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Winter 2015/16

3. Exercise sheet "Model Order Reduction"

Exercise 1

a) Show that if a single-input, single-output system (i.e., B and C are vectors) is balanced, then $|A|$ is symmetric and $|B| = |C|^T$.
Here, $|\cdot|$ denotes the matrix of elementwise absolute values.
b) Use balanced truncation to construct the reduced order model (ROM) of order $l = 1$ of the LTI system with

$$A = \begin{bmatrix} 1 & \frac{5}{4} \\ -\frac{7}{4} & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = 5.$$

Give an error bound for the $\mathbb{H}_\infty$ norm of the transfer function. Compare with the ROM generated by modal truncation.

Exercise 2

Prove the following properties of the Kronecker product and the vec-operator!
a) $(A \otimes B) \otimes C = A \otimes (B \otimes C)$ for all $A \in \mathbb{R}^{n\times k}, B^{l\times n}, C \in \mathbb{R}^{p\times q}$.
b) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$ for all $A \in \mathbb{R}^{n\times k}, B^{l\times n}, C \in \mathbb{R}^{k\times p}, D \in \mathbb{R}^{n\times q}$.
c) If $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{m\times m}$ are invertible then so is $A \otimes B$ and its inverse is given by $A^{-1} \otimes B^{-1}$.
d) $\det(A \otimes B) = \det(A)^m \det(B)^n$ for all $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{m\times m}$.
e) $\lambda(A \otimes B) = \{\lambda\mu : \lambda \in \lambda(A), \mu \in \lambda(B)\}$ for all $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{m\times m}$.
f) $\mathrm{vec}(AXB) = (B^T \otimes A)\mathrm{vec}(X)$ for all $A \in \mathbb{R}^{n\times k}, X \in \mathbb{R}^{k\times l}, B^{l\times n}$.

Exercise 3

Consider the Sylvester equation

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} X + X \begin{bmatrix} 3 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -8 & 20 \end{bmatrix}.$$

a) Is there a unique solution?
b) Find a solution without forming Kronecker products!
Hint: multiply out the products and try to use some form of back substitution.
c) Write down an algorithm that can solve the Sylvester equation $BX + XA = W$ for upper triangular matrices A, B without forming Kronecker products.
d) Widen the applicability of above algorithm to general Sylvester equations using some matrix decomposition! What complexity does this algorithm possess?