Homework 7

1. a) $a = 118$; $d = 35$

$[q = 3]$ and $[r = 13]$

iteration	q	r	$r \geq 35$
0	0	118	T
1	1	83	T
2	2	48	T
3	3	13	F

b) $a = 1024$; $d = 512$

$[q = 2]$ and $[r = 0]$

iteration	q	r	$r \geq 512$
0	0	1024	T
1	1	512	T
2	2	0	F

c) $a = 24$; $d = 5$

$[q = 4]$ and $[r = 4]$

iteration	q	r	$r \geq 5$
0	0	24	T
1	1	19	T
2	2	14	T
3	3	9	T
4	4	4	F

2. $a = 12$; $b = 9$; knowing that $\gcd(a, b) = \gcd(b, r)$ we'll need to find r. If $b = 9$; $12 - 9 = 3$ where we see that in the formula $a = bq + r$; $q = 1$ and $r = 3$ (by the division algorithm). Now, by the Euclidean Algorithm, we set $a = 9$ and $b = 3$. Therefore, $9 - 3 = 6$, $6 - 3 = 3$ and $3 - 3 = 0$;

where $q=3$ and $r=0$. Therefore, this is the end of our Euclidean algorithm where $\gcd(12,9)=3$.

3. $A=\{0,1,2,3,4,5,17\}$ and $B=\{3,7,15,17,20\}$

Find (a,b) $\in R$ such that $a\in A$, $b\in B$

a) aRB means $a|b$. We can discard combining all the prime numbers from A because they will never be divisible by an element from B except themselves. The only elements left are 0, which is divisible by all the elements from B; and 4, which is not divisible by any element from B.

Therefore, the solution is:

$(3,3)$ and $(17,17)$; ~~$R=\{(3,3),(17,17)\}$~~ and all the pairs with 0

$R=\{(0,3),(0,7),(0,15),(0,17),(0,20),(3,3),(17,17)\}$

b) aRb means that $3a+2<b$

we evaluate all the elements in A:

$3\cdot 0+2=2$

$3\cdot 1+2=5$

$3\cdot 2+2=8$

$3\cdot 3+2=11$

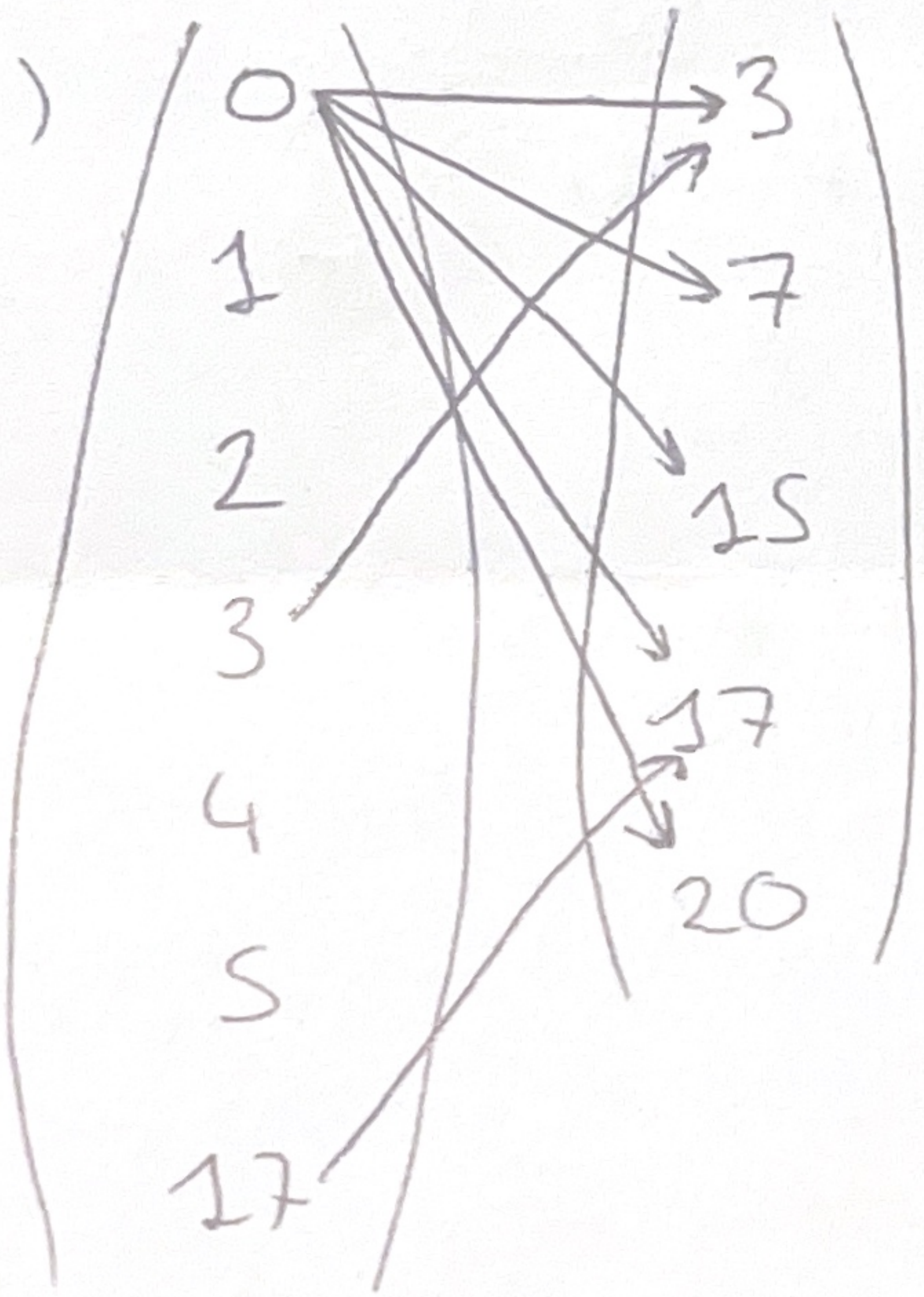
$3\cdot 4+2=14$

$3\cdot 5+2=17$

$3\cdot 17+2=53$; Now we find pairs that match

$R=\{(0,3),(0,7),(0,15),(0,17),(0,20),$
$,(1,7),(1,15),(1,17),(1,20),(2,15),$
$,(2,17),(2,20),(3,15),(3,17),(3,20),$
$,(4,15),(4,17),(4,20),(5,20)\}$

4. a)



b)