This result is less restrictive than the time-lagged explicit treatment necessary for the leap-frog scheme, which yields $\beta \leq \frac{0.5}{6}$. The stability analysis for the density equation is equivalent, except that it requires a modified definition of $\beta_\rho = \frac{\Delta t}{Re\, Pr\, \Delta z^2}$.

3.3 Numerical Boundary Conditions

3.3.1 Rayleigh Damping Sponge Layer

A wave-absorbing sponge layer is frequently used as an open boundary condition. In this model a sponge layer is used in the upper region of the finite computational domain. This layer is typically located far below the free surface of the fluid, and its purpose is simply to mimic the presence of the fluid above the computational domain. Rayleigh damping is an efficient wave-absorbing sponge layer, and is competitive with the best radiation boundary schemes for certain parameter ranges. The accuracy, however, is highly dependent upon the number of points used in the sponge layer. Durran et al. (1992) compare the wave absorbing layer to other wave permeable outflow boundary conditions. Typically, in the present model, ten percent of the total grid points ($\sim 13 - 40$) are used to form the sponge layer. The damping coefficients are suggested by Klemp and Lilly (1978) and are graphed in Figure 3.7. The method is given by Davies (1983) and Durran et al (1992).

Rayleigh damping is of the form

$$\bar{\mu}_i^{n+1} = \mu_i^{n+1} - \sigma_i \left(\mu_i^{n+1} - \mu_{io} \right) , \tag{3.56}$$

where $\bar{\mu}$ is the damped value of an arbitrary function (such as $\mathbf{u}$ or ρ), μ is the predamped value, μ_{io} is the relaxed value of the function in the sponge region (usually zero), and σ_i is the damping coefficient given for the present case by the Gaussian shape

$$\sigma_z = e^{\frac{-z^2}{2}} \quad , \quad 0 \leq z \leq 3.5 \ . \tag{3.57}$$

The two major disadvantages with the sponge layer are that it becomes computationally expensive in two- or three-dimensional problems, and it has the property that longer waves are absorbed less efficiently than short waves.

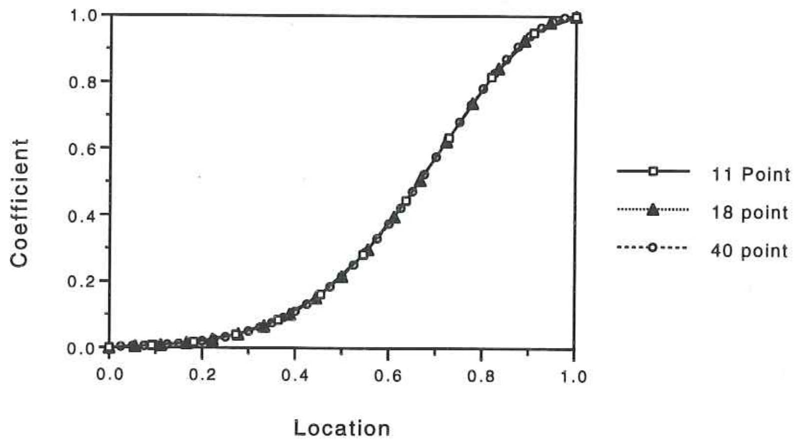

Figure 3.7: Rayleigh damping coefficients for sponge layers of Gaussian shape. The location is normalized by the thickness of the sponge layer.

Additional complexity arises in the implementation of the sponge layer because it does not damp the flow in a nondivergent manner. The divergence is evident since the coefficients σ_i are only functions of z. This problem is solved by implementing the Rayleigh damping step before the projection step. In this manner the total damping procedure is

$$\bar{u}_i^{n+1} = u_i^* - \sigma_i \left(u_i^* - u_{io}\right) - \Delta t \frac{\partial p^{n+1}}{\partial x} , \tag{3.58}$$

where the value of the pressure gradient is determined by the projection method.

3.3.2 Free-Slip Boundary Conditions

The free-slip boundary conditions require numerical implementation of at least third-order accuracy to maintain global fourth-order accuracy in the model. A second-order implementation of the condition $\left.\frac{\partial u}{\partial z}\right|_b = 0$ is simple and intuitive, i.e., $u_b^{n+1} = u_{b+1}^{n+1}$. The third- and fourth-order free-slip conditions are found by matching Taylor series coefficients to the desired order of accuracy. On a uniform mesh they are, respectively,

$$u_1 = \frac{4}{3}u_2 - \frac{1}{3}u_3 , \tag{3.59}$$

$$u_1 = \frac{18}{11}u_2 - \frac{9}{11}u_3 + \frac{2}{11}u_4 . \tag{3.60}$$