REHABILITATION AND ASSISTIVE EXOSKELETON

November 2021

Contents

1 Introduction

Exoskeletons are wearable robotic devices which are used for rehabilitation and for providing assistance in doing a task. It is powered by electric motors, pneumatics, hydraulics or combination of any of these to provide enhanced limb movement.

Cerebral palsy is a neurological disorder that affects muscles of the patient and constrains him from doing any movements of his limbs easily. This motor neuron disease is a group of disorders that affect muscle movement and coordination. The best-known solution to this problem is physical therapy and regular walking practice. In the rehabilitation routine, the physiotherapist makes the patient perform certain gait cycles at varied speeds and distances. As repeating a task with negligible variation is a great challenge for humans, exoskeleton can be used to complete the task. This can also be used in places where strength is a key factor for work. It also empowers the military of the nation by making the job of the soldiers less tedious.

Our aim is to build a lower limb exoskeleton for children capable of aiding the physical therapy process. Assistive exoskeletons are very expensive and cannot be purchased by a physiotherapy centre to provide treatment economically. With proper selection of actuators and other materials we can build it in a small budget.

2 Gait Pattern

A gait is a pattern of limb movements made during locomotion. Repetition of gait cycle leads to locomotion.

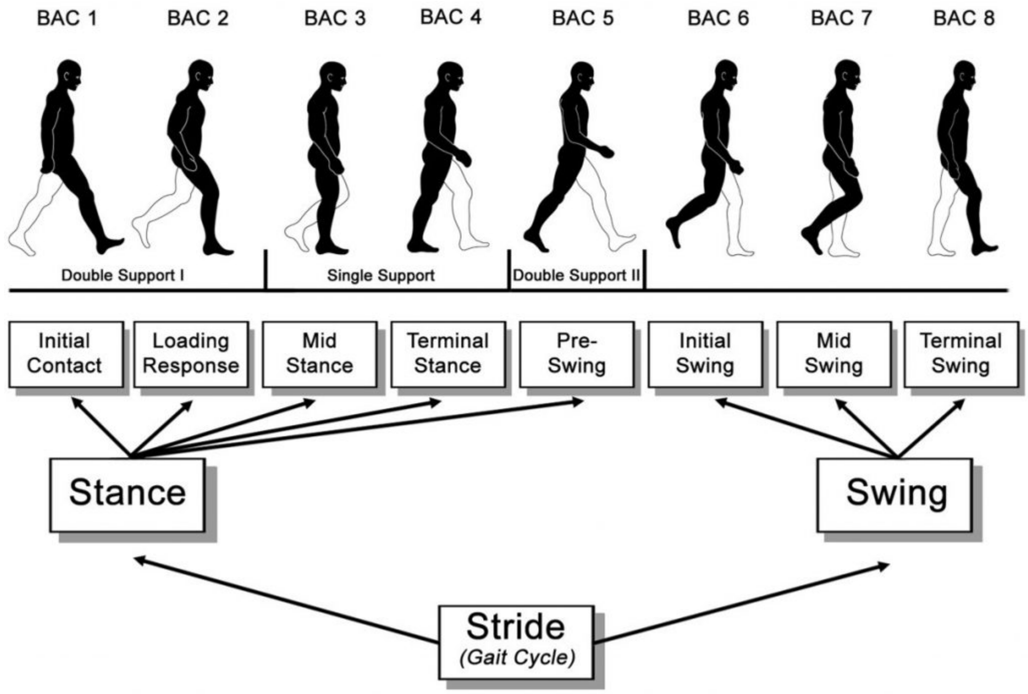

Figure 1: Gait Pattern

A complete gait cycle shown in the figure. For understanding we can define the gait cycle as the process between one foot striking the ground to the same foot again striking the ground. It is also known as one stride. The Gait cycle can be divided into two major phases:Stance phase, Swing phase

2.1 Stance phase

Stance phase is that part of a gait cycle during which the foot remains in contact with the ground. It constitutes almost 62

Initial Contact (Heel Strike): The heel of the reference foot touches the ground in front of the body.

Loading response : It begins immediately after the foot strike.During this phase body weight is transferred onto the supporting limb.

Mid-stance Phase: During mid-stance phase the reference foot contacts the ground flat-footed.

Terminal stance: Begins when the supporting heel rises from the ground and continues until the opposite heel touches the ground.

Pre-swing Phase: This phase corresponds to the loading response phase of the opposite foot.

2.2 Swing phase

Stance phase is that part of a gait cycle during which the reference foot is not in contact with the ground. It constitutes almost 38

Initial Swing Phase: The reference foot moves forward towards the opposite foot, while the knee and the hip are flexing.

Mid-swing Phase: Hip continues to flex, the reference foot pushes forward and gradually surpasses the supporting foot.

Terminal Swing Phase: During terminal swing phase, the reference foot begins landing to the ground as the respective knee and hip begin extension.

3 Data Collection

Lower limb rehabilitation assistive exoskeleton must have good human compatibility. Therefore the selection of actuators and links are the most essential parts of the design.Based on the torque provided by the actuator we need to choose it.Angular patterns can be obtained from a complete gait analysis of a healthy person.By which we can calculate torque acting on the joins. Gait analysis also provides foundation for the controlling part.

The human leg has 6 degrees of freedom (3DOF for hip, 1DOF for knee and 2DOF for ankle). The exoskeleton provides the actuation only in the sagittal plane there by restricting the degrees of freedom to 3. The orthotic lower limb device is an exoskeleton for hip and knee joints. Therefore, the data required for the analysis is the angular pattern of hip and knee joints. Different techniques used in the data acquisition process are

Use of Optical Rotary Encoders Use of IMUs (Inertial Measurement Units) Use of Imaging techniques

3.1 Optical Rotaty Encoder

An optical rotary encoder uses a reliable,defined pattern of light and dark to determine the position of the shaft and therefore the position of the link which is connected to the shaft. It converts the angular position or motion of a shaft or axle to analog or digital output signals.

In an optical rotary encoder there is a light source and sensor in between circular plates . It has continuous slots of equal width. The sensor has a phototransistor, resistor, and schmitt trigger. When light is passed through the slots phototransistor activates and it offers minimum(0) resistance, it is fed to the schmitt trigger and it produces low output and most of the voltage(5v) is dropped across the resistor. When light is blocked, the phototransistor will be in off stage and it offers maximum resistance.So nearly 5 v dropped across the phototransistor and nearly zero voltage dropped across the resistor. That 5 v fed to schmitt trigger and it gives maximum output. Square wave pattern is obtained.From this , number of pulses per unit time and corresponding angle of rotation can be calculated.

Encoder is placed at the hip and a link is connected to the shaft of the encoder then that link is attached to the thigh, from this encoder we can determine the angular pattern of the hip joint. Now another encoder is wielded to link whose other end is an encoder which is placed at the hip and a link is attached to the second encoder and that is attached to the shank, through this we can determine the angular pattern at the knee.

3.2 Inertial Measurement Unit

An Inertial Measurement Unit(IMU), is an electronic device that tracks rotational and translational movements. It is composed of 3 accelerometers, 3 gyroscopes, and depending on the heading requirement – 3 magnetometers.IMUs can measure a variety of factors, including speed, direction, acceleration, specific force, angular rate and magnetic fields surrounding the device (in the presence of a magnetometer).Each tool in an IMU is used to capture different data types: Accelerometer measures velocity and acceleration,Gyroscope measures rotation and rotational rate, Magnetometer establishes cardinal direction (directional heading)

Gyroscopes in general are devices that measure angular displacement ,angular velocity etc. Most gyroscopes only measure along a single sensitive axis.Therefore a combination of three orthogonally mounted gyroscopes is required to sense three dimensional angular motion.

This rotation about the spin axis is detected and information on this rotation is delivered to a motor or other device that applies torque in an opposite direction thereby canceling the precession and maintaining its orientation.

A plane portion is connected to the thigh and IMU is attached to it .The method of measuring angle is the same as that of encoder.

3.3 Imaging Techniques

With the development of photography and cinematography, it is possible to capture image sequences that reveal even minute details of human and animal locomotion.

Three green colour circular stickers were attached. One is at the hip and the other is at knee and the other is at the ankle. By using image processing techniques in matlab we made everything black in the image except at the location of stickers. Now we have calculated the centroids of each sticker. By using those points in every frame we have calculated the angular pattern at the hip and knee.

Let (X_1, Y_1) , (X_2, Y_2) , (X_3, Y_3) be the centroids of the markers at hip, knee and ankle joint respectively.

Tangent of the angular displacement of the thigh is given by

$$Tan(\theta) = \frac{Y_2 - Y_1}{X_2 - X_1} \tag{1}$$

So angular displacement of the thigh is

$$arctan \frac{Y_2 - Y_1}{X_2 - X_1} \tag{2}$$

3.4 Data Obtained

The angular position of hip and knee joints are recorded using the above mentioned techniques as a function of time from a healthy human sample.The minimum and maximum torque for hip and knee joints are calculated.

	Max value	Min value
Angle in degrees	14.57	-18.12
Angle in radians	0.25	-0.31
Angular velocity	1.4	-2.53
Angular acceleration	17.86	-28.39

Table 1: Data of Hip from Image Processing

	Max value	Min value
Angle in degrees	56.46	-16.52
Angle in radians	0.98	0.29
Angular velocity	3.03	-5.39
Angular acceleration	64.37	-56.28

Table 2: Data of Knee from Image Processing

	Max value	Min value
Angle in degrees	22.85	-8.56
Angle in radians	0.4	-0.15
Angular velocity	2.7	-1.3
Angular acceleration	20	-20

Table 3: Data of Hip from Encoder

	Max value	Min value
Angle in degrees	11.5	-68.5
Angle in radians	0.2	-1.2
Angular velocity	6	-10
Angular acceleration	115	-105

Table 4: Data of Knee from Encoder

Hip joint :
Maximum torque : 30Nm
Minimum torque in opposite direction: 25Nm
Knee joint :
Maximum torque : 10Nm
Minimum torque in opposite direction : 7Nm

This data is used for the dynamic analysis of the human gait. By using methods like Lagrange and Newton-Euler formulation, the dynamic analysis of the gait is done to find the torque provided by the muscles for walking.The Newton-Euler formulation is derived by the direct interpretation of Newton's Second Law of Motion, which describes dynamic systems in terms of force and momentum. In the Lagrangian formulation the system's dynamic behavior is described in terms of work and energy using generalized coordinates.

4 Mechanical Design

The exoskeleton design consists of 10 degrees of freedom(DOF) out of which 4 are active and 6 are passive. The 4 active DOF s are 2 knee joints and 2 hip joints which are revolute joints about mediolateral axis(axis perpendicular to the sagittal plane) and are controlled by DC motors. One DOF for each ankle joint(about a mediolateral axis), one DOF for each hip about an axis parallel to the anterior-posterior axis(the axis perpendicular to the frontal plane) and 2 DOF at the back of the exoskeleton about an axis parallel to the anterior-posterior axis are the 6 passive DOFs.

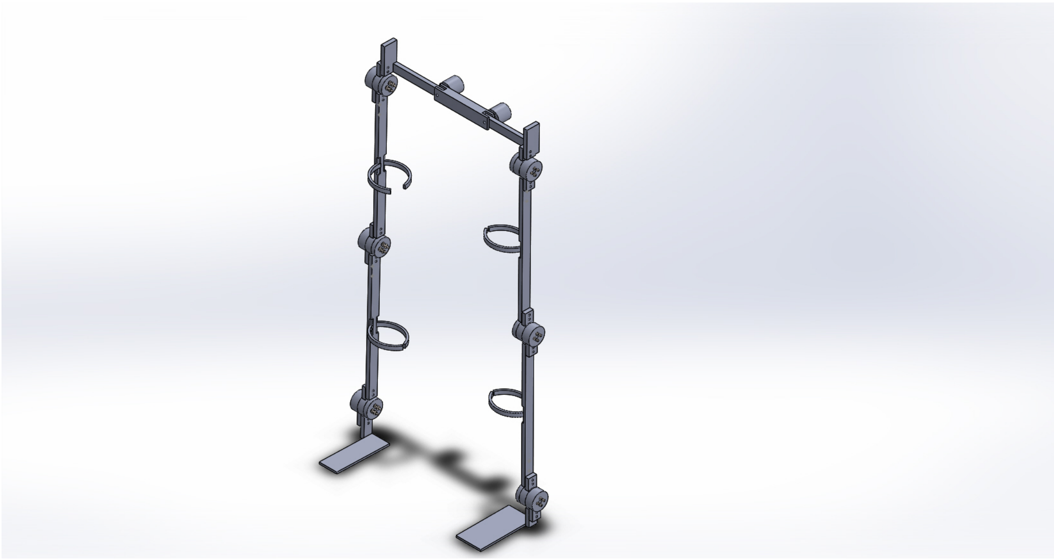

Figure 2: Lowerlimb Exoskeleton

Different Parts of the Design:-

1.There are 4 links that give structure to the exoskeleton and supports the human body. They are directly attached to the legs. Actuations are transmitted to the legs through these links

2.There are 4 joints through which the load is transmitted, out of which two joints are active and act as active joints. The joints should be able to transmit the load properly and at the same time, they should provide smooth movement to the limbs. Joint consist of 3 parts,

a) Middle Part This is the part that encloses a ball bearing and it connects to one of the links at the joint.

b) Outer Part The outer part connects to the adjacent link. It has a projection that goes halfway through the inner cylindrical portion of the ball bearing. The outer portion has a hole for the attachment of the motor shaft. In addition to this hole outer part have 4 holes for fastening with nuts and bolts.

c) Inner Part The Inner part is similar to the outer part except that it does not have a hole for motor shaft attachment.

Middle Part and Outer part fits inside the ball bearing from opposite sides and are fastened by 4 nuts and bolts. Middle Part connects the joint to one link and the Outer Part connects to the adjacent link. This type of arrangement ensures that

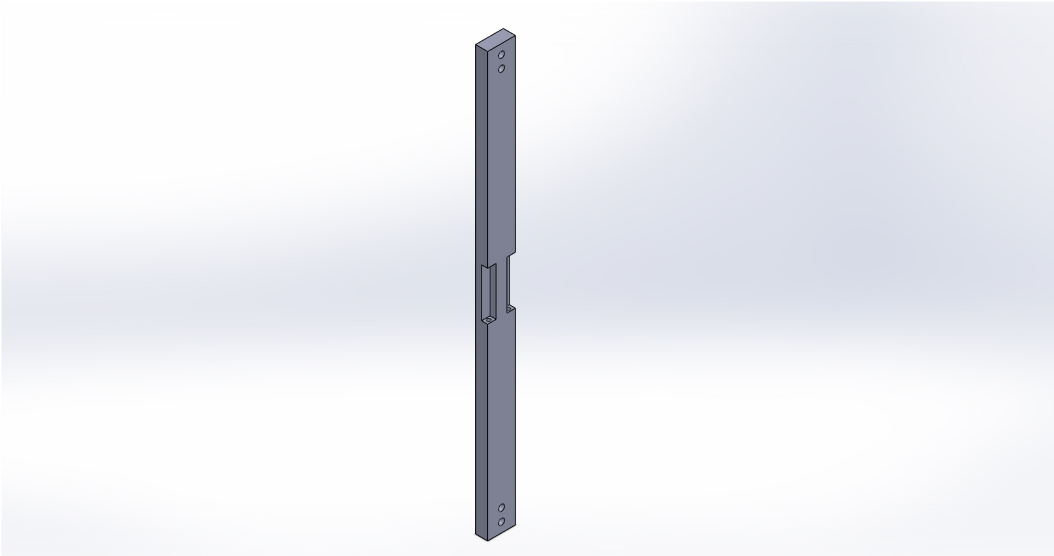

Figure 3: Link

the joint transmits the load without any failure and the relative movement between the links are smooth.

3.Hip Assembly Hip Assembly consists of 3 links and has two passive DOFs where two encoders are attached for data collection. Hip Assembly is directly attached to the body and is connected to the thigh link through 2 active joints and 2 passive joints.

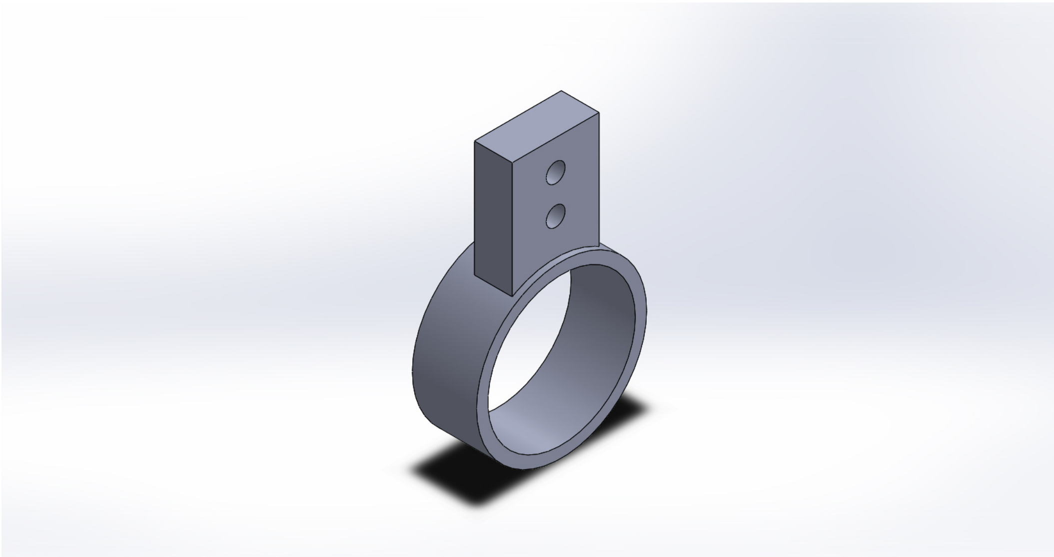

Figure 4: Joint Middle Part

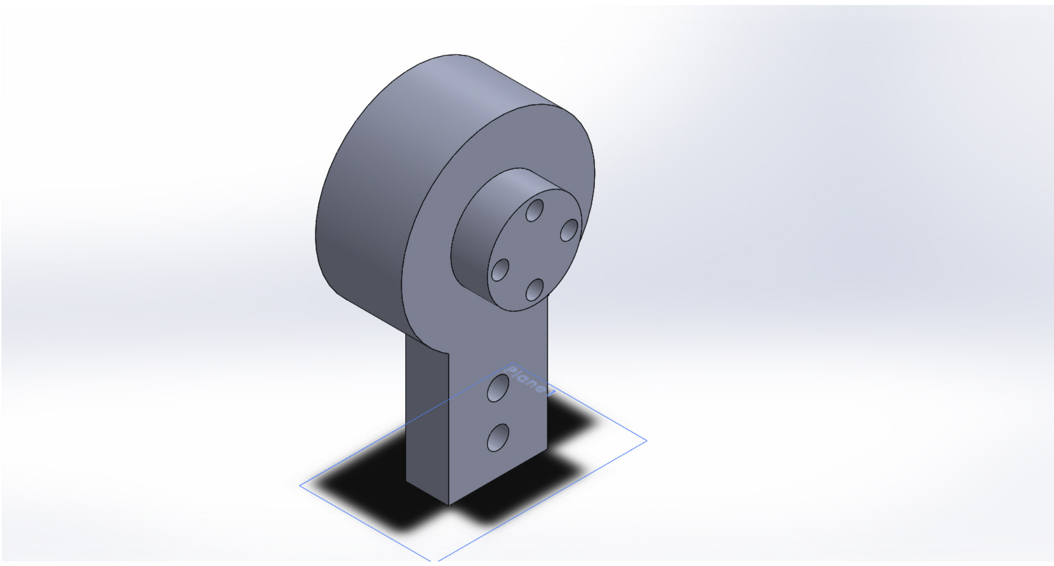

Figure 5: Joint Outer Part 1

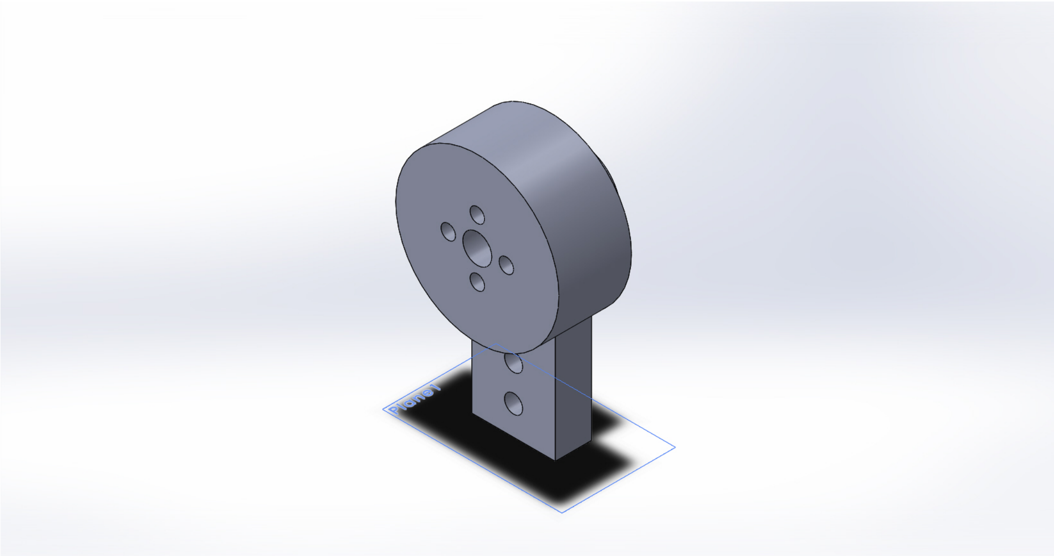

Figure 6: Joint Outer Part 2

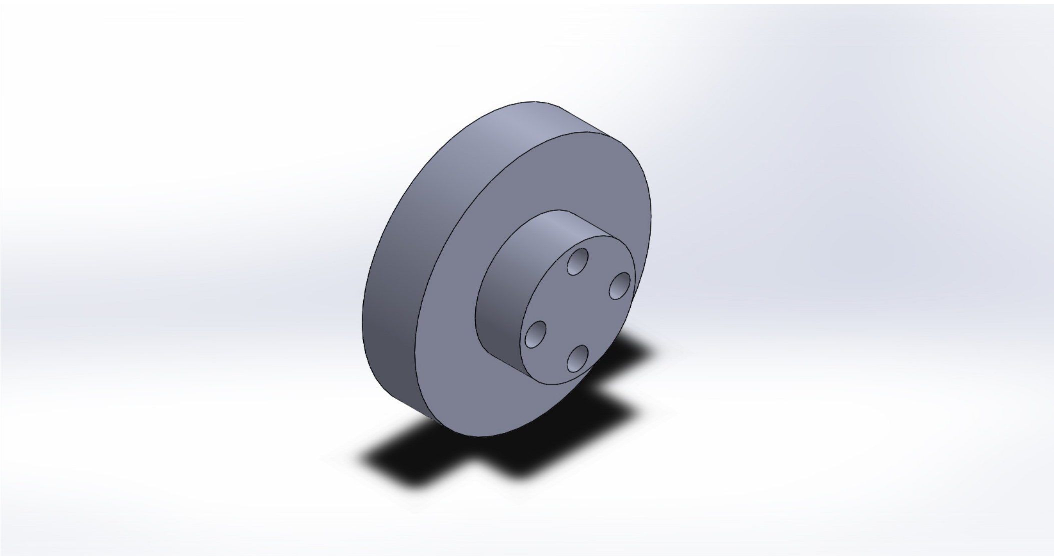

Figure 7: Joint Inner Part

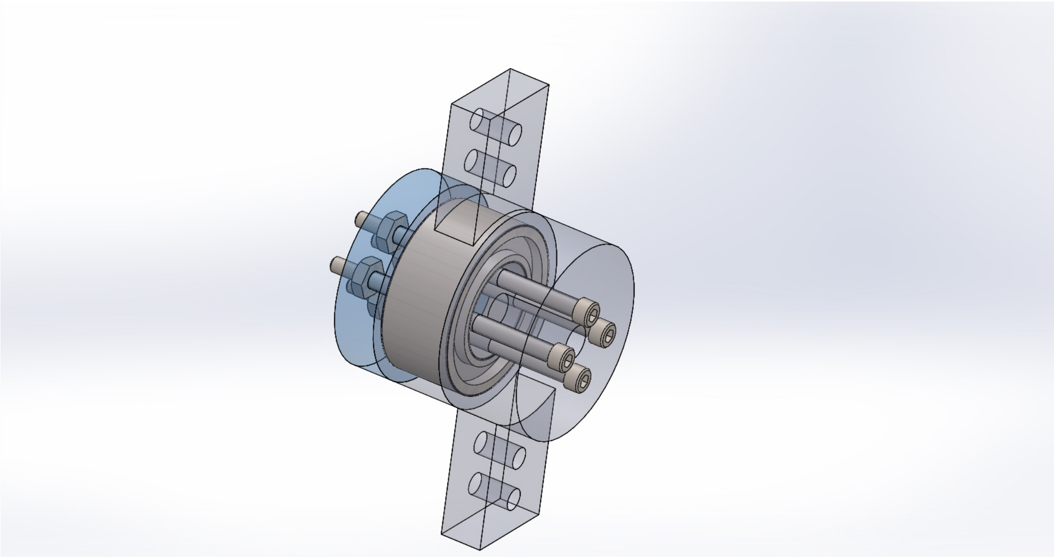

Figure 8: Joint Assembly Transparent

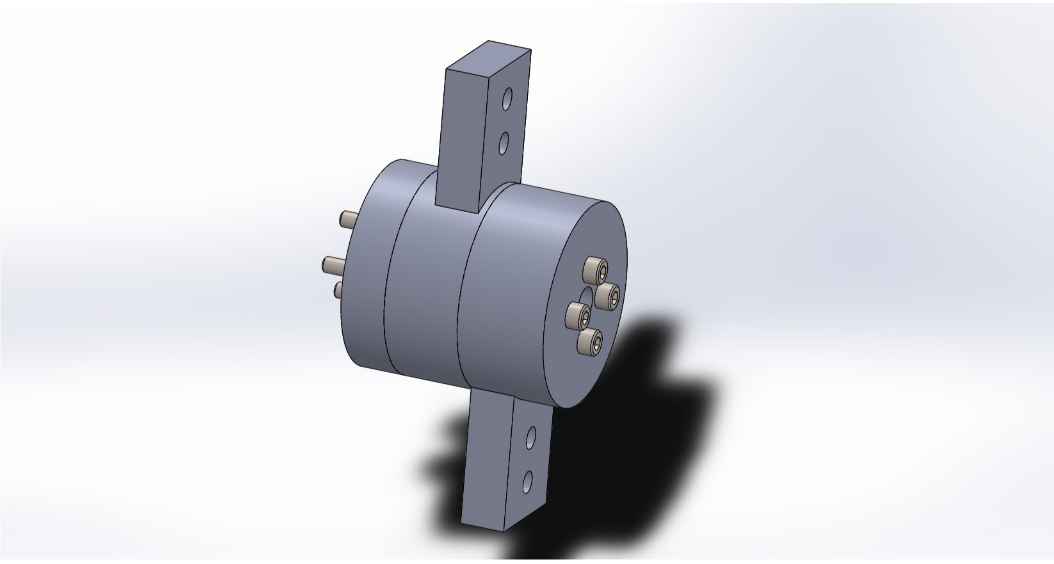

Figure 9: Joint Assembly View 1

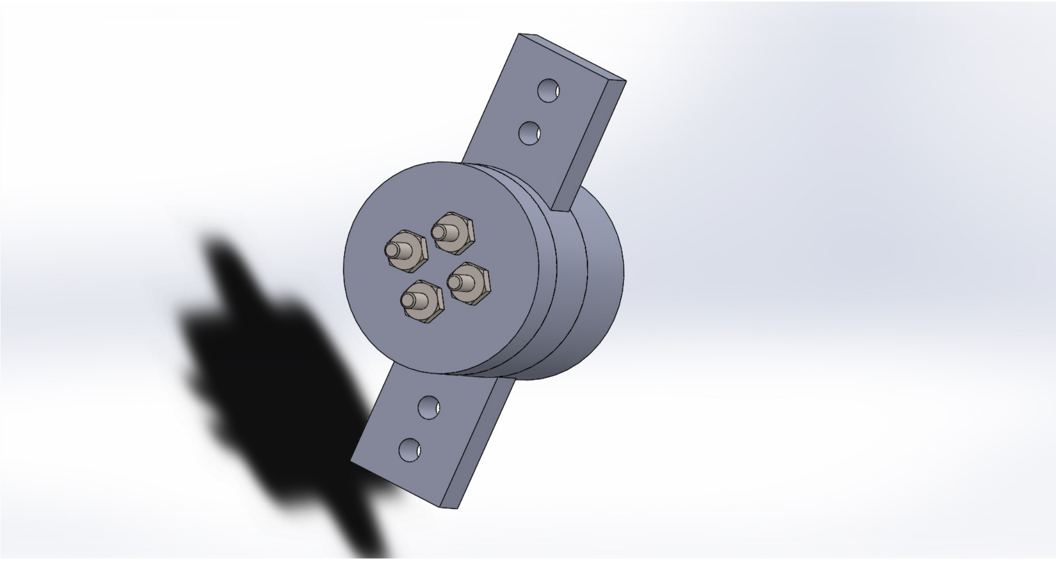

Figure 10: Joint Assembly View 2

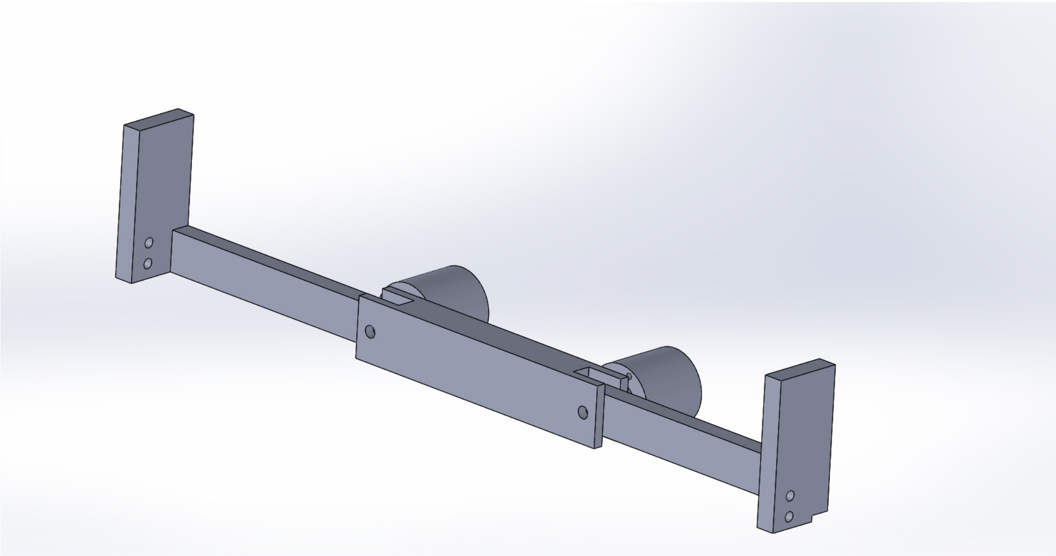

Figure 11: Hip Assembly

5 Modelling

5.1 Static Analysis

5.1.1 Forward kinematics

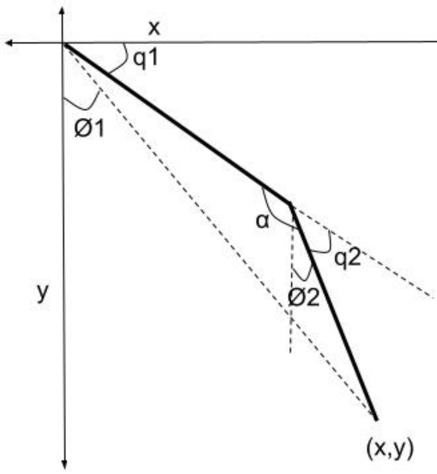

Figure 12: Forward Kinematics

We are dealing with the motion of the lower limb in the sagittal plane. Therefore constraining the motion in a 2D plane.

Length of link 1 $= L_1$
Length of link 2 $= L_2$
Angle made by link 1 $= q_1$
Angle made by link 2 $= q_2$
Task space, X $=$ [x,y]
Joint/control space, q $= [q_1, q_2]$
End-effector:

$$X = F(\theta) \quad = [x, y] = [L_1 cos(q_1) + L_2 cos(q_1 + q_2), L_1 sin(q_1) + L_2 sin(q_1 + q_2)] \quad (3)$$

5.1.2 Inverse kinematics

Consider the triangle AOB shown in the figure (13).

AB $=$ R $= \sqrt{x^2 + y^2}$
Angle AOB $= \alpha$
Apply cosine law We get

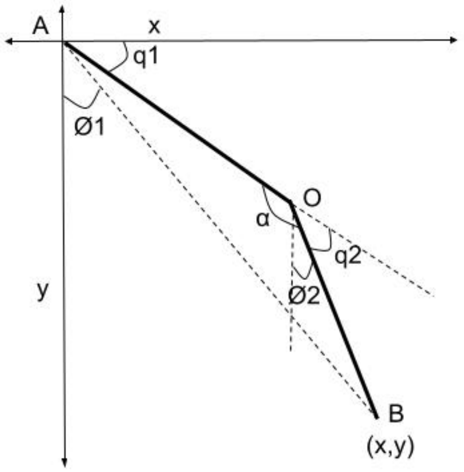

Figure 13: Inverse Kinematics

$$R^2 = L_1^2 + L_2^2 - 2L_1L_2cos(\alpha) \tag{4}$$

$$cos(\alpha) = \frac{L_1^2 + L_2^2 - X^2 - Y^2}{2L_1L_2} \tag{5}$$

$$q_2 = \pi - \alpha$$

$$cos(q_2) = -cos(\alpha)$$

$$cos(q_2) = \frac{X^2 + Y^2 - L_1^2 - L_2^2}{2L_1L_2} \tag{6}$$

$$q_2 = arccos\frac{X^2 + Y^2 - L_1^2 - L_2^2}{2L_1L_2} \tag{7}$$

In triangle OBC in figure (14)

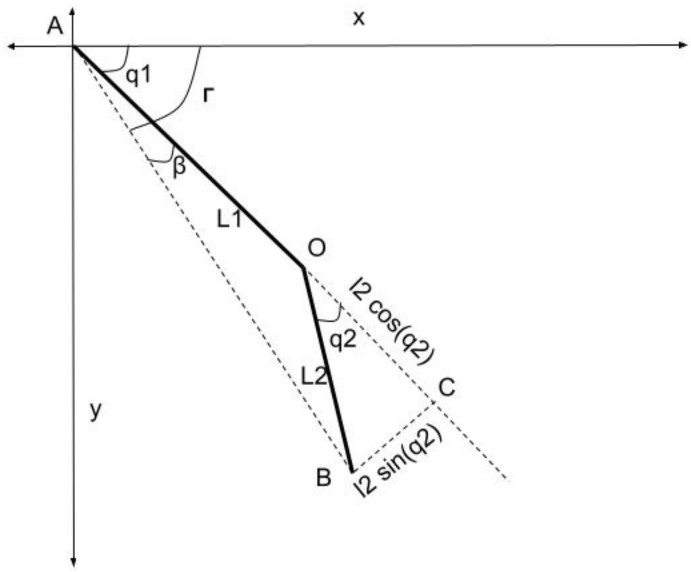

Figure 14: Inverse Kinematics

$$\begin{aligned} OC &= L_2cos(q_2) \\ BC &= L_2sin(q_2) \\ tan(\beta) &= \frac{BC}{AC} \\ tan(\beta) &= \frac{L_2sin(q_2)}{L_1 + L_2cos(q_2)} \quad (8) \\ \gamma = q_1 + \beta &= arctan\frac{y}{x} \quad (9) \\ q_1 &= arctan\frac{y}{x} - arctan\frac{L_2sin(q_2)}{L_1 + L_2cos(q_2)} \quad (10) \end{aligned}$$

From the equation 10 it is clear that q_1 also depended on q_2.
For a 2D two link manipulator there can be two solutions for q_1 and q_2.
When we solve for q_2
We get

$$cos(q_2) = X^2 + Y^2 - L_1^2 - \frac{L_2^2}{2L_1L_2} \quad (11)$$

w.k.t cos (q_2) = $cos(-q_2)$
So there is another pair of angles (q_1, q_2)
which give the same end effector (x,y).

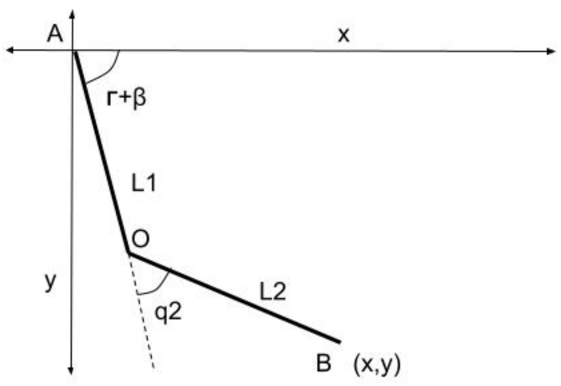

Figure 15: Inverse Kinematics

$$q_2 = -X^2 + Y^2 - L_1^2 - \frac{L_2^2}{2L_1L_2} \tag{12}$$

When we put -q_2 in the equation 10

$$q_1 = \gamma + \beta = arctan\frac{y}{x} + arctan\frac{L_2 sinq_2}{L_1 + L_2cos(q_2)} \tag{13}$$

But in our case this solution is not possible. Because the knee can only bend backwards.

We are dealing with hip ϕ_1 and knee angle ϕ_1 , that we can obtain from q_2 and q_2

$$\phi_1 = (\frac{\pi}{2} - q_1) \tag{14}$$

$$\phi_2 = (\frac{\pi}{2} - (q_1 + q_2)) \tag{15}$$

5.2 Dynamic Analysis

Consider the 2 links of the exoskeleton as shown in figure (16). Masses of link 1 and 2 are m1 and m2 respectively. X axis is considered as the reference for gravitational potential energy(PE=0).

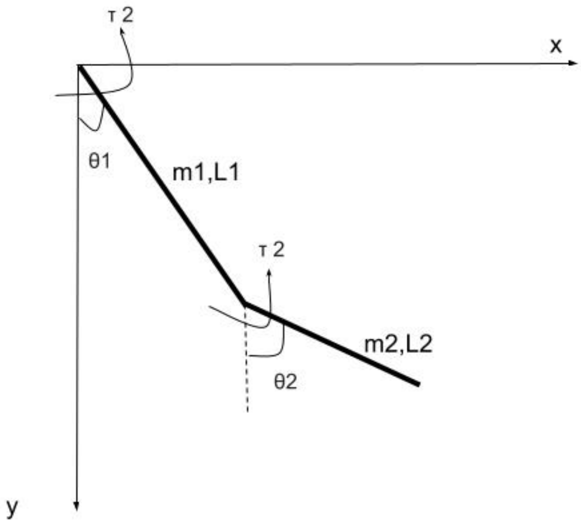

Figure 16: Dynamic Analysis

For Link 1, consider an element of length dl at a distance of l from the origin,position and velocity of that element is

$$x_1 = lsin(\theta_1) \tag{16}$$
$$y_1 = lcos(\theta_1) \tag{17}$$
$$\dot{x_1} = l\dot{\theta_1}cos(\theta_1) \tag{18}$$
$$\dot{y_1} = -l\dot{\theta_1}sin(\theta_1) \tag{19}$$
$$\tag{20}$$

Now the kinetic energy can be expressed as ,

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) \end{aligned} \tag{21}$$

The mass per unit length of link 1 is, is $m_i = m_1/L_1$. Now the kinetic energy of the elemental mass of length dl is

$$\begin{aligned}
dKE_i &= \frac{1}{2}(m_i * dl)(\dot{x_1}^2 + \dot{y_1}^2) \\
&= \frac{1}{2}m_i(\dot{x_1}^2 + \dot{y_1}^2)dl \\
&= \frac{1}{2}m_i((l\dot{\theta}_1 cos(\theta_1))^2 + (-l\dot{\theta}_1 sin(\theta_1))^2)dl \\
&= \frac{1}{2}m_i l^2\dot{\theta_1}^2 dl \\
&= \frac{1}{2}\frac{m_1}{L_1}l^2\dot{\theta_1}^2 dl \qquad (22)
\end{aligned}$$

The total kinetic energy of link 1 can be obtained by integrating (22) along its length , ie from 0 to L_1.

$$\begin{aligned}
KE_1 &= \int_0^{L_1} \frac{1}{2}\frac{m_1}{L_1}l^2\dot{\theta_1}^2\, dl \\
&= \frac{1}{6}m_1 L_1^2\dot{\theta_1}^2 \qquad (23)
\end{aligned}$$

The potential energy of link 1 with centre of mass at (x_{com1}, y_{com1}) is,

$$\begin{aligned}
PE_1 &= m_1 g y_{com1} \\
PE_1 &= -m_1 g\frac{L_1}{2}cos(\theta_1) \qquad (24)
\end{aligned}$$

To get KE and PE for link 2 , consider an element of length dl at a distance of l from the starting of link 2,position and velocity of that element is

$$x_2 = L_1 sin(\theta_1) + l sin(\theta_2) \qquad (25)$$

$$y_2 = L_1 cos(\theta_1) + l cos(\theta_2) \qquad (26)$$

$$\dot{x_2} = L_1\dot{\theta}_1 cos(\theta_1) + l\dot{\theta}_2 cos(\theta_2) \qquad (27)$$

$$\dot{y_2} = -L_1\dot{\theta}_1 sin(\theta_1) - l\dot{\theta}_2 sin(\theta_2) \qquad (28)$$

KE of the elemental mass is,

$$\begin{aligned}
dKE_2 &= \frac{1}{2}(m_i * dl)(\dot{x_2}^2 + \dot{y_2}^2) \\
&= \frac{1}{2}\frac{m_2}{L_2}(\dot{x_2}^2 + \dot{y_2}^2)dl \\
&= \frac{1}{2}\frac{m_2}{L_2}[(L_1\dot{\theta}_1 cos(\theta_1) + l\dot{\theta}_2 cos(\theta_2))^2 + (-L_1\dot{\theta}_1 sin(\theta_1) - l\dot{\theta}_2 sin(\theta_2))^2]dl \qquad (29)
\end{aligned}$$

Integrating (29) along the length of link 2 gives the total kinetic energy of link 2 as,

$$\begin{aligned}
KE_2 &= \int_0^{L_2} [\frac{1}{2}\frac{m_2}{L_2}[(L_1\dot{\theta}_1 cos(\theta_1) + L_2\dot{\theta}_2 cos(\theta_2))^2 + (-L_1\dot{\theta}_1 sin(\theta_1) - L_2\dot{\theta}_2 sin(\theta_2))^2]]\, dl \\
&= \frac{1}{2}m_2(L_1^2\dot{\theta_1}^2 + \frac{L_2^2\dot{\theta_2}^2}{3} + L_1 L_2\dot{\theta}_1\dot{\theta}_2 cos(\theta_1 - \theta_2)) \qquad (30)
\end{aligned}$$

The potential energy of link 2 with centre of mass at (x_{com2}, y_{com2}) is,

$$PE_2 = m_2 g y_{com2}$$
$$PE_2 = -m_2 g(L_1 cos(\theta_1) + \frac{L_2}{2} cos(\theta_2)) \tag{31}$$

The Lagrangian of the system is,

$$\begin{aligned}\mathcal{L} &= KE - PE \qquad (32)\\ &= (KE_1 + KE_2) - (PE_1 + PE_2)\\ &= \frac{1}{6} M_1 L_1^2 \dot{\theta_1}^2 + \frac{1}{2} M_2 [L_1^2 \dot{\theta_1}^2 + \frac{L_2^2 \dot{\theta_2}^2}{3} + L_1 L_2 \dot{\theta_1} \dot{\theta_2} cos(\theta_1 - \theta_2)]\\ &+ m_1 g \frac{L_1}{2} cos(\theta_1) + m_2 g [L_1 cos(\theta_1) + \frac{L_2}{2} cos(\theta_2)] \qquad (33)\end{aligned}$$

The Lagrangian equations for two generalised coordinates θ_1 and θ_2 with non-conservative torques τ_1 and τ_2 acting as shown in Figure 16 are ,

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{\theta_1}}) - \frac{\partial \mathcal{L}}{\partial \theta_1} = \tau_1 \tag{34}$$

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{\theta_2}}) - \frac{\partial \mathcal{L}}{\partial \theta_2} = \tau_2 \tag{35}$$

Each term for (34) and (35) is obtained from the Lagrangian of the system (33) as,

$$\frac{\partial L}{\partial \dot{\theta_1}} = \frac{1}{3} m_1 L_1^2 \dot{\theta}_1 + \frac{1}{2} m_2 [2 L_1^2 \dot{\theta}_1 + L_1 L_2 \dot{\theta}_2 cos(\theta_1 - \theta_2)] \tag{36}$$

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{\theta_1}}) = \frac{1}{3} m_1 L_1^2 \ddot{\theta}_1 + \frac{1}{2} m_2 [2 L_1^2 \ddot{\theta}_1 + L_1 L_2 (\ddot{\theta}_2 cos(\theta_1 - \theta_2) - \dot{\theta}_2 sin(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2))] \tag{37}$$

$$\frac{\partial L}{\partial \theta_1} = -\frac{1}{2} m_2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 sin(\theta_1 - \theta_2) - m_1 g \frac{L_1}{2} sin(\theta_1) - m_2 g L_1 sin(\theta_1) \tag{38}$$

Substituting above 2 into (34) gives,

$$\begin{aligned}\tau_1 &= \frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{\theta_1}}) - \frac{\partial L}{\partial \theta_1}\\ &= \ddot{\theta}_1 [\frac{1}{3} m_1 L_1^2 + m_2 L_1^2] + \ddot{\theta}_2 \frac{1}{2} m_2 L_1 L_2 cos(\theta_1 - \theta_2)\\ &+ \dot{\theta_2}^2 \frac{1}{2} m_2 sin(\theta_1 - \theta_2) L_1 L_2 + L_1 g(\frac{m_1}{2} + m_2) sin(\theta_1) \qquad (39)\end{aligned}$$

Similarly for generalised coordinate θ_2,

$$\begin{aligned}\tau_2 &= \frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{\theta_2}}) - \frac{\partial L}{\partial \theta_2}\\ &= \frac{1}{2} m_2 [\frac{2}{3} L_2^2 \ddot{\theta}_2 + L_1 L_2 \dot{\theta}_1 cos(\theta_1 - \theta_2) - L_1 L_2 \dot{\theta}_1 sin(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2)]\\ &- \frac{1}{2} m_2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 sin(\theta_1 - \theta_2) + m_2 g \frac{L_2}{2} sin(\theta_2) \qquad (40)\end{aligned}$$

Equations (39) and (40) represents the governing dynamical equations of 2 link model with torques τ_1 and τ_2 acting as shown in Figure 16.

6 Control

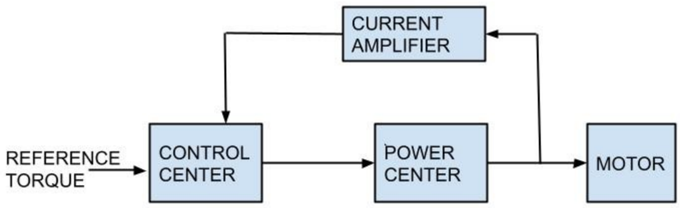

Figure 17: Control System Block Diagram

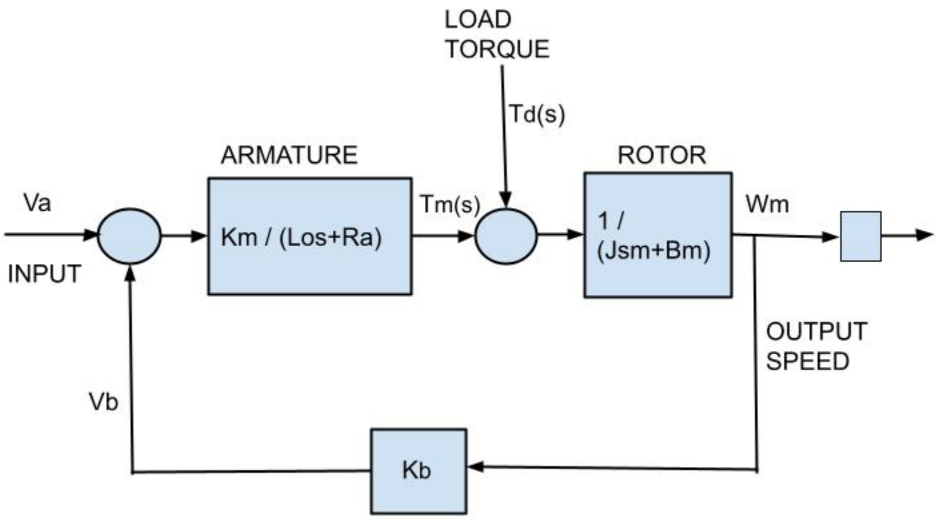

Figure 18: Motor Block Diagram

The main objective of the exoskeleton is to help in rehabilitation of the cerebral palsy affected kids. In the rehabilitation process, the patient is to be trained for two different parameters of gait. They are the step length and the speed of walking. The control system is robustly designed in order to accommodate these different parameters. The control module takes in as input the step length and the speed to generate a suitable gait pattern and precisely control the movements of each joint for a coordinated movement.

In order to coordinate the joint motion, a co-relation between the joint angle of each joint with respect to other joints is found. Having the coordinated joint angles, the controller achieves precise movement of each joint using a feedback control system to reduce the error. The speed on the joints is controlled by regulating the voltage which there by controls the angular velocity of each joint which in turn controls the speed of walking.

DC motor is a device which converts electrical energy (Direct Current) to mechanical work.It works on the principle " current carrying conductor placed in a magnetic field experiences mechanical force" and "Faraday's law of electromagnetic Induction (Lorentz;s law)".

It is widely used in industry due to its high torque and excellent speed control compatibility.

Applied voltage, E_a

Back e.m.f, E_b

Armature current, I_a

Impedance offered by armature, $L_a + R_a$

Apply KVL in loop

We get,

$$E_a = E_b + (I_a R_a + L\frac{dI_a}{dt}) \tag{41}$$

$$E_b = \frac{(Pn\phi w)}{60A} \tag{42}$$

Here P is the number of poles, n is the number of coils, A is a constant and is the magnetic flux.In today's DC motors, electromagnets are used. As a result, by having a fixed amount of current to the electromagnet, we can create a fixed amount of flux.Therefore a particular DC motor all the above things mentioned are constants.So equation can be rewritten as

$$E_b = K_b w \tag{43}$$

where w is the speed of the motor. Substituting equation 41 in equation 43

$$W = \frac{(E_a - (I_a R_a + L\frac{dI_a}{dt}))}{K_b} \tag{44}$$

Which indicates DC motor speed can be varied by changing armature voltage or supply voltage.

Torque vs Current relation in a motor Torque,

$$\tau_m = F.d \tag{45}$$

$$F = B.I_c.L \tag{46}$$

Where

B is flux density

I_c is current through Coil

L is length of the coil

$$I_c = \frac{I_a}{A} \tag{47}$$

$$\tau_m = \frac{B.I_a.L}{A} d \tag{48}$$

$$= \frac{\phi L d}{A^2} I_a \tag{49}$$

$$= K_m I_a \tag{50}$$

Going back to equation 41

$$E_a = E_b + (I_a R_a + L\frac{dI_a}{dt}) \tag{51}$$

Converting it to laplace domain

$$E_a(s) - E_b(s) = (R_a + L_a S)I_a(s) \tag{52}$$

$$I_a(s) = \frac{E_a(s) - E_b(s)}{(R_a + L_a S)} \tag{53}$$

$$T_l(s) = T_m(s) - T_d(s) \tag{54}$$

Motor output torque equation

$$I_m \frac{dw}{dt} = \tau_m - \tau_l - B_m W_m \tag{55}$$

I_m is inertia of rotor
B_m is viscous damping
In laplace domain

$$\tau_m - \tau_l = W_m(B_m + I_m S) \tag{56}$$

$$W_m = \frac{\tau_m - \tau_l}{(B_m + I_m S)} \tag{57}$$

From above expression we get transfer function of DC motor is

$$\frac{W_m(s)}{E_a(s)} = \frac{K_m}{(R_a + L_a S) + (B_m + I_m S + K_m K_b)} \tag{58}$$