In ΔDON and ΔBOM,

$\angle$DNO = $\angle$BMO (By construction)

$\angle DON = \angle BOM$ (Vertically opposite angles)

$OD = OB$ (Given)

By AAS congruence rule,

$\Delta DON \cong \Delta BOM$

$\therefore DN = BM$ ... (1)

We know that congruent triangles have equal areas.

$\therefore$ ar $(\Delta DON) =$ ar (ΔBOM) ... (2)

In ΔDNC and ΔBMA,

$\angle DNC = \angle BMA$ (By construction)

$CD = AB$ (Given)

$DN = BM$ [Using equation (1)]

$\therefore \Delta DNC \cong \Delta BMA$ (RHS congruence rule)

$\Rightarrow$ ar $(\Delta DNC) =$ ar (ΔBMA) ... (3)

On adding equations (2) and (3), we obtain

ar $(\Delta DON) +$ ar $(\Delta DNC) =$ ar $(\Delta BOM) +$ ar (ΔBMA)

Therefore, ar $(\Delta DOC) =$ ar (ΔAOB)

(ii) From (i), we have

ar $(\Delta DOC) =$ ar (ΔAOB)

$\Rightarrow$ ar $(\Delta DOC) +$ ar $(\Delta OCB) =$ ar $(\Delta AOB) +$ ar (ΔOCB)

(Adding ar (ΔOCB) to both sides)

$\Rightarrow$ ar $(\Delta DCB) =$ ar (ΔACB)

(iii) From (ii), we have

ar $(\Delta DCB) =$ ar (ΔACB)

If two triangles have the same base and equal areas, then these will lie between the same parallels.

$\therefore DA \parallel CB$ ... (4)

In ΔDOA and ΔBOC,

$\angle DOA = \angle BOC$ (Vertically opposite angles)

$OD = OB$ (Given)

$\angle ODA = \angle OBC$ (Alternate opposite angles)

By ASA congruence rule,

$\Delta DOA \cong \Delta BOC$

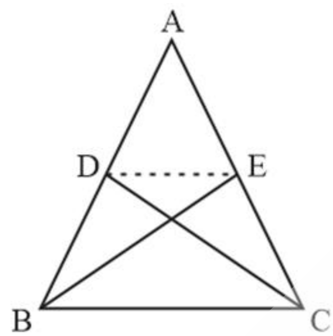
$\therefore DA = BC$ ... (5)

In quadrilateral ABCD, one pair of opposite sides is equal and parallel $(AD = BC)$

Therefore, ABCD is a parallelogram.

7. D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE ∥ BC.

Solution:

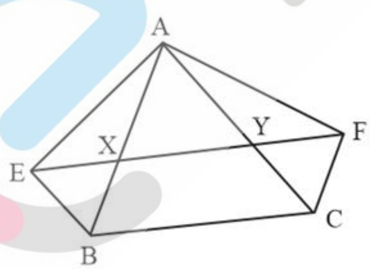


Since ΔBCE and ΔBCD are lying on a common base BC and also have equal areas, ΔBCE and ΔBCD will lie between the same parallel lines. [As per theorem]

$\therefore DE \parallel BC$

8. XY is a line parallel to side BC of a triangle ABC. If BE ∥ AC and CF ∥ AB meet XY at E and F respectively, show that ar (ABE) = ar (ACF)

Solution:



It is given that

$XY \parallel BC \Rightarrow EY \parallel BC$

$BE \parallel AC \Rightarrow BE \parallel CY$

Therefore, EBCY is a parallelogram.

It is given that

XY ∥ BC ⇒ XF ∥ BC

FC ∥ AB ⇒ FC ∥ XB

Therefore, BCFX is a parallelogram.

Parallelograms EBCY and BCFX are on the same base BC and between the same parallels BC and EF.

$\therefore$ ar (EBCY) = ar (BCFX) ... (1)

Consider parallelogram EBCY and ΔAEB

These lie on the same base BE and are between the same parallels BE and AC.

$\therefore$ ar (ΔABE) $= \frac{1}{2}$ ar (EBCY) ... (2)

Also, parallelogram BCFX and ΔACF are on the same base CF and between the same parallels CF and AB.

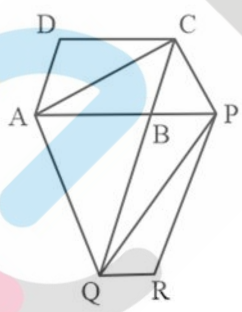
$\therefore$ ar (ΔACF) $= \frac{1}{2}$ ar (BCFX) ... (3)

From equations (1), (2), and (3), we obtain

ar (ΔABE) = ar (ΔACF)

9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that ar (ABCD) = ar (PBQR).

Solution:



Upon joining AC and PQ, we have

ΔACQ and ΔAQP are on the same base AQ and between the same parallels AQ and CP.

$\therefore$ (ΔACQ) = ar (ΔAPQ)

⇒ ar (ΔACQ) − ar (ΔABQ) = ar (ΔAPQ) − ar (ΔABQ)

$\Rightarrow$ ar (ΔABC) = ar (ΔQBP) ... (1)

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively,

$\therefore$ ar (ΔABC) $= \frac{1}{2}$ ar (ABCD) ... (2)

ar (ΔQBP) $= \frac{1}{2}$ ar (PBQR) ... (3)

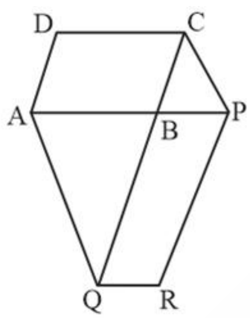
From equations (1), (2), and (3), we obtain

$\frac{1}{2}$ ar (ABCD) $= \frac{1}{2}$ ar (PBQR)

$\Rightarrow$ ar (ABCD) = ar (PBQR)

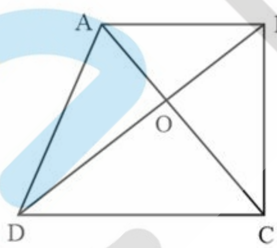
Hint:

Join AC and PQ. Now compare ar (ACQ) and ar (APQ)



10. Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

Solution:



We see that ΔDAC and ΔDBC lie on the same base DC and between the same parallels AB and CD.

$\therefore$ ar (ΔDAC) = ar (ΔDBC)

$\Rightarrow$ ar (ΔDAC) $-$ ar (ΔDOC) = ar (ΔDBC) $-$ ar (ΔDOC)

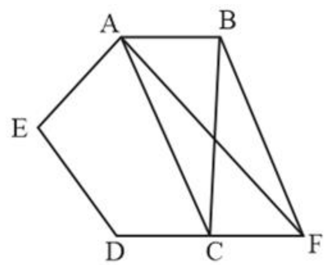
(Subtracting ar (ΔDOC) from both sides)

$\Rightarrow$ ar (ΔAOD) = ar (ΔBOC)

11. In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)



Solution:

ΔACB and ΔACF lie on the same base AC and are between the same parallels AC and BF.

$\therefore$ ar (ΔACB) = ar (ΔACF)

(ii) It can be observed that

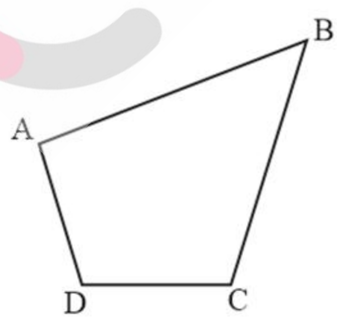
ar (ΔACB) = ar (ΔACF)

$\Rightarrow$ ar (ΔACB) + ar (ACDE) = ar (ACF) + ar (ACDE)

$\Rightarrow$ ar (ABCDE) = ar (AEDF)

12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

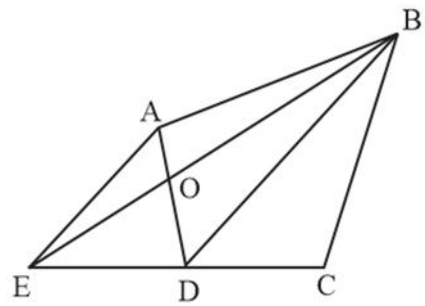
Solution:



Let quadrilateral ABCD be the actual shape of the field.

Join diagonal BD and draw a line parallel to BD through point A. Let it meet the extended side CD of ABCD at point E. Join BE and AD. Let them intersect each other at O. Then, portion ΔAOB can be cut from the original field so that the new shape of the field will be ΔBCE. (See figure)

We have to prove that the area of ΔAOB (portion that was cut so as to construct Health Centre) is equal to their of ΔDEO (portion added to the field so as to make the area of the new field so formed equal to the area of the original field)



It can be observed that ΔDEB and ΔDAB lie on the same base BD and are between the same parallels BD and AE.

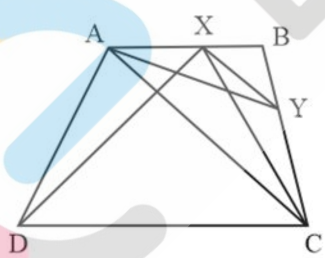
$\therefore$ ar (ΔDEB) = ar (ΔDAB)

$\Rightarrow$ ar (ΔDEB) − ar (ΔDOB) = ar (ΔDAB) − ar (ΔDOB)

$\Rightarrow$ ar (ΔDEO) = ar (ΔAOB)

13. ABCD is a trapezium with AB ∥ DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

Solution:



We see that ΔADX and ΔACX lie on the same base AX and are between the same parallels AB and DC.

$\therefore$ ar (ΔADX) = ar (ΔACX) ... (1)

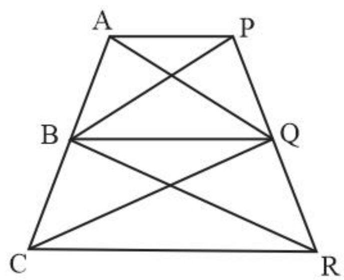
ΔACY and ΔACX lie on the same base AC and are between the same parallels AC and XY.

$\therefore$ ar (ΔACY) = ar (ACX) ... (2)

From equations (1) and (2), we obtain

ar (ΔADX) = ar (ΔACY)

14. In the given figure, AP ∥ BQ ∥ CR. Prove that ar (AQC) = ar (PBR).



Solution:

Since ΔABQ and ΔPBQ lie on the same base BQ and are between the same parallels AP and BQ,

$\therefore$ ar (ΔABQ) = ar (ΔPBQ) ... (1)

Also, ΔBCQ and ΔBRQ lie on the same base BQ and are between the same parallels BQ and CR.

$\therefore$ ar (ΔBCQ) = ar (ΔBRQ) ... (2)

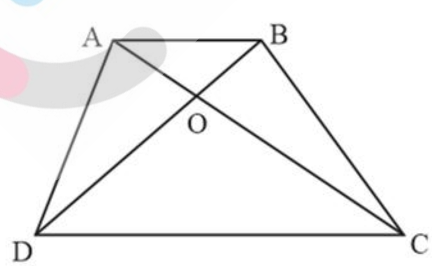
On adding equations (1) and (2), we obtain

ar (ΔABQ) + ar (ΔBCQ) = ar (ΔPBQ) + ar (ΔBRQ)

$\Rightarrow$ ar (ΔAQC) = ar (ΔPBR)

15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

Solution:



Given that,

ar (ΔAOD) = ar (ΔBOC)

$\text{ar}(\Delta AOD) + \text{ar}(\Delta AOB) = \text{ar}(\Delta BOC) + \text{ar}(\Delta AOB)$

$\Rightarrow \text{ar}(\Delta ADB) = \text{ar}(\Delta ACB)$

We know that triangles on the same base having areas equal to each other lie between the same parallels.

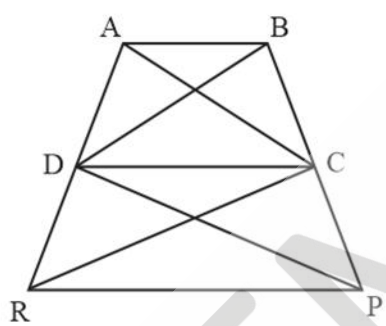
Therefore, these triangles, ΔADB and ΔACB, should be lying between the same parallels.

i.e., $AB \parallel CD$

Therefore, ABCD is a trapezium.

16. In the given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

Solution:



Given that,

$\text{ar}(\Delta DRC) = \text{ar}(\Delta DPC)$

As ΔDRC and ΔDPC lie on the same base DC and their areas are equal, therefore, they must lie between the same parallel lines.

$\therefore DC \parallel RP$

Therefore, DCPR is a trapezium.

It is also given that

$\text{ar}(\Delta BDP) = \text{ar}(\Delta ARC)$

$\Rightarrow \text{ar}(BDP) - \text{ar}(\Delta DPC) = \text{ar}(\Delta ARC) - \text{ar}(\Delta DRC)$

$\Rightarrow \text{ar}(\Delta BDC) = \text{ar}(\Delta ADC)$

Since ΔBDC and ΔADC are on the same base CD and their areas are equal, they must lie between the same parallel lines.

$\therefore AB \parallel CD$

Therefore, ABCD is a trapezium

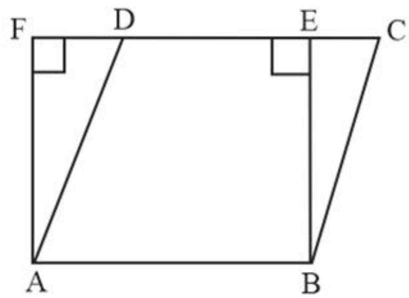
Exercise: 9.4

1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Solution:

As the parallelogram and the rectangle have the same base and equal area, these will have to lie between the same parallels.

Consider the parallelogram ABCD and rectangle ABEF as follows.



Here, it can be seen that parallelogram ABCD and rectangle ABEF are between the same parallels AB and CF.

We know that opposite sides of a parallelogram or a rectangle are of equal lengths. Therefore,

$AB = EF$… (For rectangle)

$AB = CD$…(For parallelogram)

$\therefore CD = EF$

$\Rightarrow AB + CD = AB + EF$ ... (1)

We know that, of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

$\therefore AF < AD$

And similarly, $BE < BC$

$\therefore AF + BE < AD + BC$ ... (2)

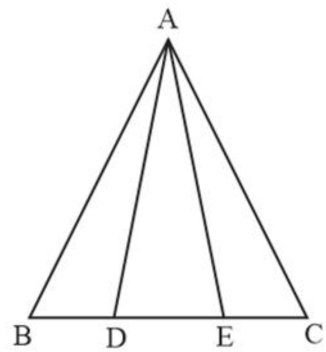
From equations (1) and (2), we obtain

$AB + EF + AF + BE < AD + BC + AB + CD$

Hence, perimeter of rectangle ABEF < Perimeter of parallelogram ABCD

2. In the following figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).

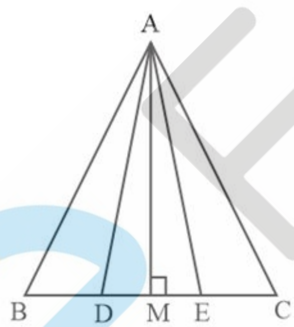
Can you answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?



[Remark: Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into n triangles of equal areas.]

Solution:

Drawing a line segment AM ⊥ BC, we obtain the following figure:



We know that,

Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Altitude

$$\text{ar}(\Delta ADE) = \frac{1}{2} \times DE \times AM$$

$$\text{ar}(\Delta ABD) = \frac{1}{2} \times BD \times AM$$

$$\text{ar}(\Delta AEC) = \frac{1}{2} \times EC \times AM$$

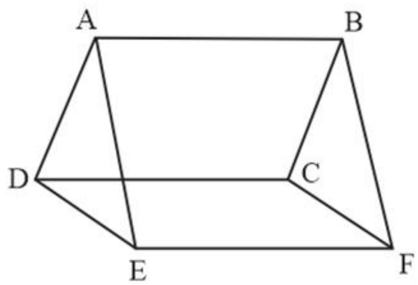
It is given that DE = BD = EC

$$\Rightarrow \frac{1}{2} \times DE \times AM = \frac{1}{2} \times BD \times AM = \frac{1}{2} \times EC \times AM$$

$$\Rightarrow ar\ (\Delta ADE) = ar\ (\Delta ABD) = ar\ (\Delta AEC)$$

It can be observed that Budhia has divided her field into 3 equal parts.

3. In the following figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Solution:

Given that ABCD is a parallelogram, we know that opposite sides of a parallelogram are equal.

$\therefore$ AD = BC ... (1)

Similarly, for parallelograms DCEF and ABFE, it can be proved that

DE = CF ... (2)

And, EA = FB ... (3)

In ΔADE and ΔBCF,

AD = BC [Using equation (1)]

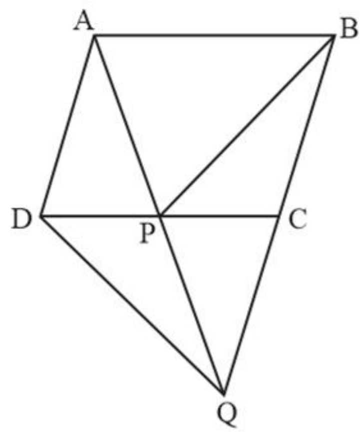
DE = CF [Using equation (2)]

EA = FB [Using equation (3)]

$\therefore \Delta ADE \cong BCF$ (SSS congruence rule)

$\Rightarrow ar\ (\Delta ADE) = ar\ (\Delta BCF)$

4. In the following figure, ABCD is parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (BPC) = ar (DPQ).

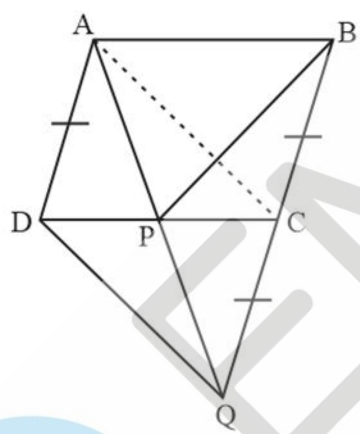


Solution:

Given that ABCD is a parallelogram, we have

AD ∥ BC and AB ∥ DC (Opposite sides of a parallelogram are parallel to each other)

Join point A to point C.



Consider ΔAPC and ΔBPC

ΔAPC and ΔBPC are lying on the same base PC and between the same parallels PC and AB. Therefore,

ar (ΔAPC) = ar (ΔBPC) ... (1)

In quadrilateral ACQD, it is given that

AD = CQ

Since ABCD is a parallelogram,

AD ∥ BC (Opposite sides of a parallelogram are parallel)

CQ is a line segment which is obtained when line segment BC is produced.

$\therefore$ AD ∥ CQ

We have,

AC = DQ and AC ∥ DQ

Hence, ACQD is a parallelogram.

Consider ΔDCQ and ΔACQ

These are on the same base CQ and between the same parallels CQ and AD. Therefore,

$\text{ar}(\Delta DCQ) = \text{ar}(\Delta ACQ)$

$\Rightarrow \text{ar}(\Delta DCQ) - \text{ar}(\Delta PQC) = \text{ar}(\Delta ACQ) - \text{ar}(\Delta PQC)$

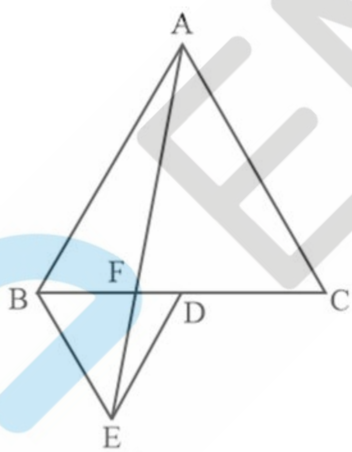
(Subtract ar (PQC) from both sides)

$\Rightarrow \text{ar}(\Delta DPQ) = \text{ar}(\Delta APC)$ ... (2)

From equations (1) and (2), we obtain

$\text{ar}(\Delta BPC) = \text{ar}(\Delta DPQ)$

5. In the following figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that



(i) $\text{ar}(BDE) = \frac{1}{4}\text{ar}(ABC)$

(ii) $\text{ar}(BDE) = \frac{1}{2}\text{ar}(BAE)$

(iii) $\text{ar}(ABC) = 2\,\text{ar}(BEC)$

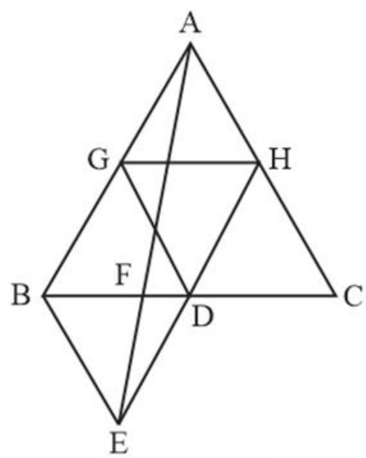
(iv) $\text{ar}(BFE) = 2\,\text{ar}(AFD)$

(v) $\text{ar}(BFE) = 2\,\text{ar}(FED)$

(vi) ar (FED) $= \frac{1}{8}$ ar (AFC)

Solution:

(i) Consider G and H to be the mid-points of side AB and AC respectively. Line segment GH is joining the mid-points. Therefore, by mid-point theorem. it will be parallel to third side BC and also its length will be half of the length of BC



$\Rightarrow \text{GH} = \frac{1}{2}$ BC and GH $\parallel$ BD

$\Rightarrow$ GH = BD = DC and GH $\parallel$ BD (D is the mid-point of BC)

Consider quadrilateral GHDB.

GH $\parallel$ BD and GH = BD

Two line segments joining two parallel line segments of equal length will also be equal and parallel to each other.

Therefore, BG = DH and BG $\parallel$ DH

Hence, quadrilateral GHDB is a parallelogram.

We know that in a parallelogram, the diagonal bisects it into two triangles of equal area.

Hence, ar (ΔBDG) = ar (ΔHGD)

Similarly, it can be proved that quadrilaterals DCHG, GDHA, and BEDG are parallelograms and their respective diagonals are dividing them into two triangles of equal area.

ar (ΔGDH) = ar (ΔCHD)… (For parallelogram DCHG)

ar (ΔGDH) = ar (ΔHAG)… (For parallelogram GDHA)

ar (ΔBDE) = ar (ΔDBG)… (For parallelogram BEDG)

$\text{ar}(\Delta ABC) = \text{ar}(\Delta BDG) + \text{ar}(\Delta GDH) + \text{ar}(\Delta DCH) + \text{ar}(\Delta AGH)$

$\text{ar}(\Delta ABC) = 4 \times \text{ar}(\Delta BDE)$

Hence, $\text{ar}(BDE) = \frac{1}{4}\text{ar}(ABC)$

(ii) $\text{ar}(\Delta BDE) = \text{ar}(\Delta AED)\ldots$ (Common base DE and DE ∥ AB)

$\text{ar}(\Delta BDE) - \text{ar}(\Delta FED) = \text{ar}(\Delta AED) - \text{ar}(\Delta FED)$

$\text{ar}(\Delta BEF) = \text{ar}(\Delta AFD)\ \ldots(1)$

$\text{ar}(\Delta ABD) = \text{ar}(\Delta ABF) + \text{ar}(\Delta AFD)$

$\text{ar}(\Delta ABD) = \text{ar}(\Delta ABF) + \text{ar}(\Delta BEF)\ \ldots$[From equation (1)]

$\text{ar}(\Delta ABD) = \text{ar}(\Delta ABE)\ldots(2)$

AD is the median in ΔABC.

$\text{ar}(\Delta ABD) = \frac{1}{2}\text{ar}(\Delta ABC)$

$= \frac{4}{2}\text{ar}(\Delta BDE)$ (As proved earlier)

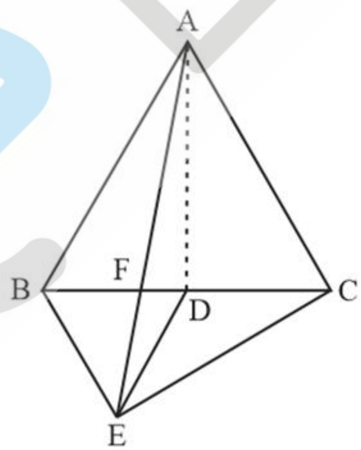
$\text{ar}(\Delta ABD) = 2\text{ar}(\Delta BDE)$ (3)

From (2) and (3), we obtain

$2\ \text{ar}(\Delta BDE) = \text{ar}(\Delta ABE)$ or

$\text{ar}(\Delta BDE) = \frac{1}{2}\text{ar}(\Delta ABE)$

(iii)



$\text{ar}(\Delta ABE) = \text{ar}(\Delta BEC)\ \ldots$ (Common base BE and BE ∥ AC)

$\text{ar}(\Delta ABF) + \text{ar}(\Delta BEF) = \text{ar}(\Delta BEC)$

Using equation (1), we obtain

$\text{ar}\ (\Delta ABF) + \text{ar}\ (\Delta AFD) = \text{ar}\ (\Delta BEC)$

$\text{ar}\ (\Delta ABD) = \text{ar}\ (\Delta BEC)$

$\frac{1}{2}\ \text{ar}\ (\Delta ABC) = \text{ar}\ (\Delta BEC)$

$\text{ar}\ (\Delta ABC) = 2\ \text{ar}\ (\Delta BEC)$

(iv) It is seen that ΔBDE and ar ΔAED lie on the same base DE and between the parallels DE and AB.

$\therefore \text{ar}\ (\Delta BDE) = \text{ar}\ (\Delta AED)$

$\Rightarrow \text{ar}\ (\Delta BDE) - \text{ar}\ (\Delta FED) = \text{ar}\ (\Delta AED) - \text{ar}\ (\Delta FED)$

$\therefore \text{ar}\ (\Delta BFE) = \text{ar}\ (\Delta AFD)$

(v) Let h be the height of vertex E, corresponding to the side BD in ΔBDE.

Let H be the height of vertex A, corresponding to the side BC in ΔABC.

In (i), we have seen that $\text{ar}(BDE) = \frac{1}{4}\ \text{ar}\ (ABC)$.

$$\therefore \frac{1}{2} \times BD \times h = \frac{1}{4}\left(\frac{1}{2} \times BC \times H\right)$$

$\Rightarrow BD \times h = \frac{1}{4}\left(\frac{1}{2} \times BC \times H\right)$

$\Rightarrow h = \frac{1}{2}H$

In (iv), we have seen that $\text{ar}\ (\Delta BFE) = \ \text{ar}\ (\Delta AFD)$.

$\therefore \text{ar}\ (\Delta BFE) = \text{ar}\ (\Delta AFD)$

$= \frac{1}{2} \times FD \times H = \frac{1}{2} \times FD \times 2h = 2\left(\frac{1}{2} \times FD \times h\right)$

$= 2\ \text{ar}\ (\Delta FED)$

Hence, $\text{ar}(BFE) = 2\text{ar}(FED)$.

(vi) $\text{ar}\ (AFC) = \text{ar}\ (AFD) + \text{ar}\ (ADC)$

$= \text{ar}(BFE) + \frac{1}{2}\text{ar}(ABC)$ [In (iv), $\text{ar}\ (BFE) = \text{ar}\ (AFE)$; AD is median of ΔABC]

$= \text{ar}(BFE) + \frac{1}{2}\ \text{ar}\ (ABC)$ [In (i), $\text{ar}\ (BDE) = \frac{1}{4}\ \text{ar}\ (ABC)$]

$= \text{ar}(BFE) + 2\ \text{ar}\ (BDE)$ … (5)

Now, by (v), $ar(BFE) = 2\ ar\ (FED) \ldots (6)$

$ar\ (BDE) = ar\ (BFE) + ar\ (FED) = 2\ ar\ (FED) + ar\ (FED) = 3\ ar\ (FED) \quad \ldots (7)$

Therefore, from equations (5), (6), and (7), we get:

$ar\ (AFC) = 2\ ar\ (FED) + 2 \times 3\ ar\ (FED) = 8ar\ (FED)$

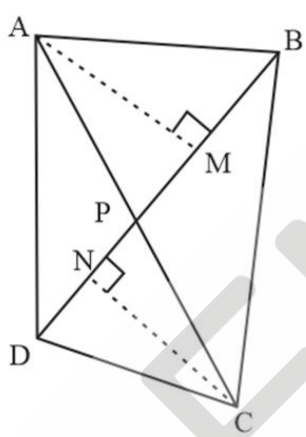
$\therefore ar\ (AFC) = 8\ ar\ (FED)$

Hence, $ar\ (FED) = \frac{1}{8} ar\ (AFC)$

6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that $ar\ (APB) \times ar\ (CPD) = ar\ (APD) \times ar\ (BPC)$

Solution:

Let us construct AM ⊥ BD and CN ⊥ BD



We know that thear of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$$ar\ (APB) \times ar\ (CPD) = \left[\frac{1}{2} \times BP \times AM\right] \times \left[\frac{1}{2} \times PD \times CN\right]$$

$$= \frac{1}{4} \times BP \times AM \times PD \times CN$$

$$ar\ (APD) \times ar\ (BPC) = \left[\frac{1}{2} \times PD \times AM\right] \times \left[\frac{1}{2} \times CN \times BP\right]$$

$$= \frac{1}{4} \times PD \times AM \times CN \times BP$$

$$= \frac{1}{4} \times BP \times AM \times PD \times CN$$

$\therefore ar\ (APB) \times ar\ (CPD) = ar\ (APD) \times ar\ (BPC)$

7. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

(i) $\text{ar (PRQ)} = \frac{1}{2}\text{ ar (ARC)}$

(ii) $\text{ar (RQC)} = \frac{3}{8}\text{ ar (ABC)}$

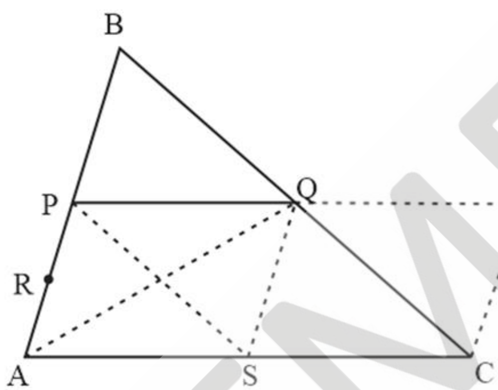
(iii) $\text{ar (PBQ)} = \text{ar (ARC)}$

Solution:

Consider a point S on AC such that S is the mid-point of AC.

Extend PQ to T such that PQ $=$ QT.

Join TC, QS, PS, and AQ.



In ΔABC, P and Q are the mid-points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

$\text{PQ} \parallel \text{AC and PQ} = \frac{1}{2}\text{ AC}$

$\Rightarrow \text{PQ} \parallel \text{AS and PQ} = \text{AS}$ (As S is the mid-point of AC)

$\therefore$ PQSA is a parallelogram. We know that diagonals of a parallelogram bisect it into equal areas of triangles.

$\therefore \text{ar}(\Delta\text{PAS}) = \text{ar}(\Delta\text{SQP}) = \text{ar}(\Delta\text{PAQ}) = \text{ar}(\Delta\text{SQA})$

Similarly, it is possible to prove that quadrilaterals PSCQ, QSCT, and PSQB are also parallelograms and therefore,

$\text{ar}(\Delta\text{PSQ}) = \text{ar}(\Delta\text{CQS})\ldots$ (For parallelogram PSCQ)

$\text{ar}(\Delta\text{QSC}) = \text{ar}(\Delta\text{CTQ})\ldots$ (For parallelogram QSCT)

$\text{ar}(\Delta\text{PSQ}) = \text{ar}(\Delta\text{QBP})\ldots$ (For parallelogram PSQB)

Thus,

ar (ΔPAS) = ar (ΔSQP) = ar (ΔPAQ) = ar (ΔSQA) = ar (ΔQSC) =
ar (ΔCTQ) = ar (ΔQBP) ... (1)

Also, ar (ΔABC) = ar (ΔPBQ) + ar (ΔPAS) + ar (ΔPQS) + ar (ΔQSC)

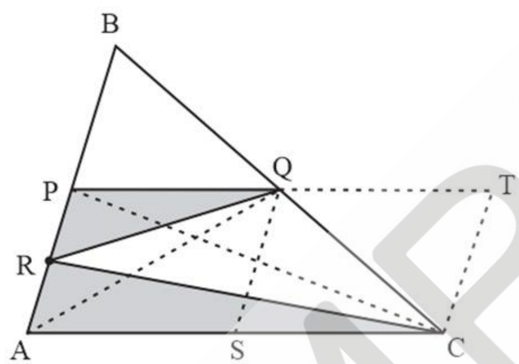
ar (ΔABC) = ar (ΔPBQ) + ar (ΔPBQ) + ar (ΔPBQ) + ar (ΔPBQ)

= ar (ΔPBQ) + ar (ΔPBQ) + ar (ΔPBQ) + ar (ΔPBQ)

= 4 ar (ΔPBQ)

$\Rightarrow$ ar $(\Delta PBQ) = \frac{1}{4}$ ar (ΔABC) ... (2)

(i) Join point P to C.



In ΔPAQ, QR is the median.

$\therefore$ ar $(\Delta PRQ) = \frac{1}{2}$ ar $(\Delta PAQ) = \frac{1}{2} \times \frac{1}{4}$ ar $(\Delta ABC) = \frac{1}{8}$ ar (ΔABC) ... (3)

In ΔABC, P and Q are the mid-points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

$$PQ = \frac{1}{2}AC$$

$AC = 2PQ \Rightarrow AC = PT$

Also, $PQ \parallel AC \Rightarrow PT \parallel AC$

Hence, PACT is a parallelogram.

ar (PACT) = ar (PACQ) + ar (ΔQTC)

= ar (PACQ) + ar (ΔPBQ)… [Using equation (1)]

$\therefore$ ar (PACT) = ar (ΔABC) ... (4)

ar $(\Delta ARC) = \frac{1}{2}$ ar (ΔPAC) (CR is the median of ΔPAC)

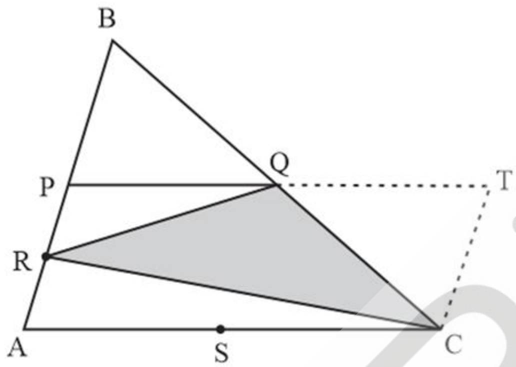
$= \frac{1}{2} \times \frac{1}{2}$ ar (PACT) (PC is the diagonal of parallelogram PACT)

$= \frac{1}{4}$ ar(ΔPACT) $= \frac{1}{4}$ ar (ΔABC)

$\Rightarrow \frac{1}{2}$ ar (ΔARC) $= \frac{1}{8}$ ar(ΔABC)

$\Rightarrow \frac{1}{2}$ ar (ΔARC) = ar(ΔPRQ) [Using equation (3)] … (5)

(ii)



ar (PACT) = ar (ΔPRQ) + ar (ΔARC) + ar (ΔQTC) + ar (ΔRQC)

Putting the values from equations (1), (2), (3), (4), and (5), we obtain

ar(ΔABC) $= \frac{1}{8}$ ar (ΔABC) $+ \frac{1}{4}$ ar (ΔABC) $+ \frac{1}{4}$ ar (ΔABC) + ar (ΔRQC)

$\Rightarrow$ ar (ΔABC) $= \frac{5}{8}$ ar (ΔABC) + ar (ΔRQC)

ar (ΔRQC) $= \left(1 - \frac{5}{8}\right)$ ar (ΔABC)

$\Rightarrow$ ar (ΔRQC) $= \frac{3}{8}$ ar (ΔABC)

(iii) In parallelogram PACT,

ar (ΔARC) $= \frac{1}{2}$ ar (ΔPAC) (CR is the median of ΔPAC)

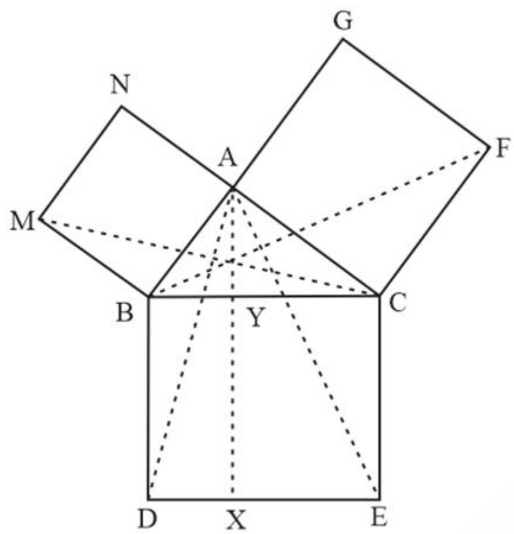
$= \frac{1}{2} \times \frac{1}{2}$ ar (PACT) (PC is the diagonal of parallelogram PACT)

$= \frac{1}{4}$ ar (ΔPACT)

$= \frac{1}{4}$ ar (ΔABC)

= ar (ΔPBQ)

8. In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that:



(i) $\Delta MBC \cong \Delta ABD$

(ii) $ar\ (BYXD) = 2\ ar\ (MBC)$

(iii) $ar\ (BYXD) = 2\ ar\ (ABMN)$

(iv) $\Delta FCB \cong \Delta ACE$

(v) $ar\ (CYXE) = 2\ ar\ (FCB)$

(vi) $ar\ (CYXE) = ar\ (ACFG)$

(vii) $ar\ (BCED) = ar\ (ABMN) + ar\ (ACFG)$

Note:

Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in class X.

Solution:

(i) Since each angle of a square is equal to 90^o.

Hence, $\angle ABM = \angle DBC = 90^o$

$\Rightarrow \angle ABM + \angle ABC = \angle DBC + \angle ABC$

$\Rightarrow \angle MBC = \angle ABD$

In ΔMBC and ΔABD,

$\angle MBC = \angle ABD$ (Proved above)

$MB = AB$ (Sides of square ABMN)

$BC = BD$ (Sides of square BCED)

$\therefore \Delta MBC \cong \Delta ABD$ (SAS congruence rule)

(ii) We have

$\Delta MBC \cong \Delta ABD$

$\Rightarrow$ ar $(\Delta MBC) =$ ar (ΔABD) ... (1)

It is given that $AX \perp DE$ and $BD \perp DE$ (Adjacent sides of square BDEC)

$\Rightarrow BD \parallel AX$ (Two lines perpendicular to same line are parallel to each other)

ΔABD and parallelogram BYXD are on the same base BD and between the same parallels BD and AX.

$\therefore$ ar$(\Delta ABD) = \frac{1}{2}$ ar (BYXD)

ar(BYXD) $= 2$ ar (ΔABD)

ar (BYXD) $= 2$ar (ΔMBC) [Using equation (1)] ... (2)

(iii) ΔMBC and parallelogram ABMN are lying on the same base MB and between same parallels MB and NC.

$\therefore$ ar$(\Delta MBC) = \frac{1}{2}$ar(ABMN)

2 ar $(\Delta MBC) =$ ar (ABMN)

ar (BYXD) $=$ ar (ABMN) [Using equation (2)] ... (3)

(iv) We know that each angle of a square is 90^o.

$\therefore \angle FCA = \angle BCE = 90^o$

$\Rightarrow \angle FCA + \angle ACB = \angle BCE + \angle ACB$

$\Rightarrow \angle FCB = \angle ACE$

In ΔFCB and ΔACE,

$\angle FCB = \angle ACE$

$FC = AC$ (Sides of square ACFG)

$CB = CE$ (Sides of square BCED)

$\Delta FCB \cong \Delta ACE$ (SAS congruence rule)

(v) It is given that $AX \perp DE$ and $CE \perp DE$ (Adjacent sides of square BDEC)

Hence, $CE \parallel AX$ (Since, two lines perpendicular to the same line are parallel to each other)

Consider ΔACE and parallelogram CYXE

ΔACE and parallelogram CYXE are on the same base CE and between the same parallels CE and AX.

$\therefore \text{ar}(\Delta ACE) = \frac{1}{2}\text{ar}\,(CYXE)$

$\Rightarrow \text{ar}\,(CYXE) = 2\,\text{ar}\,(\Delta ACE)$ ... (4)

We have previously proved that

$\Delta FCB \cong \Delta ACE$

ar (ΔFCB) $\cong$ ar (ΔACE) ... (5)

On comparing equations (4) and (5), we obtain

ar (CYXE) = 2 ar (ΔFCB) ... (6)

(vi) Consider ΔFCB and parallelogram ACFG

ΔFCB and parallelogram ACFG are lying on the same base CF and between the same parallels CF and BG.

$\therefore \text{ar}(\Delta FCB) = \frac{1}{2}\text{ar}\,(ACFG)$

$\Rightarrow \text{ar}\,(ACFG) = 2\,\text{ar}\,(\Delta FCB)$

$\Rightarrow \text{ar}\,(ACFG) = \text{ar}\,(CYXE)$ [Using equation (6)] ... (7)

(vii) From the figure, it is evident that

ar (BCED) = ar (BYXD) + ar (CYXE)

$\Rightarrow$ ar (BCED) = ar (ABMN) + ar (ACFG) [Using equations (3) and (7)]