Multiclass Support Vector Machine exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page (http://vision.stanford.edu/teaching/cs231n/assignments.html)](http://vision.stanford.edu/teaching/cs231n/assignments.html) on the course website.

In this exercise you will:

- implement a fully-vectorized **loss function** for the SVM
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

In [1]:

```
# Run some setup code for this notebook.
import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

# This is a bit of magic to make matplotlib figures appear inline in the
# notebook rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

CIFAR-10 Data Loading and Preprocessing

In [2]:

```
# Load the raw CIFAR-10 data.
cifar10_dir = 'D:\sufe/人工智能/spring1819_assignment1/assignment1/cs231n/datasets/cifar-10-python/c

# Cleaning up variables to prevent loading data multiple times (which may cause memory issue)
try:
   del X_train, y_train
   del X_test, y_test
   print('Clear previously loaded data.')
except:
   pass

X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

In [3]:

```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```

In [4]:

```
# Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Train data shape:  (49000, 32, 32, 3)
Train labels shape:  (49000,)
Validation data shape:  (1000, 32, 32, 3)
Validation labels shape:  (1000,)
Test data shape:  (1000, 32, 32, 3)
Test labels shape:  (1000,)
```

In [5]:

```
# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
print('dev data shape: ', X_dev.shape)
```

```
Training data shape:  (49000, 3072)
Validation data shape:  (1000, 3072)
Test data shape:  (1000, 3072)
dev data shape:  (500, 3072)
```

In [6]:

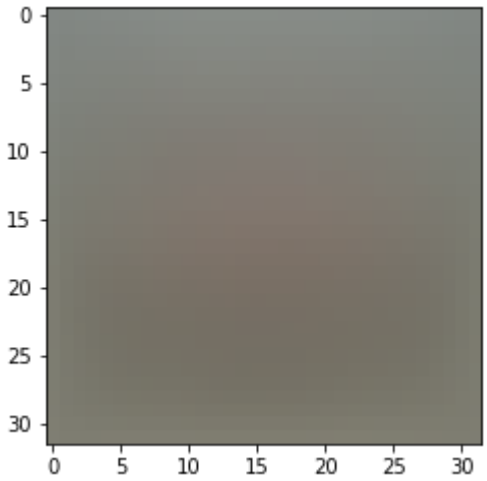
```
# Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean image
plt.show()

# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image

# third: append the bias dimension of ones (i.e. bias trick) so that our SVM
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)
```

```
[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082
 131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]
```




```
(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

SVM Classifier

Your code for this section will all be written inside **cs231n/classifiers/linear_svm.py**.

As you can see, we have prefilled the function `compute_loss_naive` which uses for loops to evaluate the multiclass SVM loss function.

In [7]:

```
# Evaluate the naive implementation of the loss we provided for you:
from cs231n.classifiers.linear_svm import svm_loss_naive
import time

# generate a random SVM weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001

loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.000005)
print('loss: %f' % (loss, ))
```

```
loss: 9.207639
```

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function `svm_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

In [8]:

```
# Once you've implemented the gradient, recompute it with the code below
# and gradient check it with the function we provided for you

# Compute the loss and its gradient at W.
loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.0)

# Numerically compute the gradient along several randomly chosen dimensions, and
# compare them with your analytically computed gradient. The numbers should match
# almost exactly along all dimensions.
from cs231n.gradient_check import grad_check_sparse
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad)

# do the gradient check once again with regularization turned on
# you didn't forget the regularization gradient did you?
loss, grad = svm_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad)
```

```
numerical: -8.052323 analytic: -8.052323, relative error: 1.515876e-11
numerical: -5.386503 analytic: -5.335221, relative error: 4.783068e-03
numerical: 46.065120 analytic: 46.065120, relative error: 1.983391e-12
numerical: 10.347004 analytic: 10.418619, relative error: 3.448721e-03
numerical: -21.919426 analytic: -21.899566, relative error: 4.532433e-04
numerical: 30.803461 analytic: 30.803461, relative error: 1.617901e-11
numerical: 23.642841 analytic: 23.642841, relative error: 5.567353e-14
numerical: -8.979176 analytic: -8.979176, relative error: 5.701580e-12
numerical: 9.493934 analytic: 9.493934, relative error: 4.135943e-13
numerical: 8.871639 analytic: 8.956802, relative error: 4.776837e-03
numerical: 1.525908 analytic: 1.529851, relative error: 1.290385e-03
numerical: 7.023453 analytic: 7.017941, relative error: 3.925456e-04
numerical: 1.348772 analytic: 1.350532, relative error: 6.523615e-04
numerical: 5.983245 analytic: 5.928392, relative error: 4.605001e-03
numerical: 20.901930 analytic: 20.804677, relative error: 2.331841e-03
numerical: 20.717090 analytic: 20.740276, relative error: 5.592720e-04
numerical: 18.675800 analytic: 18.672488, relative error: 8.867513e-05
numerical: 6.766306 analytic: 6.814697, relative error: 3.563123e-03
numerical: -0.954882 analytic: -1.022985, relative error: 3.443264e-02
numerical: -3.924383 analytic: -3.930129, relative error: 7.315494e-04
```

Inline Question 1

It is possible that once in a while a dimension in the gradcheck will not match exactly. What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a gradient check could fail? How would change the margin affect of the frequency of this happening? *Hint: the SVM loss function is not strictly speaking differentiable*

YourAnswer: In zero loss function is not differentiable so the numerical may be fail to check.

In [9]:

```
# Next implement the function svm_loss_vectorized; for now only compute the loss;
# we will implement the gradient in a moment.
tic = time.time()
loss_naive, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from cs231n.classifiers.linear_svm import svm_loss_vectorized
tic = time.time()
loss_vectorized, _ = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# The losses should match but your vectorized implementation should be much faster.
print('difference: %f' % (loss_naive - loss_vectorized))
```

```
Naive loss: 9.207639e+00 computed in 0.177897s
Vectorized loss: 9.207639e+00 computed in 0.587671s
difference: -0.000000
```

In [10]:

```
# Complete the implementation of svm_loss_vectorized, and compute the gradient
# of the loss function in a vectorized way.

# The naive implementation and the vectorized implementation should match, but
# the vectorized version should still be much faster.
tic = time.time()
_, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss and gradient: computed in %fs' % (toc - tic))

tic = time.time()
_, grad_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss and gradient: computed in %fs' % (toc - tic))

# The loss is a single number, so it is easy to compare the values computed
# by the two implementations. The gradient on the other hand is a matrix, so
# we use the Frobenius norm to compare them.
difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('difference: %f' % difference)
```

```
Naive loss and gradient: computed in 0.168514s
Vectorized loss and gradient: computed in 0.015625s
difference: 0.000000
```

Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss.

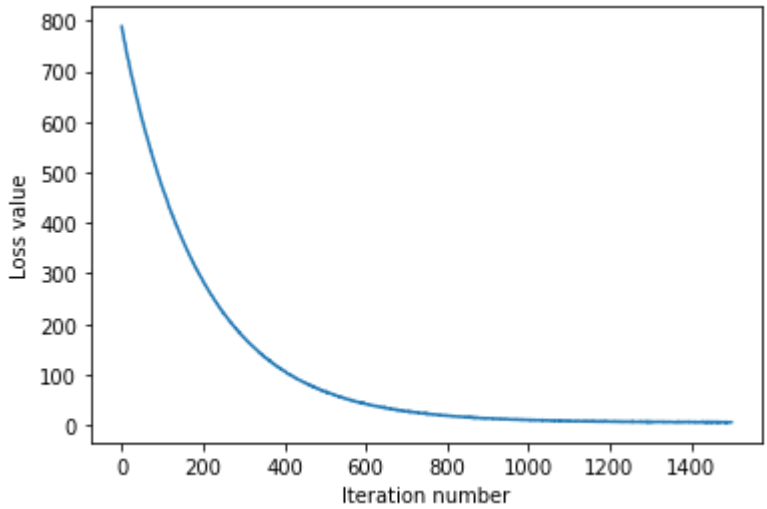
In [11]:

```
# In the file linear_classifier.py, implement SGD in the function
# LinearClassifier.train() and then run it with the code below.
from cs231n.classifiers import LinearSVM
svm = LinearSVM()
tic = time.time()
loss_hist = svm.train(X_train, y_train, learning_rate=1e-7, reg=2.5e4,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took %fs' % (toc - tic))
```

```
iteration 0 / 1500: loss 788.484168
iteration 100 / 1500: loss 470.476430
iteration 200 / 1500: loss 284.315863
iteration 300 / 1500: loss 173.604461
iteration 400 / 1500: loss 107.190669
iteration 500 / 1500: loss 66.198773
iteration 600 / 1500: loss 42.145588
iteration 700 / 1500: loss 27.150341
iteration 800 / 1500: loss 18.544021
iteration 900 / 1500: loss 13.932274
iteration 1000 / 1500: loss 10.807558
iteration 1100 / 1500: loss 8.280959
iteration 1200 / 1500: loss 7.255413
iteration 1300 / 1500: loss 6.064130
iteration 1400 / 1500: loss 6.054853
That took 269.229949s
```

In [12]:

```
# A useful debugging strategy is to plot the loss as a function of
# iteration number:
plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```



In [13]:

```
# Write the LinearSVM.predict function and evaluate the performance on both the
# training and validation set
y_train_pred = svm.predict(X_train)
print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))
y_val_pred = svm.predict(X_val)
print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))
```

```
training accuracy: 0.378816
validation accuracy: 0.387000
```

In [19]:

```
# Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of about 0.39 on the validation set.

#Note: you may see runtime/overflow warnings during hyper-parameter search.
# This may be caused by extreme values, and is not a bug.

learning_rates = [1e-7, 5e-5]
regularization_strengths = [2.5e4, 5e4]

# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the fraction
# of data points that are correctly classified.
results = {}
best_val = -1   # The highest validation accuracy that we have seen so far.
best_svm = None # The LinearSVM object that achieved the highest validation rate.

################################################################################
# TODO:                                                                        #
# Write code that chooses the best hyperparameters by tuning on the validation #
# set. For each combination of hyperparameters, train a linear SVM on the      #
# training set, compute its accuracy on the training and validation sets, and  #
# store these numbers in the results dictionary. In addition, store the best   #
# validation accuracy in best_val and the LinearSVM object that achieves this  #
# accuracy in best_svm.                                                        #
#                                                                              #
# Hint: You should use a small value for num_iters as you develop your         #
# validation code so that the SVMs don't take much time to train; once you are #
# confident that your validation code works, you should rerun the validation   #
# code with a larger value for num_iters.                                      #
################################################################################
# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

for learning_rate in learning_rates:
    for regularization_strength in regularization_strengths:
        svm = LinearSVM()
        loss_hist = svm.train(X_train, y_train, learning_rate=learning_rate, reg=regularization_str
                    num_iters=1000, verbose=True)
        y_train_pred = svm.predict(X_train)
        y_val_pred = svm.predict(X_val)
        y_train_acc = np.mean(y_train_pred==y_train)
        y_val_acc = np.mean(y_val_pred==y_val)
        results[(learning_rate,regularization_strength)] = [y_train_acc, y_val_acc]
        if y_val_acc > best_val:
            best_val = y_val_acc
            best_svm = svm


# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
                lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val)
```

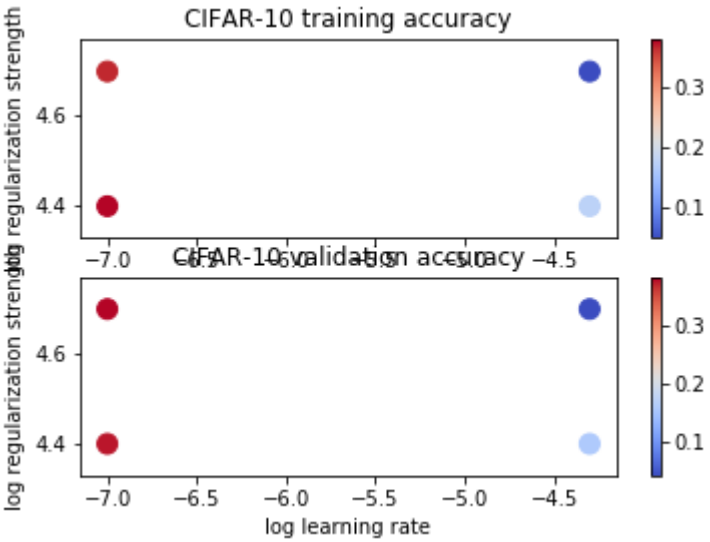
```
iteration 0 / 1000: loss 782.932409
iteration 100 / 1000: loss 465.641702
iteration 200 / 1000: loss 283.566194
iteration 300 / 1000: loss 171.077748
iteration 400 / 1000: loss 105.631311
iteration 500 / 1000: loss 65.696312
iteration 600 / 1000: loss 41.568842
iteration 700 / 1000: loss 27.743106
iteration 800 / 1000: loss 18.162112
iteration 900 / 1000: loss 13.179842
iteration 0 / 1000: loss 1565.381478
iteration 100 / 1000: loss 570.571744
iteration 200 / 1000: loss 211.511528
iteration 300 / 1000: loss 80.514224
iteration 400 / 1000: loss 33.009016
iteration 500 / 1000: loss 15.489619
iteration 600 / 1000: loss 9.253721
iteration 700 / 1000: loss 7.228497
iteration 800 / 1000: loss 6.194568
iteration 900 / 1000: loss 5.860734
iteration 0 / 1000: loss 782.614940
iteration 100 / 1000: loss 2080.160344
iteration 200 / 1000: loss 1684.686949
iteration 300 / 1000: loss 1601.870073
iteration 400 / 1000: loss 1397.265847
iteration 500 / 1000: loss 1368.648298
iteration 600 / 1000: loss 1465.032583
iteration 700 / 1000: loss 1612.470008
iteration 800 / 1000: loss 1472.785296
iteration 900 / 1000: loss 1398.053889
iteration 0 / 1000: loss 1579.847080
iteration 100 / 1000: loss 7845211833042028542007346792998958530 56.000000
iteration 200 / 1000: loss 1296750684318662904266817427312736852986862100526752040376240249049081446 40.000000
iteration 300 / 1000: loss 2143425025438585955168191625644838031821069194231615300996140134134851761255996298325396466561256173224958361 6.000000
iteration 400 / 1000: loss 3542909901829217083968003884145272460684084466987264493479766479803788514968031401681113959807554297633356879984547265169897510142984844071665664.000000
iteration 500 / 1000: loss 5856146318862301435023601324274873810996557890610485898057286839658039786066224985831642266980411065860416682884884021730581233348342248339391721073072437686403675260016661752709 12.000000
iteration 600 / 1000: loss 9679740850936722620154227344651345012449948663416953737656949991541727734340584478988472135499322633137146143815524928184911759730266391987069482766003598634649636716345992638752198561508609771697806286948290002944 0.000000
iteration 700 / 1000: loss 1599983638378355391919376144208842148129112860023094966869733114630898479688949027299674141931041943643411999268161251388576986061533313300028146604386523369871506107432507903877780609126522302775405995207616822619389021636804353250851957901138015027 2.000000
iteration 800 / 1000: loss 2644644812810986915938009944855462404777129753975287291751933877887768464048026450359138648978028821490146042475045590234813269748203522057288983698008015790837480379340699993804717138761392645104529825891066059354346971490507707325722706611458189841049042145726008053640440032100614144.000000
iteration 900 / 1000: loss inf
lr 1.000000e-07 reg 2.500000e+04 train accuracy: 0.380469 val accuracy: 0.379000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.370000 val accuracy: 0.385000
lr 5.000000e-05 reg 2.500000e+04 train accuracy: 0.176531 val accuracy: 0.160000
lr 5.000000e-05 reg 5.000000e+04 train accuracy: 0.047122 val accuracy: 0.040000
best validation accuracy achieved during cross-validation: 0.385000
```

In [20]:

```
# Visualize the cross-validation results
import math
x_scatter = [math.log10(x[0]) for x in results]
y_scatter = [math.log10(x[1]) for x in results]

# plot training accuracy
marker_size = 100
colors = [results[x][0] for x in results]
plt.subplot(2, 1, 1)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap=plt.cm.coolwarm)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 training accuracy')

# plot validation accuracy
colors = [results[x][1] for x in results] # default size of markers is 20
plt.subplot(2, 1, 2)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap=plt.cm.coolwarm)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 validation accuracy')
plt.show()
```



In [21]:

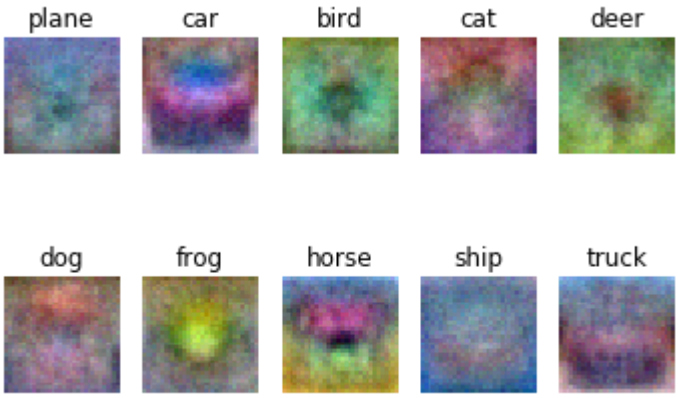
```
# Evaluate the best svm on test set
y_test_pred = best_svm.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('linear SVM on raw pixels final test set accuracy: %f' % test_accuracy)
```

```
linear SVM on raw pixels final test set accuracy: 0.360000
```

In [22]:

```
# Visualize the learned weights for each class.
# Depending on your choice of learning rate and regularization strength, these may
# or may not be nice to look at.
w = best_svm.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)
w_min, w_max = np.min(w), np.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```



Inline question 2

Describe what your visualized SVM weights look like, and offer a brief explanation for why they look they way that they do.

YourAnswer: They look like some obsure signals. Because it more like a correlation detection, so pictures will fall into the category with which they are more relevant.