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1. Exercise sheet "Model Order Reduction"

Exercise 1

Show that the Laplace transform of e^{At} is given by

$$\mathcal{L}(e^{At})(s) = (sI - A)^{-1} \quad \text{for } Re(s) > \max_{\lambda \in \mathrm{eig}(A)} \mathrm{Re}(\lambda),$$

where $\mathrm{eig}(A)$ denotes the set of eigenvalues of A.

Exercise 2

Let $[\,A,\, B,\, C,\, D\,]$ be a realization of the transfer function $G(s) \in R_p^{p,m}(s)$.

a) Construct a realization of $G^T(s)$.

b) Let D be invertible. Construct a realization of $G^{-1}(s)$.
Hint: Use the Sherman-Morrisson-Woodbury formula. Alternatively consider the inverse of

$$\begin{aligned}\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} &= \begin{bmatrix} I & 0 \\ A_{21}A_{11}^{-1} & I \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} - A_{21}A_{11}^{-1}A_{12} \end{bmatrix} \\ &= \begin{bmatrix} A_{11} - A_{12}A_{22}^{-1}A_{21} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ A_{22}^{-1}A_{21} & I \end{bmatrix}.\end{aligned}$$

Exercise 3

Let $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{n,m}$, and $\mathcal{S} = [B, AB, A^2B, \ldots, A^{n-1}B]$. Show that if $\mathrm{rank}(\mathcal{S}) = n$, then the matrix

$$P(t) := \int_0^t e^{A\tau} BB^T e^{A^T\tau} \mathrm{d}\tau$$

is symmetric positive definite for every $t > 0$.

Exercise 4

Show that if $(A, C) \in \mathbb{R}^{n,n} \times \mathbb{R}^{p,n}$ is observable, then the matrix

$$Q(t) := \int_0^t e^{A^T\tau} C^T C e^{A\tau} \mathrm{d}\tau$$

is symmetric positive definite for every $t > 0$.

Exercise 5 (image compression using the SVD)

Write an MATLAB program `imagecompressionsvd(image,k)` which

- loads a matrix X of image informations,
- computes a best rank-k approximation Y to X (wrt. the 2 norm),
- shows the original and compressed images as well as the error image $X - Y$ (use `subplot`), and
- prints the amount of numbers needed to store the original and compressed images.

Test your program on the images clown, gatlin, durer, mandrill, earth and for different values of k.
Hint. You will need the `load` command. For example `load clown` will provide a matrix X of image information (in this case of dimension 200×320 corresponding to the resolution of the image) and a matrix `map` containing colormap information. Showing the image on screen is done by the commands `colormap(map);image(X)`.