Model order reduction

LECTURE NOTES

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1 Introduction: What is model order reduction

- rigorous definition is difficult/ impossible
- MOR is the approximation of large-scale systems of equations by small scale ones with similar solution properties

Here, we consider the MOR of dynamical systems of the form

$$\begin{aligned} &\dot{x}(t) := f(x(t), u(t), t) \quad x(t_0) = x_0 \quad t \in I \subset \mathbb{R} \quad I = [t_0, t_f] \\ &y(t) = g(x(t), u(t), t) \end{aligned}$$

with

$$\begin{aligned} &x(\dot{}) : I \to \mathbb{R}^n && \text{state (function)} \\ &u(\dot{}) : I \to \mathbb{R}^m && \text{input (function)} \\ &y(\dot{}) : I \to \mathbb{R}^p && \text{output (function)} \\ &f : \mathbb{R}^n \times \mathbb{R}^m \times I \to \mathbb{R}^n && \text{dynamics of system} \\ &g : \mathbb{R}^n \times \mathbb{R}^m \times I \to \mathbb{R}^p && \text{output map} \end{aligned}$$

Note. *if u is fixed we can calculate u, then y. In this lecture we only consider systems where the pde is solvable.*

usually the input can be chosen by a system operator in order to influence the system state. The output consists of the system parameters that are servable or that are of interest to the operator.

The task of MOR can be described as

- find functions

$$\tilde{f} : \mathbb{R}^l \times \mathbb{R}^m \times I \to \mathbb{R}^n$$
$$\tilde{g} : \mathbb{R}^l \times \mathbb{R}^m \times I \to \mathbb{R}^p$$

  with $l << n$ and $\tilde{x}_0 \in \mathbb{R}^l$ such that the output $\tilde{y}$ of the small system

$$\dot{\tilde{x}}(t) := f(\tilde{x}(t), u(t), t) \quad \tilde{x}(t_0) = x_0$$
$$\tilde{y}(t) = g(\tilde{x}(t), u(t), t)$$

  is very close to y for most (or all admissable) input functions $u : I \to \mathbb{R}^m$

- meaning of very close will become clear later
- purpose of MOR is to aid numerical simulation of large scale systems, which is computationally very demanding for large n, but cheap for small l.

Example. *Consider the linear system with $I = (0, \infty]$*

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 1 \\ \frac{1}{10} \end{pmatrix} u(t), \quad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The state variable $x_1(t)$, $x_2(t)$ are mostly decoupled, so using the formular of variation of the constant, we have

$$\begin{aligned} y(t) = 2x_1(t) + x_2(t) &= 2 \int_0^t e^{-(t-\tau)} u(\tau)\tau + \frac{1}{10} \int_0^t e^{-2(t-\tau)} u(\tau)\tau \\ &\sim 2 \int_0^t e^{-(t-\tau)} u(\tau)\tau \end{aligned}$$

2 Heranführende Kapitel

3 Hauptteil der Arbeit I

4 Hauptteil der Arbeit II

5 Fazit und Ausblick