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4. Exercise sheet "Model Order Reduction"

Exercise 1

Prove Theorem 5.4 for $s_0 = \infty$.

Theorem 5.4 Let $[A, B, C, D]$ be a controlable LTI-SISO-system of order n with moments $\{M_k(s_0)\}_k$ for some given $s_0 \in (\mathbb{C} \cup \{\infty\}) \setminus \operatorname{eig}(A)$. Let $\ell \in \{1, \ldots, n\}$ and $\tilde{T} \in \mathbb{C}^{\ell,\ell}$ be invertible. Let $T_1 := S_\ell(s_0)\tilde{T} \in \mathbb{C}^{n,\ell}$ and $W_1 \in \mathbb{C}^{\ell,n}$ such that $W_1 T_1 = I_\ell$. Let $[\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}] := [W_1 A T_1, W_1 B, C T_1, D]$ with moments $\{\tilde{M}_k(s_0)\}_k$. Then $M_k(s_0) = \tilde{M}_k(s_0)$ for $k = 1, \ldots, \ell - 1$.

Exercise 2

a) Does the proof of Theorem 5.4 still hold for SIMO (single input, multiple output) systems? If not, can it be fixed?
b) Does the proof of Theorem 5.4 still hold for MISO (multiple input, single output) systems? If not, can it be fixed?
c) Does the proof of Theorem 5.4 still hold for MIMO (multiple input, multiple output) systems? If not, can it be fixed?
d) For $m \neq p$ would you prefer the moment matching method from Theorem 5.4 or the one from Theorem 5.5?

Exercise 3

Let $[A, B, C, D]$ be an LTI-SISO-system of order n. Let $s_0, s_1 \in \mathbb{C} \setminus \operatorname{eig}(A), s_0 \neq s_1$. Let $\ell_0, \ell_1 \in \mathbb{N}$ such that $\ell := \ell_0 + \ell_1 \leq n$.
a) Assume $T_1 := [S_{\ell_0}(s_0), S_{\ell_1}(s_1)]$ has full rank, let W_1 be any left inverse of T_1 and set $[\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}] := [W_1 A T_1, W_1 B, C T_1, D]$. Prove that then
$M_k(s_0) = \tilde{M}_k(s_0)$ for $k = 1, \ldots, \ell_0 - 1$ and $M_k(s_1) = \tilde{M}_k(s_1)$ for $k = 1, \ldots, \ell_1 - 1$.
b) Assume that A is diagonalizable. Prove

$$\operatorname{span}[(s_0 I - A)^{-1} B, (s_1 I - A)^{-1} B] = \operatorname{span}[(s_0 I - A)^{-1} B, (s_1 I - A)^{-1} (s_0 I - A)^{-1} B]$$

and

$$\begin{aligned}\operatorname{span}[(s_0 I - A)^{-1} B, (s_0 I - A)^{-2} B, (s_1 I - A)^{-1} B, (s_1 I - A)^{-2} B]& \\ = \operatorname{span}[(s_0 I - A)^{-1} B,& \\ (s_1 I - A)^{-1} (s_0 I - A)^{-1} B,& \\ (s_0 I - A)^{-1} (s_1 I - A)^{-1} (s_0 I - A)^{-1} B,& \\ (s_1 I - A)^{-1} (s_0 I - A)^{-1} (s_1 I - A)^{-1} (s_0 I - A)^{-1} B]&.\end{aligned}$$

Notation

$S_\ell(s_0) = [(s_0 I - A)^{-1} B, \ldots, (s_0 I - A)^{-\ell} B], \quad S_\ell(\infty) = [B, AB, \ldots, A^{\ell-1} B]$