Technische Universität Berlin
Fakultät II – Institut für Mathematik
C. Schröder

Winter 2015/16

2. Exercise sheet "Model Order Reduction"

Exercise 1

Let $A, Q \in \mathbb{R}^{n,n}$ with Q invertible be such that

$$Q^{-1}AQ = \begin{bmatrix} A_1 & A_{12} \\ 0 & A_2 \end{bmatrix}$$

where $A_1 \in \mathbb{R}^{r,r}$ for some $r \in \{1, \dots, n-1\}$. Let $B \in \mathbb{R}^{n,m}$ and define

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} := Q^{-1}B$$

where $B_1 \in \mathbb{R}^{r,m}$.
a) Show that if (A, B) is controllable, then also (A_2, B_2) is controllable.
b) Is then also also (A_1, B_1) controllable? Give a proof or a counter example!

Exercise 2

Consider an LTI system $[A, B, C, D]$ of size n with transfer function $G(s)$. Let Q be invertible such that A, B, C are put into the forms

$$Q^{-1}AQ = \begin{bmatrix} A_{c\bar{o}} & A_{12} & A_{13} & A_{14} \\ 0 & A_{co} & A_{23} & A_{24} \\ 0 & 0 & A_{\bar{c}\bar{o}} & A_{34} \\ 0 & 0 & 0 & A_{\bar{c}o} \end{bmatrix}, \quad Q^{-1}B = \begin{bmatrix} B_1 \\ B_2 \\ 0 \\ 0 \end{bmatrix}, \quad CQ = [0, C_2, 0, C_4]$$

where (A_{co}, B_2) is controllable and (A_{co}, C_2) is observable. Show that $[A_{co}, B_2, C_2, D]$ is a minimal realization of $G(s)$.

Exercise 3

Prove!
Lemma Let $A \in \mathbb{R}^{n,n}$. Let $X_I \in \mathbb{R}^{n,n}$ be a symmetric positive definite solution of $A^T X_I + X_I A + I = 0$. Then $\lambda(A) \subset \{z \in \mathbb{C} | \mathrm{Re}(z) \le -(2\|X_I\|)^{-1}\}$.
Hint: the proof is a short extension of a proof in the lecture.

Exercise 4

Show that the observability Grammian Q of an asymptotically stable system $[A, B, C, D]$ can be written as

$$Q = \frac{1}{2\pi} \int_{-\infty}^{\infty} (-\imath\omega I - A^T)^{-1} C^T C (\imath\omega I - A)^{-1} \mathrm{d}\omega.$$

Hint: Parsevals identity for Fourier transforms is helpful here.

Exercise 5

For $G \in \mathbb{R}^{p,m}(s)$ prove that $G \in \mathbb{H}_{\infty}^{p,m}$ if and only if G is proper and has no poles in $\overline{\mathbb{C}^-}$.