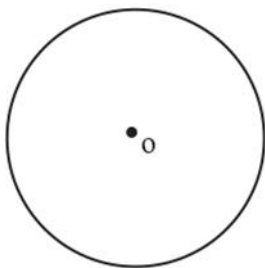


CBSE NCERT Solutions for Class 9 Mathematics Chapter 10**Back of Chapter Questions****Exercise: 10.1**

1. The center of a circle lies in _____ of the circle. (exterior/interior)

Solution:

The centre of a circle lies in the interior of the circle.



2. A point, whose distance from the center of a circle is greater than its radius lies in of the circle. (exterior/ interior)

Solution:

A point, whose distance from the centre of a circle is greater than its radius lies in the exterior of the circle.

3. The longest chord of a circle is a _____ of the circle.

Solution:

The circle has its longest chord passing through its centre and known as diameter of the circle.

4. An arc is a _____ when its ends are the ends of a diameter.

Solution:

An arc is a semicircle when its ends are the ends of a diameter.

5. Segment of a circle is the region between an arc and _____ of the circle.

Solution:

Segment of a circle is the region between an arc and chord of the circle.

6. A circle divides the plane, on which it lies, in _____ parts.

Solution:

Circle divides the plane, on which it lies, in three parts i.e. one part inside the circle, 2nd part on the circle and third part outside the circle.

7. Line segment joining the center of any point on the circle is a radius of the circle.

(A) True

(B) False

Solution: (A)

True, all the points on circle are at equal distance from the centre of circle, and this equal distance is called as radius of circle.

8. A circle has only finite number of equal chords.

(A) True

(B) False

Solution: (B)

False, on a circle there are infinite points. So, we can draw infinite number of chords of given length. Hence, a circle has infinite number of equal chords.

9. If a circle is divided into three equal arcs, each is a major arc.

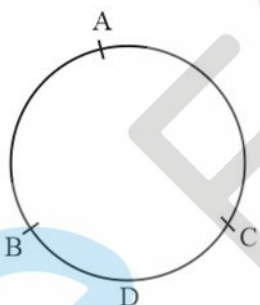
(A) True

(B) False

Solution: (B)

False, consider three arcs of same length as AB, BC and CA. Now we may see that for minor arc BDC, CAB is major arc.

So AB, BC and CA are minor arcs of circle.



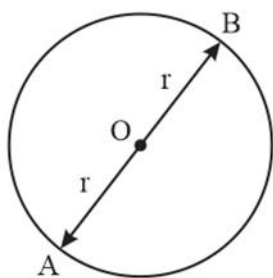
10. A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(A) True

(B) False

Solution: (A)

True, Let AB be a chord which is twice as long as its radius. In this situation our chord will be passing through centre of circle and we know any chord passing through the centre of the circle is known as the diameter of circle.



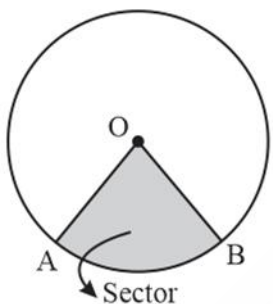
11. Sector is the region between the chord and its corresponding arc.

(A) True

(B) False

Solution: (B)

False, sector is the region between an arc and two radii joining the centre to the end points of the arc.



12. A circle is a plane figure.

(A) True

(B) False

Solution: (A)

True, a circle is a two dimensional figure and it can also be referred as plane figure

Exercise: 10.2

13. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centers.

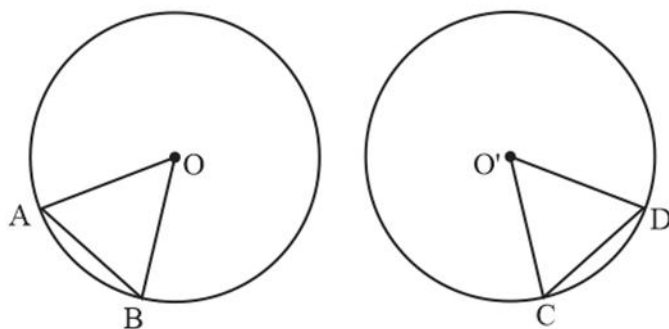
Solution:

A circle is a collection of points which are equidistant from a fix point known as the centre of the circle and this equal distance is called as radius of circle.

Now, if we try to superimpose two circles of equal radius one each other, both circles will cover each other.

So, two circles are congruent if they have equal radius.

Now consider two congruent circles having centre O and O' and two chords AB and CD of equal length.



In $\triangle AOB$ and $\triangle CO'D$

$AB = CD$ (chords of same length)

$OA = O'C$ (radii of congruent circles)

$OB = O'D$ (radii of congruent circles)

$\therefore \triangle AOB \cong \triangle CO'D$ (By SSS congruence rule)

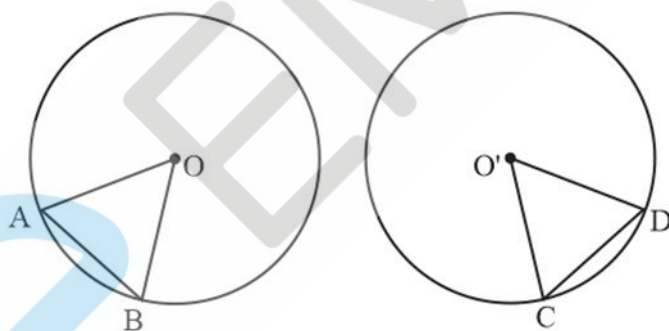
$\Rightarrow \angle AOB = \angle CO'D$ (by CPCT)

Hence equal chords of congruent circles subtend equal angles at their centres.

14. Prove that if chords of congruent circles subtend equal angles at their centers, then the chords are equal.

Solution:

Consider two congruent circles with centres as O and O' .



In $\triangle AOB$ and $\triangle CO'D$

$\angle AOB = \angle CO'D$ (given)

$OA = O'C$ (radii of congruent circles)

$OB = O'D$ (radii of congruent circles)

$\therefore \triangle AOB \cong \triangle CO'D$ (By SAS rule)

$\Rightarrow AB = CD$ (by CPCT)

Hence, if chords of congruent circles subtend equal angles at their centres then chords are equal.

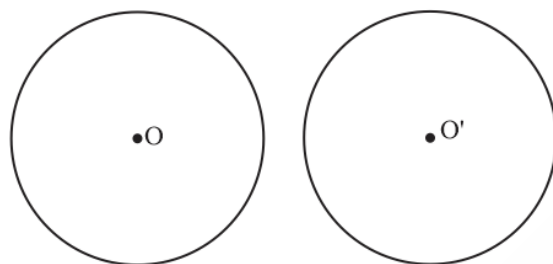
Exercise: 10.3

15. Draw different pair of circles. How many points does each pair have in common? What is the maximum number of common points?

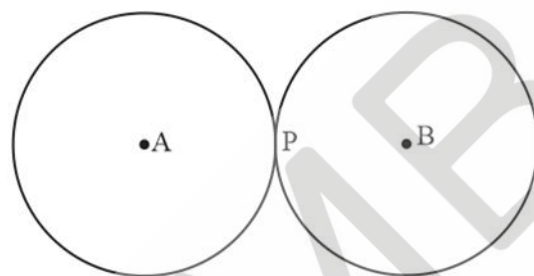
Solution:

Consider the following pair of circles:

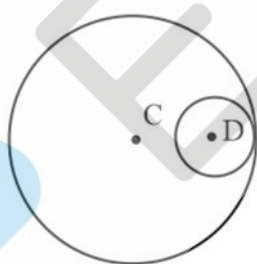
- (i) Circles don't intersect each other at any point, so circles are not having any point in common.



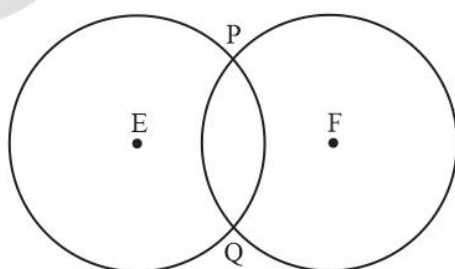
- (ii) Circles touch each other only at exactly one point P, so there is only 1 point in common.



- (iii) Circles touch each other at 1-point X only. So the circles have 1 point in common.



- (iv) These circles intersect each other at two points P and Q. So the circles have two points in common.



We may observe that there can be maximum 2 points in common between 2 circles.

16. Suppose you are given a circle, give a construction to find its center.

Solution:

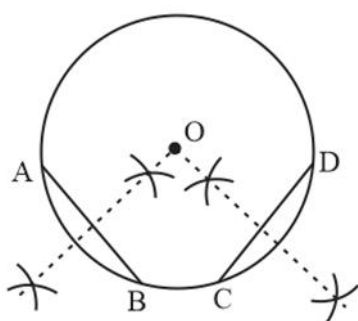
Following are the steps to find the centre of a circle:

Step1: Draw the given circle centred at point O.

Step2: Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these chords.

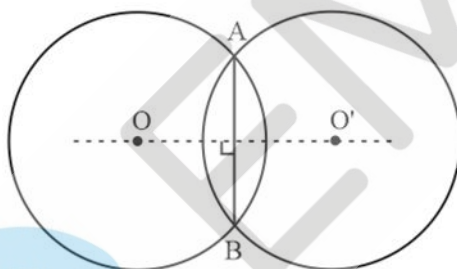
Step3: Let these perpendicular bisectors meet at point O.

Now, O is the centre of given circle.



17. If two circles intersect at two points, prove that their centers lie on the perpendicular bisector of the common chord.

Solution:



Let two circles centred at point O and O' intersect each other at point A and B respectively.

Now join AB. AB is the chord for circle centred at O, so perpendicular bisector of AB will pass through O.

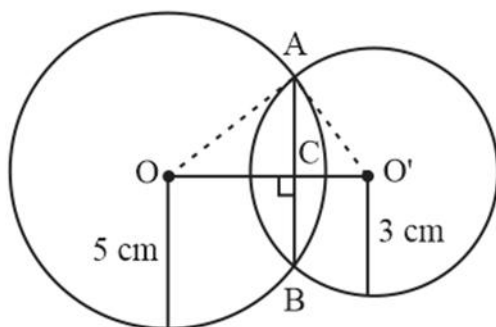
And AB is also the chord of circle centred at O' , so, perpendicular bisector of AB will also pass through O' .

As you can see, centres of these circles lie on the perpendicular bisector of common chord.

Exercise: 10.4

18. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

Solution:



Radius of circle centred at O and O' be 5 cm and 3 cm respectively.

$$OA = OB = 5 \text{ cm}$$

$$O'A = O'B = 3 \text{ cm}$$

OO' will be the perpendicular bisector of chord AB.

$$\therefore AC = CB$$

$$\text{Given that } OO' = 4 \text{ cm}$$

$$\text{Let } OC = x.$$

$$\text{So, } O'C = 4 - x$$

In $\triangle OAC$

$$OA^2 = AC^2 + OC^2 \text{ (using Pythagoras theorem)}$$

$$\Rightarrow 5^2 = AC^2 + x^2$$

$$\Rightarrow 25 - x^2 = AC^2 \quad \dots (1)$$

In $\triangle O'AC$

$$O'A^2 = AC^2 + O'C^2 \text{ (using Pythagoras theorem)}$$

$$\Rightarrow 3^2 = AC^2 + (4 - x)^2$$

$$\Rightarrow 9 = AC^2 + 16 + x^2 - 8x$$

$$\Rightarrow AC^2 = -x^2 - 7 + 8x \quad \dots (2)$$

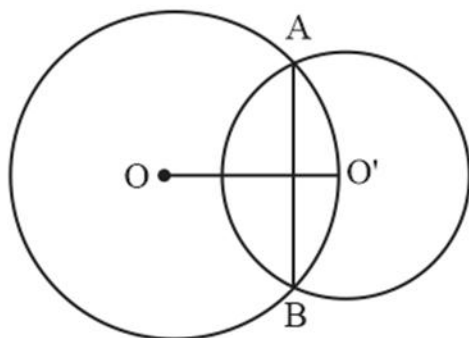
From equations (1) and (2),

$$25 - x^2 = -x^2 - 7 + 8x$$

$$8x = 32$$

$$x = 4$$

So, the common chord will pass through the centre of smaller circle i.e. O'. and hence it will be diameter of smaller circle.



Now, $AC^2 = 25 - x^2$ (from equation 1)

$$= 25 - 4^2$$

$$= 25 - 16$$

$$= 9$$

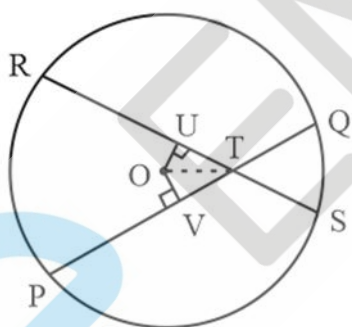
$$\therefore AC = 3 \text{ cm}$$

The length of the common chord $AB = 2 AC = (2 \times 3) \text{ cm} = 6 \text{ cm}$.

19. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution:

Let PQ and RS are two equal chords of a given circle and they are intersecting each other at point T.



Draw perpendiculars OV and OU on the chords PQ and RS respectively.

In $\triangle OVT$ and $\triangle OUT$

$$OV = OU \quad (\text{Equal chords of a circle are equidistant from the centre})$$

$$\angle OVT = \angle OUT = 90^\circ$$

$$OT = OT \quad (\text{common})$$

$$\therefore \triangle OVT \cong \triangle OUT \quad (\text{SAS rule})$$

$$\text{This gives, } VT = UT \quad (\text{by CPCT}) \dots (1)$$

It is given that

$$PQ = RS \quad \dots (2)$$

$$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}RS$$

$$\Rightarrow PV = RU \quad \dots (3)$$

Add equations (1) and (3), we get

$$PV + VT = RU + UT$$

$$\Rightarrow PT = RT \quad \dots (4)$$

Subtracting equation (4) from equation (2), we get

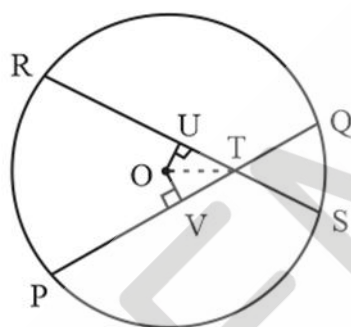
$$PQ - PT = RS - RT$$

$$\Rightarrow QT = ST \quad \dots (5)$$

Equations (4) and (5) shows that the corresponding segments of chords PQ and RS are congruent to each other.

20. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution:



Let PQ and RS are two equal chords of a given circle and they are intersecting each other at point T.

Draw perpendiculars OV and OU on these chords.

In $\triangle OVT$ and $\triangle OUT$

$$OV = OU \quad (\text{Equal chords of a circle are equidistant from the centre})$$

$$\angle OVT = \angle OUT \quad (\text{Each } 90^\circ)$$

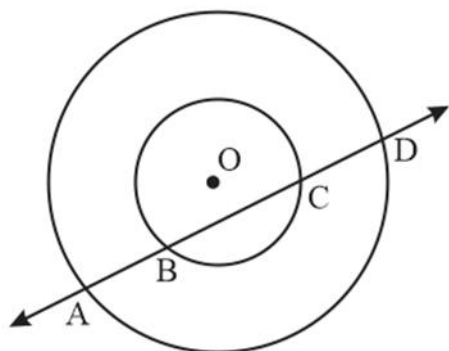
$$OT = OT \quad (\text{common})$$

$$\therefore \triangle OVT \cong \triangle OUT \quad (\text{SAS rule})$$

$$\therefore \angle OTV = \angle OTU \quad (\text{by CPCT})$$

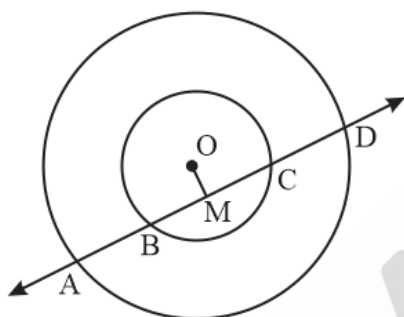
Hence, the line joining the point of intersection to the centre makes equal angles with the chords.

21. If a line intersects two concentric circles (circles with the same centre) with center O at A, B, C and D, prove that $AB = CD$.



Solution:

Let us draw a perpendicular OM on line AD.



In the given figure, BC is chord of smaller circle and AD is chord of bigger circle.

We know that the perpendicular drawn from centre of circle bisects the chord.

$$\therefore BM = MC \quad \dots (1)$$

$$\text{And } AM = MD \quad \dots (2)$$

Subtract equations (1) from (2), we get

$$AM - BM = MD - MC$$

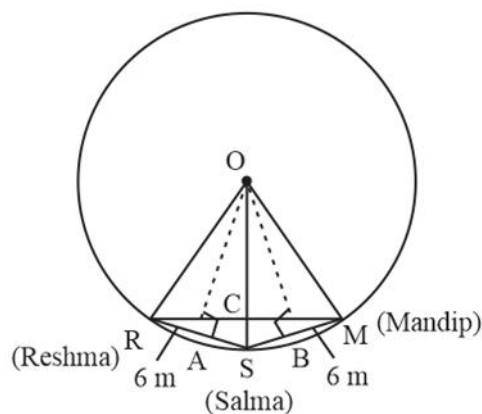
$$\Rightarrow AB = CD$$

22. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Solution:

Draw perpendiculars OA and OB on RS and SM respectively.

Let R, S and M be the position of Reshma, Salma and Mandip respectively.



$$AR = AS = \frac{6 \text{ cm}}{2} = 3 \text{ cm}$$

$$OR = OS = OM = 5 \text{ m} \quad (\text{radii of circle})$$

In ΔOAR

$$OA^2 + AR^2 = OR^2 \quad (\text{Pythagoras Theorem})$$

$$0A^2 + (3m)^2 = (5m)^2$$

$$OA^2 = (25 - 9) \text{ m}^2 = 16 \text{ m}^2$$

$$OA = 4 \text{ m}$$

We know that in an isosceles triangle, altitude divides the base

So, In Δ RSM

$\angle RCS = 90^\circ$ and $RC = CM$

$$\text{Area of } \triangle ORS = \frac{1}{2} \times OA \times RS$$

$$\frac{1}{2} \times \text{RC} \times \text{OS} = \frac{1}{2} \times 4 \times 6$$

$$RC \times 5 = 24$$

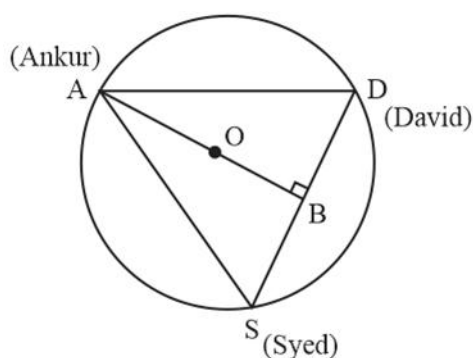
$$RC = 4.8$$

$$RM = 2RC = 2(4.8) = 9.6$$

So, distance between Reshma and Mandip is 9.6 m.

23. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution:



Given that $AS = SD = DA$

\Rightarrow ASD is an equilateral triangle.

OA (radius) = 20 m.

We know that the medians of equilateral triangle pass through the circumcentre of the equilateral triangle ABC.

We also know that median intersect each other at 2: 1.

Since, AB is the median of equilateral triangle ABC

$$\Rightarrow \frac{OA}{OB} = \frac{2}{1}$$

$$\Rightarrow \frac{20 \text{ m}}{OB} = \frac{2}{1}$$

$$\Rightarrow OB = 10 \text{ m.}$$

$$\therefore AB = OA + OB = (20 + 10) \text{ m} = 30 \text{ m.}$$

In $\triangle ABD$

$$AD^2 = AB^2 + BD^2 \quad (\text{using Pythagoras theorem})$$

$$AD^2 = (30)^2 + \left[\frac{SD}{2}\right]^2$$

$$AD^2 = (30)^2 + \left[\frac{AD}{2}\right]^2 \quad (SD = AD)$$

$$AD^2 = 900 + \frac{1}{4} AD^2$$

$$\frac{3}{4} AD^2 = 900$$

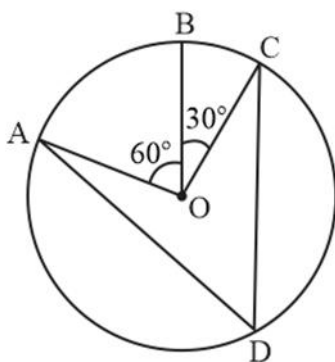
$$AD^2 = 1200$$

$$AD = 20\sqrt{3}$$

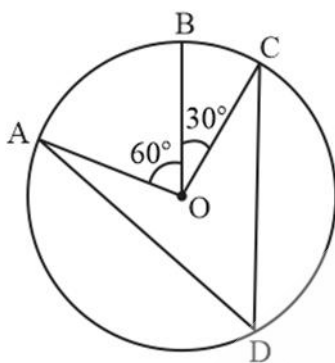
So, length of string of each phone will be $20\sqrt{3}$ m.

Exercise: 10.5

1. In the given figure, A, B and C are three points on a circle with center O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.



Solution:



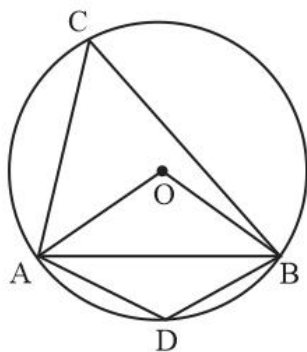
$$\begin{aligned}\angle AOC &= \angle AOB + \angle BOC \\ &= 60^\circ + 30^\circ \\ &= 90^\circ\end{aligned}$$

We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.

$$\begin{aligned}\therefore \angle ADC &= \frac{1}{2} \angle AOC \\ &= \frac{1}{2} (90^\circ) \\ &= 45^\circ\end{aligned}$$

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution:



In $\triangle OAB$

$AB = OA = OB = \text{radius}$

$\therefore \triangle OAB$ is an equilateral triangle.

So, each interior angle of the $\triangle OAB = 60^\circ$

$\therefore \angle AOB = 60^\circ$

$= 90^\circ$

We know that angle subtended by an arc at centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \angle ACB = \frac{1}{2} \angle AOB$

$= \frac{1}{2} (60^\circ)$

$= 30^\circ$

Now,

In cyclic quadrilateral ACBD

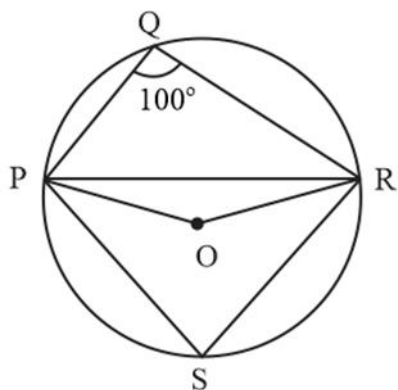
$\angle ACB + \angle ADB = 180^\circ$ (Opposite angle in cyclic quadrilateral)

$\angle ADB = 180^\circ - 30^\circ = 150^\circ$

So, angle subtended by this chord at a point on major arc and minor arc are 30° and 150° respectively.

3. In the given figure, $\angle PQR = 100^\circ$, where P, Q and R are the points on a circle with center O. Find $\angle OPR$.

Solution:



Let PR be the chord of circle.

Take any point S on the major arc of circle.

Now PQRS is a cyclic quadrilateral.

$$\angle PQR + \angle PSR = 180^\circ \quad (\text{Opposite angles of cyclic quadrilateral})$$

$$\angle PSR = 180^\circ - 100^\circ = 80^\circ$$

We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.

$$\therefore \angle POR = 2\angle PSR = 2(80^\circ) = 160^\circ$$

In $\triangle POR$

$$OP = OR \quad (\text{radii of same circle})$$

$$\therefore \angle OPR = \angle ORP \quad (\text{Angles opposite equal sides of a triangle}) \dots (i)$$

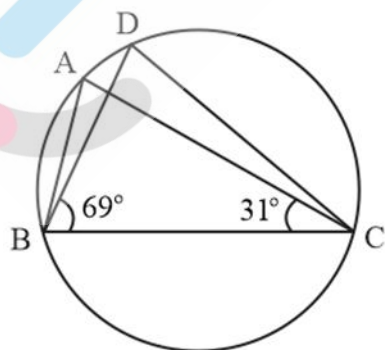
$$\angle OPR + \angle ORP + \angle POR = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$2\angle OPR + 160^\circ = 180^\circ \quad (\text{using equation (i)})$$

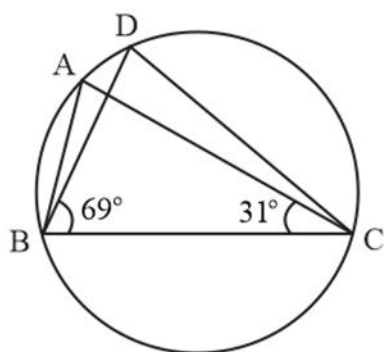
$$2\angle OPR = 180^\circ - 160^\circ = 20^\circ$$

$$\therefore \angle OPR = 10^\circ$$

4. In the given figure, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.



Solution:



In $\triangle ABC$

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ \quad (\text{Angle sum property of a triangle})$$

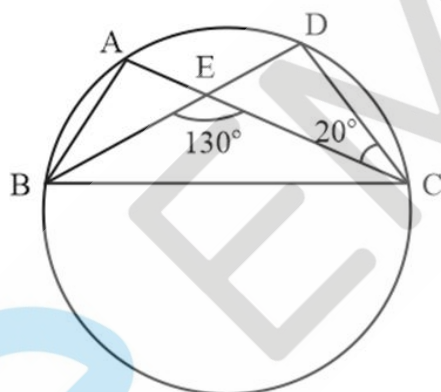
$$\angle BAC + 69^\circ + 31^\circ = 180^\circ$$

$$\angle BAC = 180^\circ - 100^\circ$$

$$\angle BAC = 80^\circ$$

$$\angle BDC = \angle BAC = 80^\circ \quad (\text{Angles in same segment of circle are equal})$$

5. In the given figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.



Solution:

In $\triangle CDE$

$$\angle CDE + \angle DCE = \angle CEB \quad (\text{Exterior angle})$$

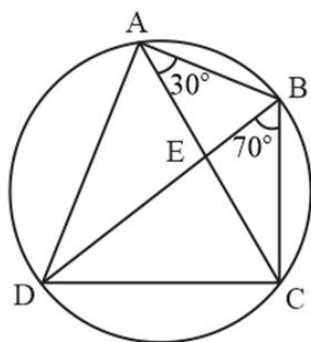
$$\angle CDE + 20^\circ = 130^\circ$$

$$\angle CDE = 110^\circ$$

$$\text{But } \angle BAC = \angle CDE \quad (\text{Angles in same segment of circle})$$

$$\angle BAC = 110^\circ$$

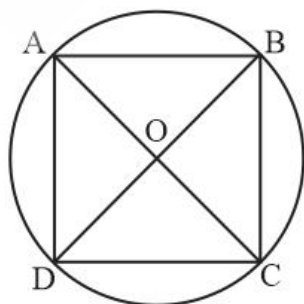
6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC = 30^\circ$, find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Solution:

For chord CD

 $\angle CBD = \angle CAD$ (Angles in same segment) $\angle CAD = 70^\circ$ $\angle BAD = \angle BAC + \angle CAD = 30^\circ + 70^\circ = 100^\circ$ $\angle BCD + \angle BAD = 180^\circ$ (Opposite angles of a cyclic quadrilateral) $\angle BCD + 100^\circ = 180^\circ$ $\angle BCD = 80^\circ$ In $\triangle ABC$ $AB = BC$ (given) $\therefore \angle BCA = \angle CAB$ (Angles opposite to equal sides of a triangle) $\angle BCA = 30^\circ$ We have $\angle BCD = 80^\circ$ $\angle BCA + \angle ACD = 80^\circ$ $30^\circ + \angle ACD = 80^\circ$ $\angle ACD = 50^\circ$ $\angle ECD = 50^\circ$

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution:

Let ABCD a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.

Now, Consider BD as a chord.

We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.

$$\therefore \angle BAD = \frac{1}{2} \angle BOD$$

$$= \frac{1}{2} (180^\circ)$$

$$= 90^\circ$$

$$\angle BCD + \angle BAD = 180^\circ \quad (\text{Cyclic quadrilateral})$$

$$\angle BCD = 180^\circ - 90^\circ = 90^\circ$$

Now, Consider AC as a chord.

We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.

$$\therefore \angle ADC = \frac{1}{2} \angle AOC$$

$$= \frac{1}{2} (180^\circ)$$

$$= 90^\circ$$

$$\angle ADC + \angle ABC = 180^\circ \quad (\text{Cyclic quadrilateral})$$

$$90^\circ + \angle ABC = 180^\circ$$

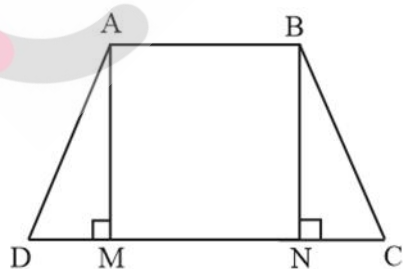
$$\angle ABC = 90^\circ$$

Since, each interior angle of cyclic quadrilateral is of 90° .

Hence it is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution:



Consider a trapezium ABCD with $AB \parallel DC$ and $BC = AD$.

Draw $AM \perp DC$ and $BN \perp DC$

In $\triangle AMD$ and $\triangle BNC$

$$AD = BC \quad (\text{Given})$$

$$\angle AMD = \angle BNC \quad (\text{By construction each is } 90^\circ)$$

$$AM = BN \quad (\perp \text{ distance between two parallel lines is same})$$

$$\therefore \triangle AMD \cong \triangle BNC \quad (\text{By RHS congruence rule})$$

$$\therefore \angle ADC = \angle BCD \quad (\text{CPCT}) \quad \dots (1)$$

Since, $\angle BAD$ and $\angle ADC$ are on same side of transversal AD

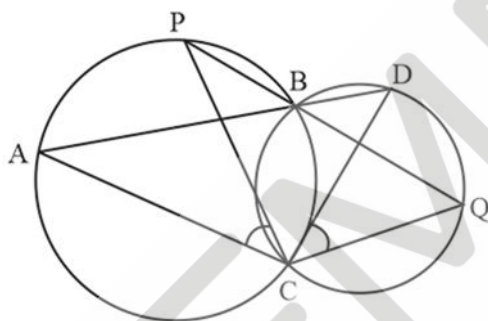
$$\therefore \angle BAD + \angle ADC = 180^\circ \quad \dots (2)$$

$$\angle BAD + \angle BCD + 180^\circ \quad [\text{Using equation (1)}] \dots (3)$$

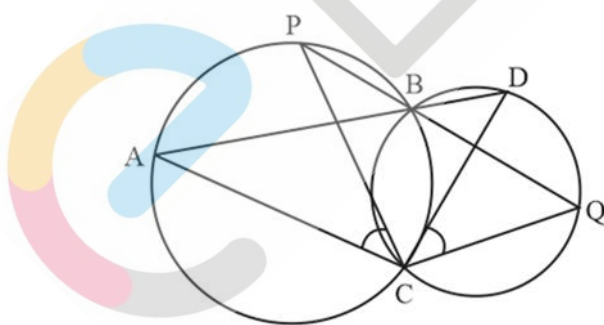
Equation (3) shows that the opposite angles are supplementary.

\therefore ABCD is a cyclic quadrilateral.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$.



Solution:



Join chords AP and DQ

For chord AP

$$\angle PBA = \angle ACP \quad (\text{Angles in same segment}) \quad \dots (1)$$

For chord DQ

$$\angle DBQ = \angle QCD \quad (\text{Angles in same segment}) \quad \dots (2)$$

Since, the line segments ABD and PBQ are intersecting at B.

$$\therefore \angle PBA = \angle DBQ \quad (\text{Vertically opposite angles}) \quad \dots (3)$$

From equations (1), (2) and (3), we have

$$\angle ACP = \angle QCD$$

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Solution:

Consider a $\triangle ABC$

Two circles are drawn taking AB and AC as diameter.

Let they intersect each other at D and let D does not lie on BC.

Now, Join AD

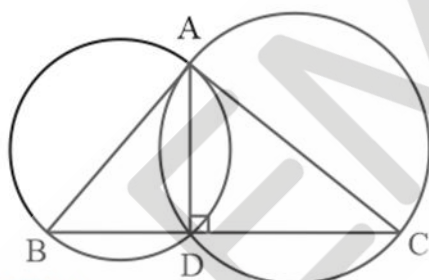
$$\angle ADB = 90^\circ \quad (\text{Angle subtend by semicircle})$$

$$\angle ADC = 90^\circ \quad (\text{Angle subtend by semicircle})$$

$$\angle BDC = \angle ADB + \angle ADC = 90^\circ + 90^\circ = 180^\circ$$

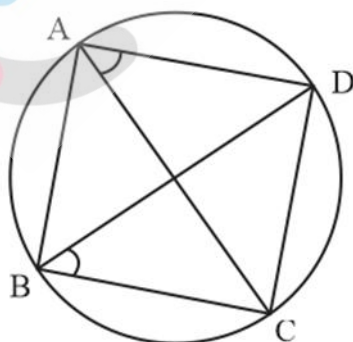
Hence BDC is straight line and our assumption was wrong.

Thus, Point D lies on third side BC of $\triangle ABC$



11. ABC and ADC are two right triangle with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Solution:



In $\triangle ABC$

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$\Rightarrow 90^\circ + \angle BCA + \angle CAB = 180^\circ$$

$$\Rightarrow \angle BCA + \angle CAB = 90^\circ \quad \dots (1)$$

In $\triangle ADC$

$$\angle CDA + \angle ACD + \angle DAC = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$\Rightarrow 90^\circ + \angle ACD + \angle DAC = 180^\circ$$

$$\Rightarrow \angle ACD + \angle DAC = 90^\circ \quad \dots (2)$$

Adding equations (1) and (2), we have

$$\angle BCA + \angle CAB + \angle ACD + \angle DAC = 180^\circ$$

$$\Rightarrow (\angle BCA + \angle ACD) + (\angle CAB + \angle DAC) = 180^\circ$$

$$\angle BCD + \angle DAB = 180^\circ \quad \dots (3)$$

But it is given that

$$\angle B + \angle D = 90^\circ + 90^\circ = 180^\circ \quad \dots (4)$$

From equations (3) and (4), we can see that quadrilateral ABCD is having sum of opposite angles is 180° .

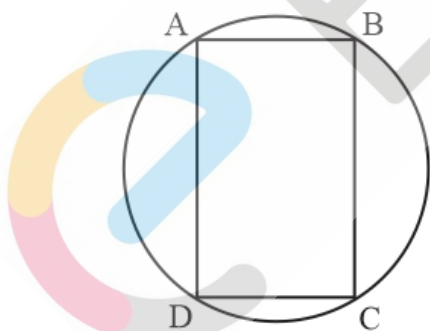
So, it is a cyclic quadrilateral.

Consider chord CD.

$$\text{Now, } \angle CAD = \angle CBD \quad (\text{Angles in same segment})$$

12. Prove that a cyclic parallelogram is a rectangle.

Solution:



Let ABCD be a cyclic parallelogram.

$$\angle A + \angle C = 180^\circ \quad (\text{Opposite angle of cyclic quadrilateral}) \quad \dots (1)$$

We know that opposite angles of a parallelogram are equal

$$\therefore \angle A = \angle C \text{ and } \angle B = \angle D$$

From equation (1)

$$\angle A + \angle C = 180^\circ$$

$$\angle A + \angle A = 180^\circ$$

$$\Rightarrow 2\angle A = 180^\circ$$

$$\angle A = 90^\circ$$

Since, parallelogram ABCD is having its one of interior angles as 90° , so, it is a rectangle.

