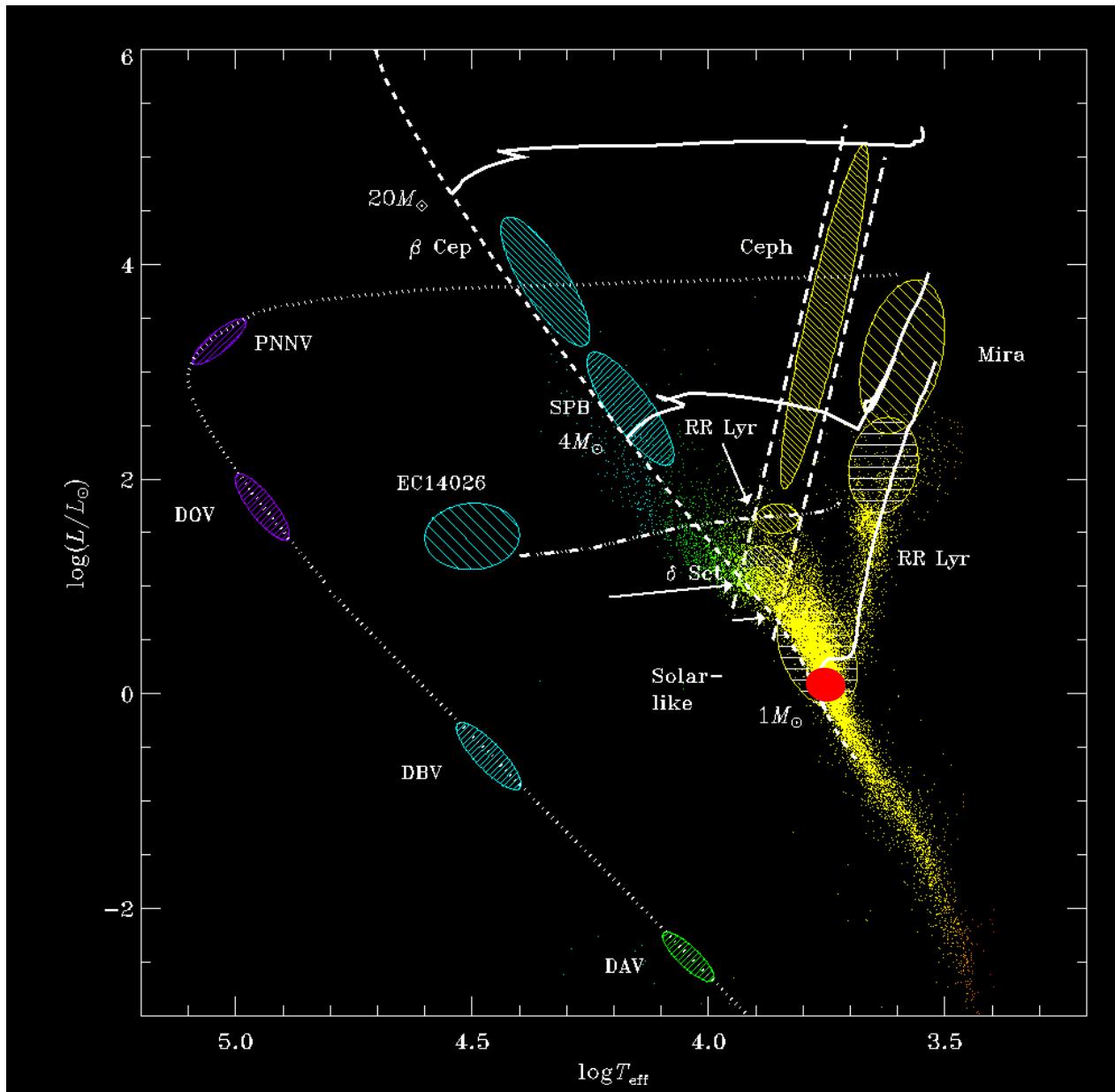


Theory of Stellar Oscillations

Lecture 3 Helioseismology

Ilídio Lopes
[\(ilidio.lopes@ist.utl.pt\)](mailto:ilidio.lopes@ist.utl.pt)

The Sun



Three types of modes

- G(avity) Modes – restoring force is buoyancy – internal gravity waves
- P(ressure) Modes – restoring force is pressure
- F(undamental) Modes – restoring force is buoyancy modified by density interface – surface gravity waves

Physics of pulsations : Stars

Spherical Coordinates

$$\nabla^2 \psi \square \left(\frac{\omega^2}{c^2} - \frac{\omega_c^2}{c^2} - \frac{N^2}{\omega^2} \nabla_h^2 \right) \psi \square 0$$

∇_h^2 Horizontal Laplacian operator

c- sound speed,
acoustic waves (**p-modes**)

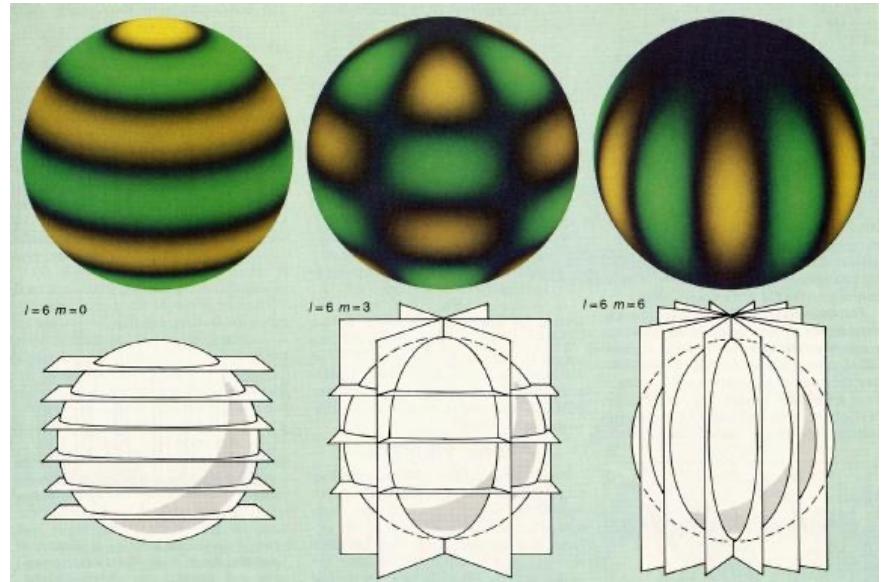
N- Buoyancy frequency,
gravity waves (**g-modes**)

Quantification (Numbers-integers):
1D wave equation – (n)
3D wave equation – (n,l,m)

n,l,m integers

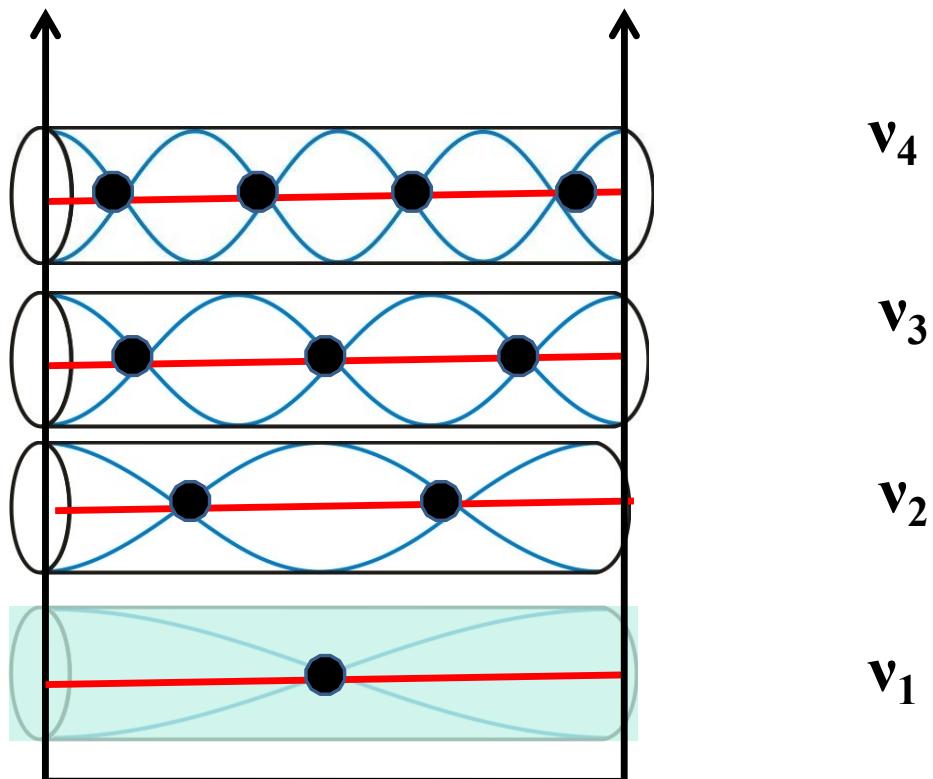
Stellar Oscillations (stratified sphere):
3D wave-like equation (Gough 2001)

$$\psi(r, \theta, \phi, t) \propto r^{-1} \Psi_r(r) Y_l^m(\theta, \phi) e^{-i\omega t}$$



Quantification (Numbers):
radial eigen-fuction, n- number of nodes
Spherical harmonics – (l,m)

Physics of pulsations : Basics



Propagation Diagram
(Finite potential well)

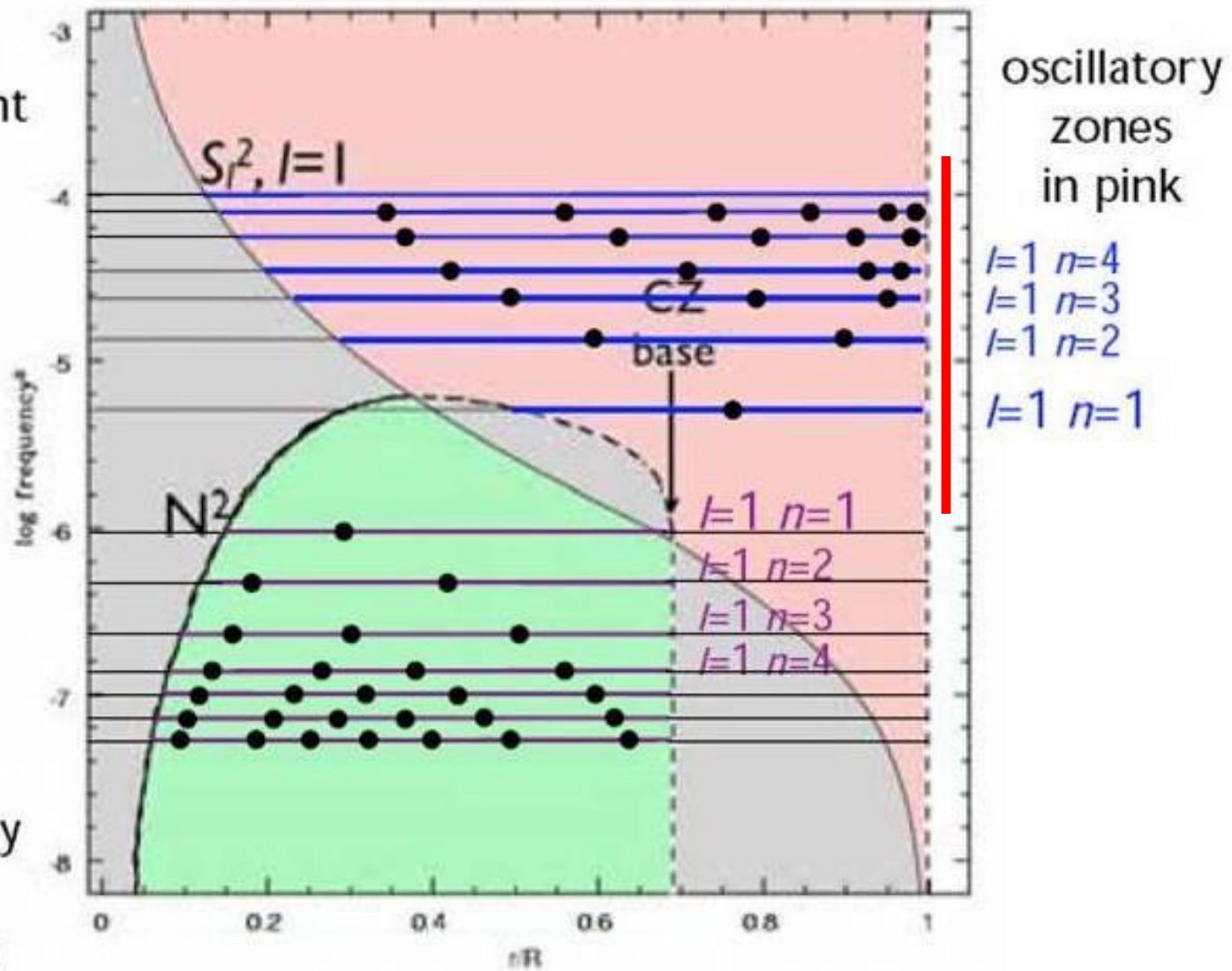
Physics of pulsations : Stars

evanescent
zones
in gray

Acoustic
frequency

$$S_l^2 \propto \frac{l(l+1)}{r^2} c^2$$

oscillatory
zones
in green

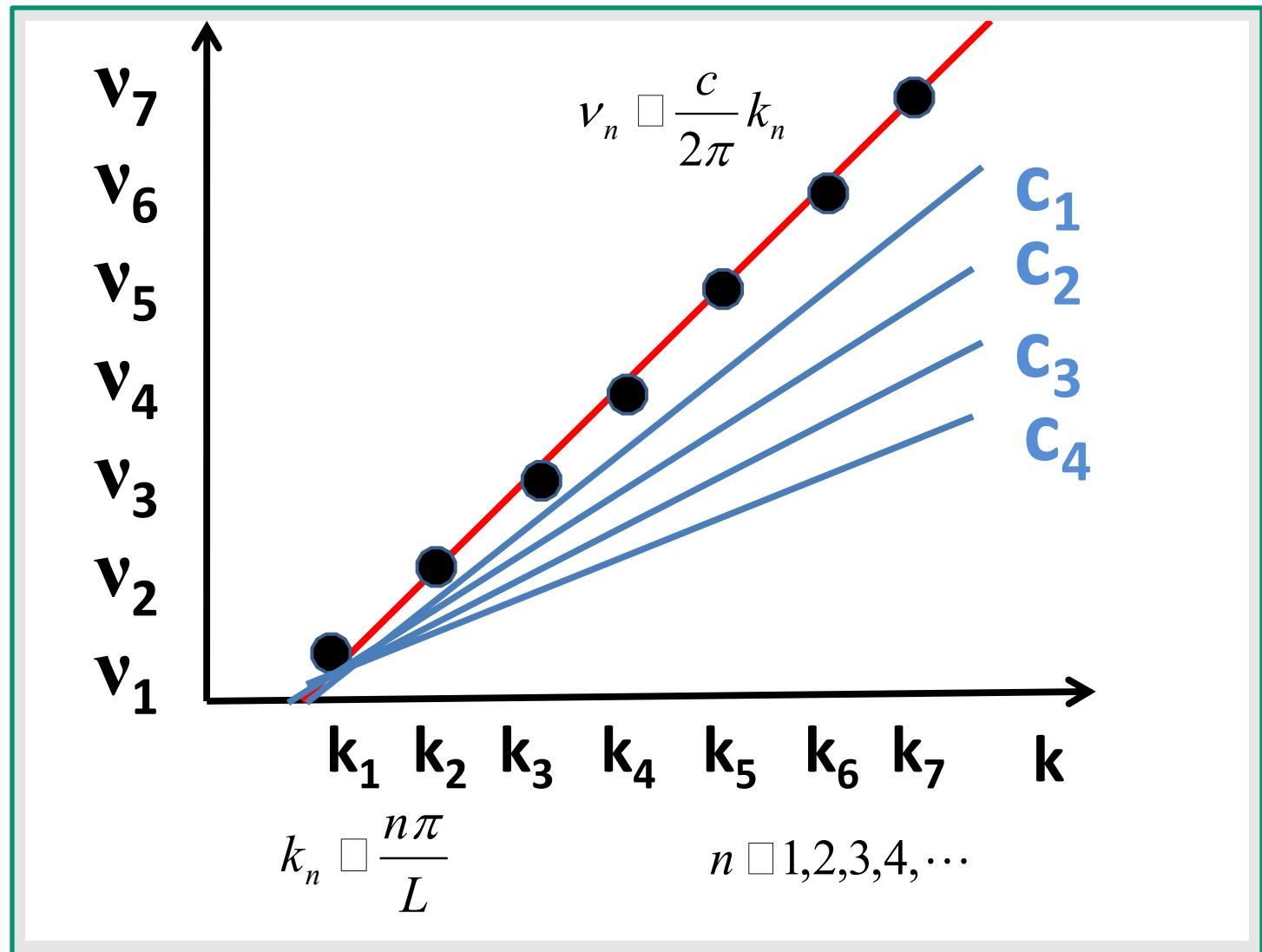


A wave of frequency ω can propagate only if

$k_r^2 \neq 0, \omega^2 \neq S_l^2$ and N^2 (p modes)

$k_r^2 \neq 0, \omega^2 \neq S_l^2$ and N^2 (g modes)

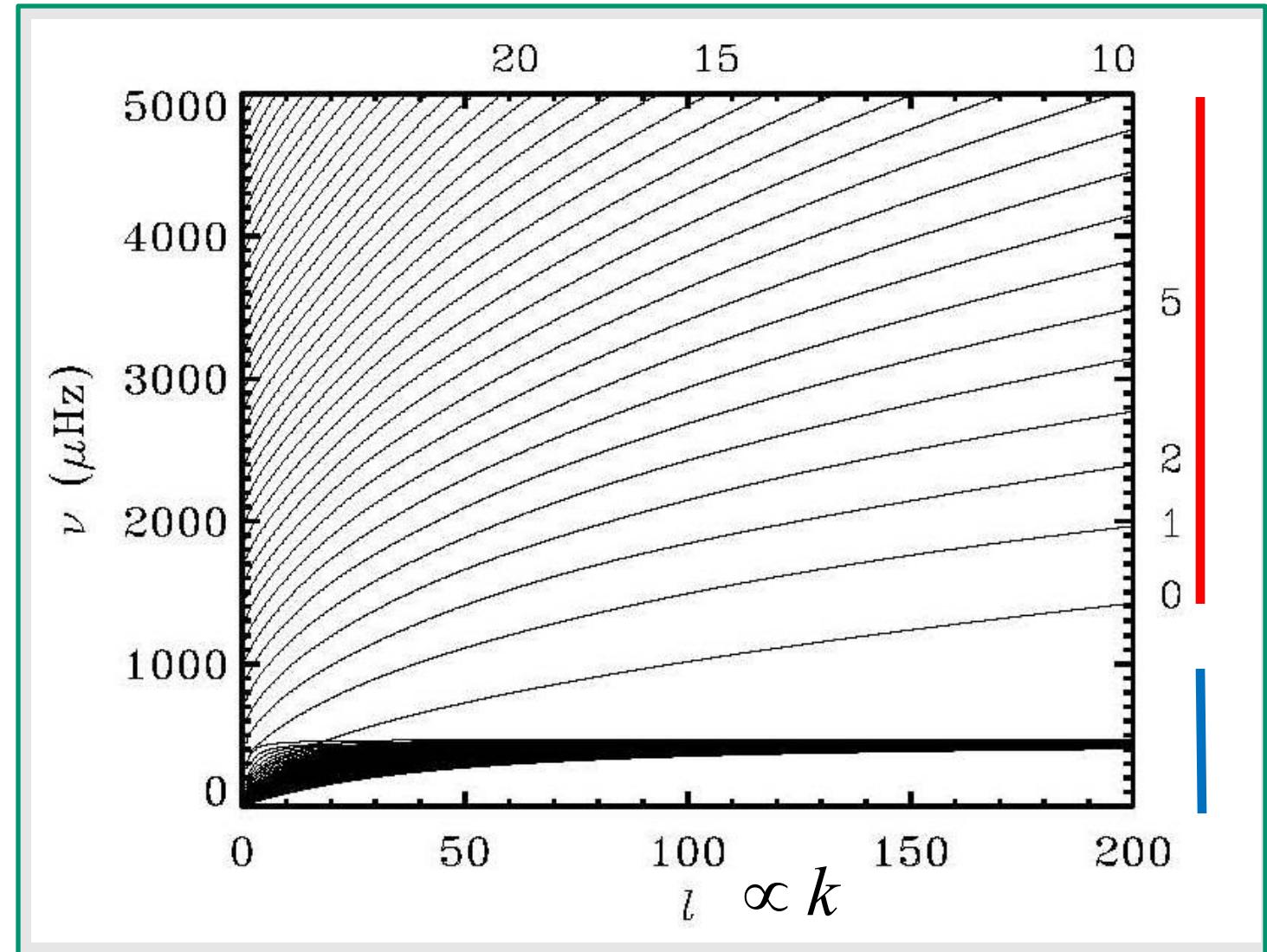
PHYSICS OF PULSATIONS : BASICS



Dispersion relation (open tube)

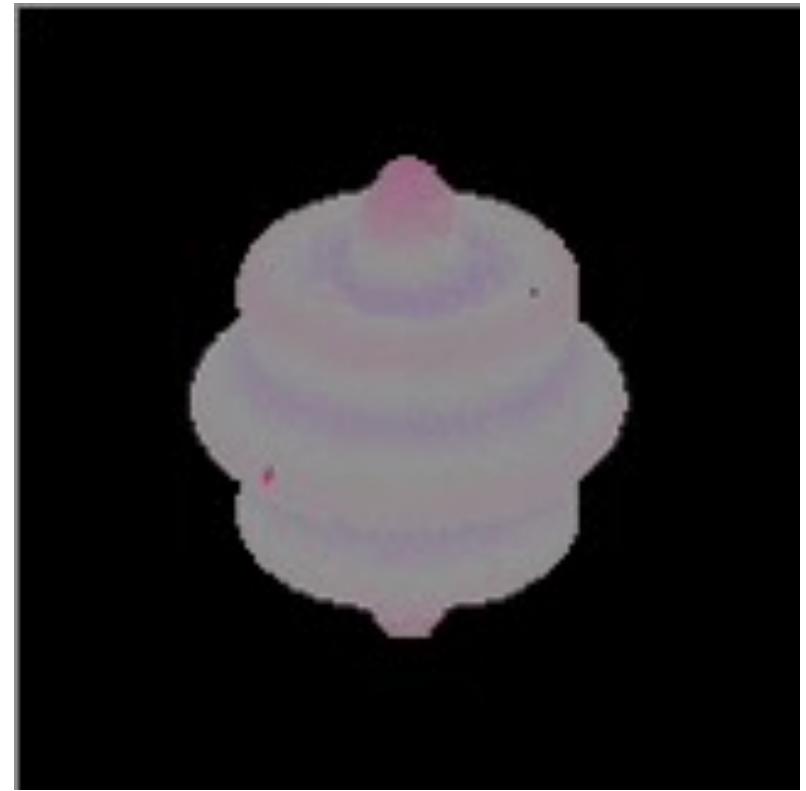
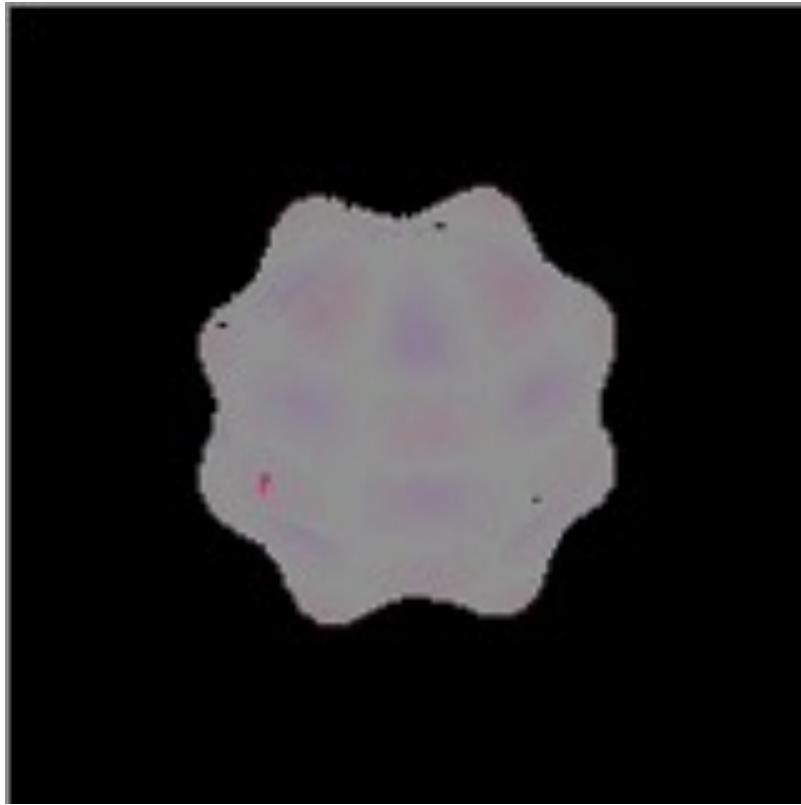
PHYSICS OF PULSATIONS: STARS

Dispersion relation



Numerical computation of oscillations for the solar standard model

Acoustic Modes of Oscillations



Mode $l=8$ $m=4$

Rodrigo Lopes, Oporto 2008, Stellar
Structure and Evolution 1.9

Mode $l=8$ $m=0$

The $\ell=20$ $m=16$ Mode



Physics of pulsations : Basics

Any **perturbation (oscillation) of equilibrium structure** (air in a tube, equilibrium structure inside a star) can be expressed as a finite sum of harmonic functions.

A generic perturbation (in space and time):

$$\psi(x, t) \propto \sum_n \sin(k_n x) \cos(\omega_n t)$$

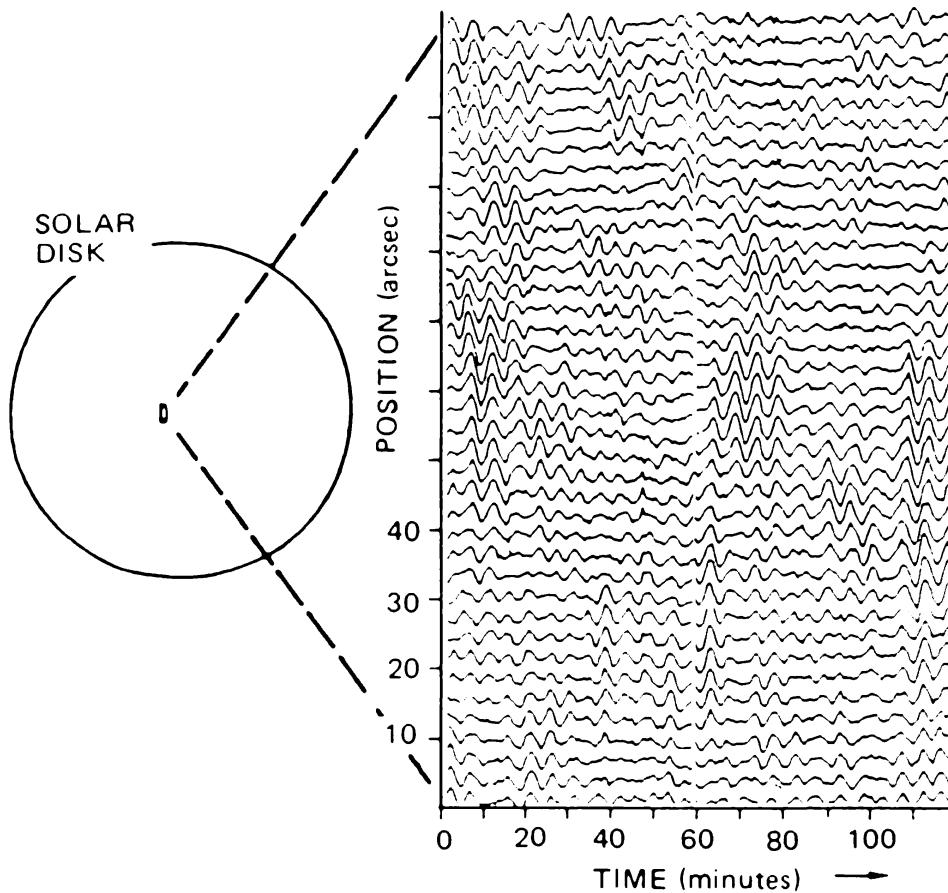
$$\psi(r, \theta, \phi, t) \propto \sum_{n,l,m} r^{-1} \Psi_{r,n}(r) Y_l^m(\theta, \phi) e^{-i\omega_{n,l,m} t}$$

Therefore, the **oscillation spectrum** characterizes the perturbations of the equilibrium structure of the star.

Helioseismology: Discovery of THE Pulsating Sun

Five-min oscillations: A local phenomenon in the solar atmosphere?

Musman & Rust
(1970; Solar Phys. 13, 261)



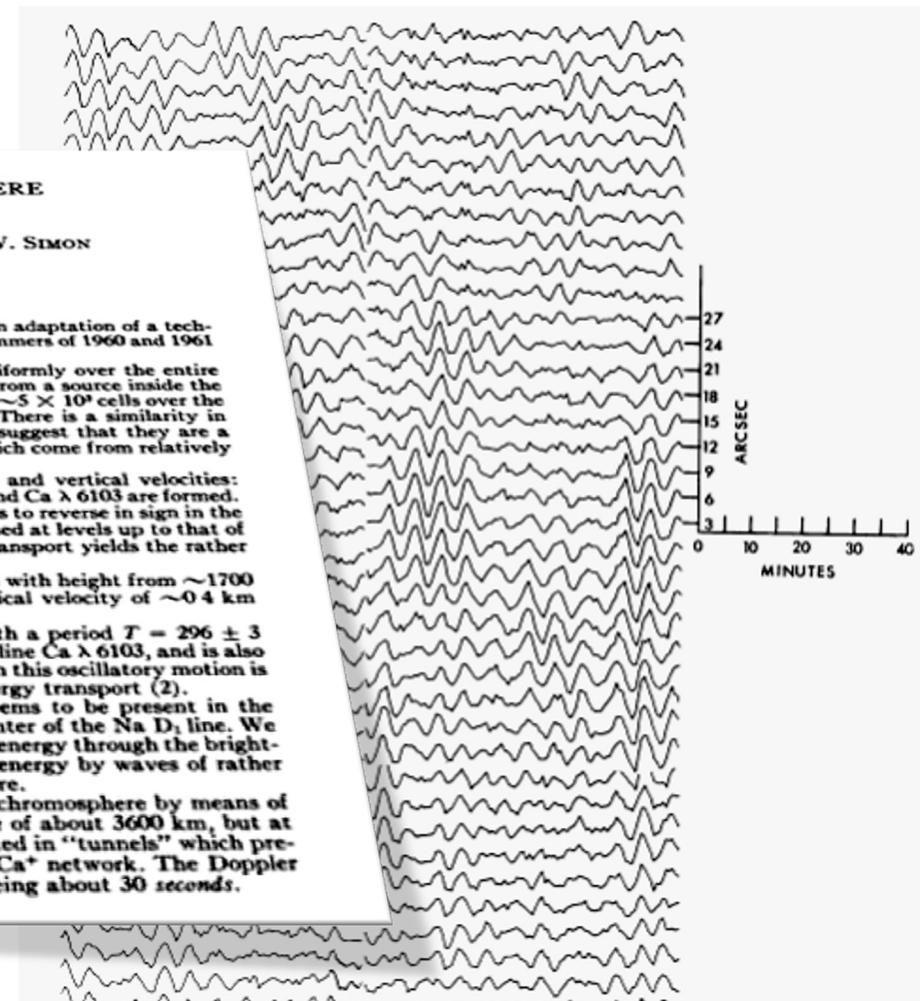
HELIOSEISMOLOGY: DISCOVERING A PULSATING SUN

Many theories of wave physics postulated:
- gravity-acoustic waves or MHD waves?

A puzzle ?

Where was the region of propagation?

Discovery in 1960 by Leighton et al.
that the solar surface is rising and
falling with a 5 minute period.



VELOCITY FIELDS IN THE SOLAR ATMOSPHERE I. PRELIMINARY REPORT*

ROBERT B. LEIGHTON, ROBERT W. NOYES, AND GEORGE W. SIMON
California Institute of Technology, Pasadena, California

Received October 16, 1961

ABSTRACT

Velocity fields in the solar atmosphere have been detected and measured by an adaptation of a technique previously used for measuring magnetic fields. Data obtained during the summers of 1960 and 1961 have been partially analyzed and yield the following principal results:

1. Large "cells" of horizontally moving material are distributed roughly uniformly over the entire solar surface. The motions within each cell suggest a (horizontal) outward flow from a source inside the cell. Typical diameters are 1.6×10^4 km; spacings between centers, 3×10^4 km ($\sim 5 \times 10^3$ cells over the solar surface); r.m.s. velocities of outflow, 0.5 km sec $^{-1}$; lifetimes, 10^4 - 10^5 sec. There is a similarity in appearance to the Ca $^{+}$ network. The appearance and properties of these cells suggest that they are a surface manifestation of a "supergranulation" pattern of convective currents which come from relatively great depths inside the sun.

2. A distinct correlation is observed between local brightness fluctuations and vertical velocities: bright elements tend to move upward, at the levels at which the lines Fe λ 6102 and Ca λ 6103 are formed. In the line Ca λ 6103, the correlation coefficient is ~ 0.5 . This correlation appears to reverse in sign in the height range spanned by the Doppler wings of the Na D₁ line and remains reversed at levels up to that of Ca $^{+}$ λ 8542. At the level of Ca λ 6103, an estimate of the mechanical energy transport yields the rather large value 2 W cm $^{-2}$.

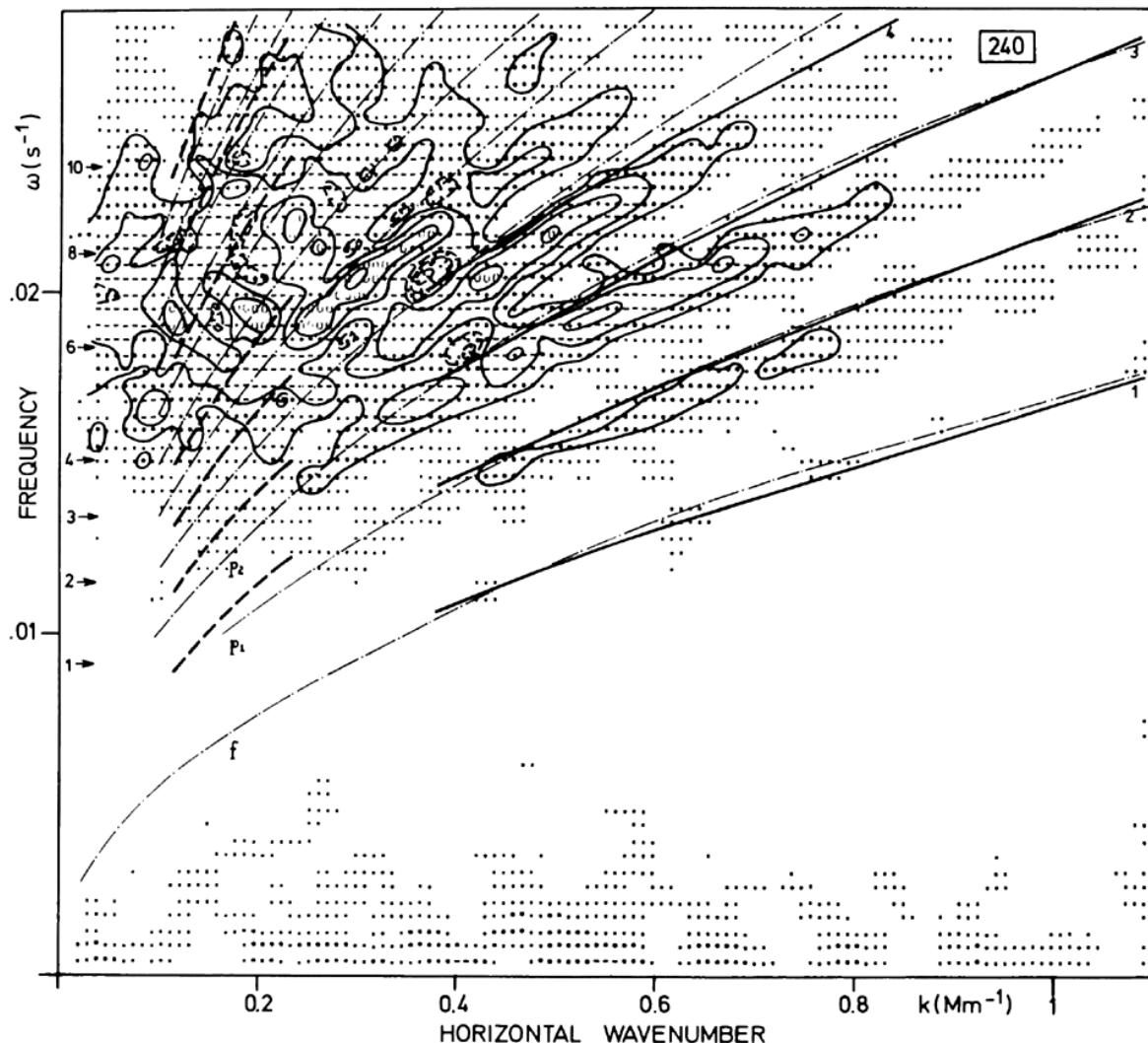
3. The characteristic "cell size" of the vertical velocities appears to increase with height from ~ 1700 km at the level of Fe λ 6102 to ~ 3500 km at that of Na λ 5896. The r.m.s. vertical velocity of ~ 0.4 km sec $^{-1}$ appears nearly constant over this height range.

4. The vertical velocities exhibit a striking repetitive time correlation, with a period $T = 296 \pm 3$ sec. This quasi-sinusoidal motion has been followed for three full periods in the line Ca λ 6103, and is also clearly present in Fe λ 6102, Na λ 5896, and other lines. The energy contained in this oscillatory motion is about 160 J cm $^{-2}$; the "losses" can apparently be compensated for by the energy transport (2).

5. A similar repetitive time correlation, with nearly the same period, seems to be present in the brightness fluctuations observed on ordinary spectroheliograms taken at the center of the Na D₁ line. We believe that we are observing the transformation of potential energy into wave energy through the brightness-velocity correlation in the photosphere, the upward propagation of this energy by waves of rather well-defined frequency, and its dissipation into heat in the lower chromosphere.

6. Doppler velocities have been observed at various heights in the upper chromosphere by means of the H α line. At great heights one finds a granular structure with a mean size of about 3600 km, but at lower levels one finds predominantly downward motions, which are concentrated in "tunnels" which presumably follow magnetic lines of force and are geometrically related to the Ca $^{+}$ network. The Doppler field changes its appearance very rapidly at higher levels, typical lifetimes being about 30 seconds.

Oscillation modes of the Sun

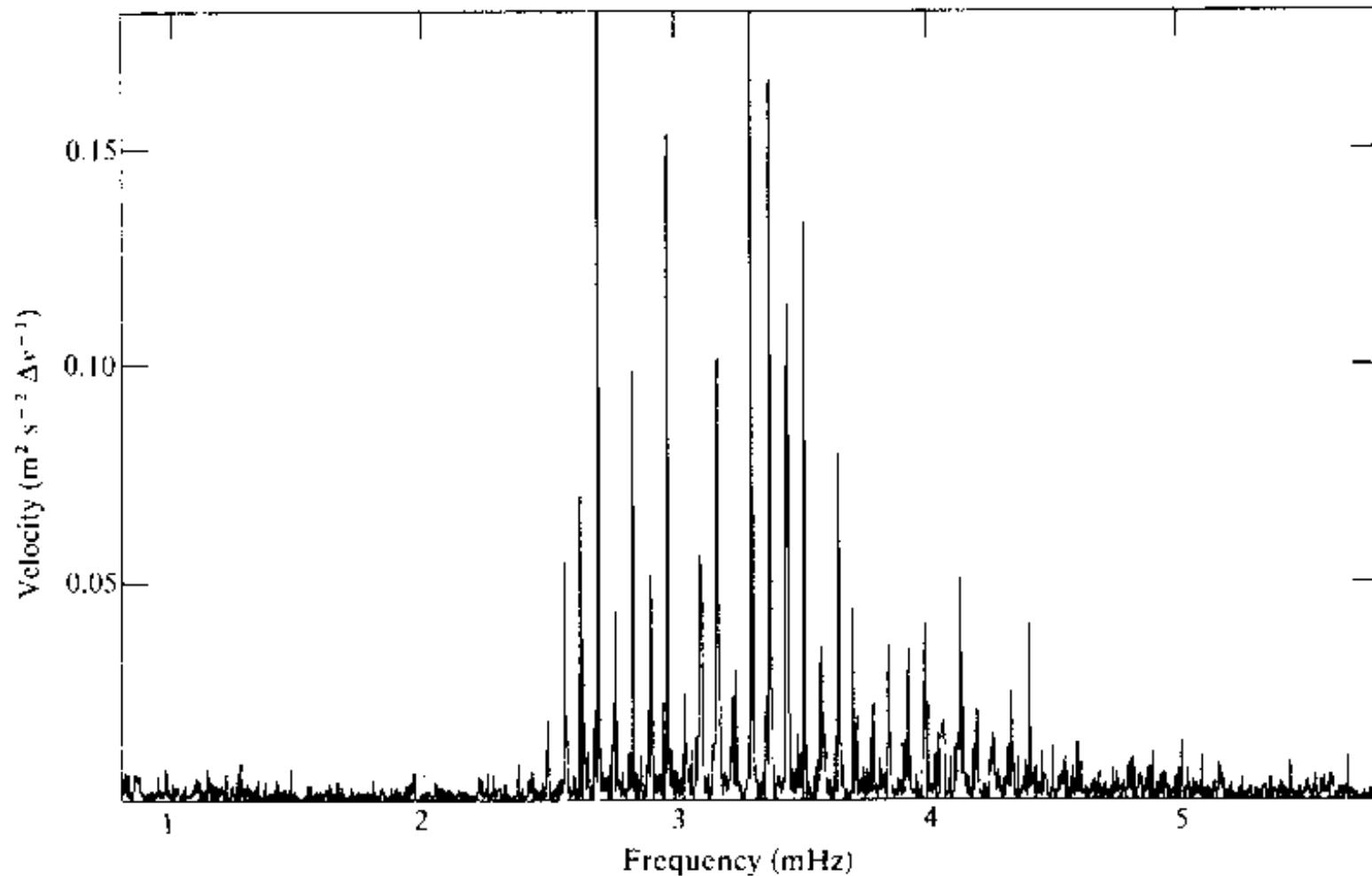


Models:

- Ulrich (1970): —
- Wolff (1972): - - -
- Ando & Osaki (1975): - - - -

Deubner (1975; Astron. Astrophys. 44, 371)

The start of global helioseismology



Grec et al. (1980; Nature 288, 541)

HELIOSEISMOLOGY: 70'S -ORIGIN OF SOLAR OSCILLATIONS

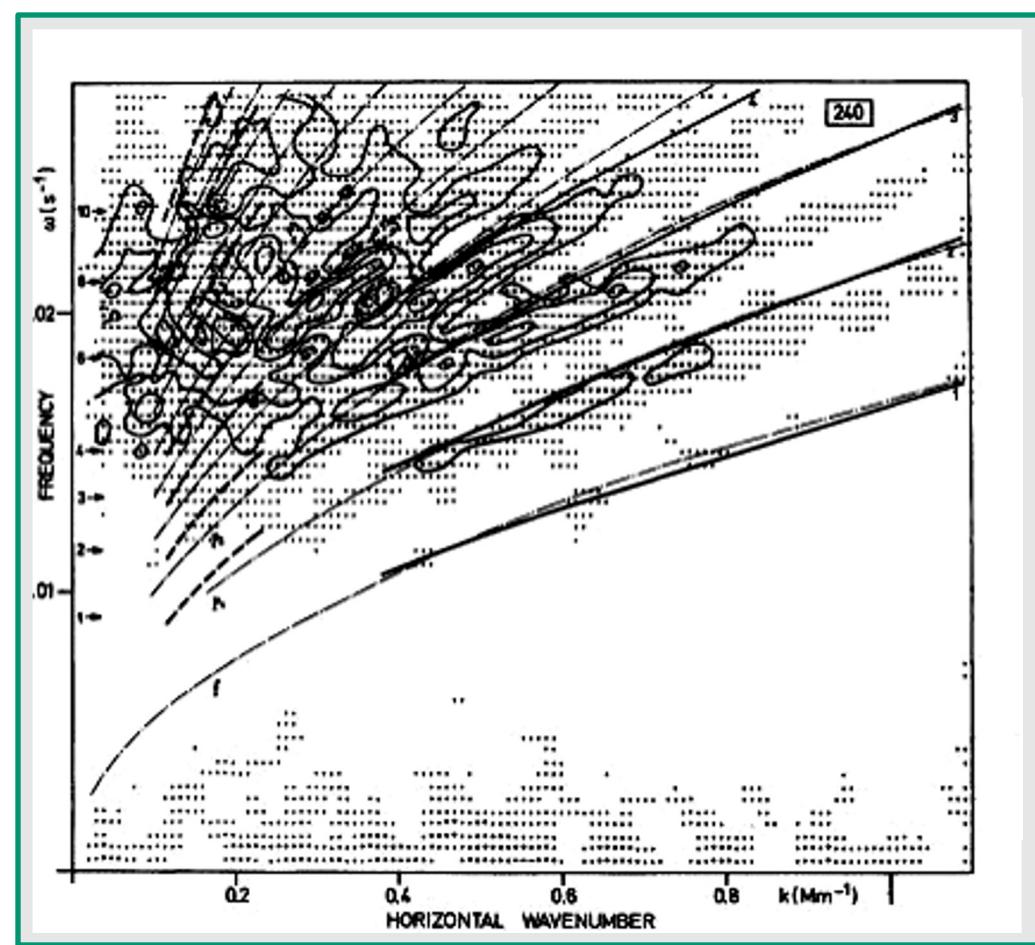
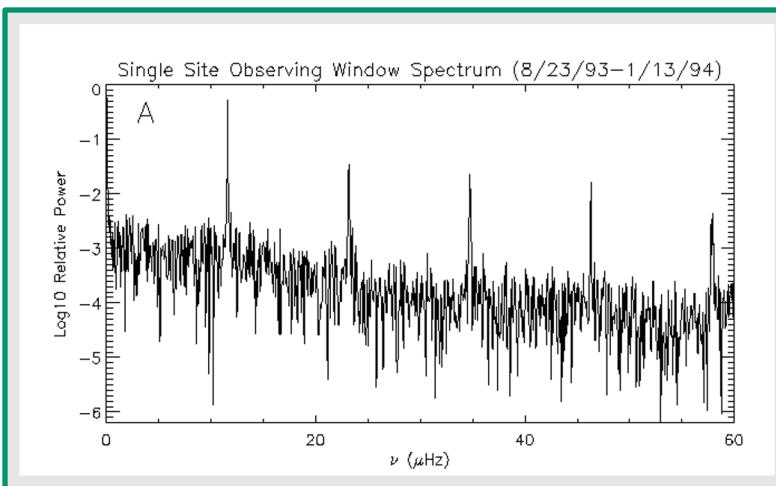
R. Ulrich 1975

- Acoustic waves trapped within the internal temperature gradient.
- Dispersion relation between frequency and wavelength : $\omega = \omega (\kappa)$
- Max amplitude 20 cm/s

An Observational Problem:

The Sun sets at a single terrestrial site,
producing periodic time series gaps.

The solar acoustic spectrum is convolved with the
temporal window spectrum, contaminating the
solar spectrum with many spurious peaks.



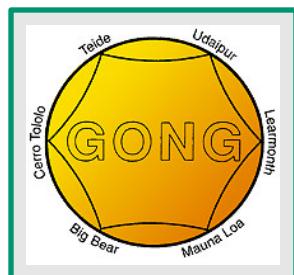
Solution:

Antarctica – max 6 month duration
Network – BiSON, IRIS, GONG
Space – SoHO: MDI, GOLF, VIRGO

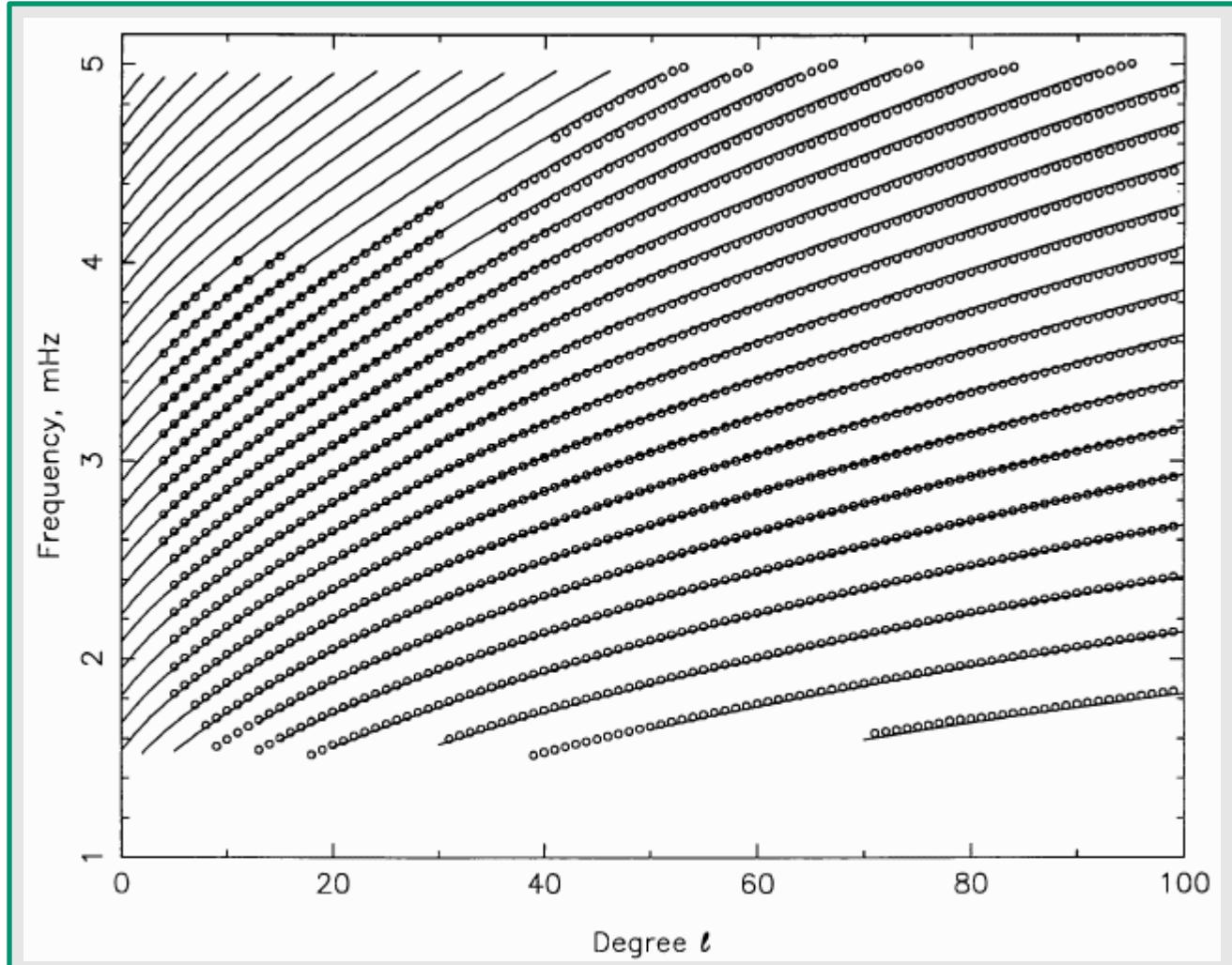
HELIOSEISMOLOGY: 80-90'S – GLOBAL NETWORK OF OBSERVATION OF SOLAR OSCILLATIONS



BISON (University of Birmingham, UK), 6-site network of single-pixel instruments, data since 1976, completed 1992. Observations of global modes up to $l=4$.

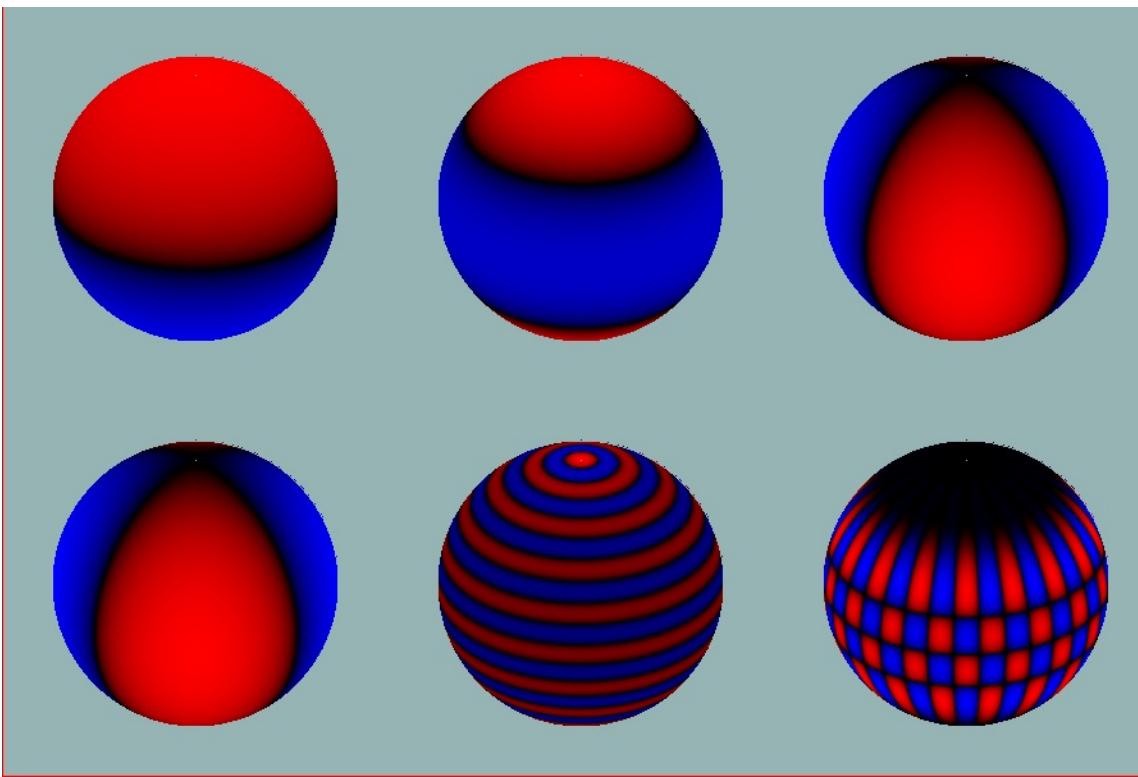


GONG (National Solar Observatory-Tucson), Six stations around the world for continued coverage: (i) 256x256 pixels 1995-2001, (ii) 1024 pixels since 2001.

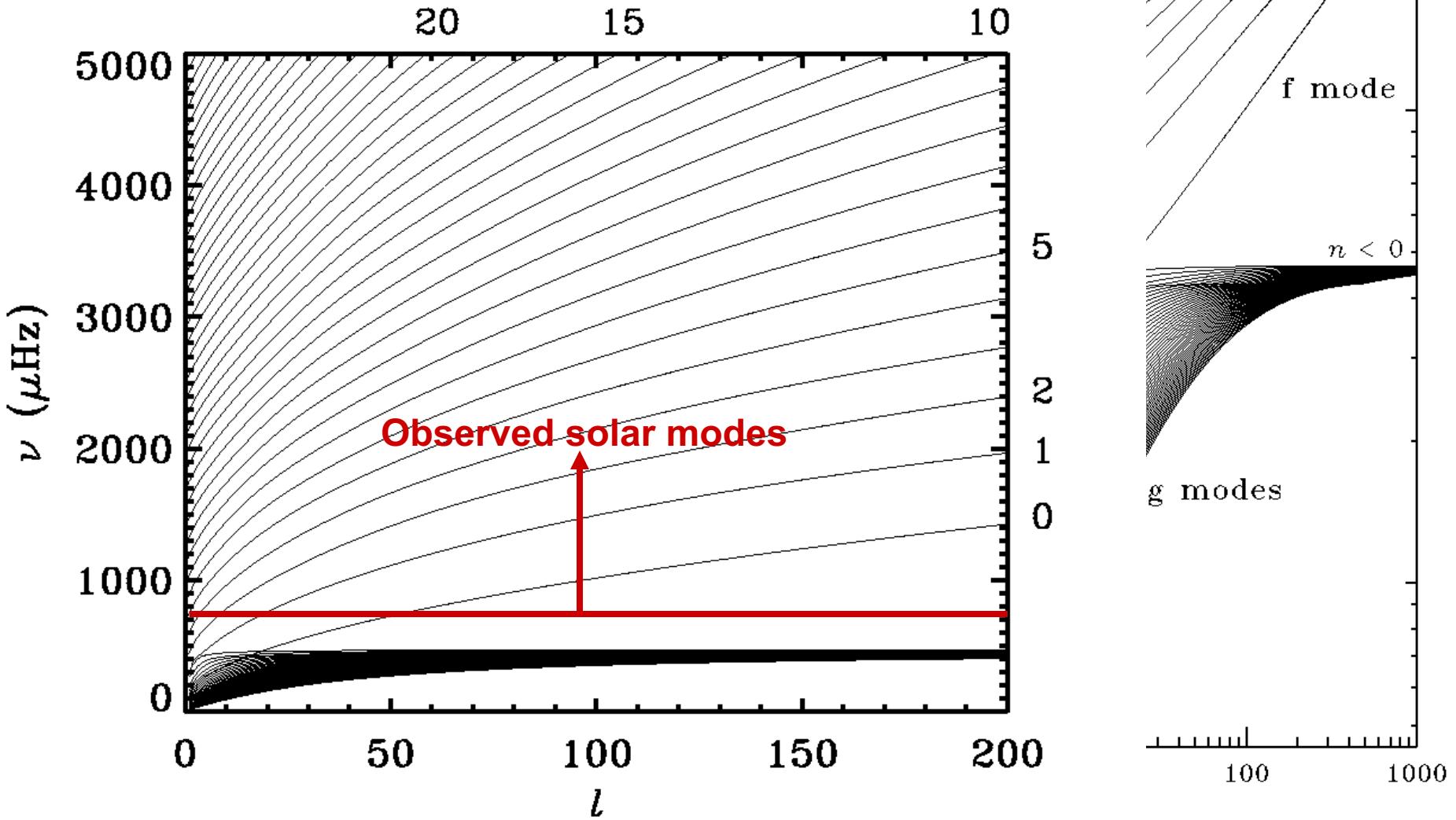


Crucial advantages

- Access to modes of degree up to 1000+
- Observations over very extended period (more than 10 years nearly continuously)
- Well-determined global parameters



Frequencies of solar model

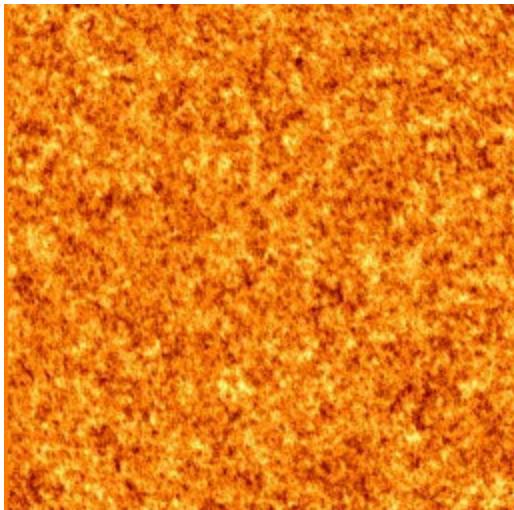
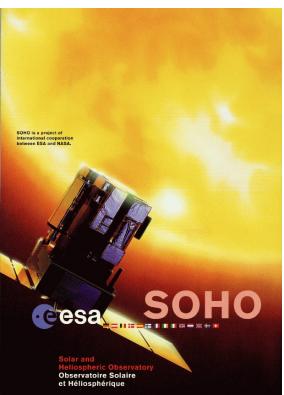


Helioseismology : Space Instruments

- SoHO: Solar and Heliospheric Observatory (2/12/1995) :
- Three seismic Instruments: MDI , GOLF, VIRGO

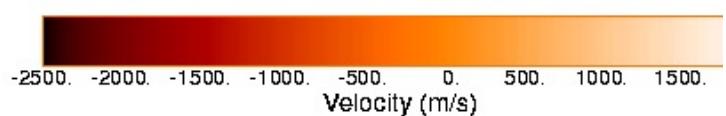
MDI-full-disk Dopplergram sequence
shows solar "5-minute" oscillations.

MDI Dopplergrams from high
resolution field show solar
oscillations. This data was
observed with a 12-second cadence.



Single Dopplergram

(30-MAR-96 19:54:00)

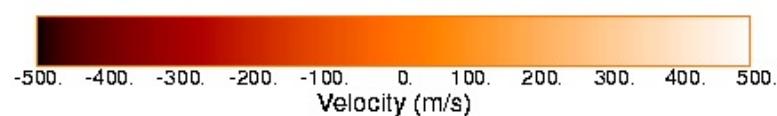


SOI / MDI

Stanford Lockheed Institute for Space Research

Single Dopplergram Minus 45 Images Average

(30-MAR-96 19:54:00)



SOI / MDI

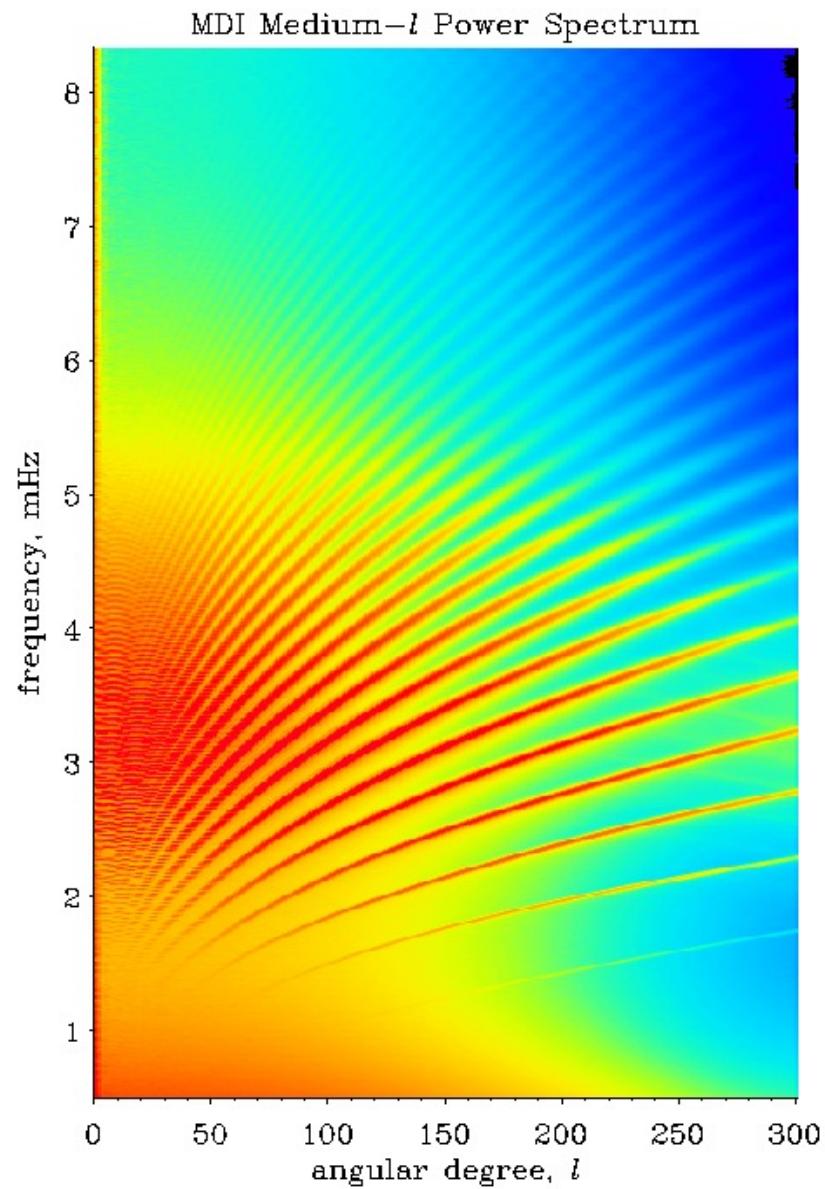
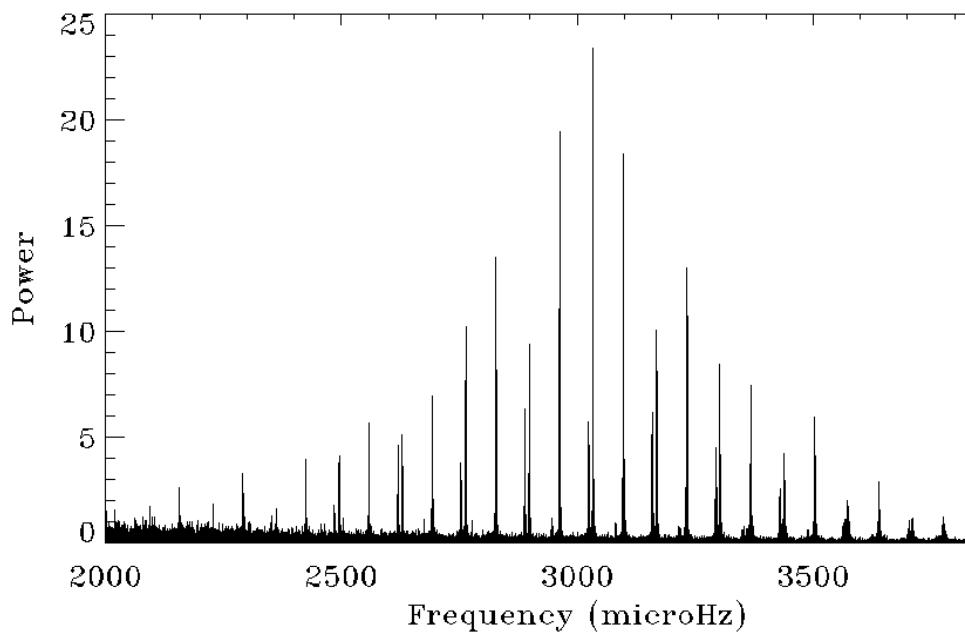
Stanford Lockheed Institute for Space Research

Data on solar oscillations

Observations:

MDI on SOHO

VIRGO on SOHO (whole-disk):



Observing the Sun from Space: MDI/SoHO

Global modes of resonance:

- Millions of modes of oscillation excited by near-surface turbulent convection.
- Acoustic modes with similar wave speed probe similar depths.
- MDI-SOHO measures Dopplergrams every minute since 1996.
- 5-minute p modes have a very low amplitude \sim velocity 10 cm/s
- $\Delta L/L \sim 10^{-6}$
- incoherent superposition of 10 million modes

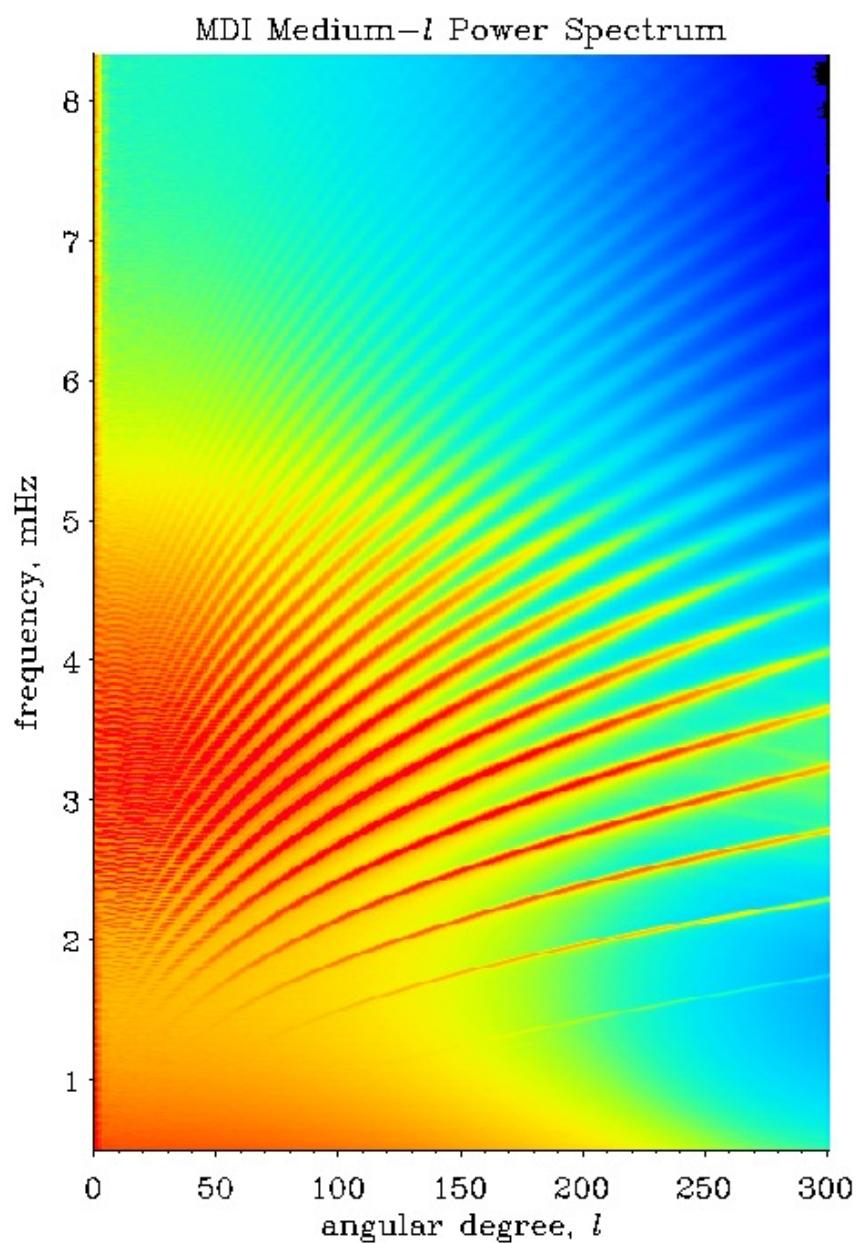
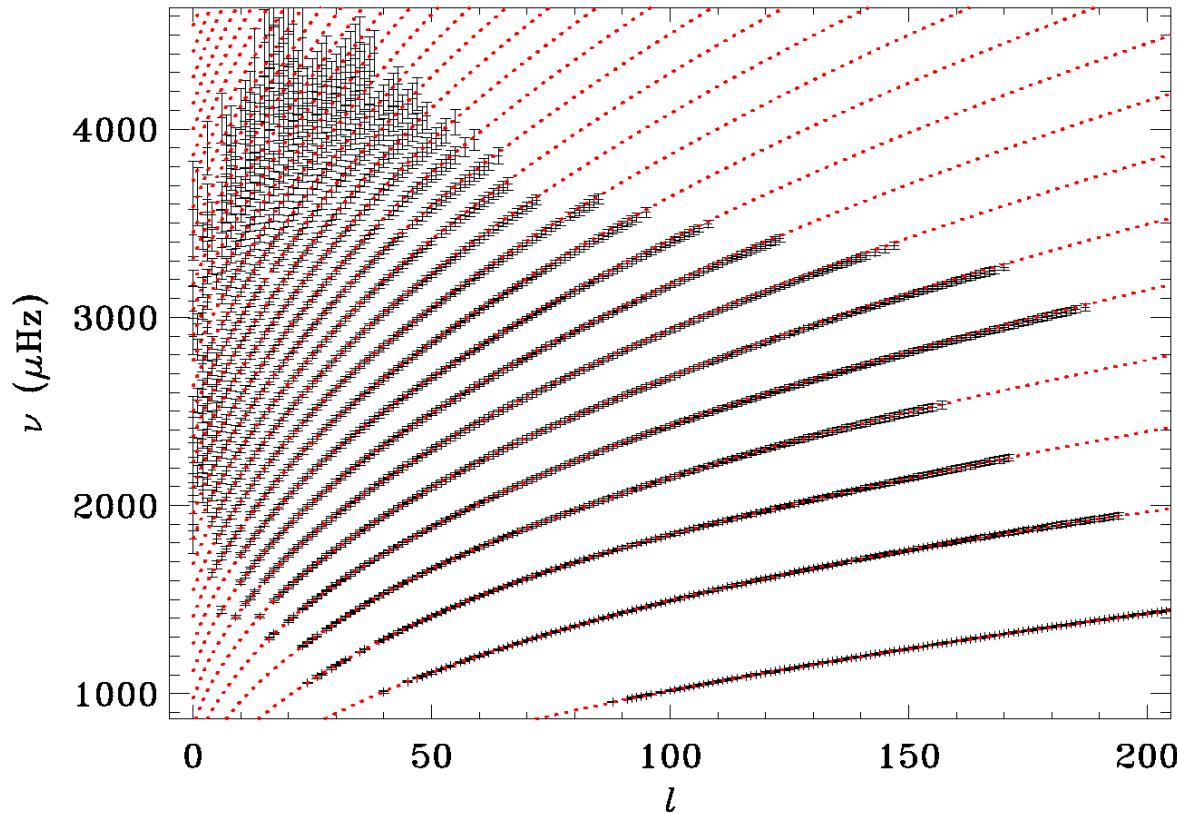


Figure: $\omega = \omega(k)$ or $1-v$ diagram from MDI high-cadence full disk data. (i) Shows mode frequencies up to 10 mHz and $l=1000$. (ii) m-averaged medium-*l*: Power spectrum .

Observed frequencies



m-averaged frequencies from MDI instrument on SOHO
1000 σ error bars

Stellar Seismology: Discovering Gravity Modes

OBSERVING THE SUN FROM SPACE: GOLF/SOHO ACOUSTIC MODES

Lazrek et al. (1997)

FIRST RESULTS ON P-MODES FROM GOLF EXPERIMENT

9

I. Lopes Co-I GOLF Team

The p-mode Fourier spectrum from GOLF, using a 690-day time series of calibrated velocity signal, which exhibits an excellent signal to noise ratio.

The low-frequency range of the P-modes from above spectrum, showing low-n order modes.

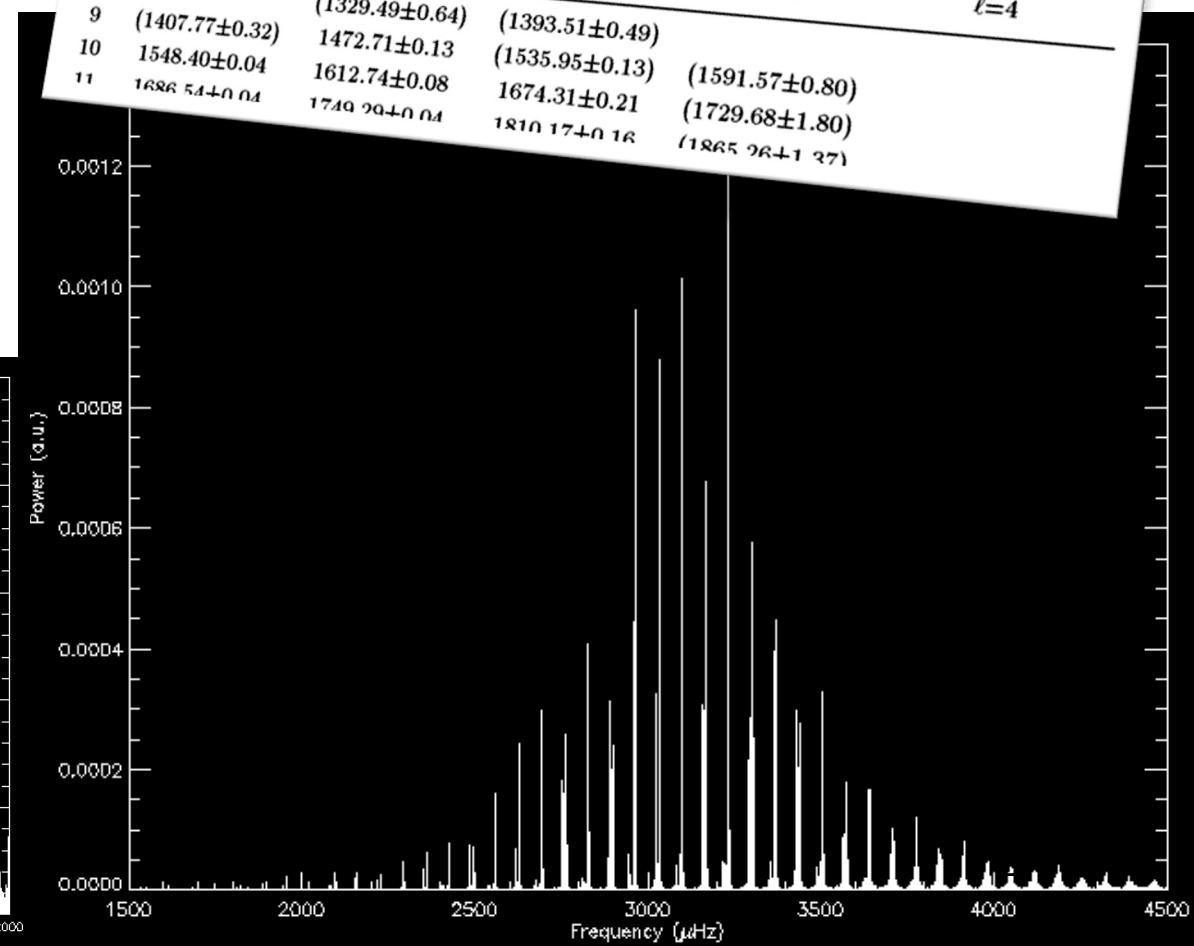
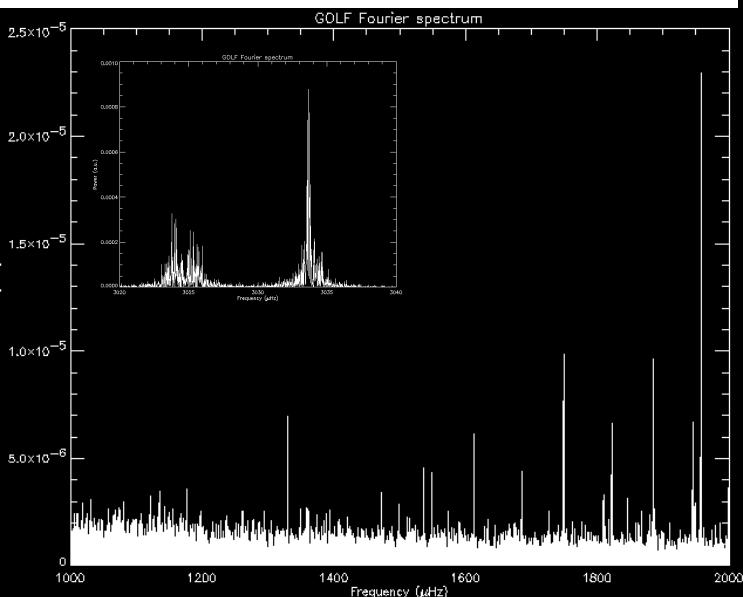


TABLE I
Frequencies and errors, in μHz , as measured by GOLF instrument. Bracketted frequencies are possible identifications with $S/N < 1$ (see text).

n	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
8					
9	(1407.77±0.32)	(1329.49±0.64)	(1393.51±0.49)		
10	1548.40±0.04	1472.71±0.13	(1535.95±0.13)	(1591.57±0.80)	
11	1686.54±0.04	1612.74±0.08	1674.31±0.21	(1729.68±1.80)	(1865.96±1.37)

Observing the Sun from Space: GOLF/SoHO GraVity Modes

THE ASTROPHYSICAL JOURNAL, 604:455–468, 2004 March 20
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LOOKING FOR GRAVITY-MODE MULTIPLETS WITH THE GOLF EXPERIMENT ABOARD SOHO

S. TURCK-CHIÈZE,¹ R. A. GARCÍA,¹ S. COUVIDAT,^{1,2} R. K. ULRICH,³ L. BERTELLO,³ F. VARADI,³ A. G. KOSOVICHEV,² A. H. GABRIEL,⁴
 G. BERTHOMIEU,⁵ A. S. BRUN,¹ I. LOPEZ,^{6,7} P. PALLE,⁸ J. PROVOST,⁵ J. M. ROBILLOT,⁹ AND T. ROCA CORTÉS⁸

Received 2002 April 26; accepted 2003 December 2

+ GOLF Team (2004)

List of gravity modes candidates

THE g -MODE PATTERNS WITH MORE THAN 90% CONFIDENCE
 WITH THE THEORETICAL CENTRAL VALUE FOR 1290 DAYS OF GOLF OBSERVATION

Order	Seismic Model (μ Hz)	Periodogram (μ Hz)	Multitaper (μ Hz)
$l = 1$			
$n = -3$	153.25
$n = -2$	191.55	196.94/198.01	...
$n = -1$	262.73	...	262.15
$l = 2$			
$n = -6$	151.26	144.63/145.62/146.60	...
$n = -5$	170.46
$n = -4$	194.06	...	193.86/195.25
$n = -3$	222.02	218.95/220.10/220.70	218.95/220.10/221.28
$n = -2$	256.09	251.37/252.5	251.14/252.48
$n = -1$	296.38
$l = 3$			
$n = -9$	148.33	146.79/149.99	...
$n = -8$	161.72
$n = -7$	177.46	...	171.56/172.26/173.56
$n = -6$	195.93	...	193.86/195.25
$n = -5$	217.07	218.95/220.11/220.73	218.96/220.72
$n = -4$	238.35
$n = -3$	261.31	251.37/252.5	262.18
$n = -2$	296.50
$n = -1$	340.07	...	337.56/338.01/338.94

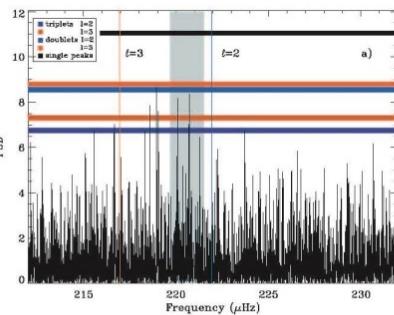


Fig. 4a

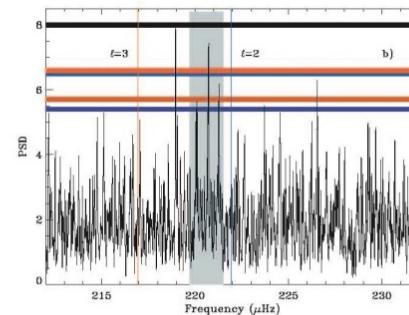


Fig. 4b

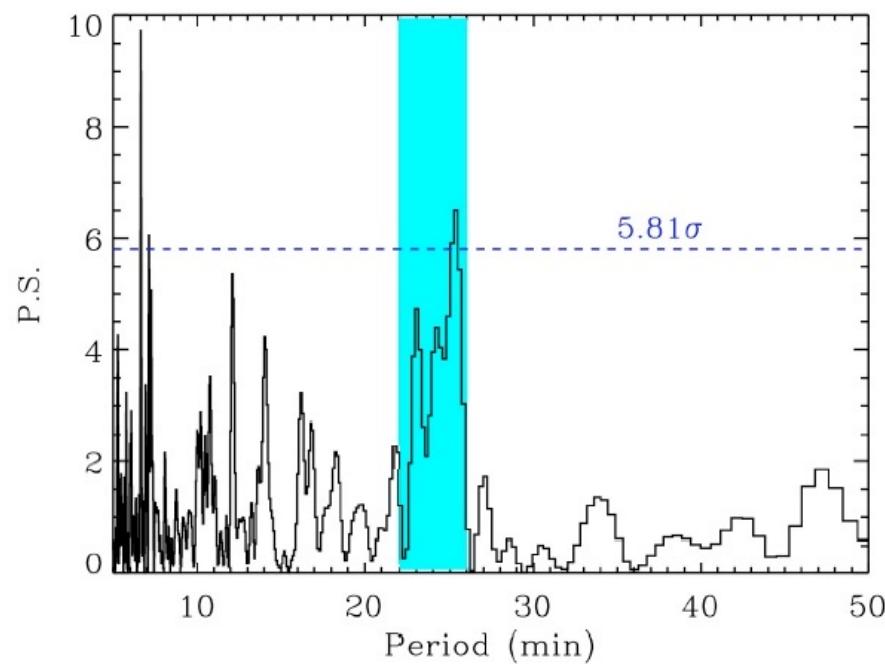
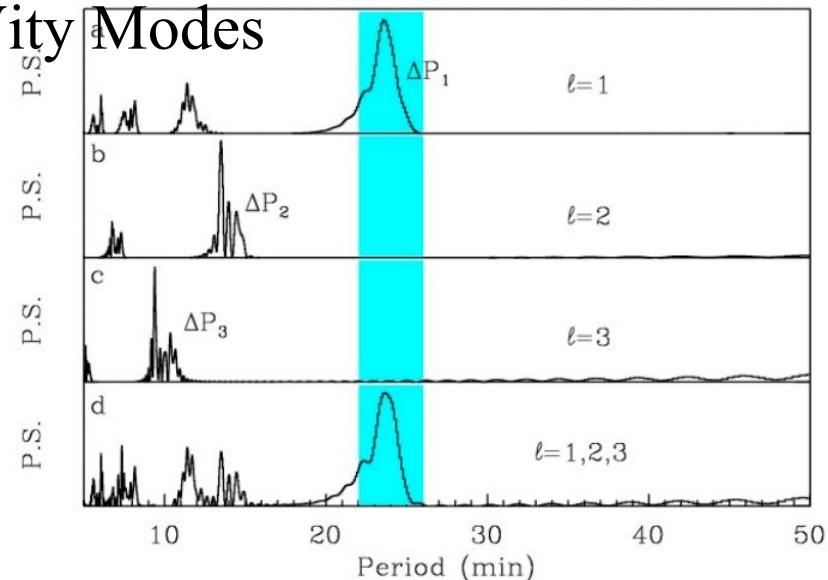
FIG. 4.—Example of the class A pattern. (a) Periodograms of 1290 days with zero-padding by a factor of 3, centered on the predicted $l = 2$, $n = -3$. (b) MT estimate. The 90% confidence levels of not being noise are shown by the black line for single peaks, the blue (light or dark) lines for respectively doublets and triplets corresponding to $l = 2$, and the red (light or dark) lines for respectively doublets and triplets corresponding to $l = 3$. The gray area on the plots corresponds to the region where a structure has been identified in one or several methods.

Observing the Sun from Space: GOLF/SoHO GraVity Modes

- Analysis uses:
 - very long time series (10 years)
 - assumed internal rotation
 - estimated observational SNR
-

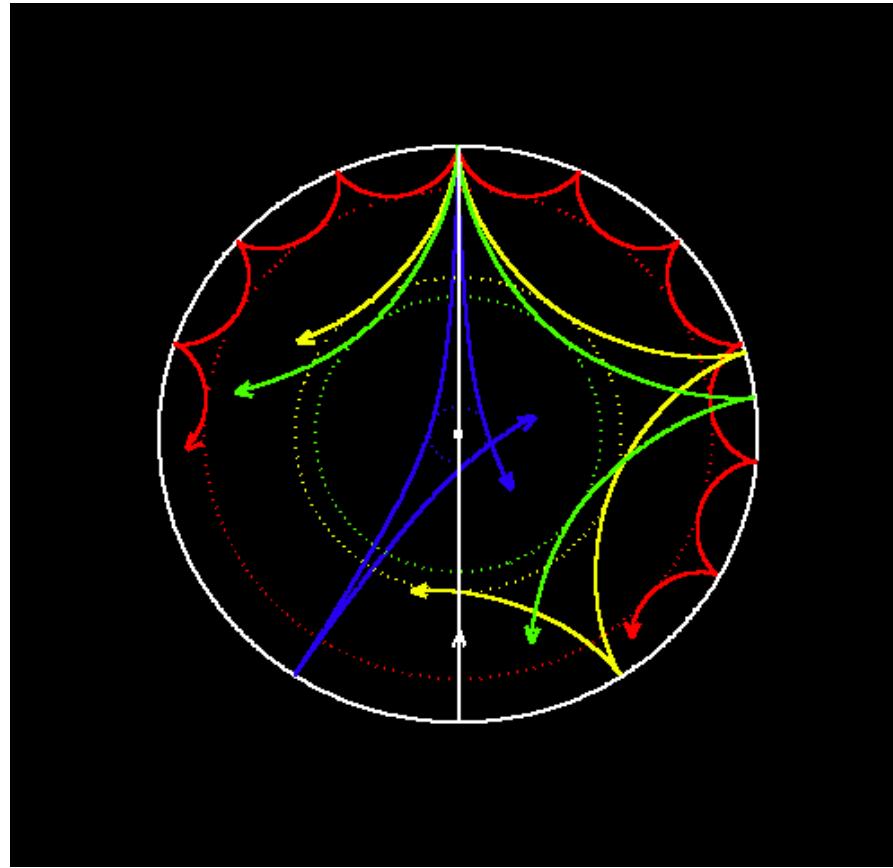


Garcia et al.
+GOLF Team,
Science, June
15, 2007



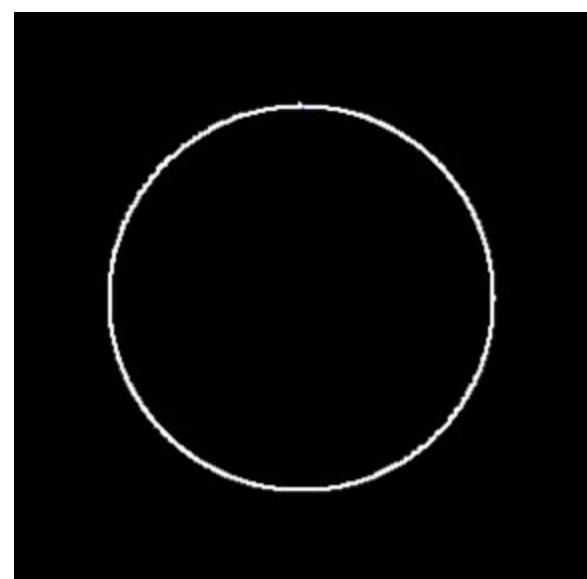
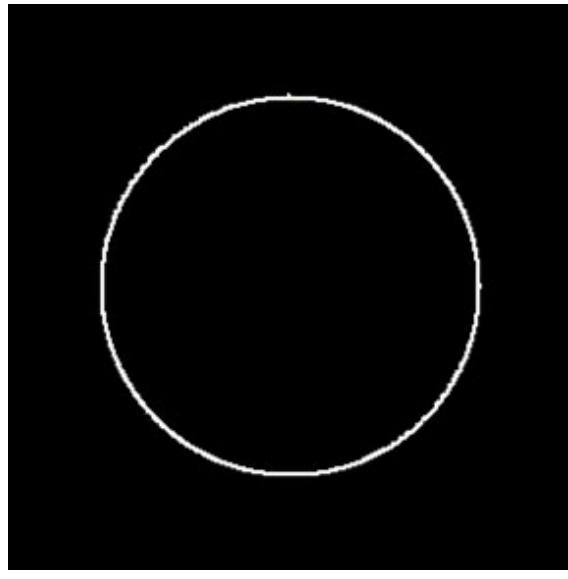
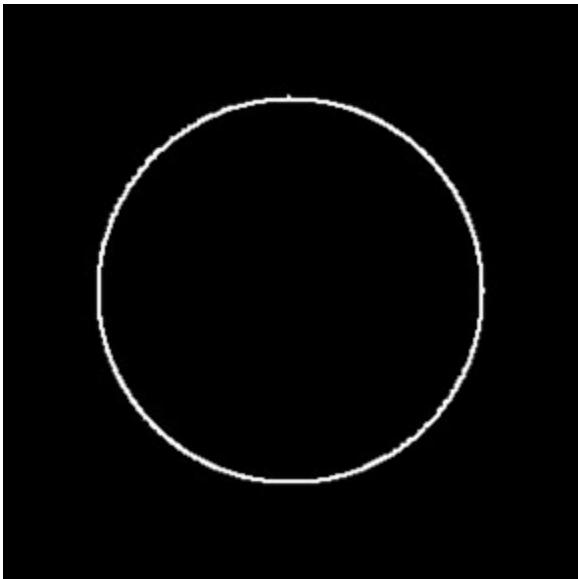
Stellar Seismology: Probing fundamental Physics

Rays



$$k_r = \left[\frac{\omega^2}{c^2} - \frac{l(l+1)}{r^2} \right]^{1/2}, \quad \frac{c(r_t)}{r_t} = \frac{\omega}{\sqrt{l(l+1)}}$$

Rays



$$k_r = \left[\frac{\omega^2}{c^2} - \frac{l(l+1)}{r^2} \right]^{1/2},$$

$$\frac{c(r_t)}{r_t} = \frac{\omega}{\sqrt{l(l+1)}}$$

Turning points

$$\frac{c^2(r_t)}{r_t^2} = \frac{\omega^2}{l(l+1)}$$

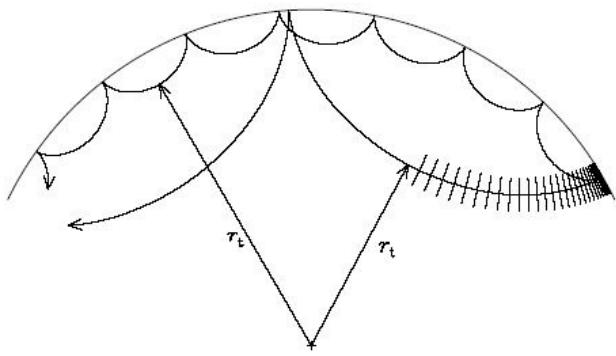


Figure 5.4: Propagation of acoustic waves, corresponding to modes with $l = 30$, $\nu = 3\text{ mHz}$ (deeply penetrating rays) and $l = 100$, $\nu = 3\text{ mHz}$ (shallow penetrating rays). The lines orthogonal to the former path of propagation illustrate the wave fronts.

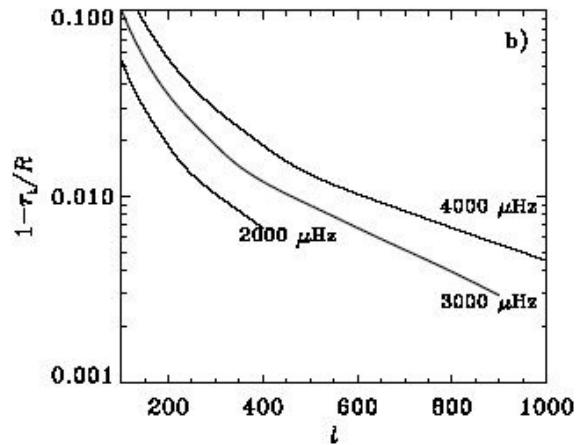
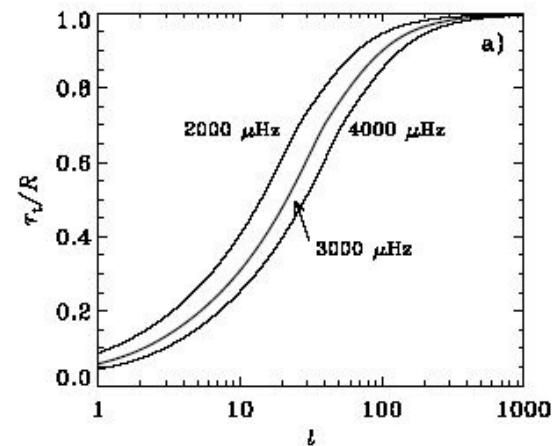
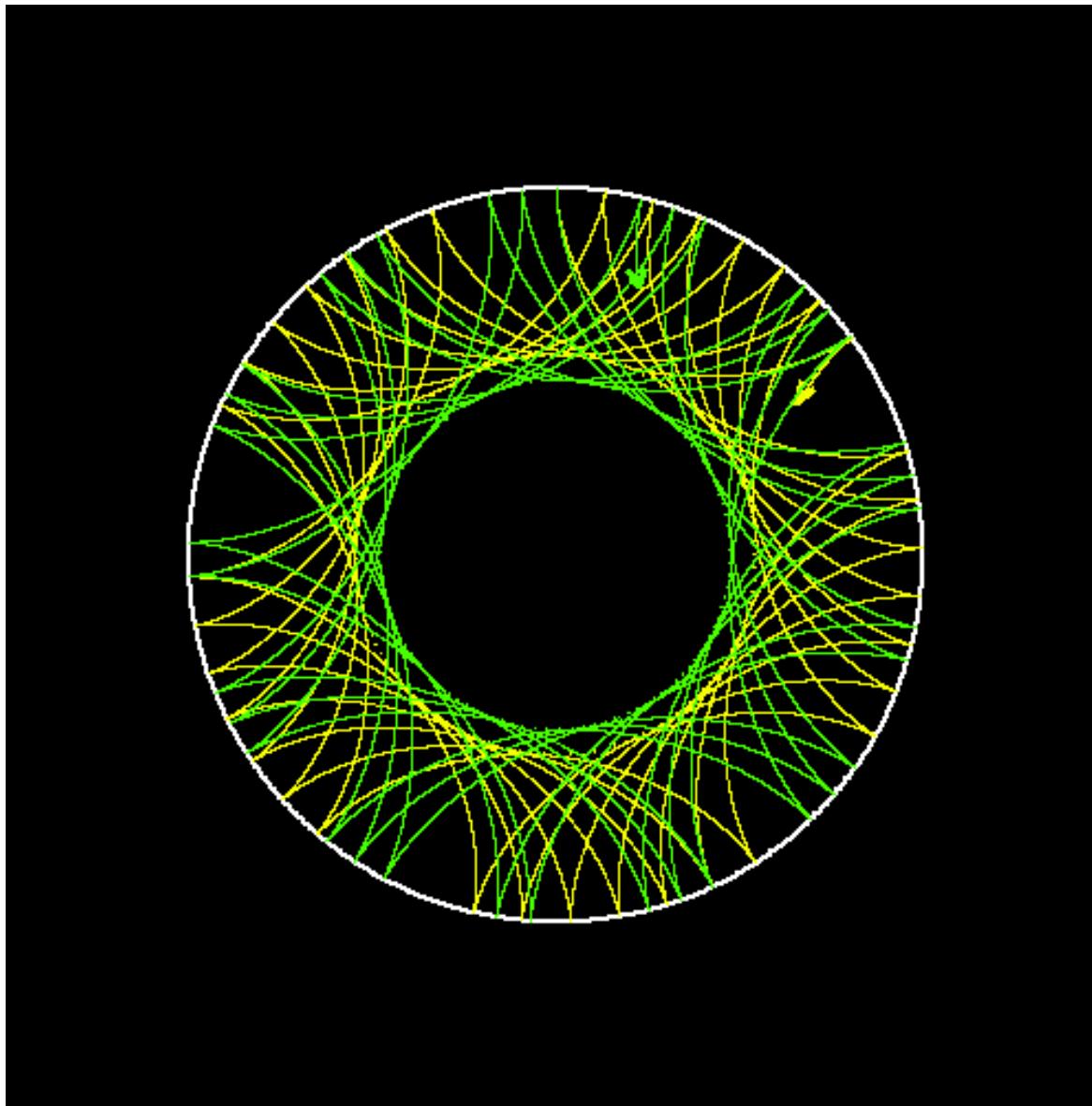
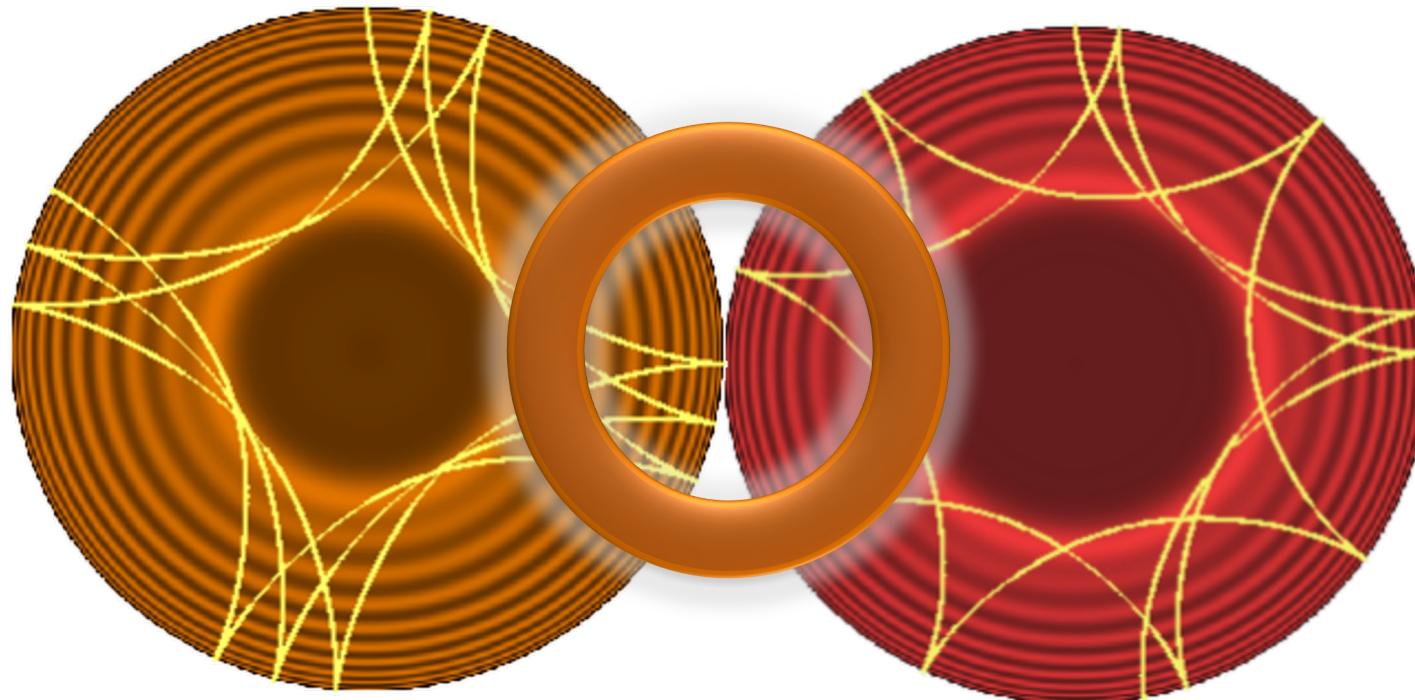


Figure 5.5: The location r_t (a) of the inner turning point, and the depth of penetration $R - r_t$ (b), in units of the solar radius R , for p modes in a standard solar model. The results are shown as functions of degree l , for three typical frequencies.

Inversion with rays

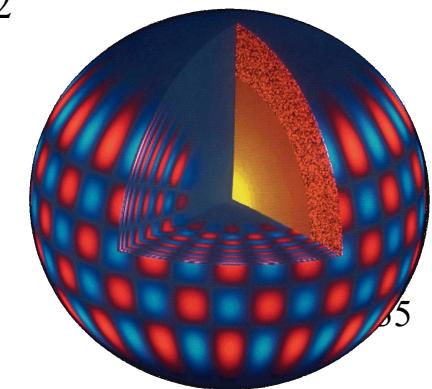


SOLAR OSCILLATIONS: INVERSIONS



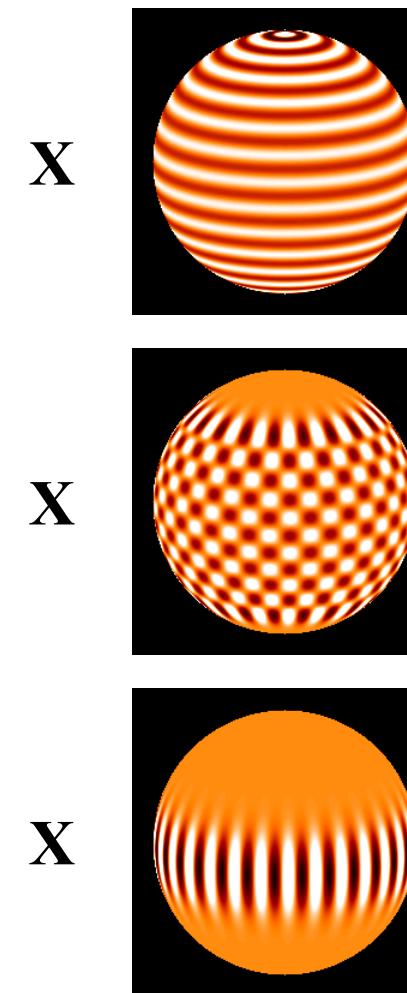
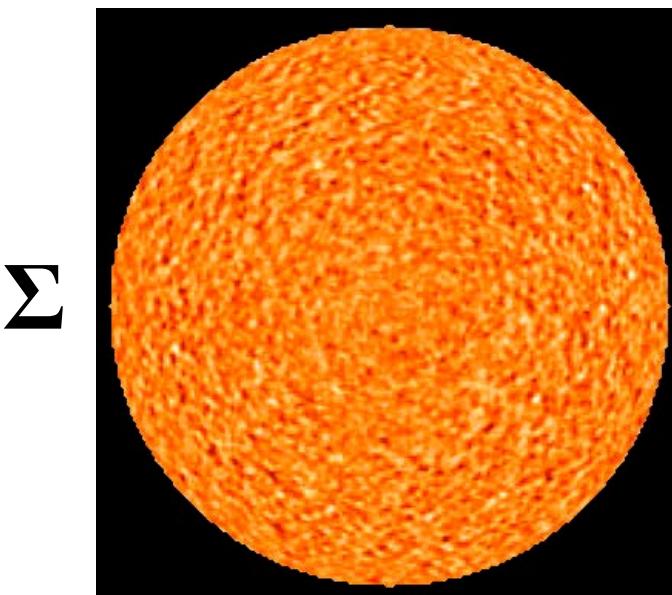
$$\omega_1 \quad \delta\omega = -$$

$$\omega_2$$

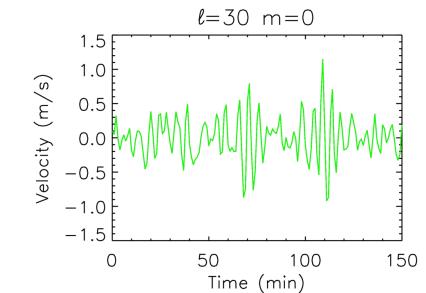


- Modes of different $\omega_{l,n}$ cover different depths, towards the Sun's interior.

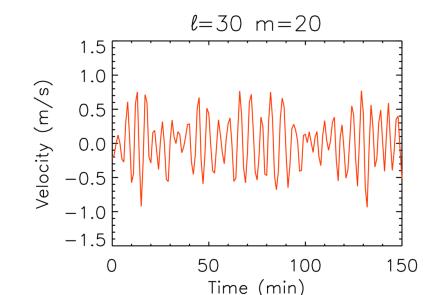
Observing Time Series



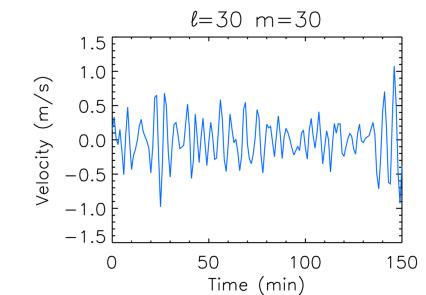
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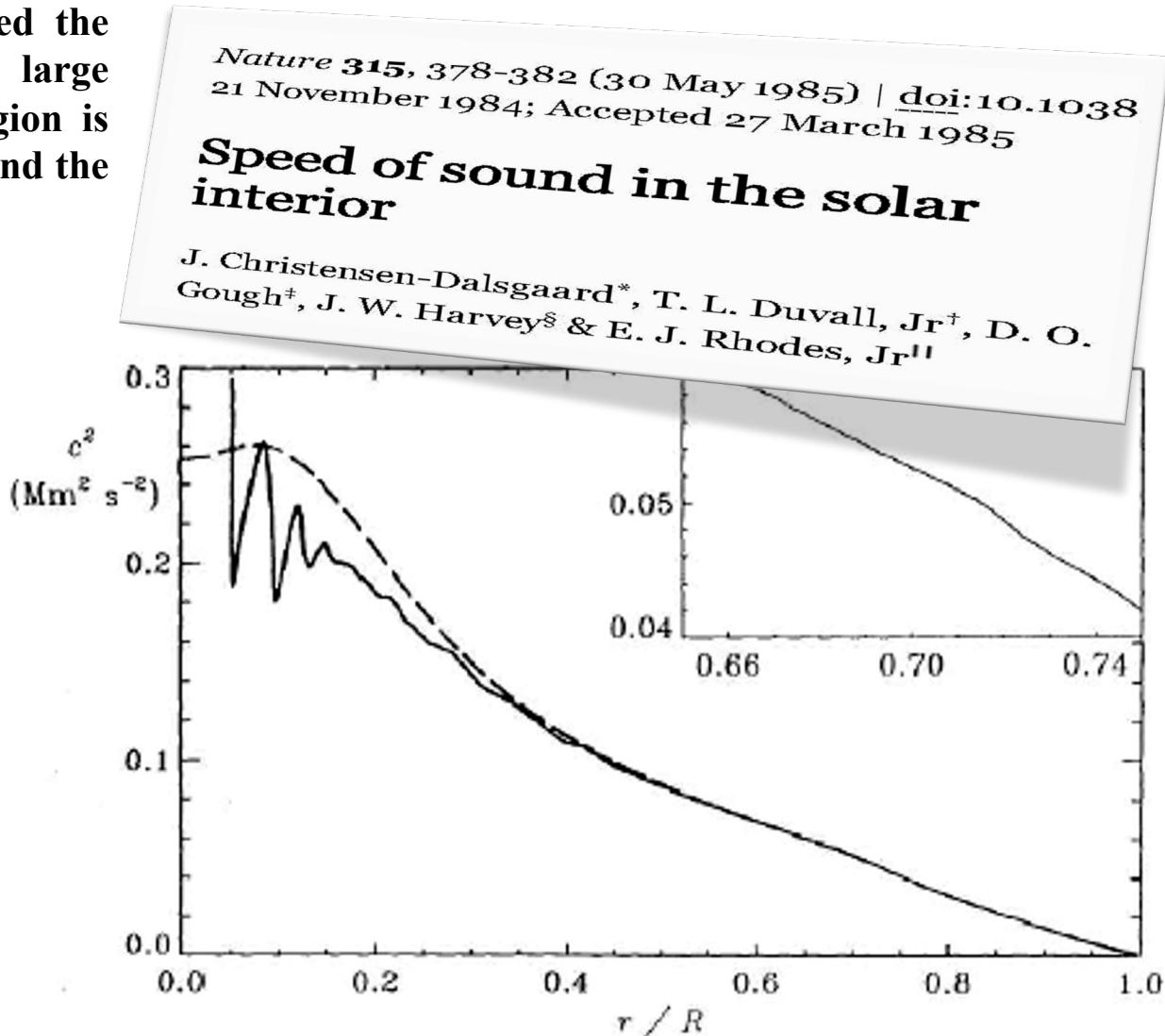
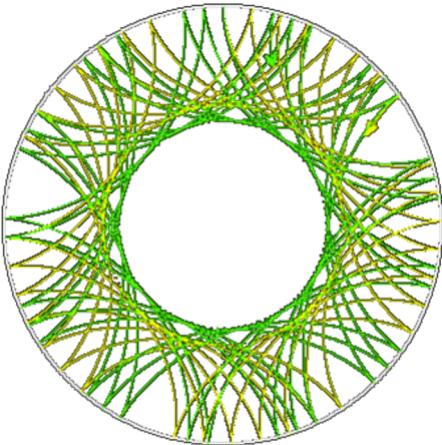


=



SOLAR OSCILLATIONS: SOUND SPEED INVERSION

- The first result of the asymptotic sound inversion that confirmed the standard solar model. The large discrepancy in the central region is due to inaccuracy of the data and the asymptotic approximation.



Stellar Structure AND Evolution Probing Fundamental Physics

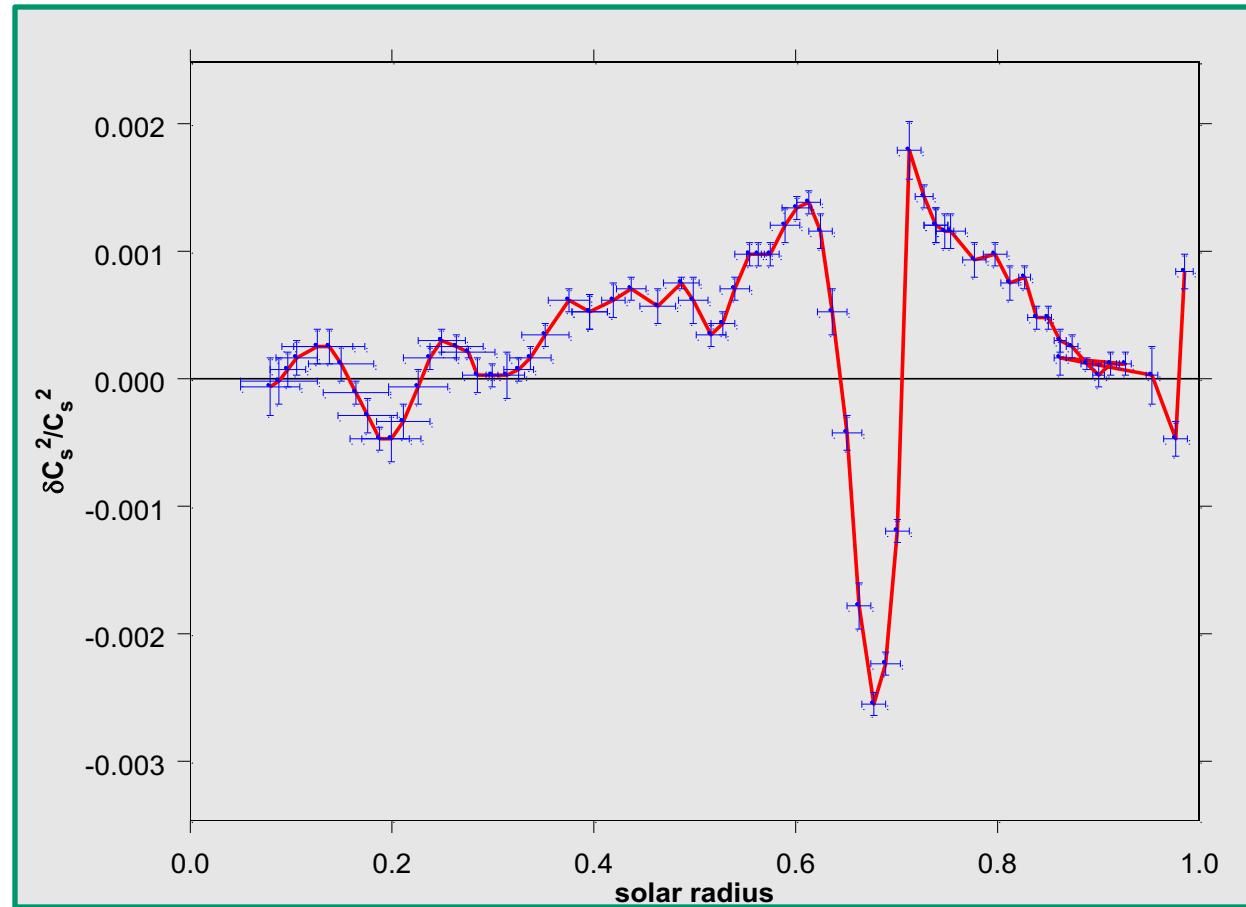
- 1906 - [Arthur Eddington](#): statistical study of stellar motions
- 1908 - [Henrietta Leavitt](#): Cepheid period-luminosity relation
- 1910 - [E.Hertzsprung](#) & [H.N.Russell](#); HR Diagram (spectral classification)
- 1924 - [Arthur Eddington](#): [main sequence](#) star mass-luminosity relationship
- 1929 - [George Gamow](#): [hydrogen fusion](#) as the energy source for stars
- 1938 - [Hans Bethe](#) and [Carl von Weizsäcker](#) pp-chain and CNO cycle
- 1952 - [Walter Baade](#) Cepheid I and Cepheid II variable stars
- 1953 - [Fred Hoyle](#) predicts a [carbon](#)-12 resonance - [triple alpha](#) reactions
- 1961 - [Chūshirō Hayashi](#) : Hayashi track of fully convective stars
- 1963 - [Fred Hoyle](#) and [William A. Fowler](#): supermassive stars
- 1964 - [Subrahmanyan Chandrasekhar](#):
GR theory of stellar pulsations and evolution;

STANDARD PICTURE OF STELLAR EVOLUTION SOLAR STANDARD MODEL

STELLAR STRUCTURE AND EVOLUTION PROBING FUNDAMENTAL PHYSICS

Improve Microscopic Physics Description

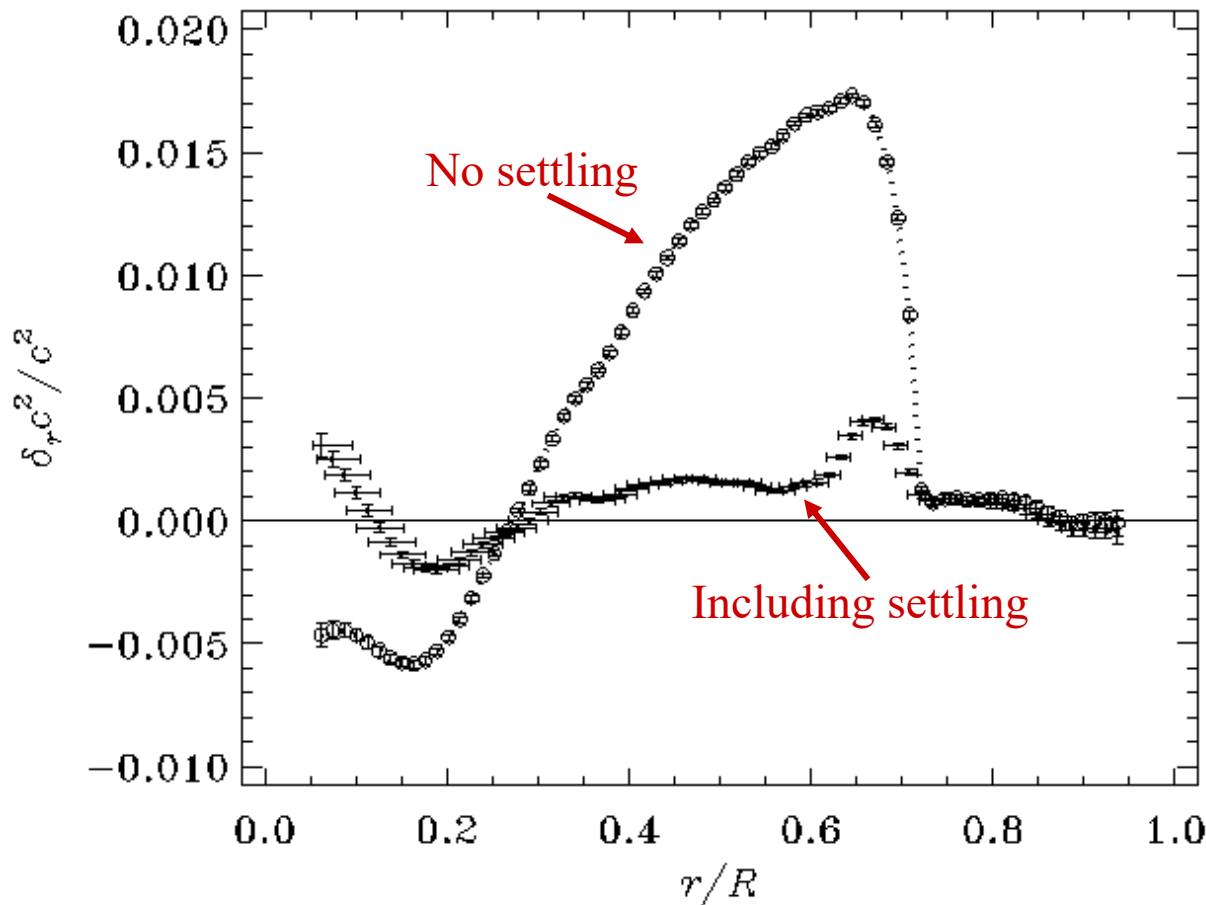
- Present features in the sound speed: (i) Surface: Interaction between the acoustic waves, the convection and radiation in the super-adiabatic region. Asymmetry of the background state due to rotation. (ii) Tachocline: time-evolution due to microscopic diffusion and turbulence. (iii) Intermediate region: The chemical abundance and the opacities:
 - The nuclear region ?
 - The central core ?



C_s^2 Inversion (MDI+GOLF data) Turck-Chieze et al.³⁹ (GOLF, 1997); CENTRA Standard Model

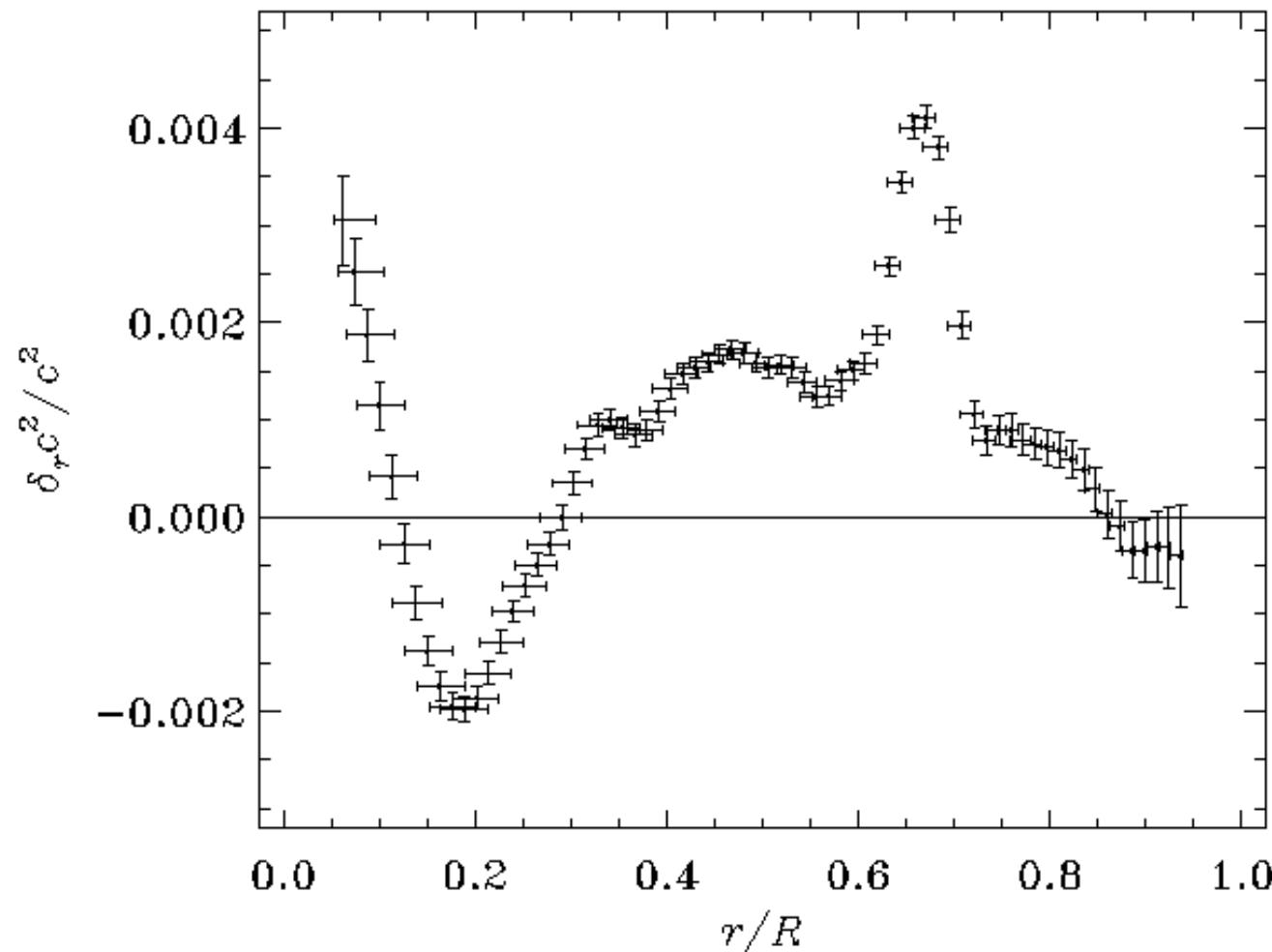
The solar internal sound speed

Sun - model



The solar internal sound speed

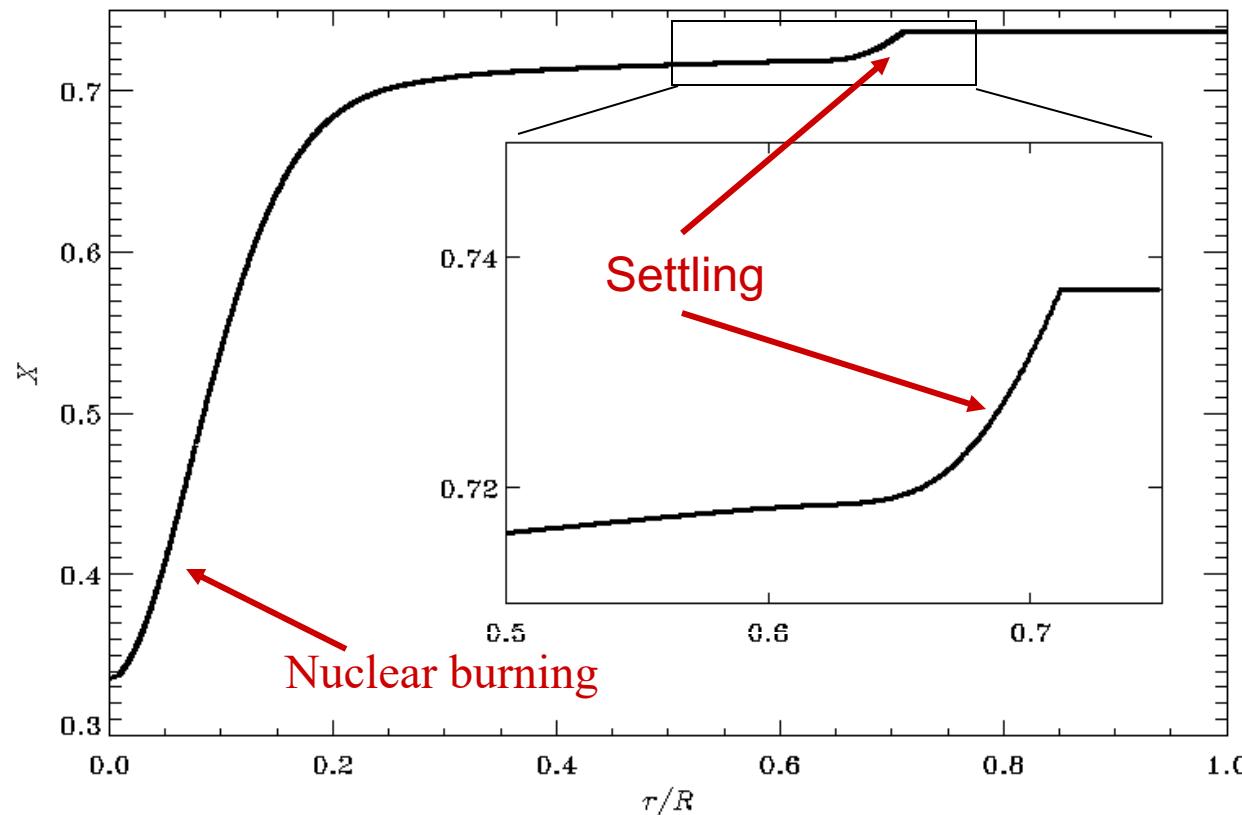
Sun - model



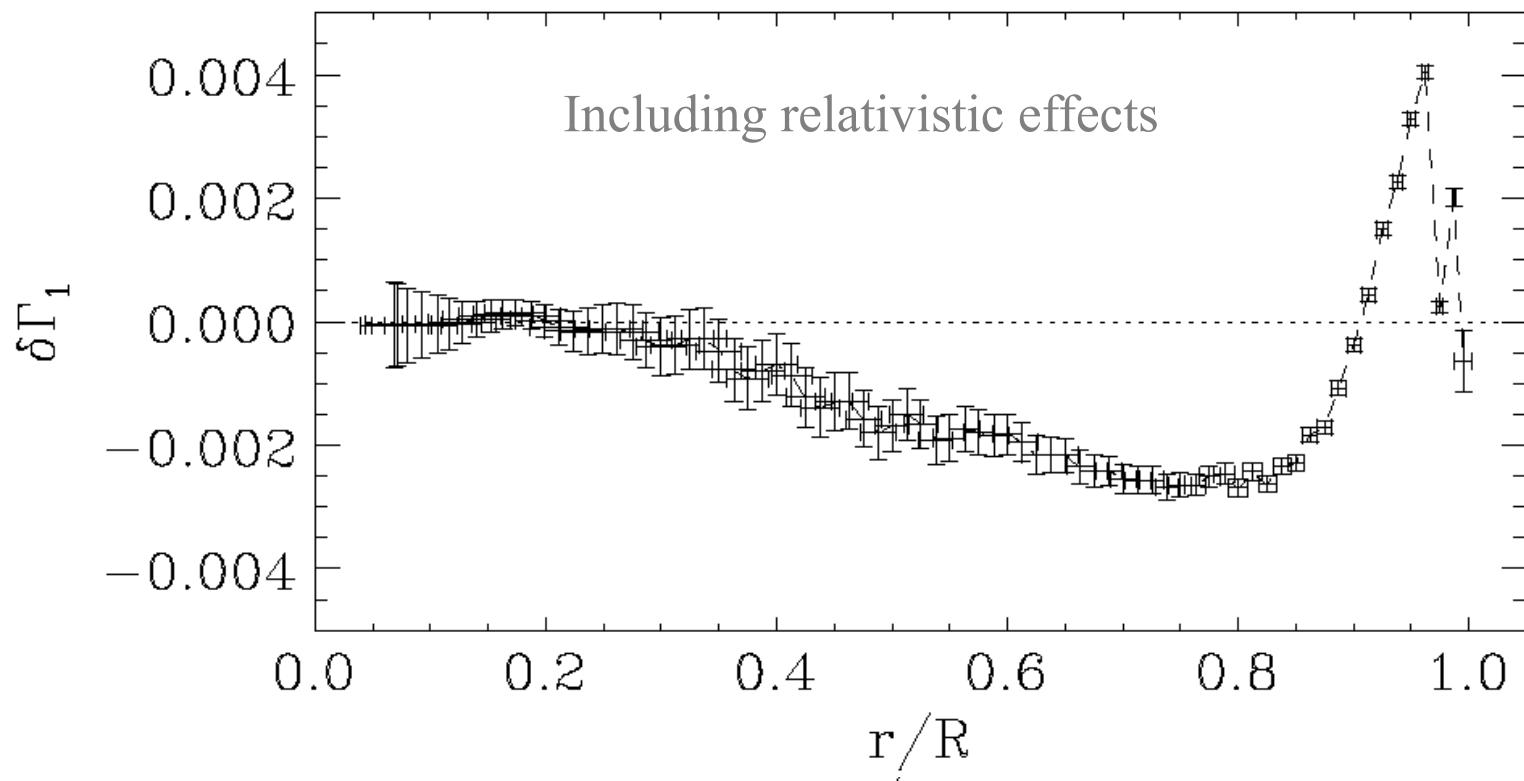
Basu et al. (1997; MNRAS 292, 243)

Changes in composition

The evolution of stars is controlled by the changes in their interior composition:



Relativistic electrons in the Sun

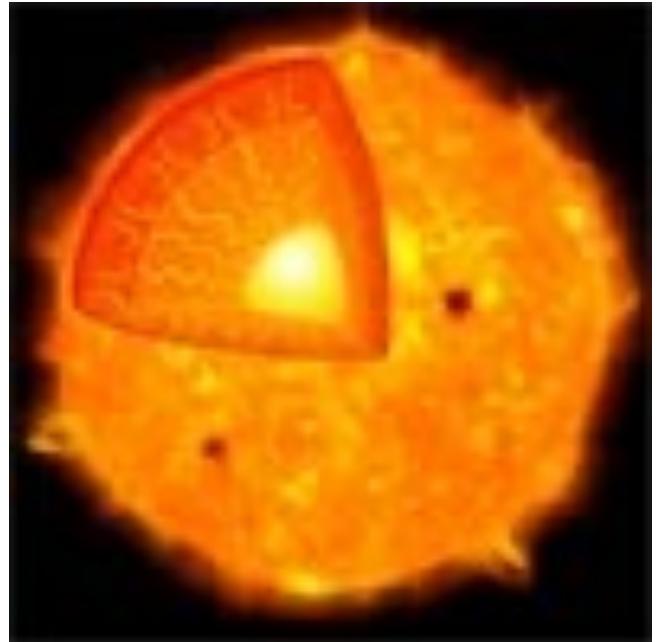


Elliot & Kosovichev (1998; ApJ 500, L199)

Neon discovery solves mystery of sun's interior

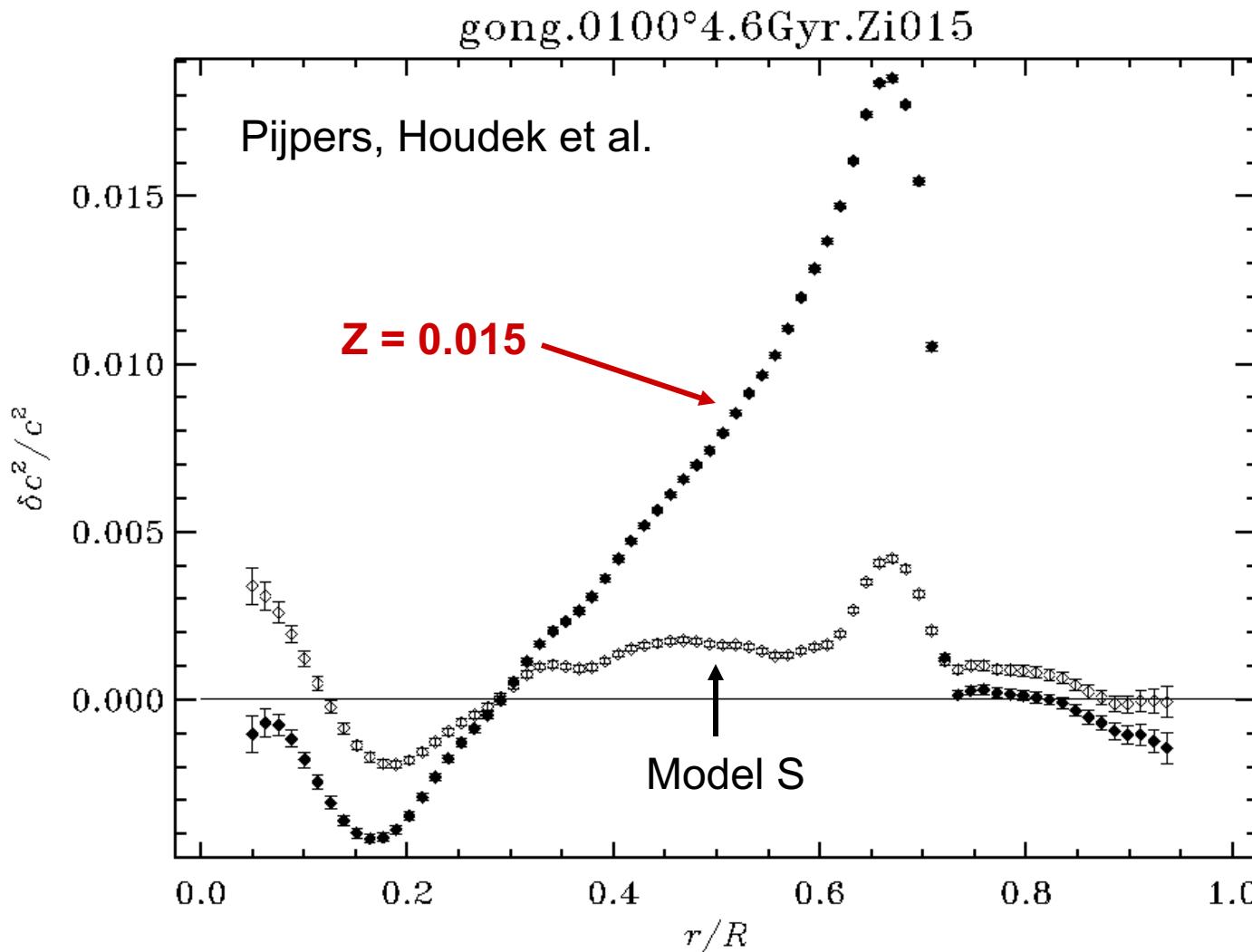
NASA's Chandra X-ray Observatory survey of nearby sun-like stars suggests there is nearly three times more neon in the sun and local universe than previously believed. If true, this would solve a critical problem with understanding how the sun works.

[FULL STORY](#)

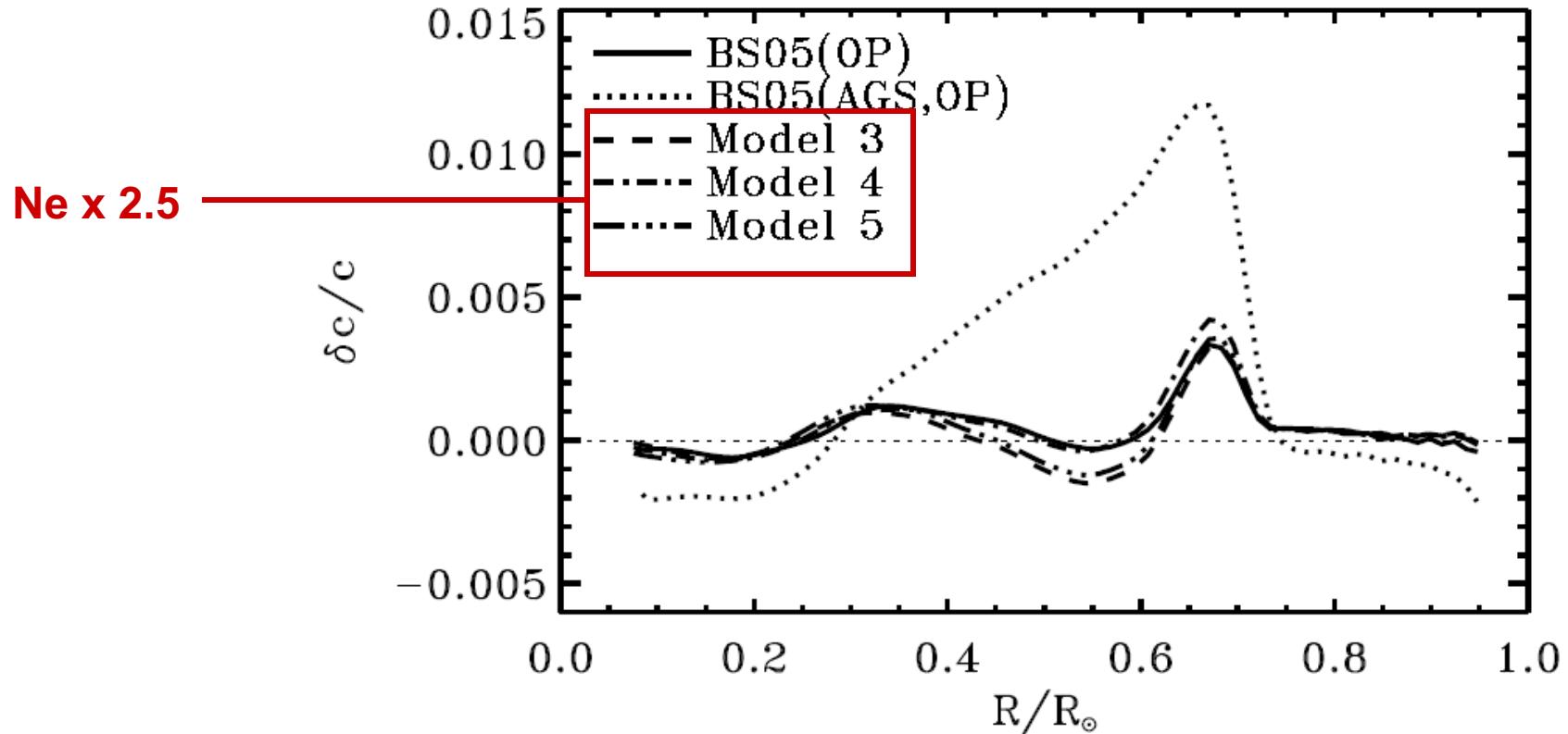


(Spaceflight Now – 14 Aug 2005)

Revision of solar surface abundances



The neon story



Bahcall et al. (2005; ApJ, in the press [astro-ph/0502563])

Drake & Testa (2005; Nature 436, 525): X-ray observations of nearby stars indicate such a neon increase

**STELLAR SEISMOLOGY:
PROBING FUNDAMENTAL PHYSICS
THE SOLAR STANDARD MODEL
AND THE SOLAR NEUTRINOS**

STELLAR STRUCTURE AND EVOLUTION PROBING FUNDAMENTAL PHYSICS

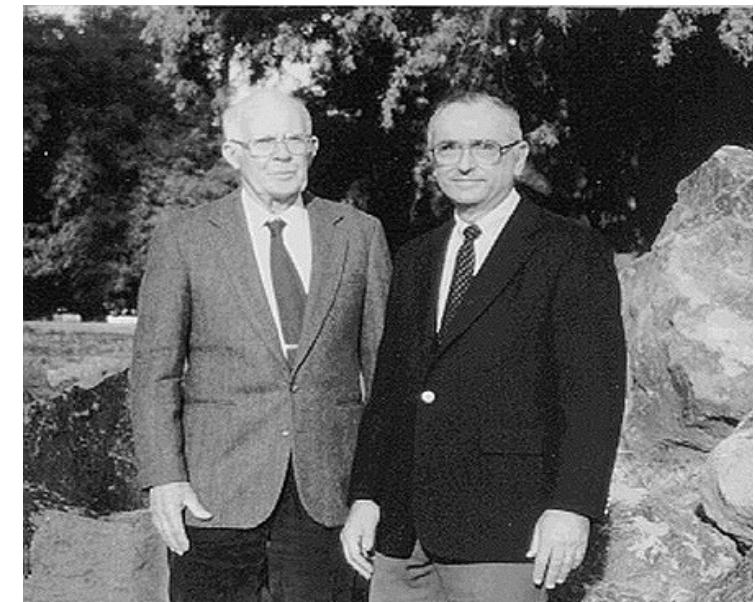
The solar neutrino problem

A 40 year long journey

In 1963 J Bahcall and R Davis, based on ideas from Bruno Pontecorvo, started an exploration of the Sun by means of solar neutrinos.

A long detour: the “solar neutrino puzzle”

All experiments, performed at Homestake, Kamiokande, Gran Sasso (Gallex) and Baksan (Sage), exploring different parts of the solar spectrum (B,pp+Be..) and sensitive to ν_e , reported a neutrino deficit (of 1/3; disappearance) with respect to Standard Solar Model.



Was the SSM wrong?

Was nuclear physics wrong?

Were all experiments wrong?

Or did something happen to neutrinos during their trip from Sun to Earth?

STELLAR STRUCTURE AND EVOLUTION PROBING FUNDAMENTAL PHYSICS

*TOWARD A UNIFIED CLASSICAL MODEL OF THE SUN: ON THE SENSITIVITY
OF NEUTRINOS AND HELIOSEISMOLOGY TO THE MICROSCOPIC PHYSICS*

SYLVAIN TURCK-CHIÈZE AND ILIDIO LOPES¹

Service d'Astrophysique, DAPNIA, Centre d'Etudes de Saclay, 91191 Gif-sur-Yvette, France
Received 1992 August 3; accepted 1992 October 29

Turck-Chieze & Lopes (1993)

How Does the Sun Shine?

J.N. Bahcall, M. Fukugita, and P.I. Krastev

*School of Natural Sciences, Institute for Advanced Study
Princeton, NJ 08540*

pp not CNO reactions. However, helioseismological measurements indicate [14] that the solar sound velocity (closely related to the density profile) does not differ significantly (less than or of order a percent) from the standard model profile, as far as the helioseismological measurements have probed (down to about 10% of the solar radius). So, if there were a



“...Helioseismological measurements indicate [14, which is Turck-Chièze and Lopez (1993) – with a Z because John thought I was Spanish] that the solar sound velocity does not differ significantly from the standard model profile, as far as the helioseismological measurements have probed ”

John cite us ~! (10%)

STELLAR STRUCTURE AND EVOLUTION PROBING FUNDAMENTAL PHYSICS

**Resolution of the solar neutrino problem
(18 June 2001)**

VOLUME 87, NUMBER 7

PHYSICAL REVIEW LETTERS

13 AUGUST 2001

Measurement of the Rate of $\nu_e + d \rightarrow p + p + e^-$ Interactions Produced by ^8B Solar Neutrinos at the Sudbury Neutrino Observatory

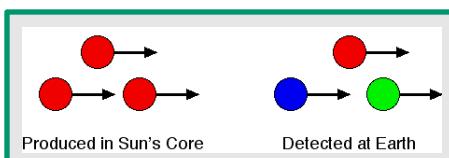
O. R. Ahmad,¹⁵ R. C. Allen,¹¹ T. C. Andersen,¹² J. D. Anglin,⁷ G. Bühler,¹¹ J. C. Barton,^{13,*} E. W. Beier,¹⁴ +al.

The SNO (Sudbury Neutrino Observatory) results for the ^8B solar neutrino flux

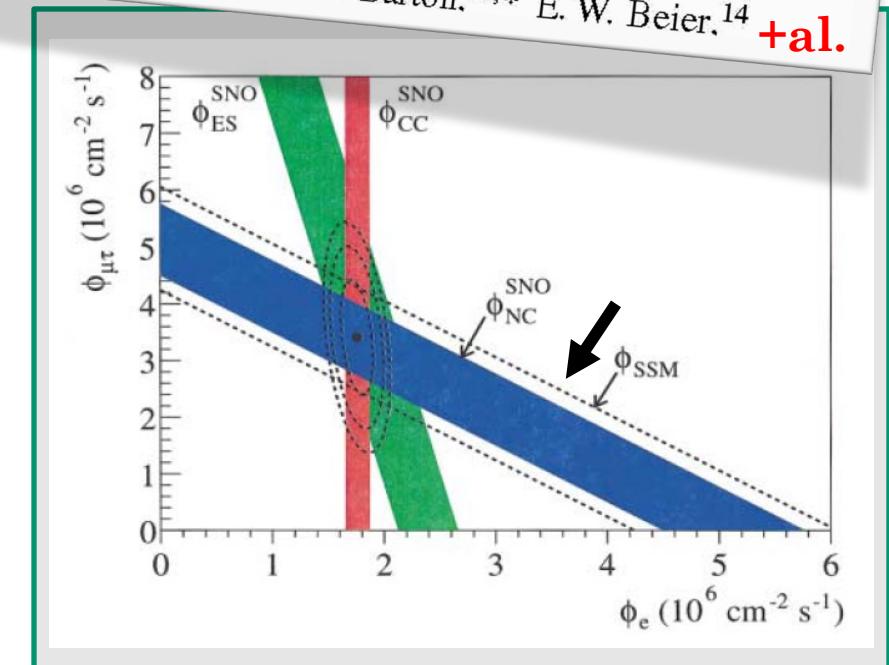
$$\phi(\nu_x) = 5.44 \pm 0.99 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$$

$$\phi(\nu_{\mu\tau}) = 3.69 \pm 1.13 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$$

Neutrino Oscillations



SNO Results show: (i)- detected $\sim 1/3$ of electron v's predicted by SSM; (ii)- 2/3 of the electron v's were converted into other flavors before they reached Earth ;(iii)-the total number is conserved

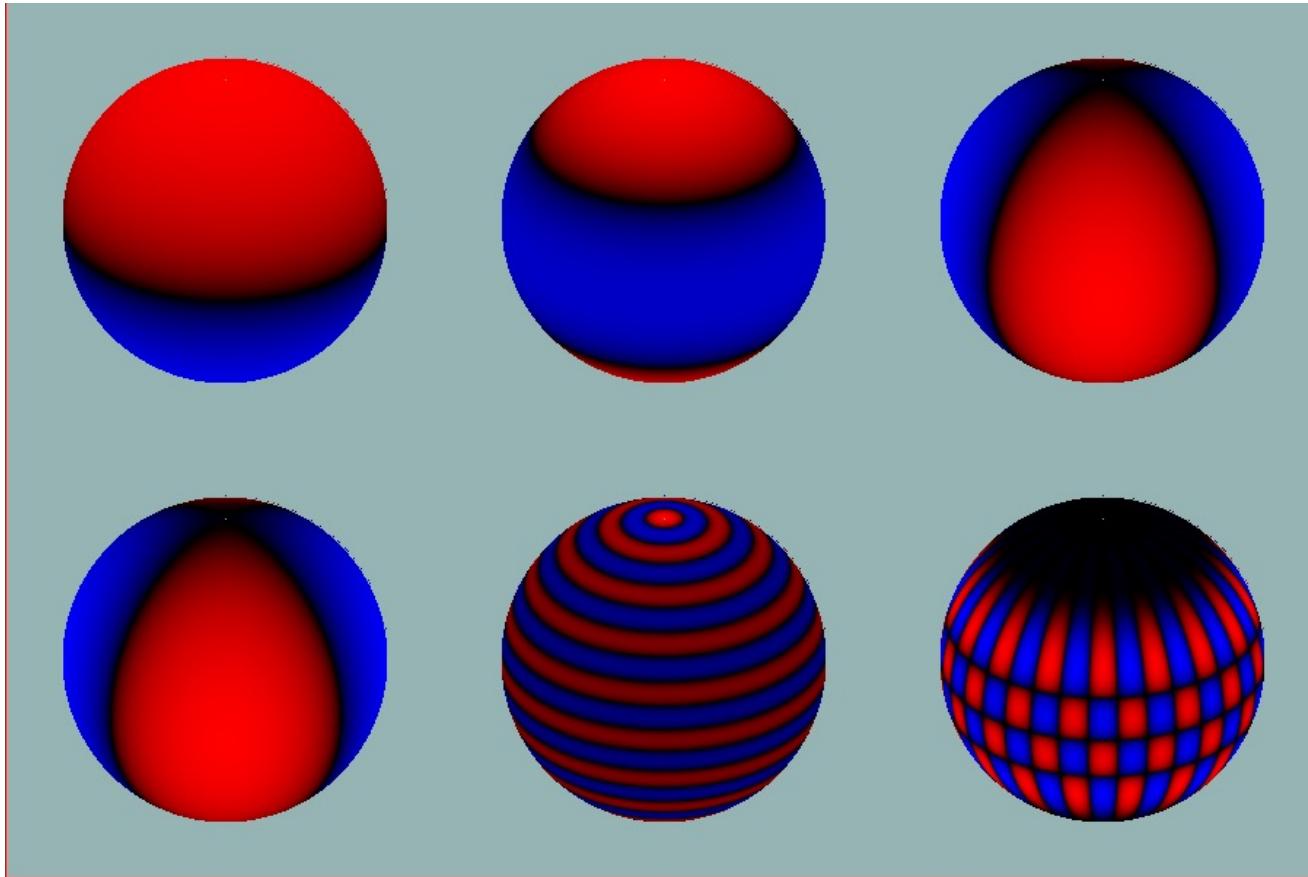


**Ray(mond) Davis: Nobel Prize of Physics 2002
(Nobel Lecture – Helioseismology : temperature correct in the SSM)**

END OF LAST LECTURE

**STELLAR SEISMOLOGY:
PROBING FUNDAMENTAL PHYSICS
THE DIFFERENTIAL ROTATION AND THE ORIGIN
OF THE SOLAR MAGNETIC FIELD**

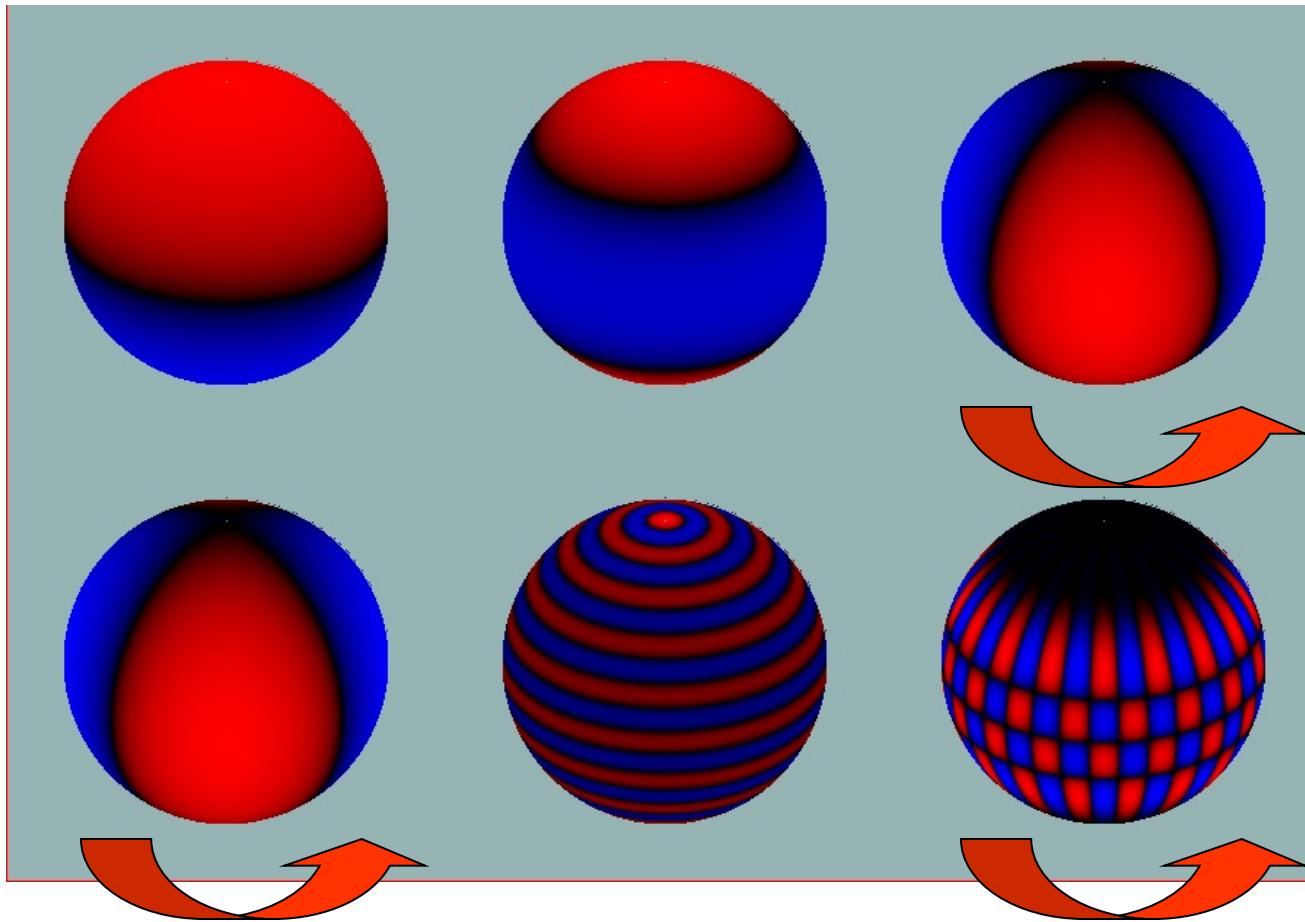
Spherical harmonics



Note: dependence on θ and t :

$$\Re \{ \exp(im\phi) \exp(-i\omega t) \} = \cos(m\phi - \omega t) \quad \text{i.e., running wave in } \phi$$

Rotational splitting



$$\omega_{nlm} = \omega_{nl0} + m\langle \Omega \rangle$$

Simple rotational splitting

In rotating frame:

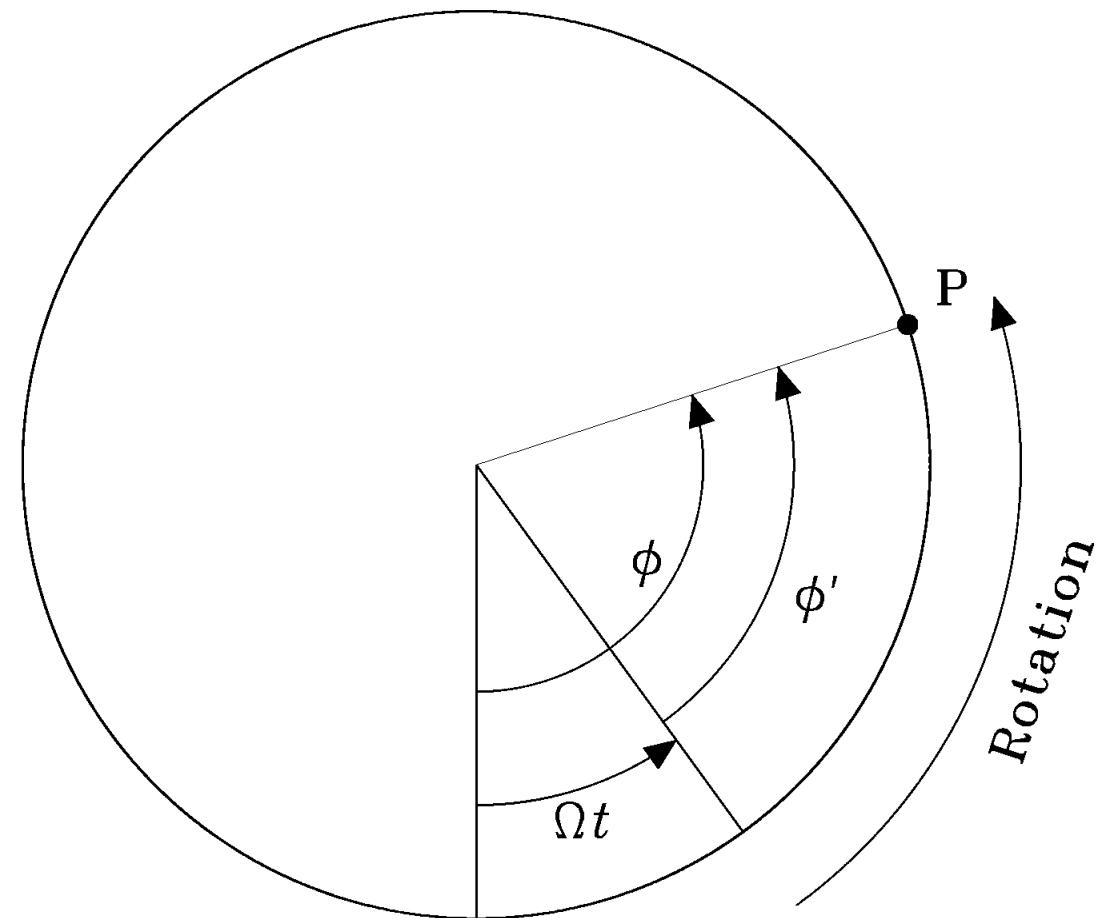
$$\cos(m\phi' - \omega_0 t)$$

In inertial frame:

$$\cos[m(\phi - \Omega t) - \omega_0 t]$$

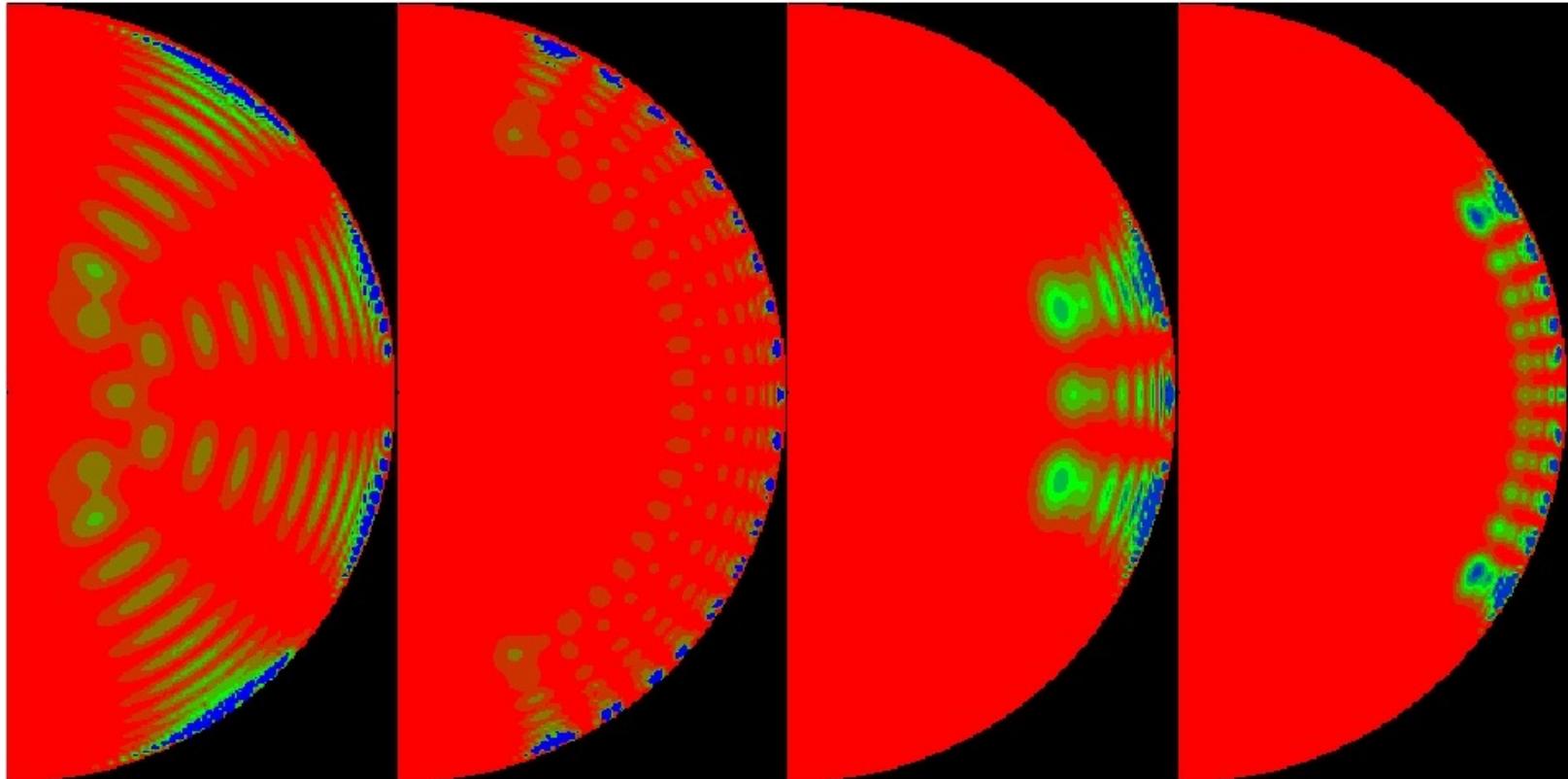
$$= \cos(m\phi - \omega_m t)$$

with $\omega_m = \omega_0 + m\Omega$

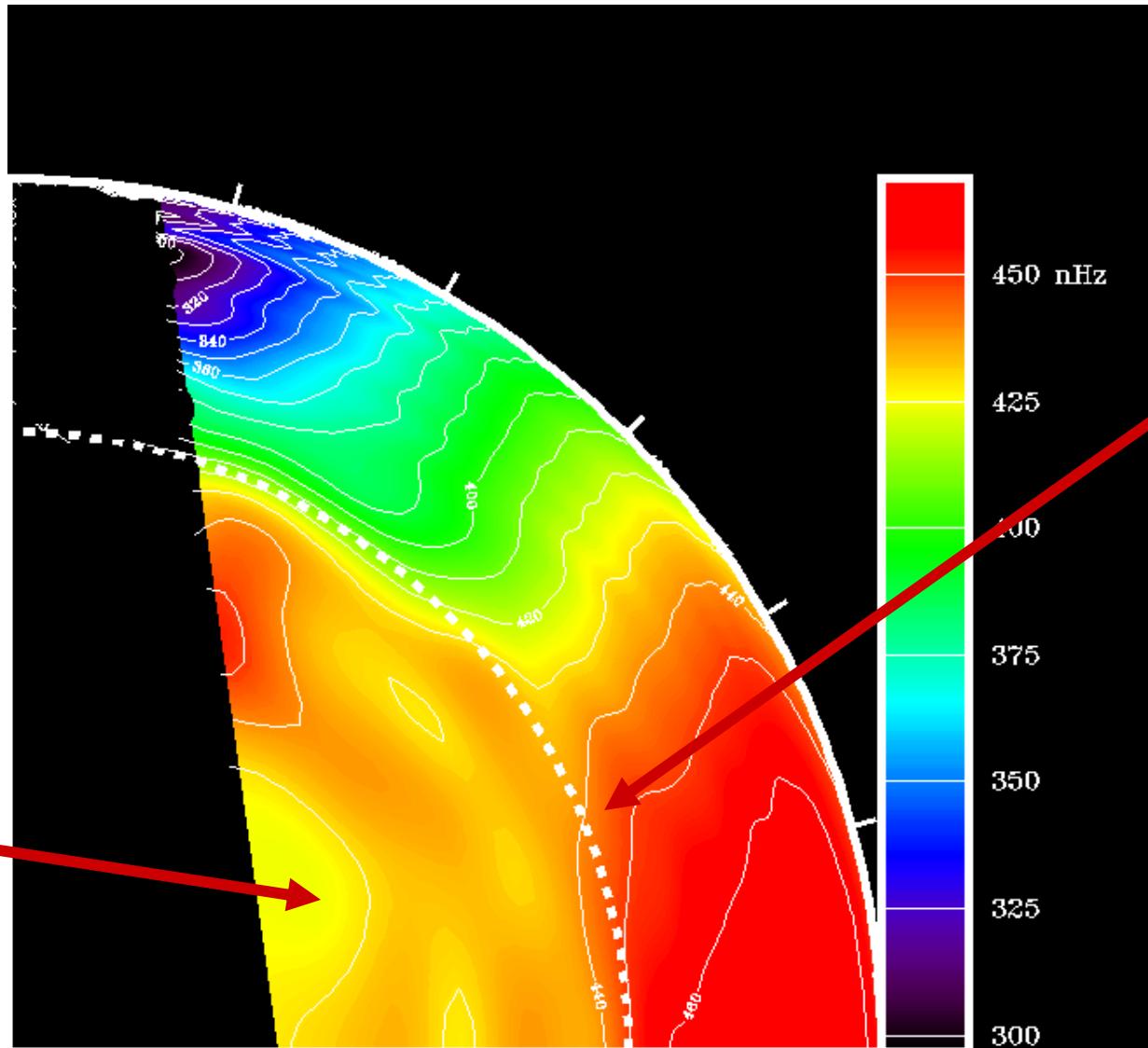


Kernels for rotational splitting

$$\omega_{nlm} = \omega_{nl0} + m \int_0^R \int_0^\pi K_{nlm}(r, \theta) \Omega(r, \theta) r dr d\theta$$

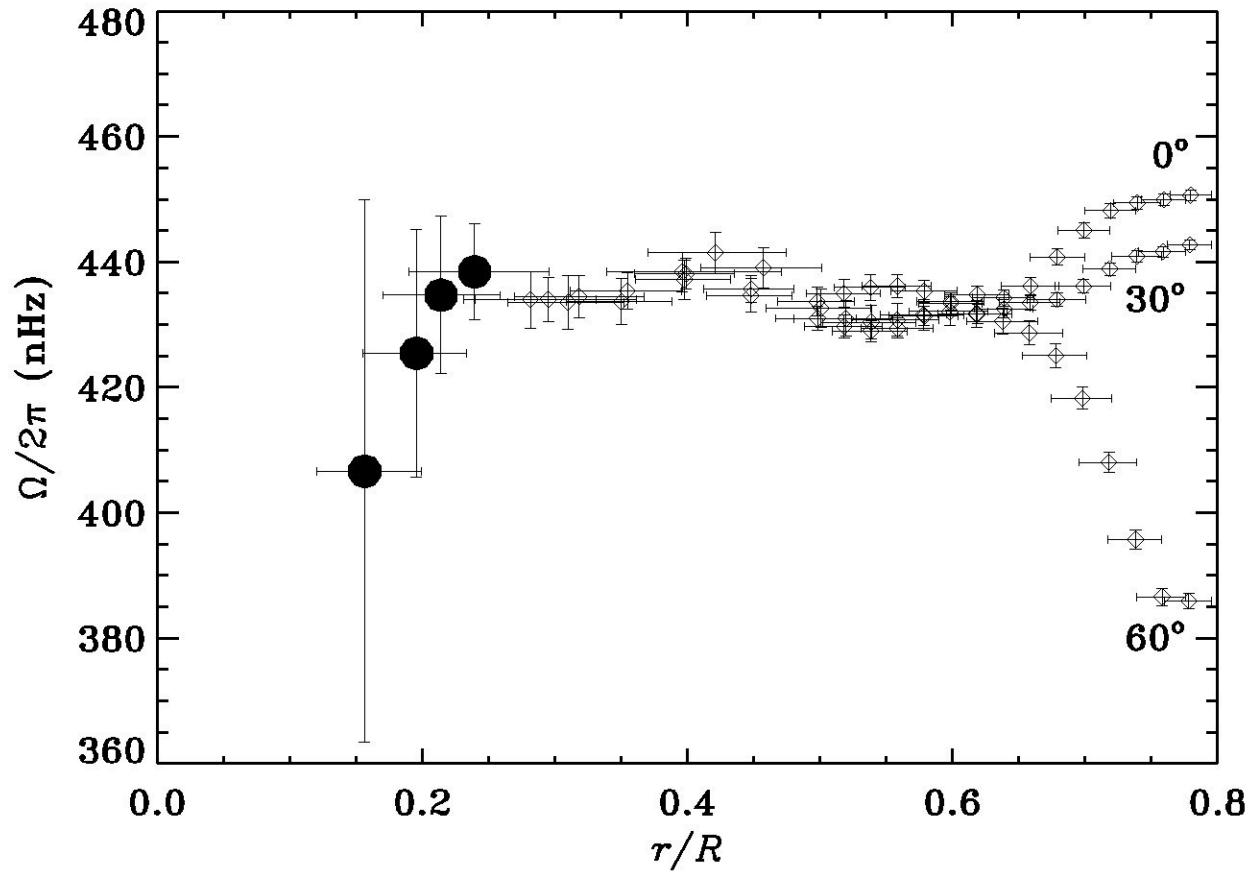


Inferred solar internal rotation



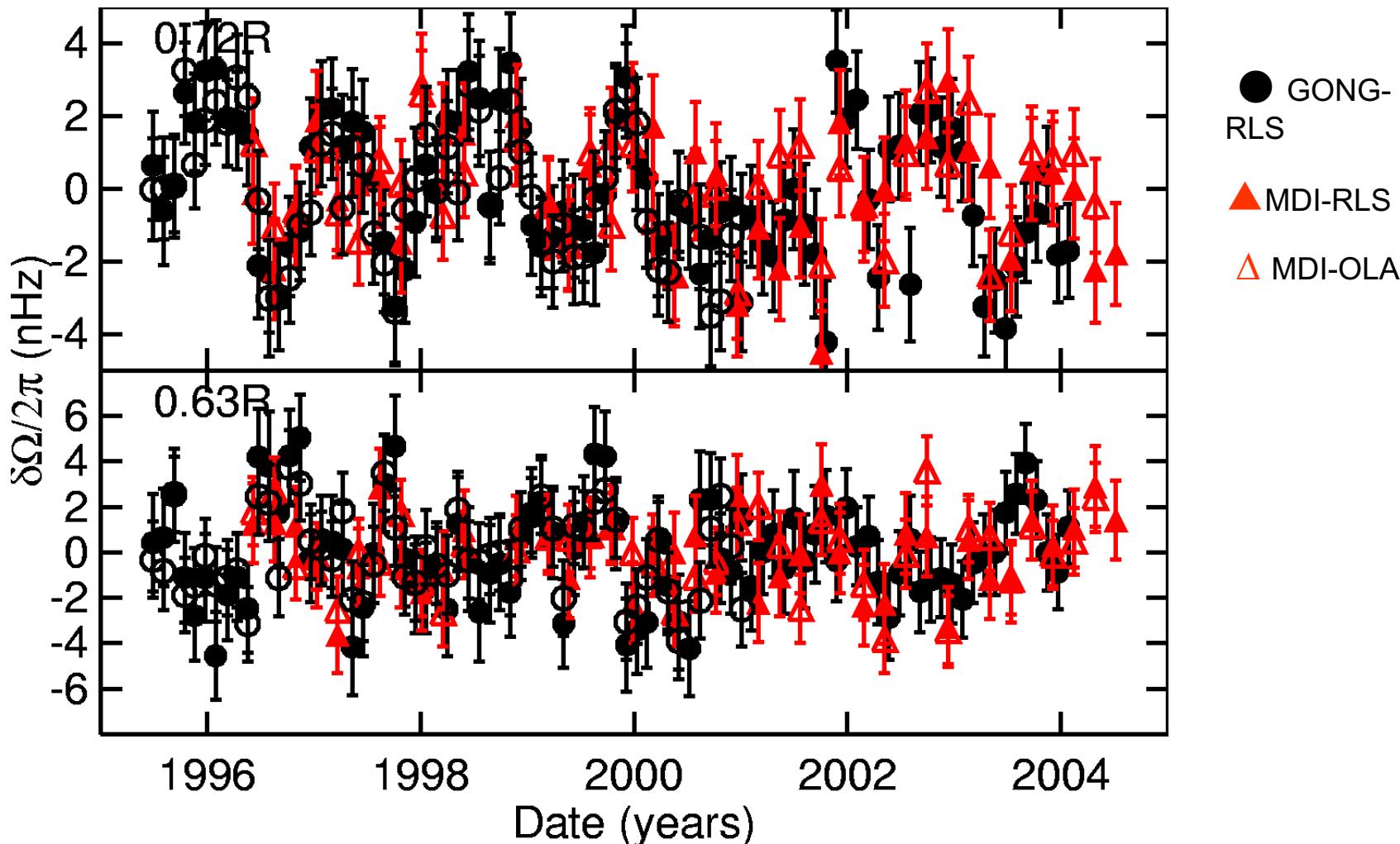
Schou et al. (1998; ApJ 505, 390)

Rotation of the solar interior



BiSON and LOWL data; Chaplin et al. (1999; MNRAS 308, 405)

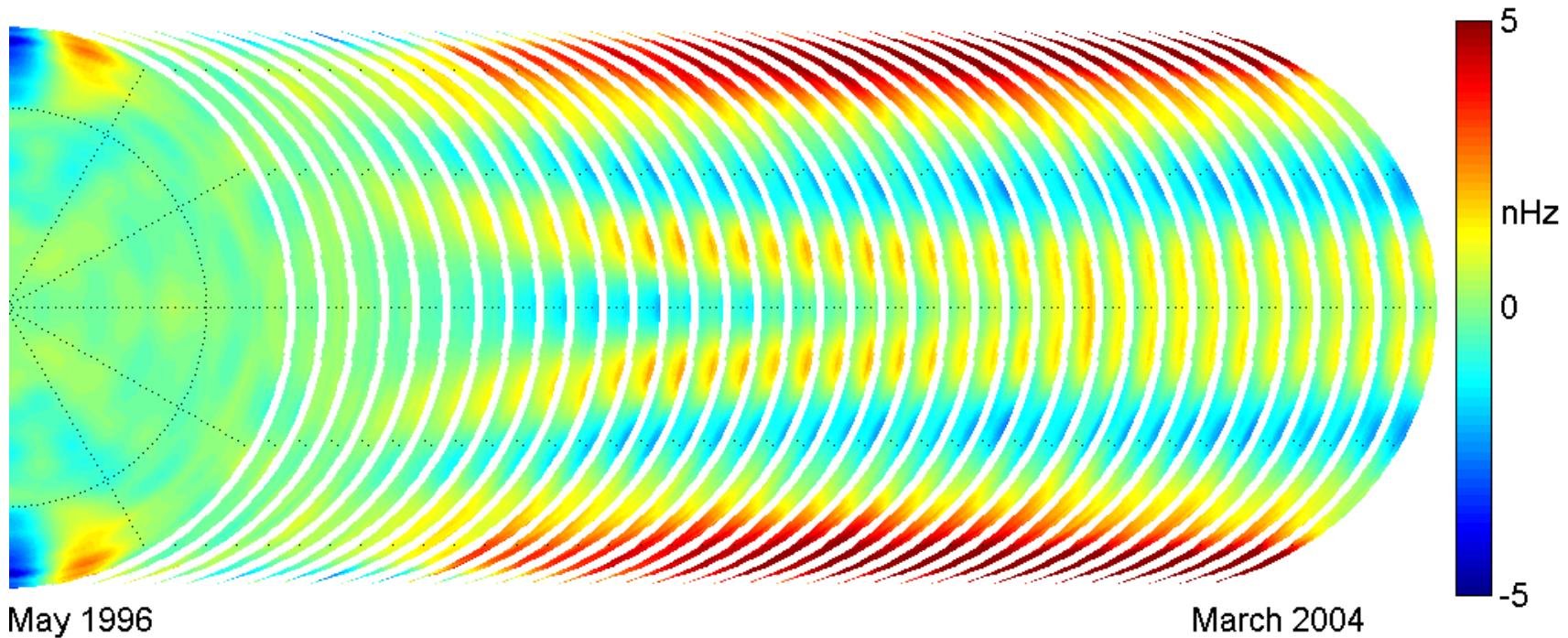
Tachocline oscillations



See Howe et al. (2000; Science 287, 2456)

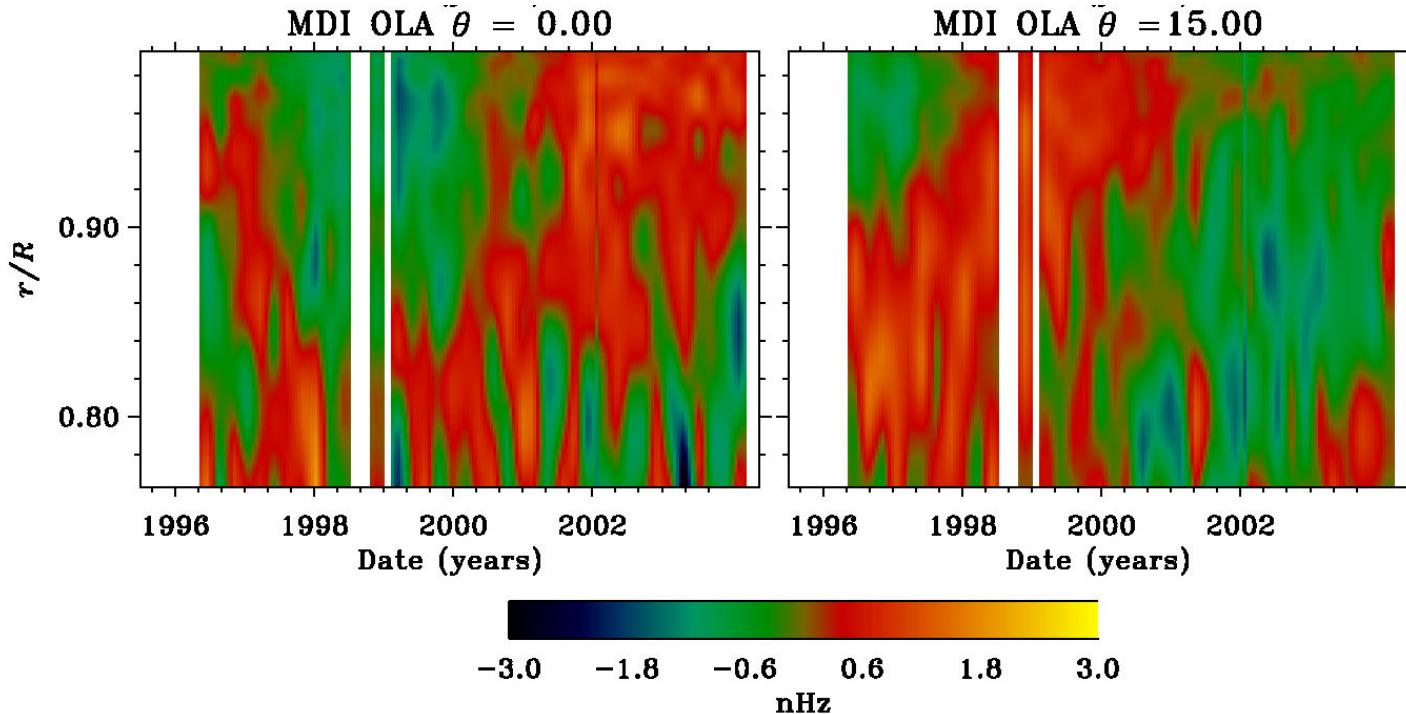
Zonal flows

Rotation rate - average value at solar minimum



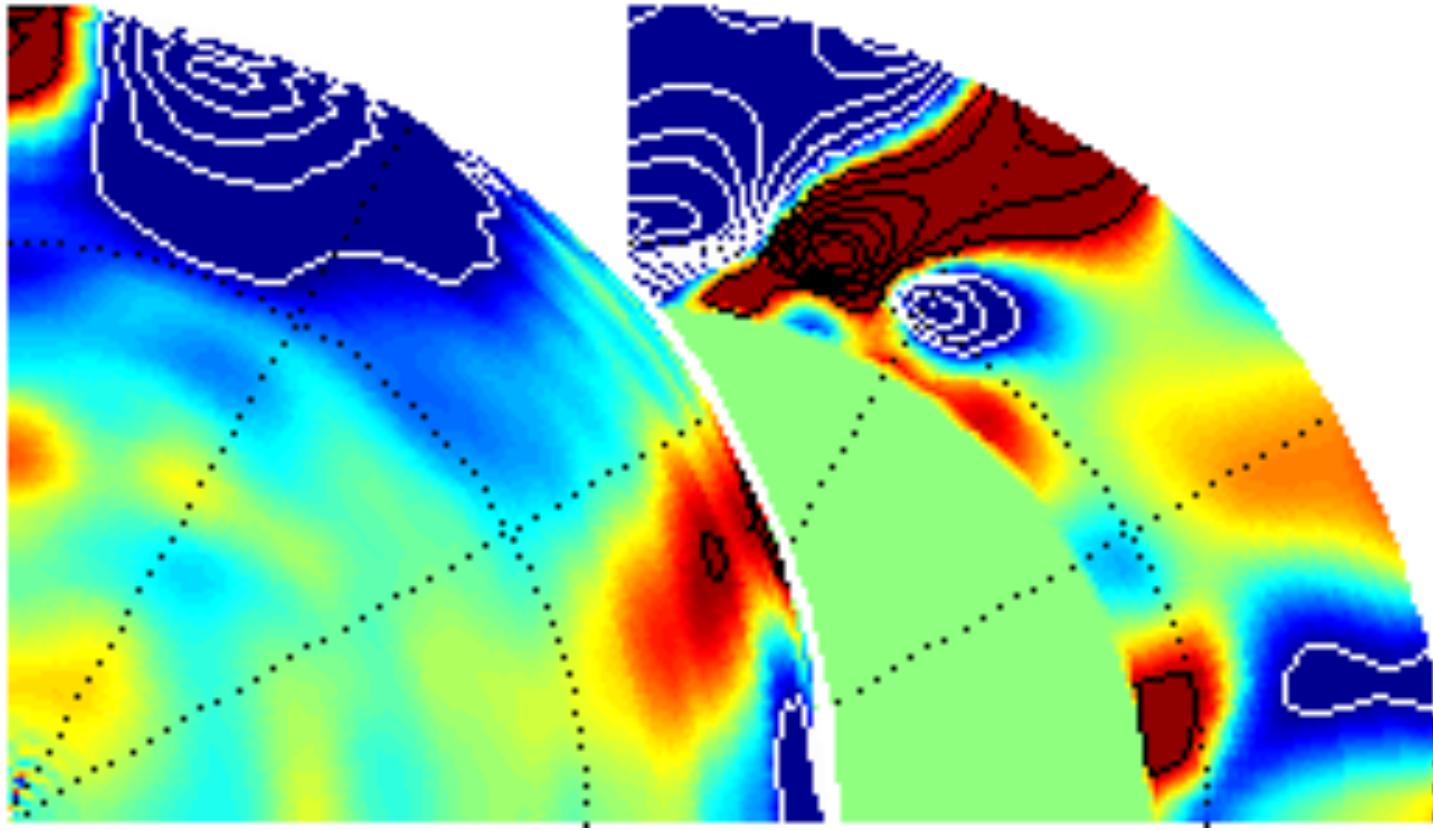
Vorontsov et al. (2002; Science 296, 101)

Radial development of zonal flows



Howe et al., (ApJ, in the press)

Observed and modelled dynamics



6 1/2 year MDI inversion,
enforcing 11-yr periodicity

Vorontsov et al. (2002;
Science 296, 101)

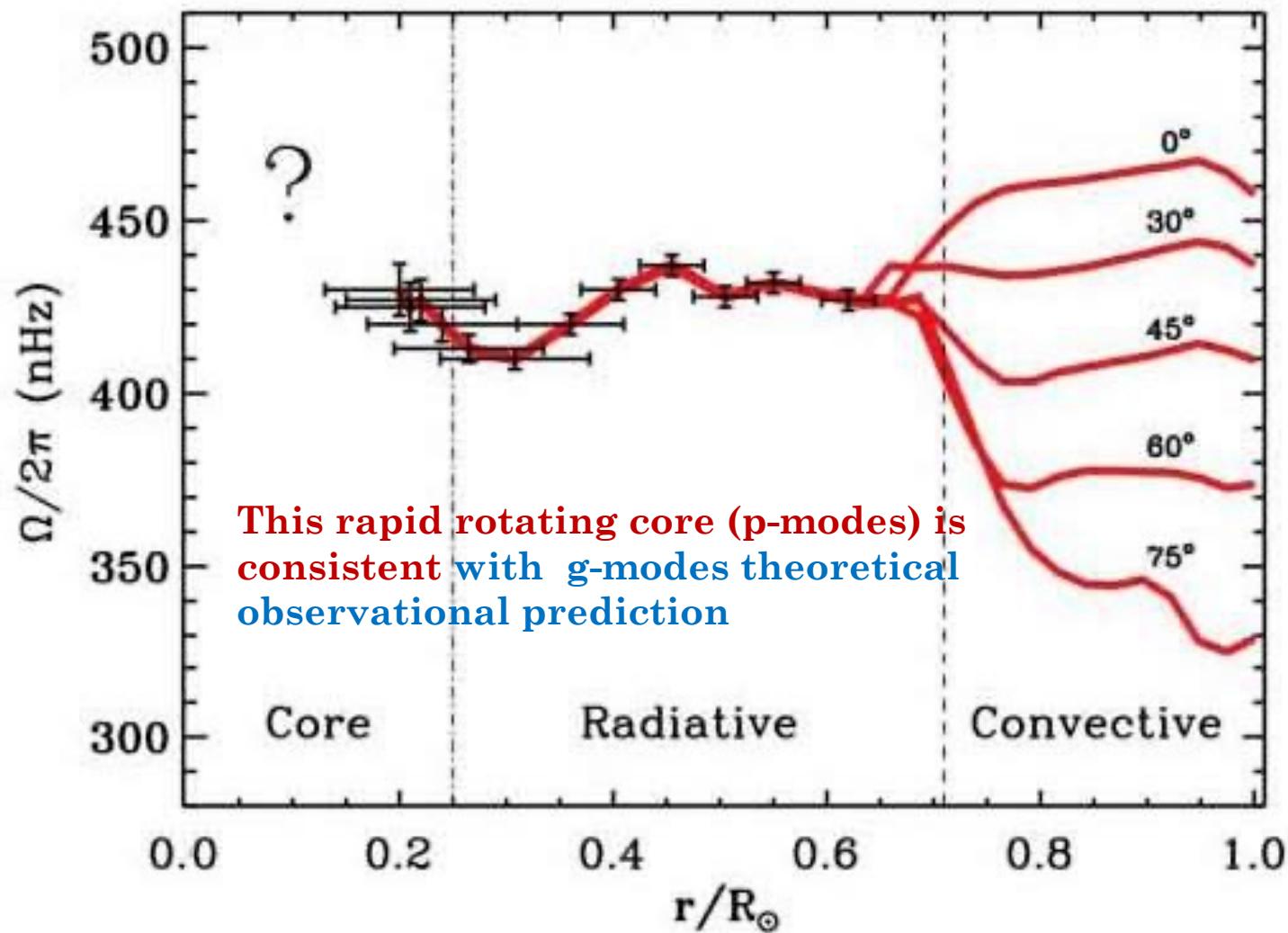
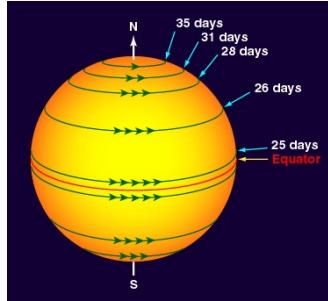
Non-linear mean-field solar dynamo
models

Covas, Tavakol and Moss (2001;
Astron. Astrophys 371, 718)

Probing the dynamics of the solar interior

Helioseismology shows the internal dynamics of the Sun:
differential rotation and tachocline

GOLF (2088 days~6 years) +MDI



Towards a better understanding of the origin and evolution of the solar magnetic cycle

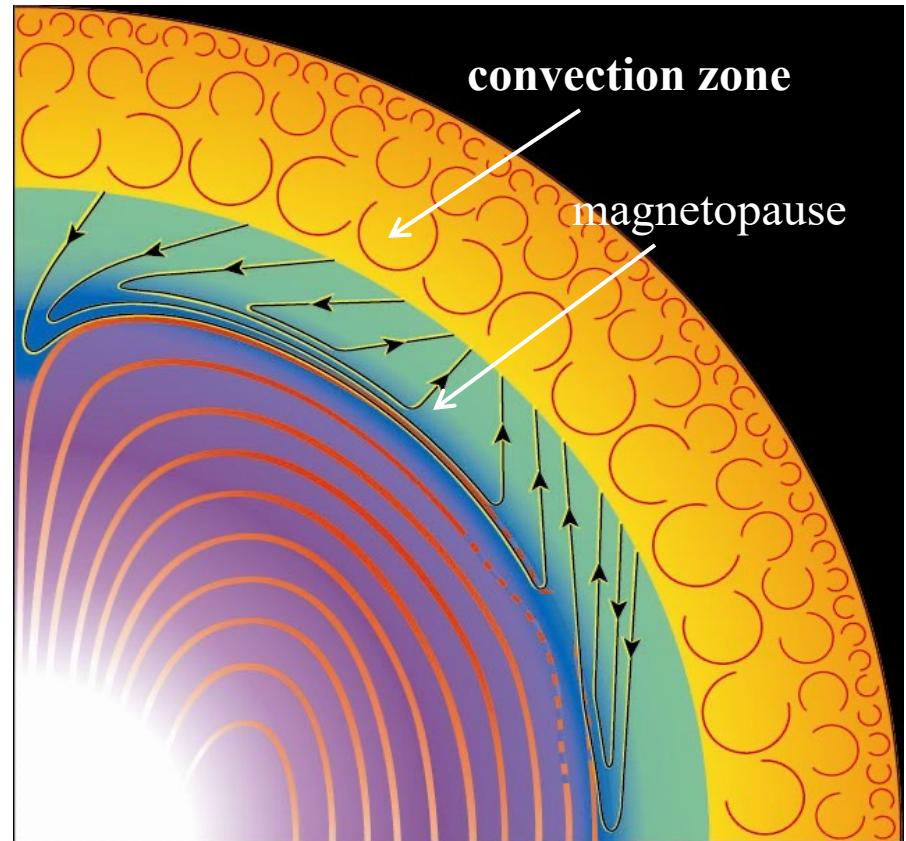
Gough, McIntyre (Nature 1998)

**Solar Dynamo (Seismology):
Differential rotation + Meridional
circulation
(near the surface)**

- Probing the existence and dimensions of the solar tachocline.
- Emerge a clear picture about the origin of the solar magnetic cycle.

Tachocline maintained by angular momentum transported by meridional circulation and Lorentz forces.

Shear responsible for production of strong toroidal field.

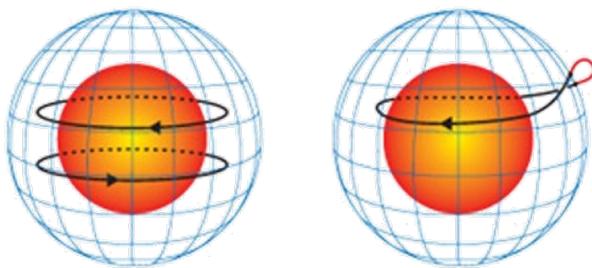


Helioseismology has shown that solar rotation is almost constant with radius in convection zone: thin stably stratified shear layer (tachocline) at the base.⁶⁴

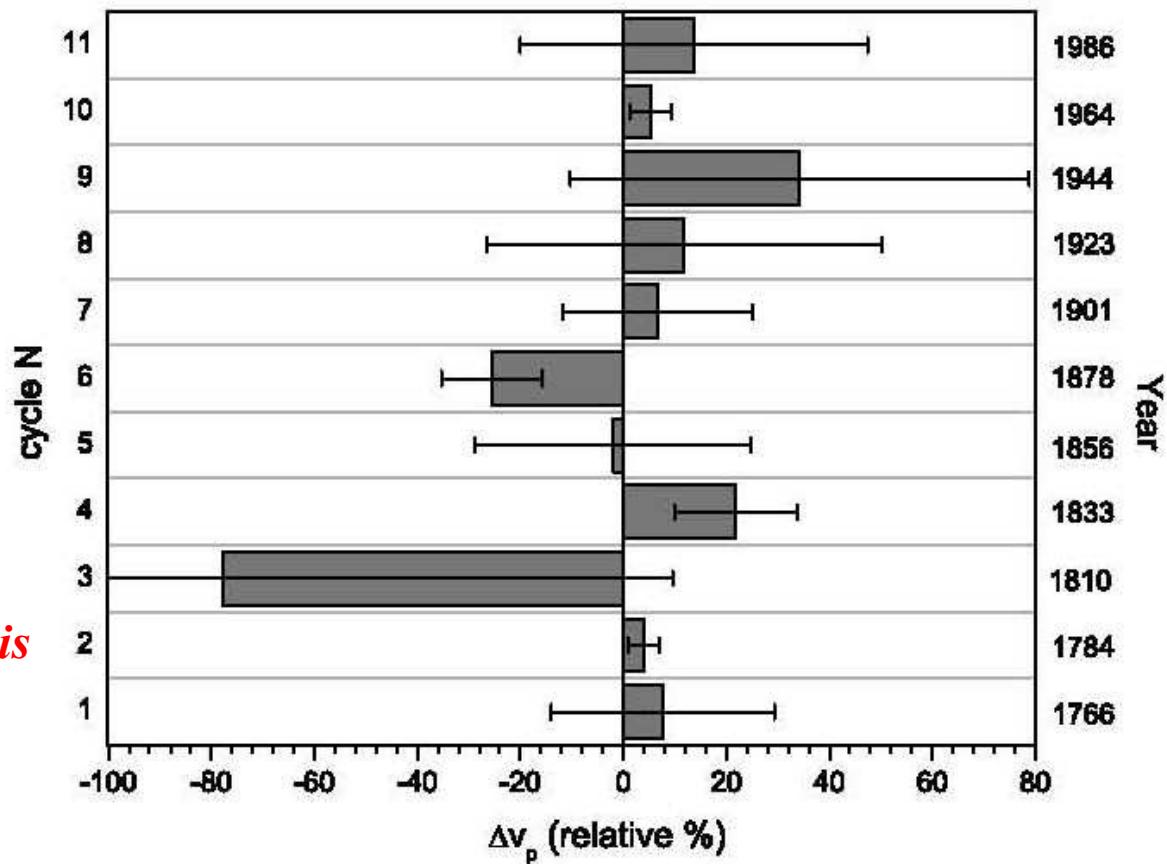
Solar Dynamo: Probing the evolution of the solar magnetic cycle throughout the centuries

- First attempt to infer/**invert** the mean velocity of meridional circulation throughout the last 3 centuries.

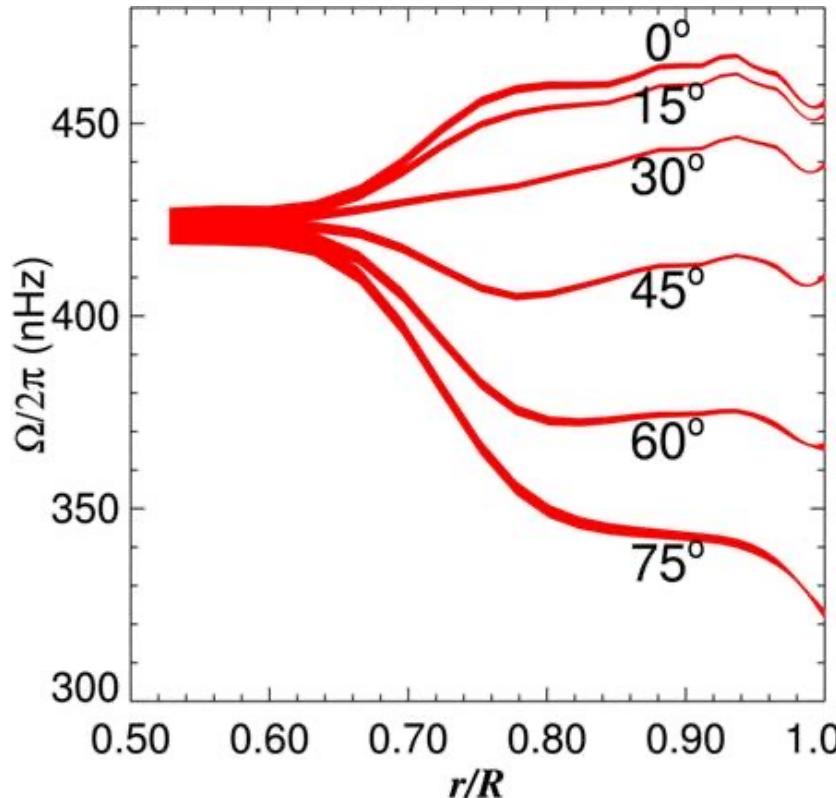
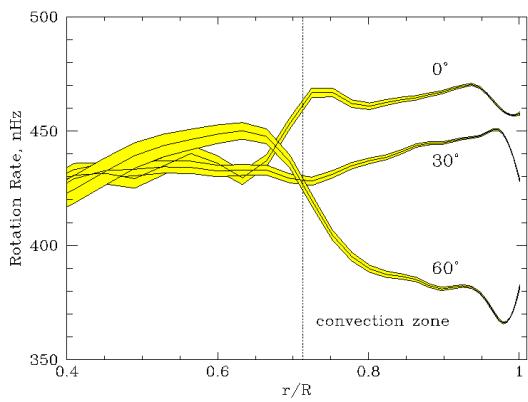
Passos & Lopes 2008



Dário Passos received a
Gulbenkian Award in 2007
'Earth and Space Science', for his
Ph.D. research project

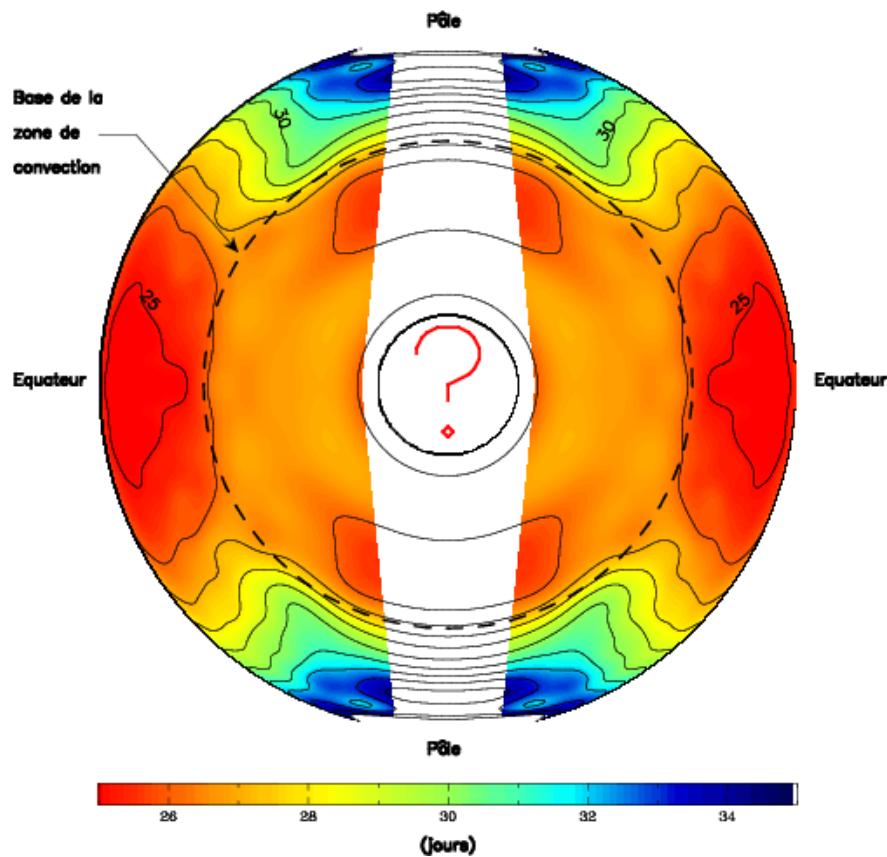


Helioseismology: differential rotation



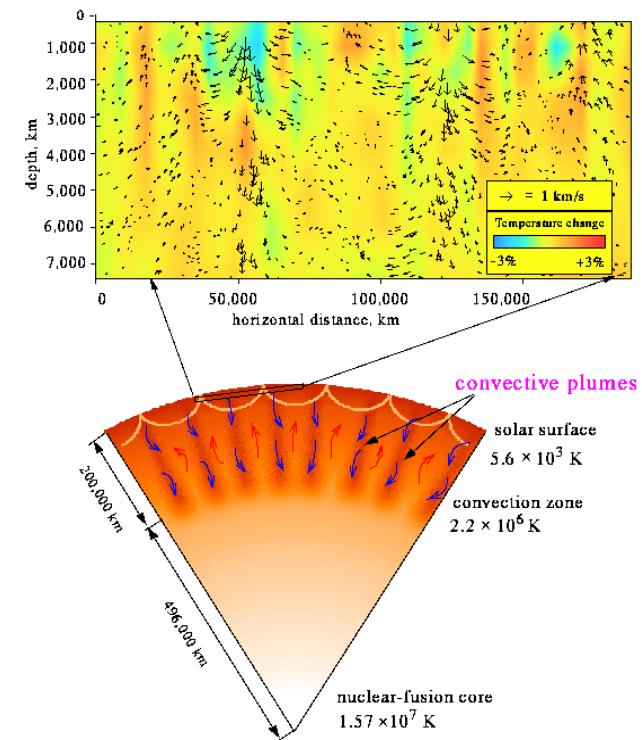
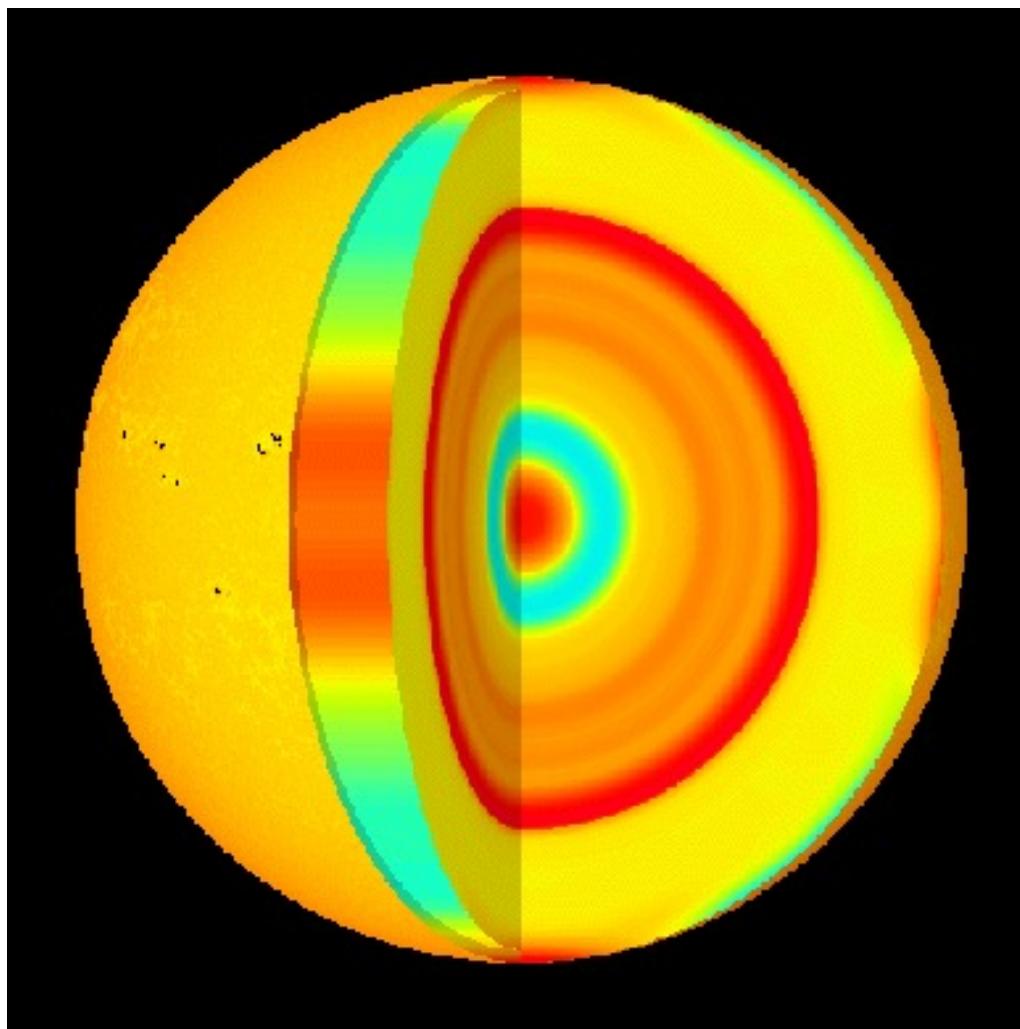
008, Stellar
rotation 1.66

ROTATION INTERNE DU SOLEIL



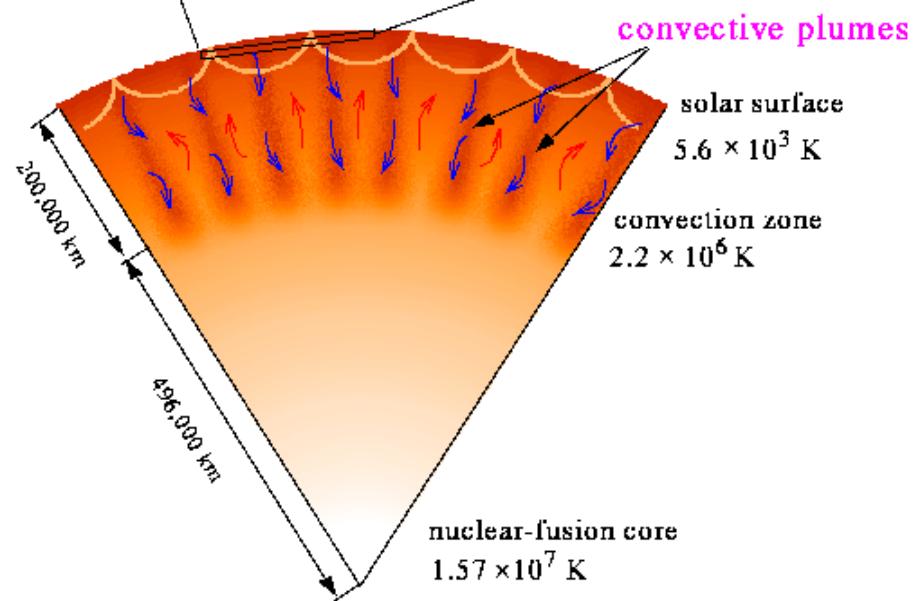
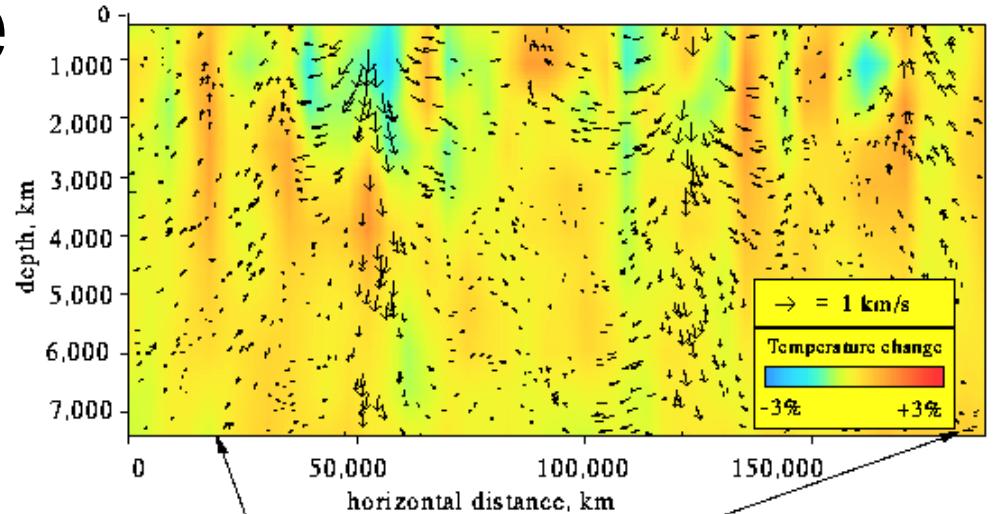
Time-distance

Internal Flows



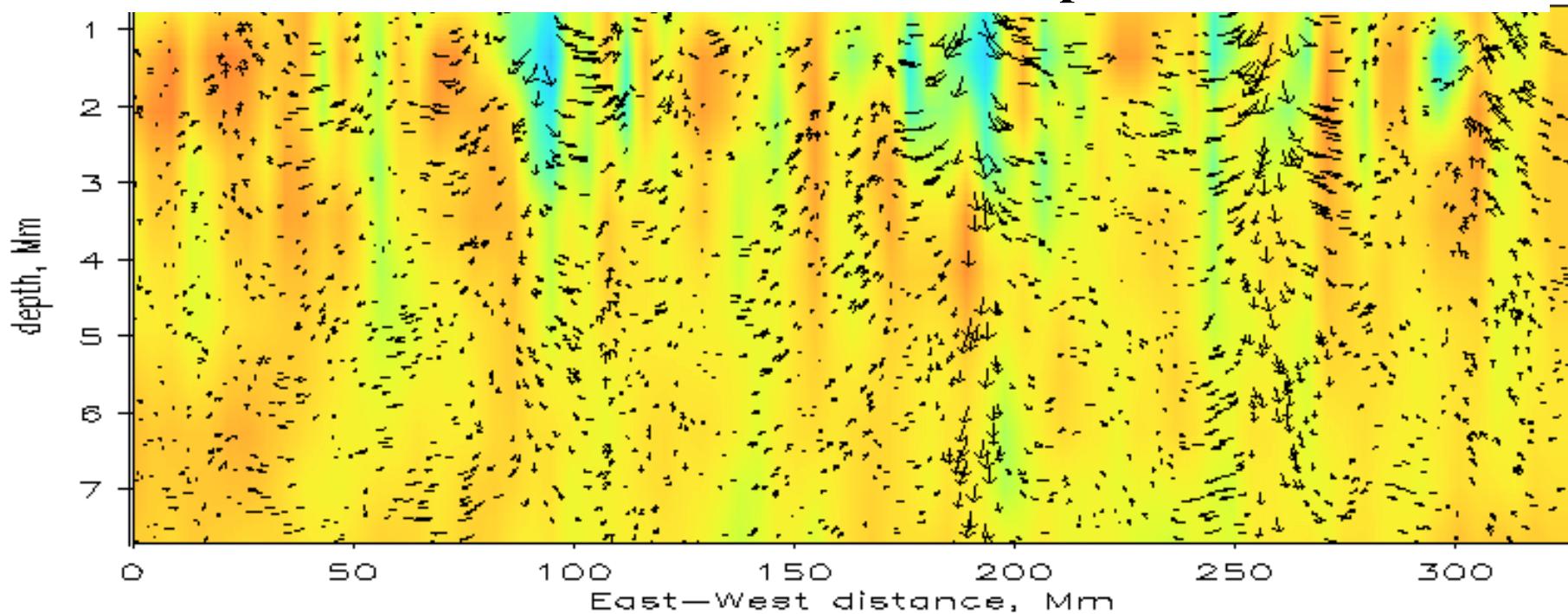
Convective Flows Below The Sun's Surface

Time-distance

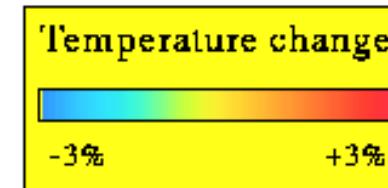


Tomografia Solar :

• Movimentos da conveccao Abaixo da Superfice do Sol

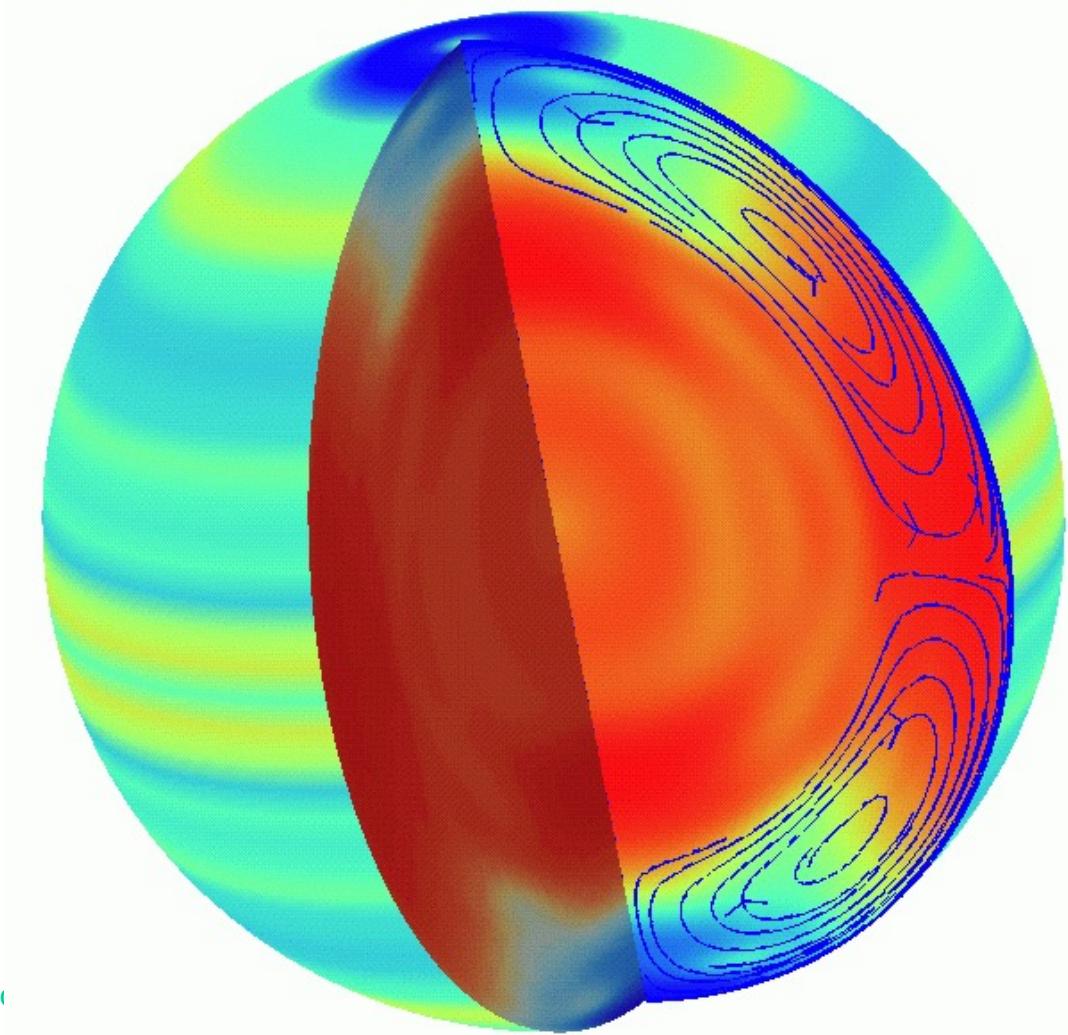


← = 1 km/s



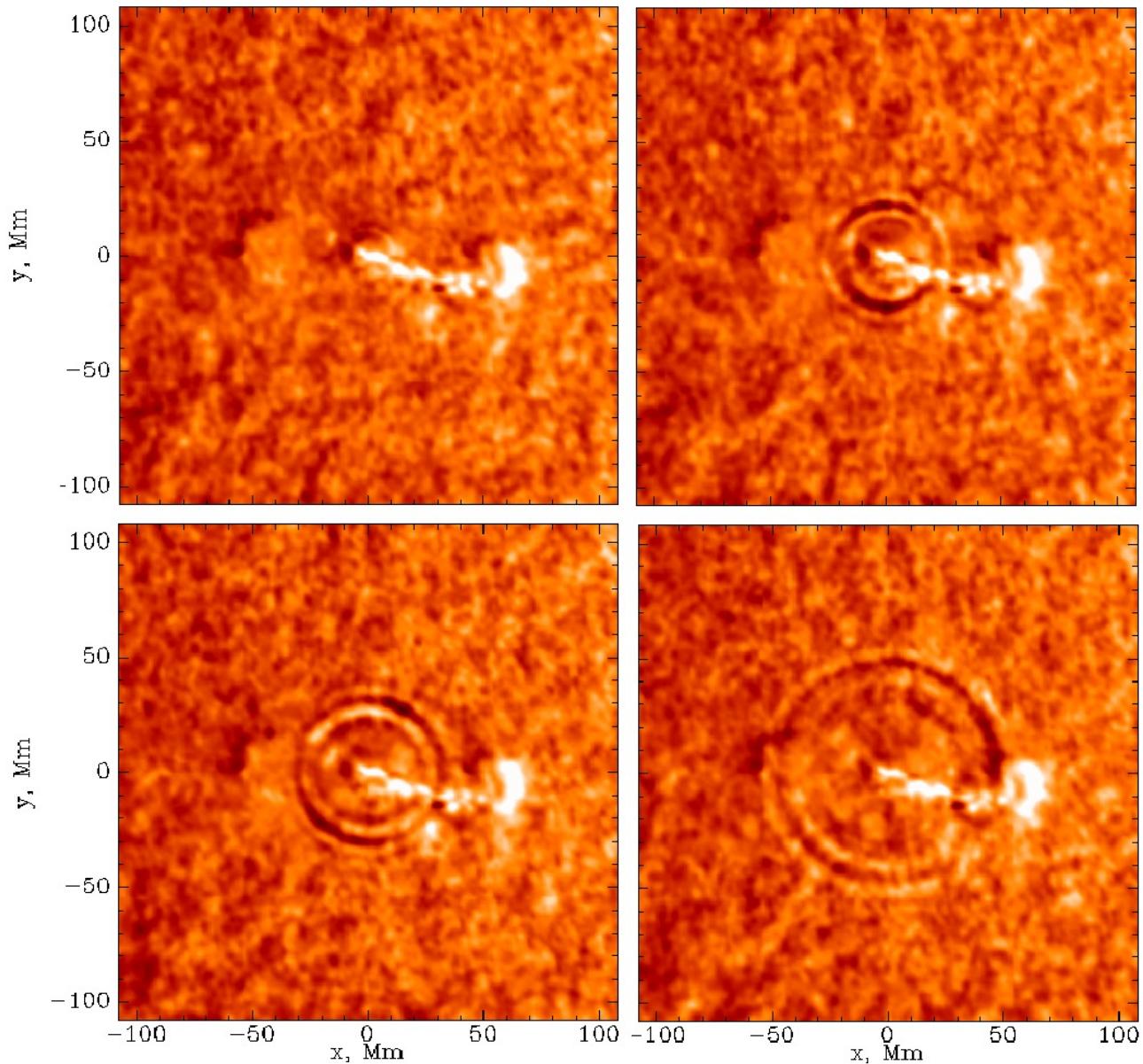
- Cote Vertical : 1% da parte exterior do Sol
- Variações da temperatura
- Campo de velocidade

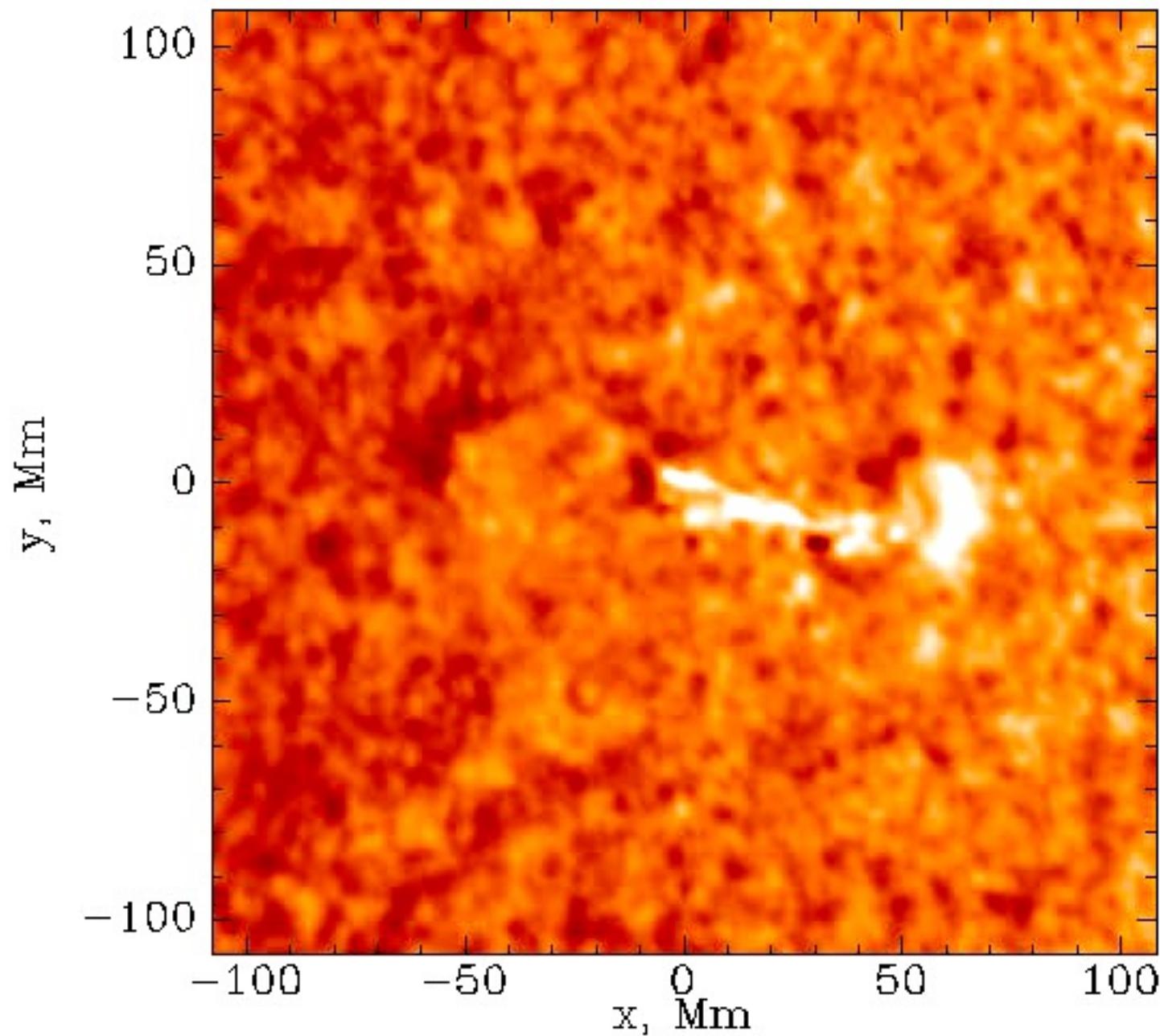
‘Tempo Solar’



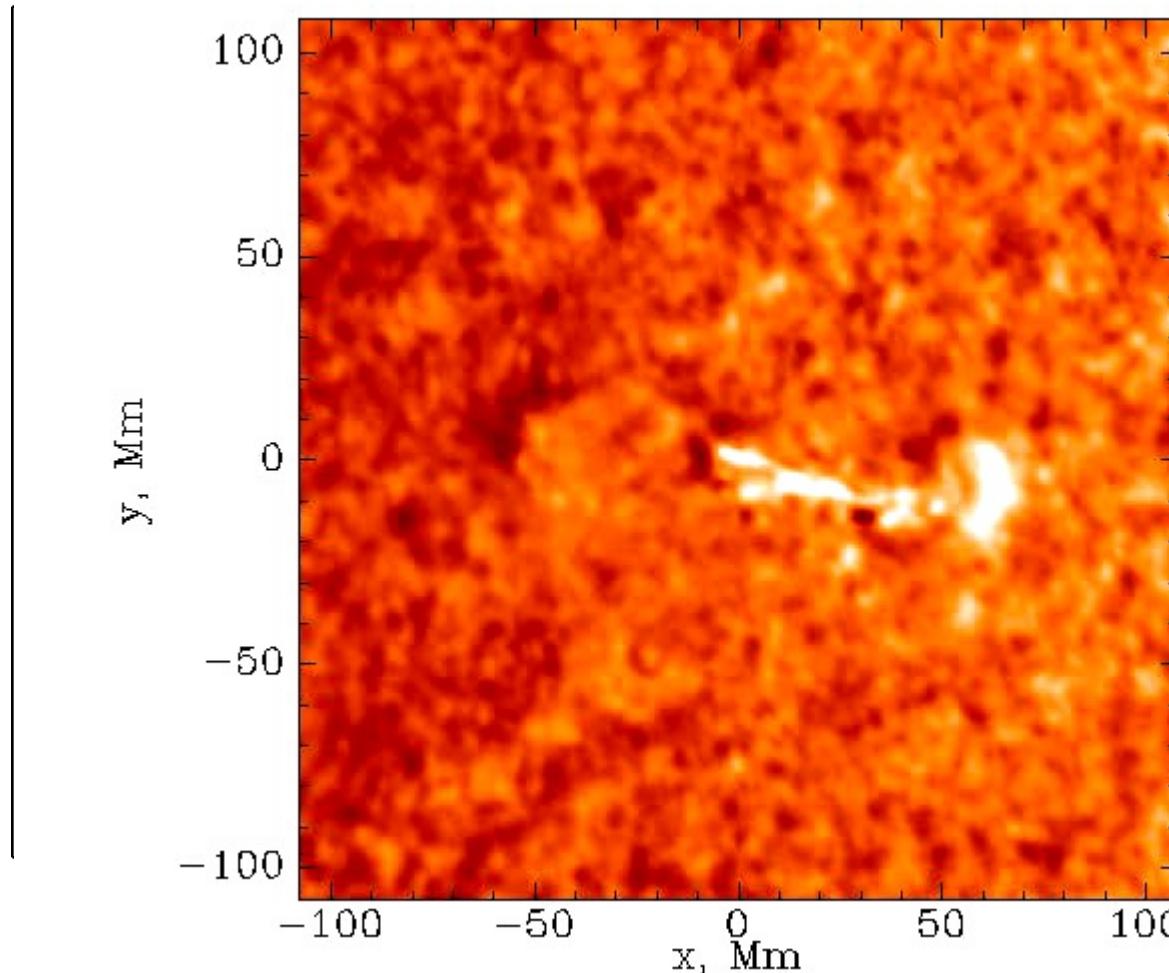
IIIc

Sismologia dos “Sunspots”





Sismologia dos “Sunspots”

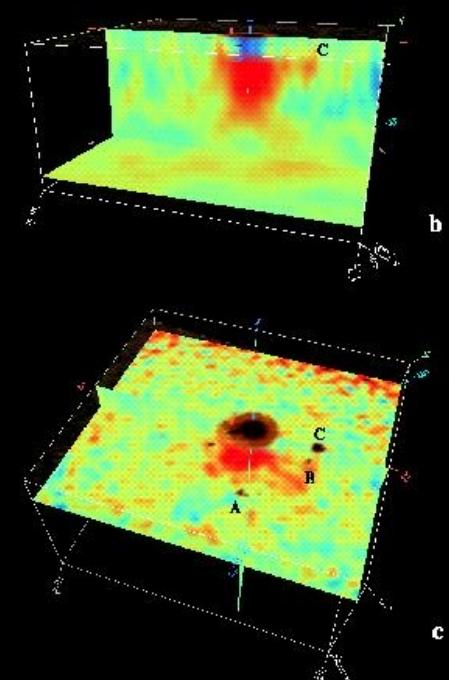
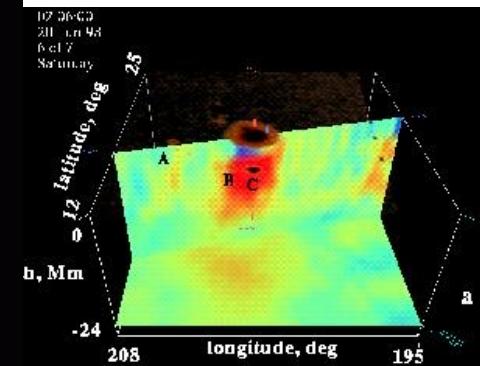
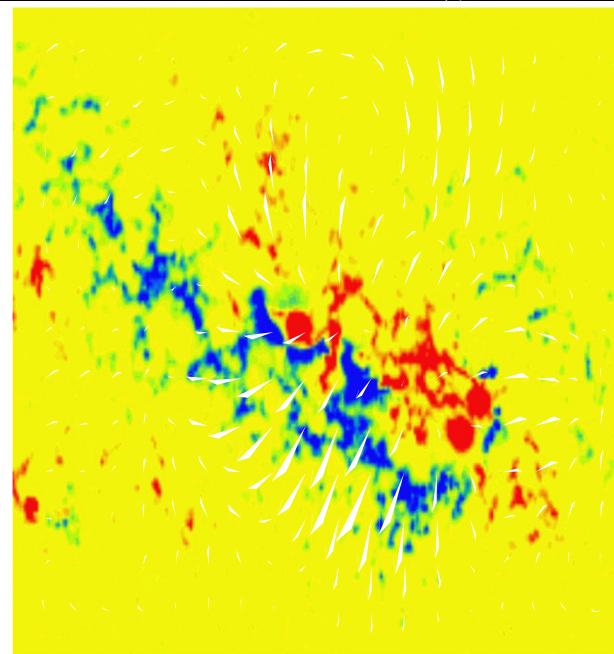
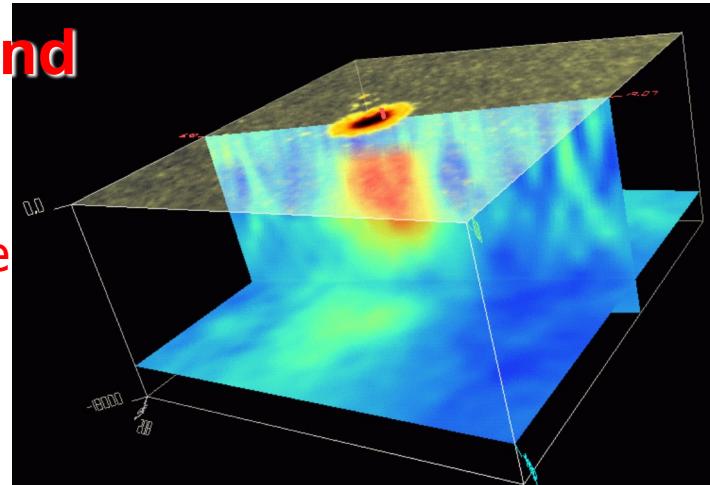
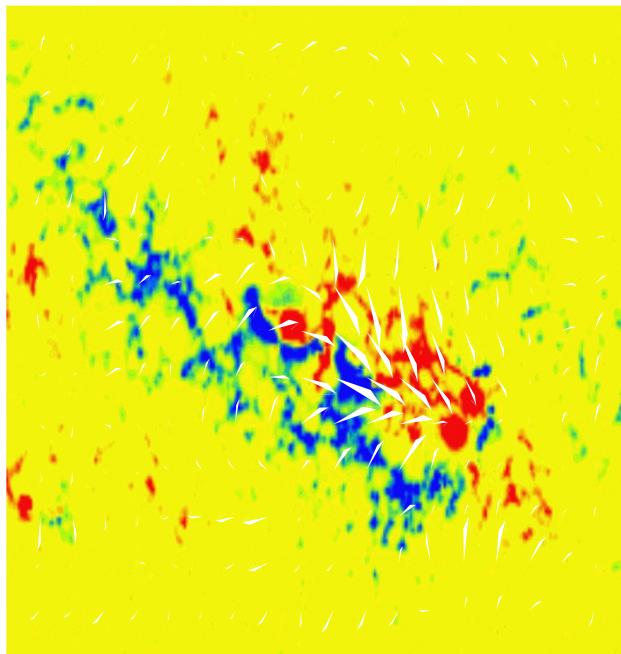


Sub-surface Structures

Structure of Sunspot

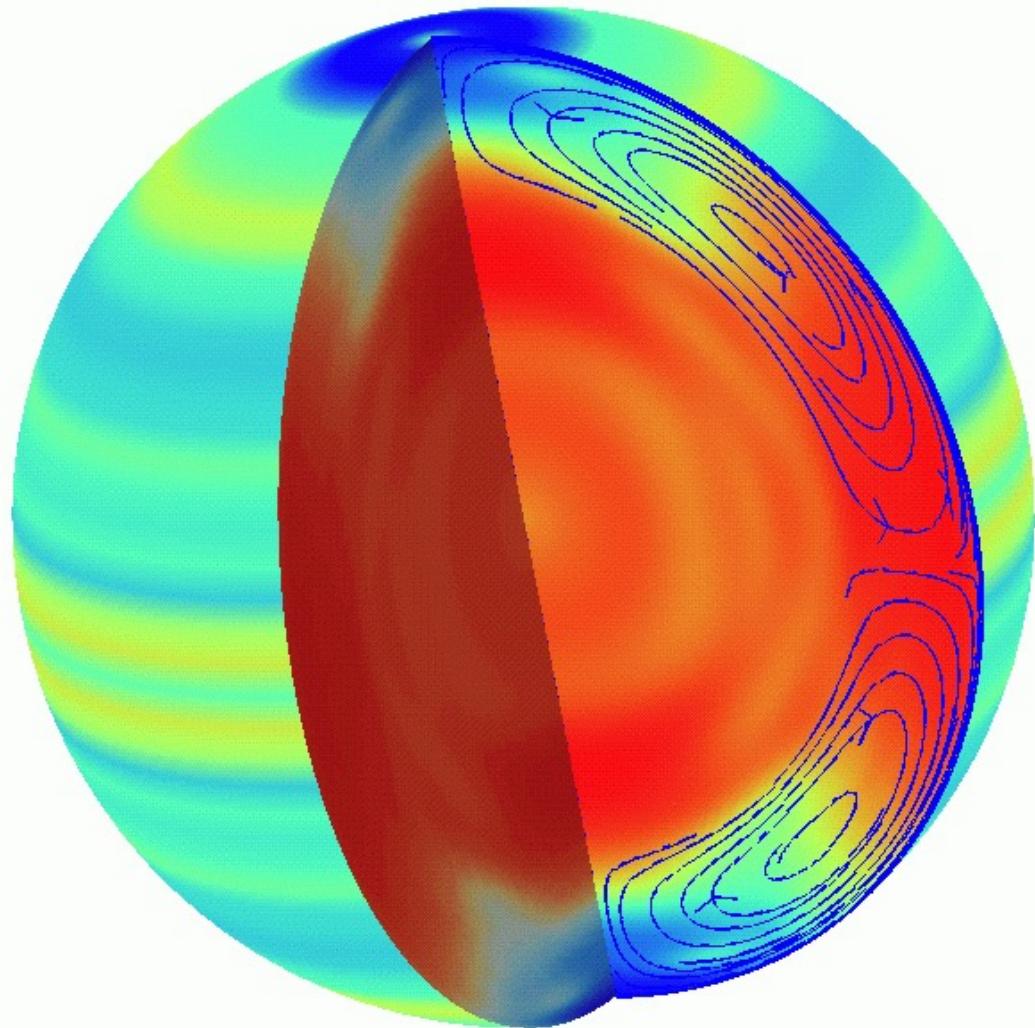
Large-scale flows around active regions:

- Converging 40 m/s flow toward the neutral line in the upper layers
- diverging flow below 9 Mm

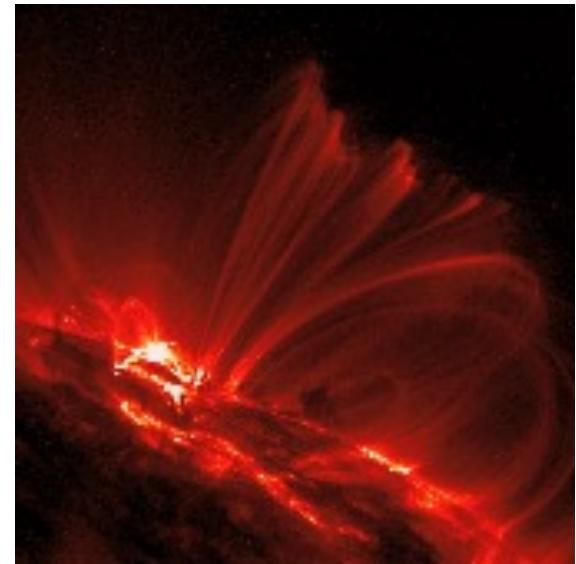
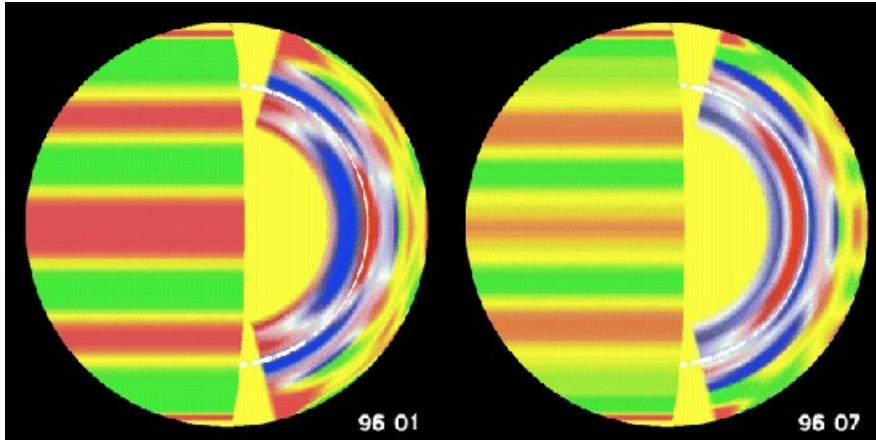


Solar rotation and polar flows of the Sun

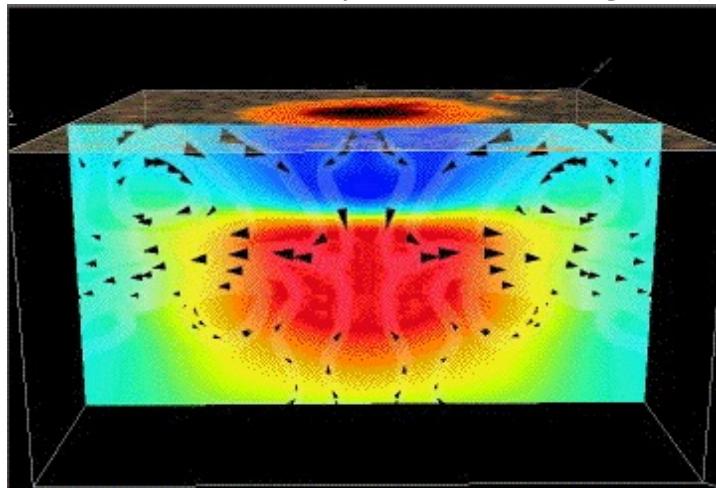
- Measurements by the **Michelson Doppler Imager (MDI)**. The left side of the image represents the difference in rotation speed between various areas on the Sun.
- Red-yellow is faster than average and blue is slower than average.
- The light orange bands are zones that are moving slightly faster than their surroundings.
- The new SOHO observations indicate that these extend down approximately 20,000 km into the Sun. Sunspots, caused by disturbances in the solar magnetic field, tend to form at the edge of these bands.
- The **cutaway reveals** rotation speed inside the Sun. The large dark red band is a **massive fast flow of hot**, electrically charged gas called plasma beneath the solar equator. Additionally, a newly discovered, but much more subtle, **plasma stream+** can be seen in the **cutaway at the poles**. They are the **light blue areas** embedded in the **slower moving dark blue regions**. Finally, the **blue lines** in the **cutaway** at the right represent the **surface flow** from the **equator to the poles** of the Sun which, as SOHO observations have revealed for the first time, extends to a depth of at least 26,000 km (**4% of the solar radius**), so that it is likely to be an important factor in solar dynamics, although the **flow speed (10-20 m/s)** is small compared to random motions at the surface (1 km/s). The return flow indicated at the bottom of the convection zone is from a simple model and has not been observed yet.



MDI/SOHO reveals the interior and explains surface activity



MDI shows how the dynamo changes (1.3y)



Sunspots are footpoints of emerging magnetic flux tubes

MDI shows how magnetic elements form sunspots

Lecture 3

END

Theory of Stellar Oscillations and Planetary Formation

Lecture 4 Astroseismology

Ilídio Lopes
[\(ilidio.lopes@ist.utl.pt\)](mailto:(ilidio.lopes@ist.utl.pt))

Physics of pulsations : Basics

Any **perturbation (oscillation) of equilibrium structure** (air in a tube, equilibrium structure inside a star) can be expressed as a finite sum of harmonic functions.

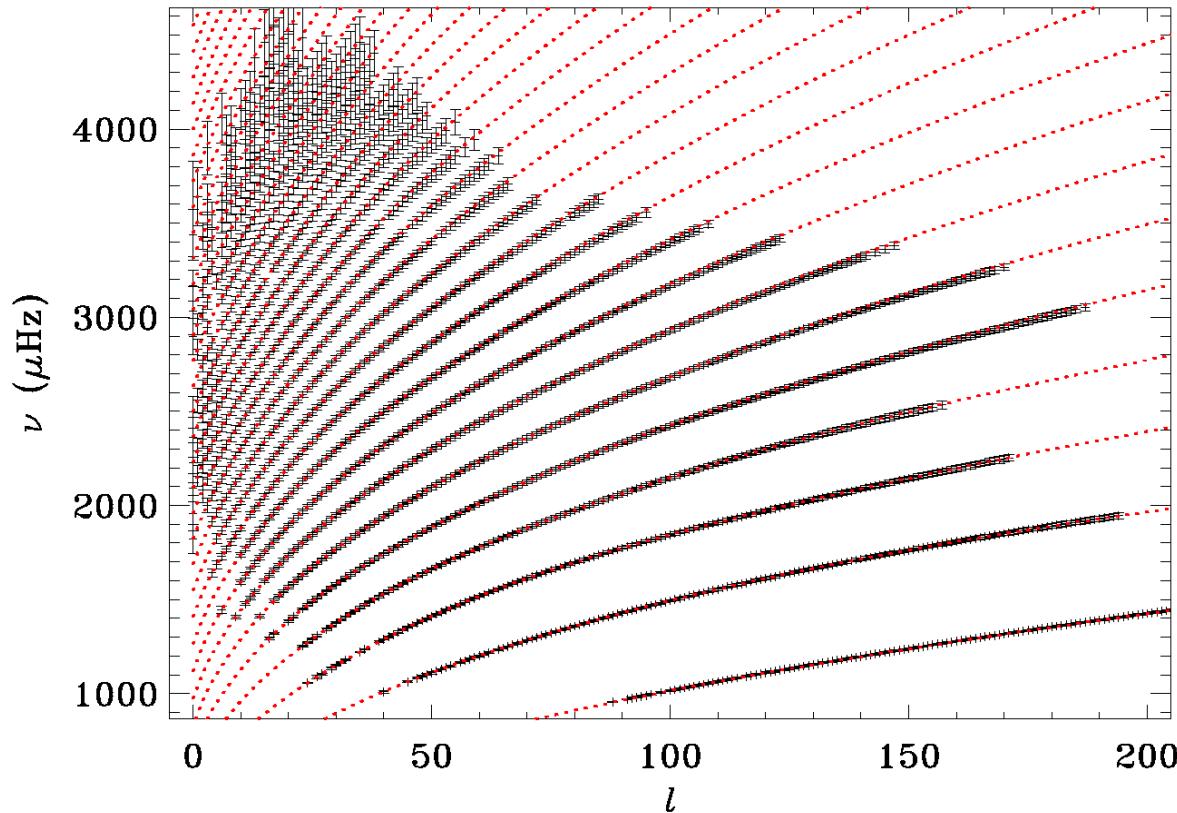
A generic perturbation (in space and time):

$$\psi(x, t) \propto \sum_n \sin(k_n x) \cos(\omega_n t)$$

$$\psi(r, \theta, \phi, t) \propto \sum_{n,l,m} r^{-1} \Psi_{r,n}(r) Y_l^m(\theta, \phi) e^{-i\omega_{n,l,m} t}$$

Therefore, the **oscillation spectrum** characterizes the perturbations of the equilibrium structure of the star.

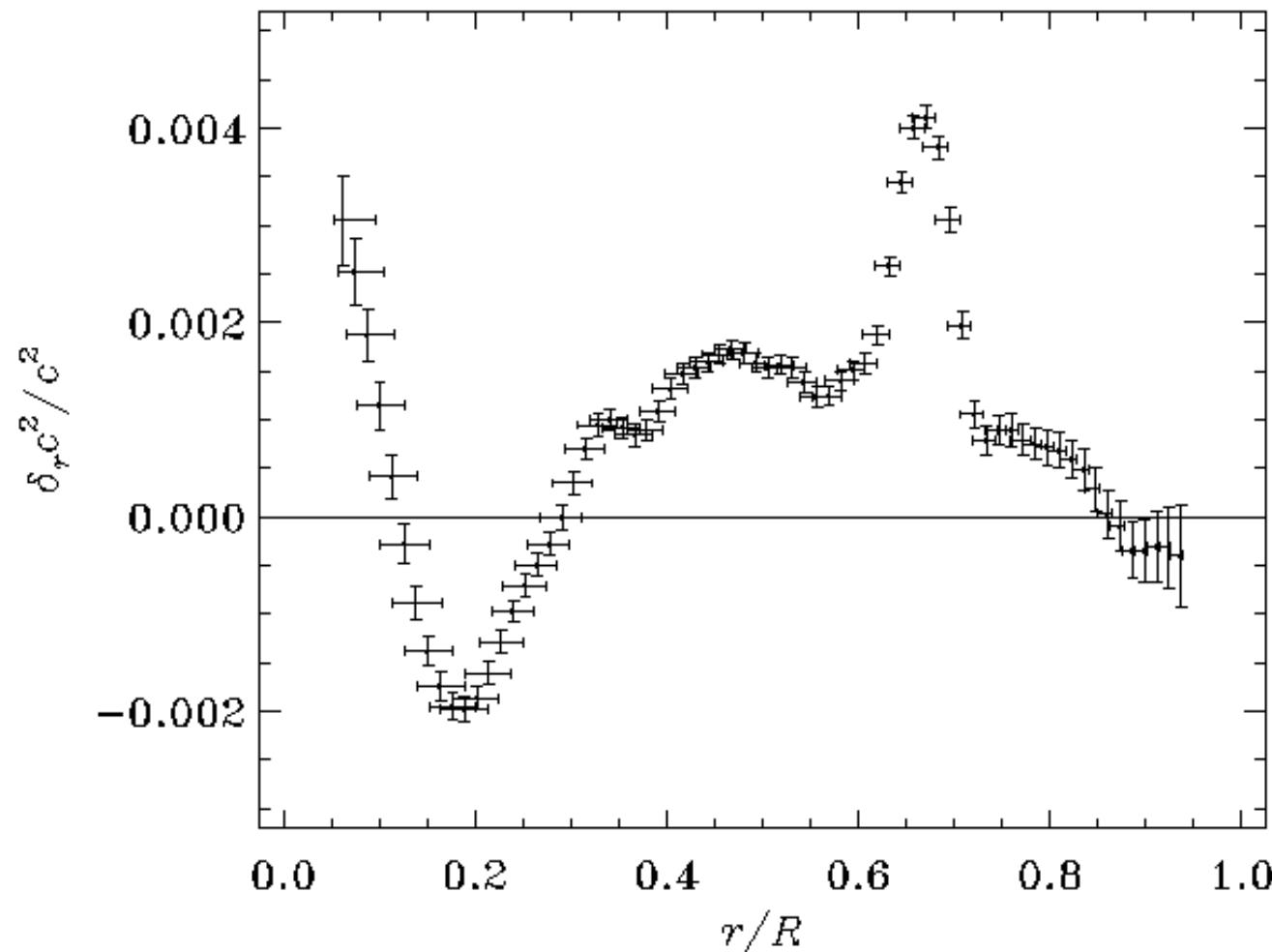
Observed frequencies



m-averaged frequencies from MDI instrument on SOHO
1000 σ error bars

The solar internal sound speed

Sun - model

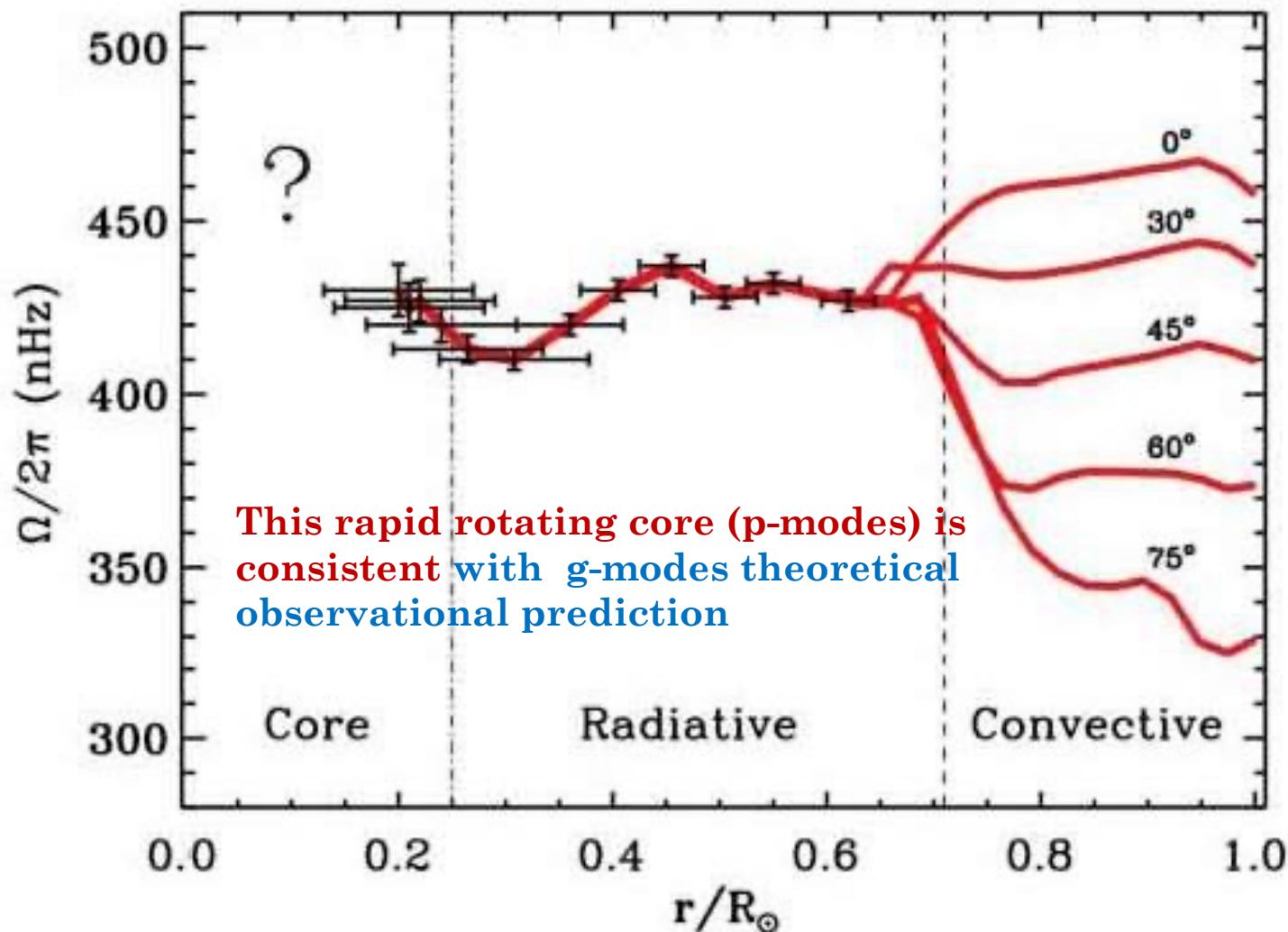
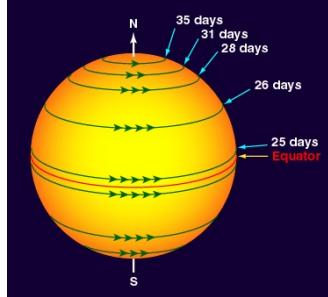


Basu et al. (1997; MNRAS 292, 243)

Probing the dynamics of the solar interior

Helioseismology shows the internal dynamics of the Sun: differential rotation and tachocline

GOLF (2088 days~6 years) +MDI



OBSERVING THE SUN FROM SPACE: GOLF/SOHO ACOUSTIC MODES

Lazrek et al. (1997)

FIRST RESULTS ON P-MODES FROM GOLF EXPERIMENT

9

I. Lopes Co-I GOLF Team

The p-mode Fourier spectrum from GOLF, using a 690-day time series of calibrated velocity signal, which exhibits an excellent signal to noise ratio.

The low-frequency range of the P-modes from above spectrum, showing low-n order modes.

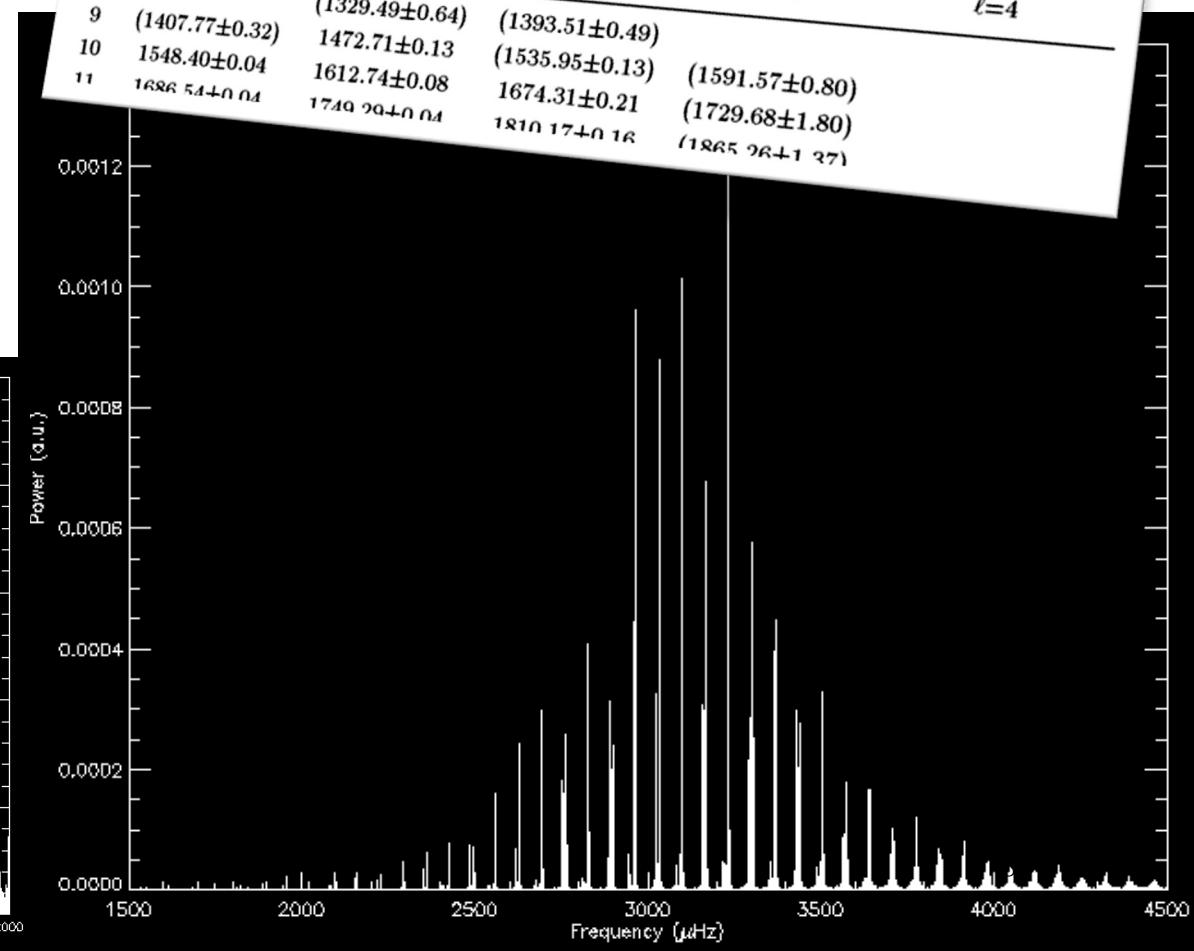
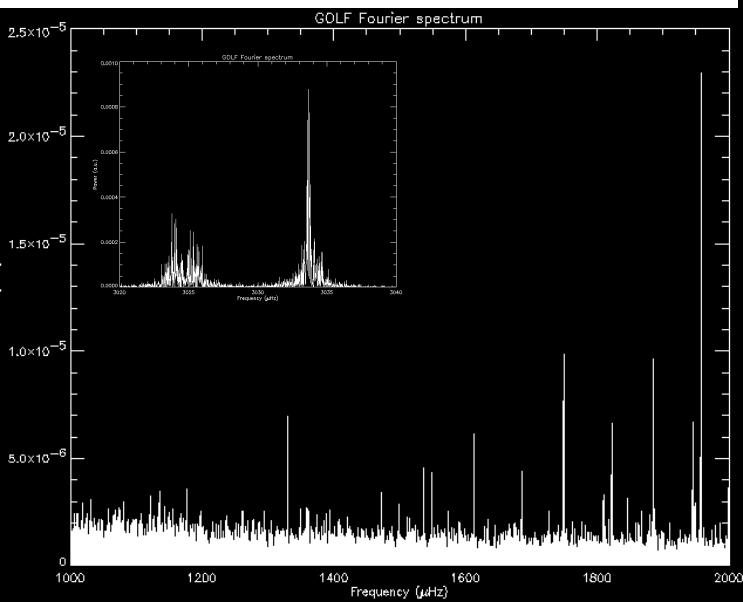
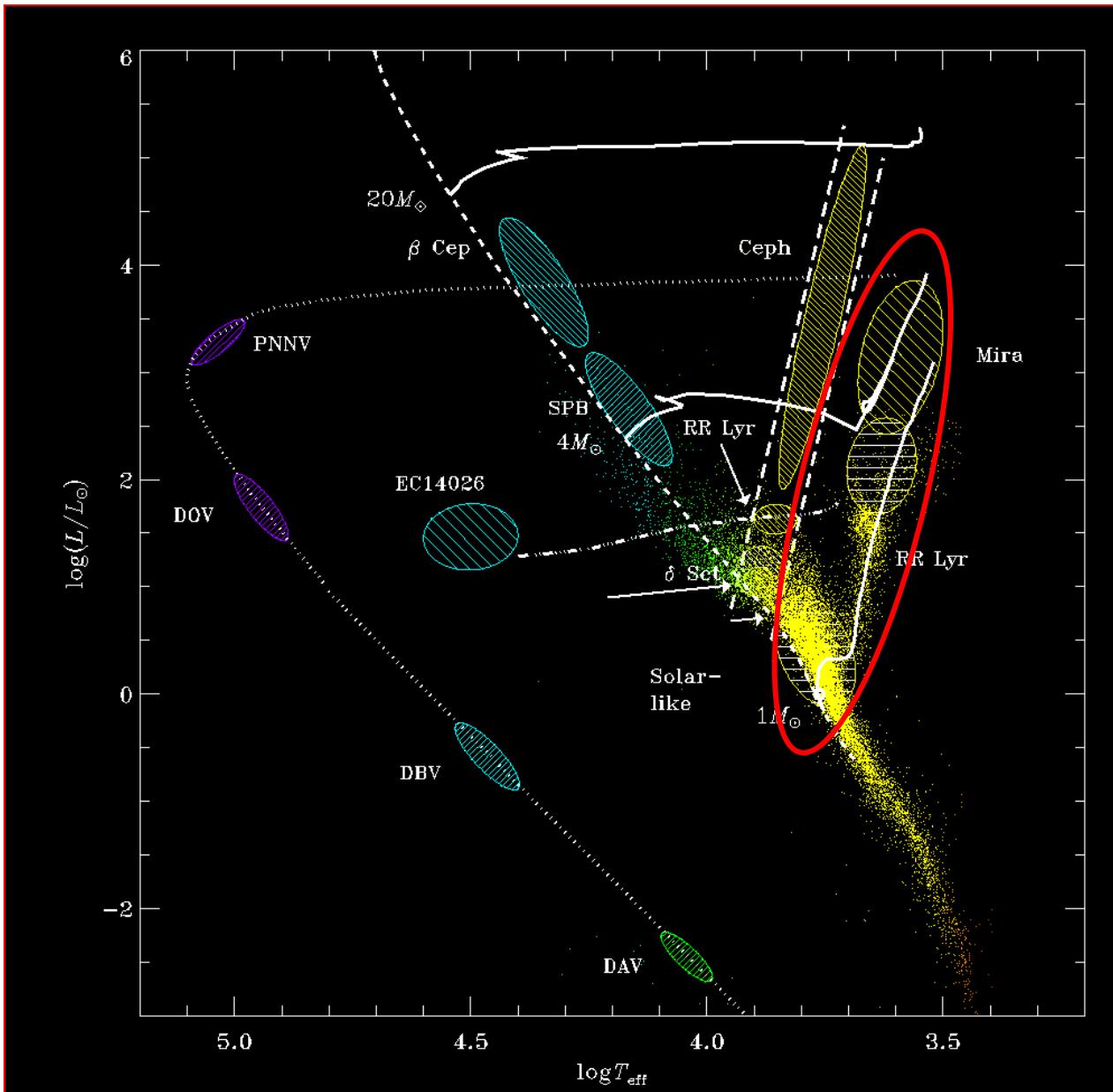


TABLE I
Frequencies and errors, in μHz , as measured by GOLF instrument. Bracketted frequencies are possible identifications with $S/N < 1$ (see text).

n	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
8					
9	(1407.77±0.32)	(1329.49±0.64)	(1393.51±0.49)		
10	1548.40±0.04	1472.71±0.13	(1535.95±0.13)	(1591.57±0.80)	
11	1686.54±0.04	1612.74±0.08	1674.31±0.21	(1729.68±1.80)	(1865.96±1.37)

Solarlike pulsators



Characteristics of solar-like oscillations

- Intrinsically damped by the effects of convection
- Stochastically excited by the effects of convection
- **Typically very small amplitudes (20 cm/sec or 5 ppm for main-sequence stars)**

Found in stars that are not very solar-like

Mode damping

Damping rate: amplitude / $\exp(\eta t)$, with

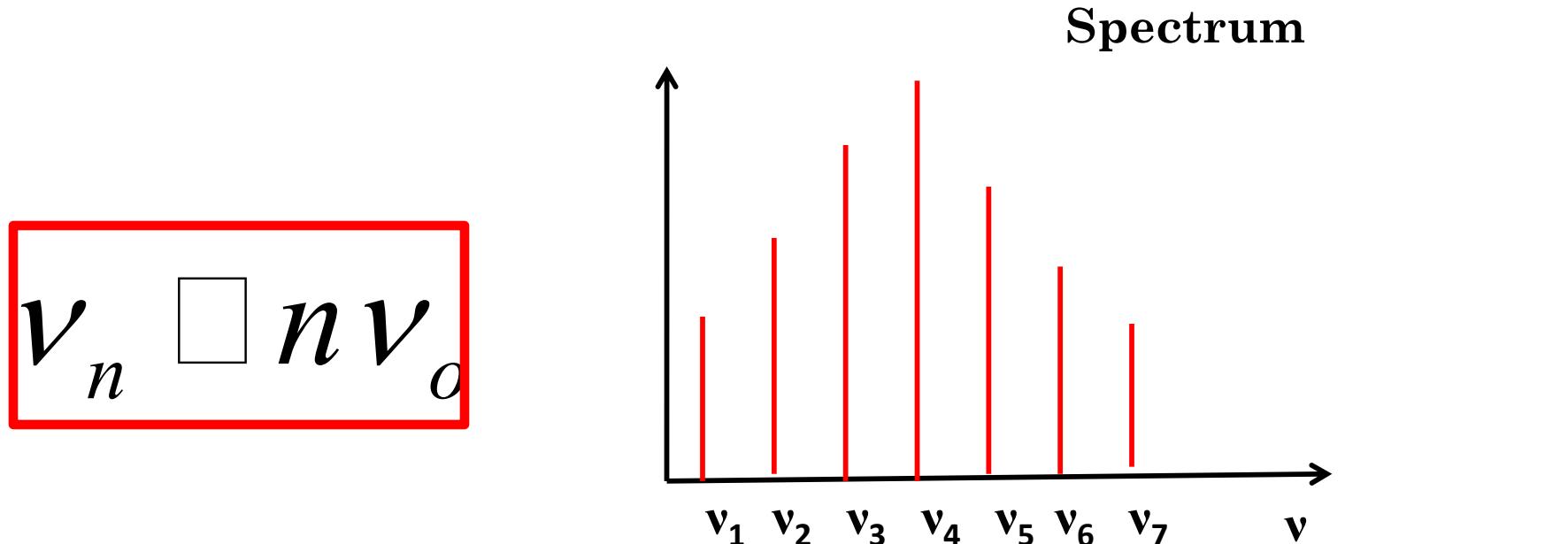
$$\begin{aligned}\eta \simeq & -\frac{1}{2\omega} \frac{\text{Im} \left[\int_V \frac{\delta\rho^*}{\rho} \delta p_t dV \right]}{\int_V \rho |\boldsymbol{\delta r}|^2 dV} \\ & + \frac{1}{2\omega^2} \frac{\text{Re} \left[\int_V \frac{\delta\rho^*}{\rho} (\Gamma_3 - 1) \delta(\rho\epsilon - \text{div F}) dV \right]}{\int_V \rho |\boldsymbol{\delta r}|^2 dV}.\end{aligned}$$

Perturbation in convective flux and effect of turbulent pressure perturbation δp_t appear to yield negative η , i.e., damped modes

Astroseismology: Pulsating staRs

Physics of pulsations : Basics

Stellar Seismology : open tube (flute)



$$\nu_o \square \frac{c}{2L}$$

$$\Delta\nu_n \square \nu_n - \nu_{n-1} \square \nu_o$$

$n \square 1, 2, 3, 4, \dots$

Large separation

Physics of pulsations : Stars

p - mode eigenfrequency (low l and $l \ll n$):

$$\nu_{l,n} = \left(n + \frac{1}{2} l \varepsilon \right) \nu_o \sqrt{A l(l+1) - B \frac{\nu_o^2}{\nu_{l,n}}} \dots \quad (\text{Tassoul 1980})$$

Large separation

$$\Delta \nu_{l,n} = \nu_{l,n} - \nu_{l,n-1} = \nu_o$$

sensitive to sound speed
in the surface

$$\nu_o = \left(2 \int_0^R \frac{dr}{c} \right)^{-1} \quad c = \text{const.} \quad \nu_o = \frac{c}{2R}$$

Small separation

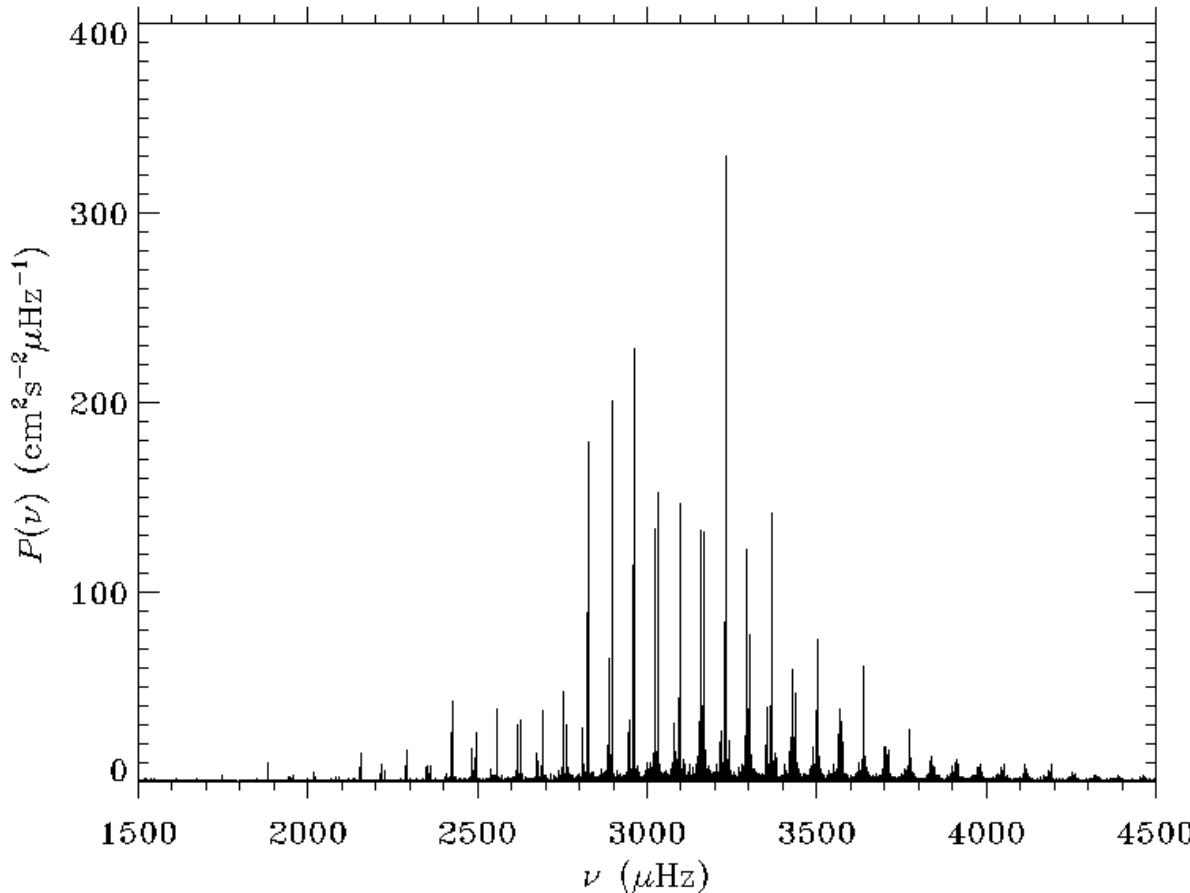
$$\delta \nu_{l,n} = \nu_{l,n} - \nu_{l+2,n-1} \approx \frac{6 \nu_o^2}{(n + l/2) \varepsilon} A$$

sensitive to sound speed
gradients in the interior

$$A = \frac{1}{2\pi\omega_o} \left[\frac{c(R)}{R} - \int_0^R \frac{1}{2} \frac{dc}{dr} dr \right]$$

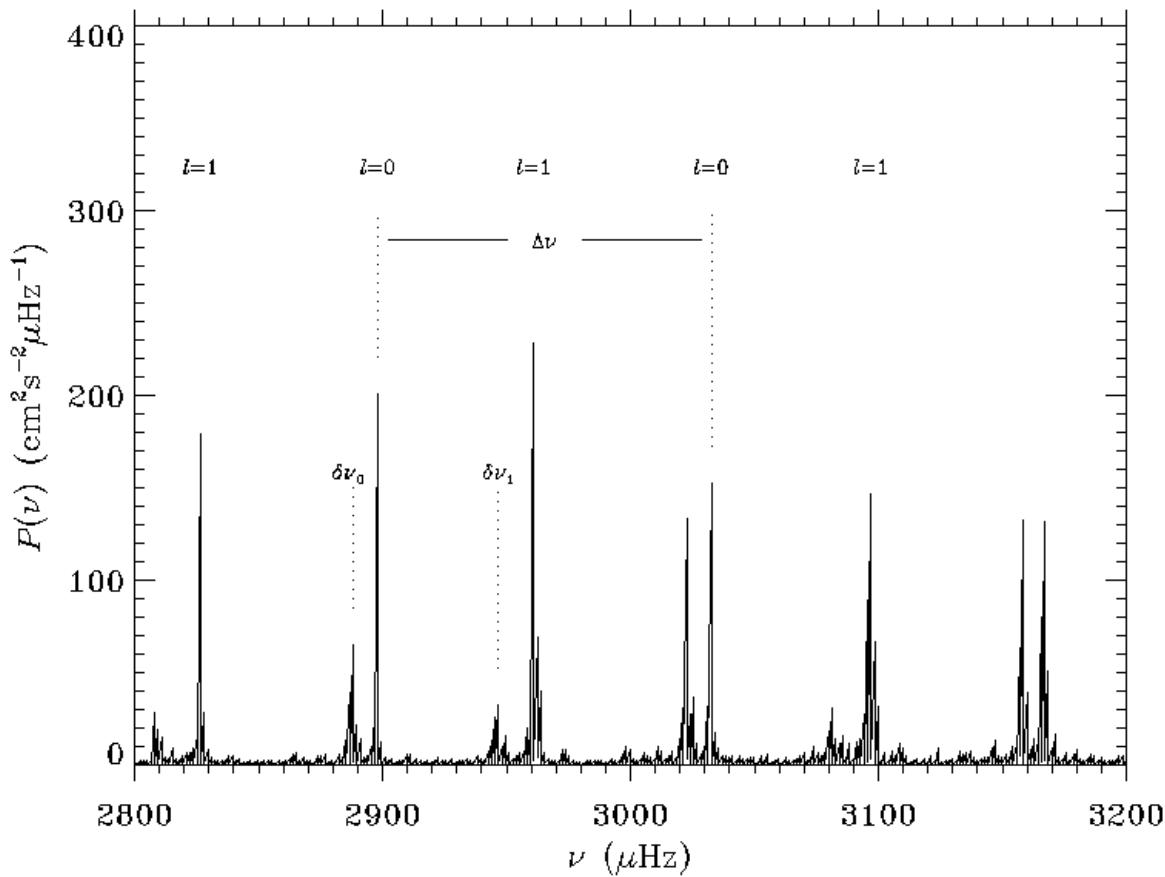
Asymptotics of low-degree p modes

Low-degree modes: $\nu_{nl} \sim \Delta\nu \left(n + \frac{l}{2} + \alpha \right) + \epsilon_{nl}$



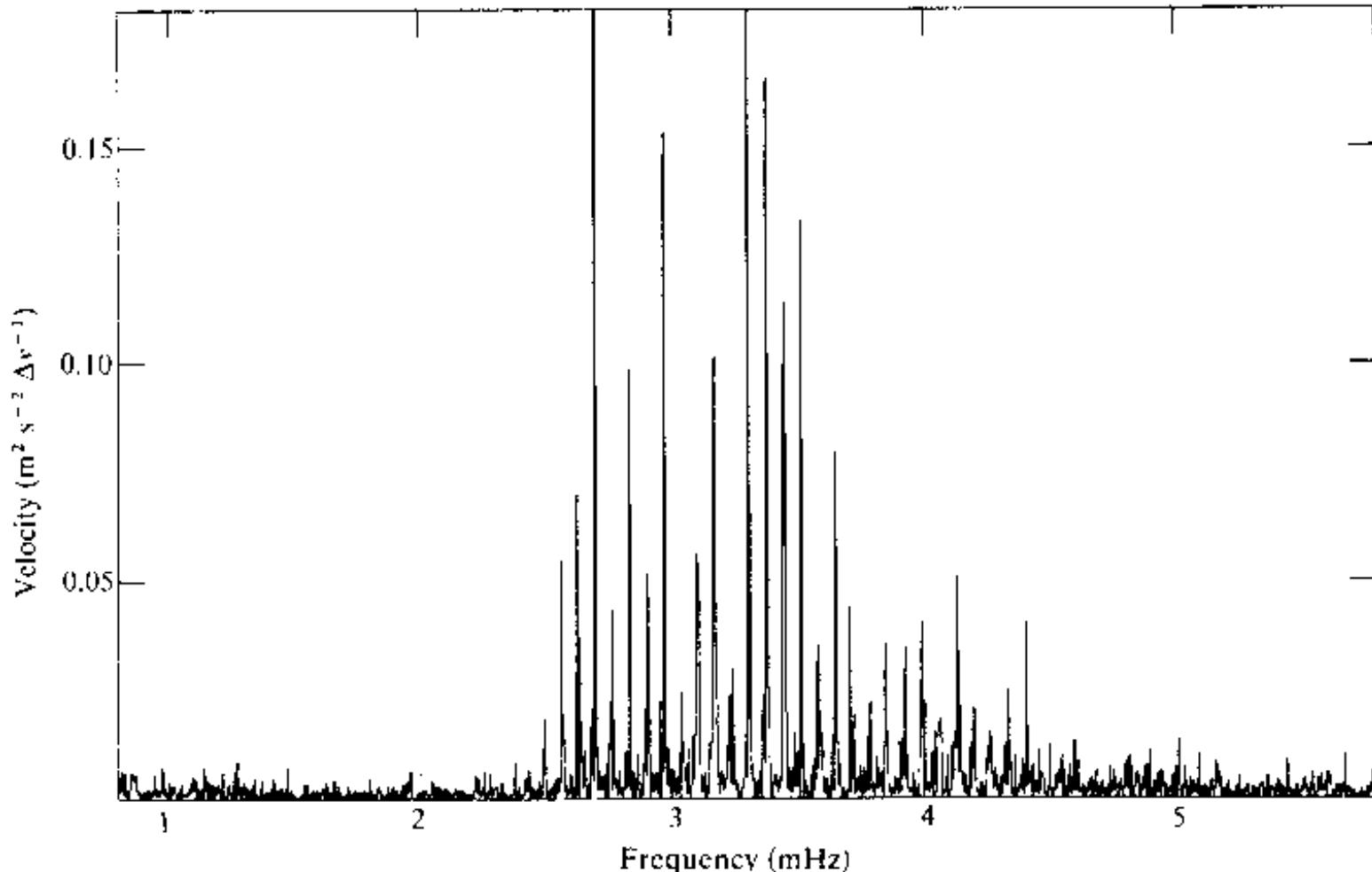
$$\Delta\nu_{nl} = \nu_{nl} - \nu_{n-1,l} \simeq \Delta\nu$$

Small frequency separations

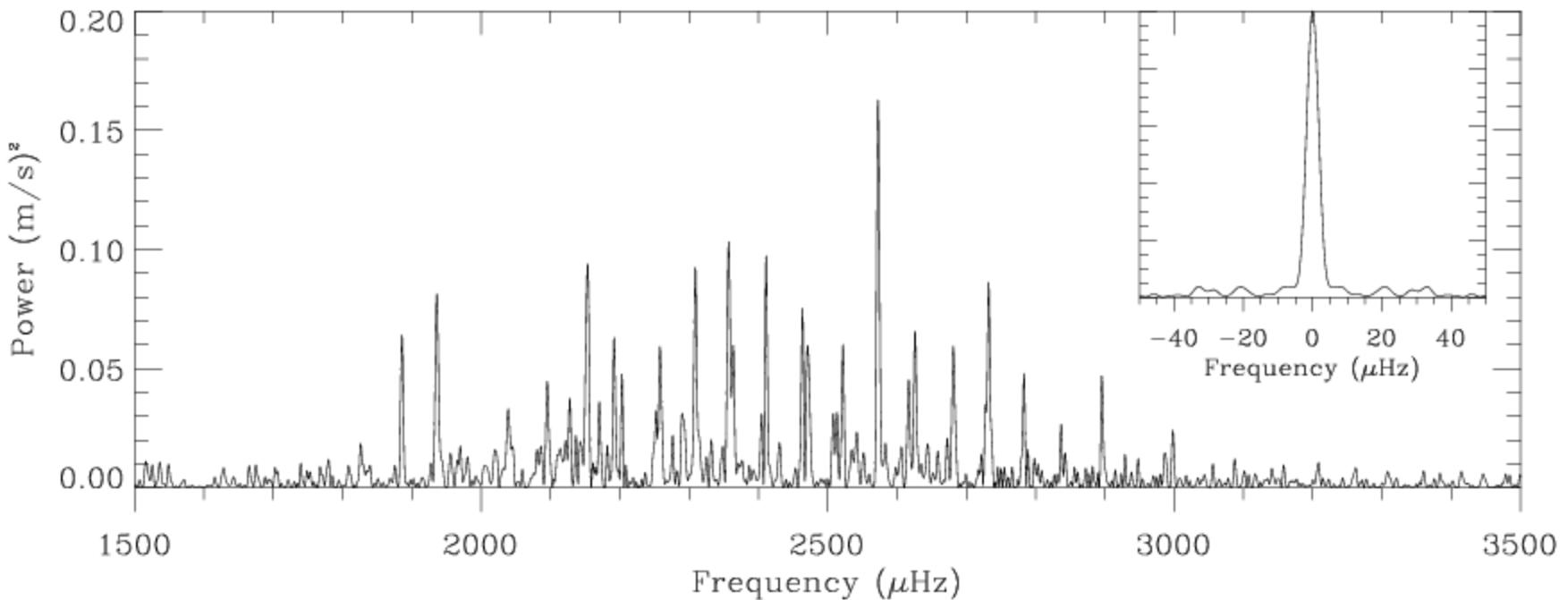


$$\delta\nu_{nl} = \nu_{nl} - \nu_{n-1, l+2} \simeq -(4l+6) \frac{\Delta\nu}{4\pi^2 \nu_{nl}} \int_0^R \frac{dc}{dr} \frac{dr}{r}$$

25 years ago:the start of global helioseismology

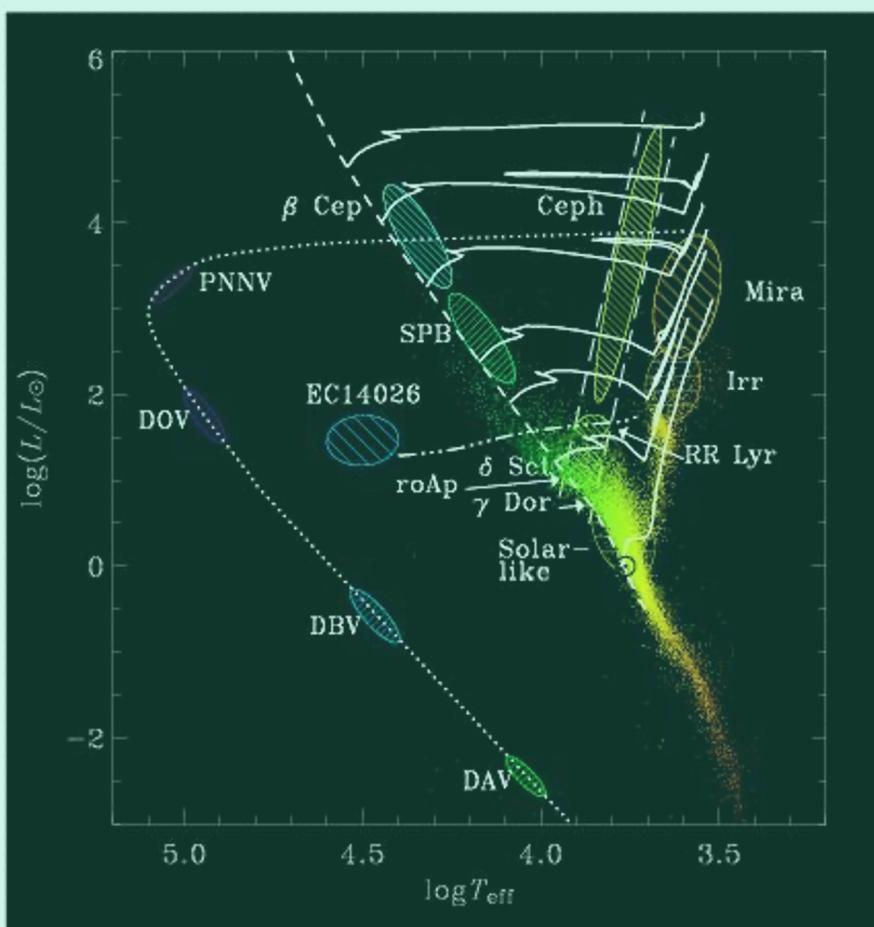


Now: the start of solar-like asteroseismology

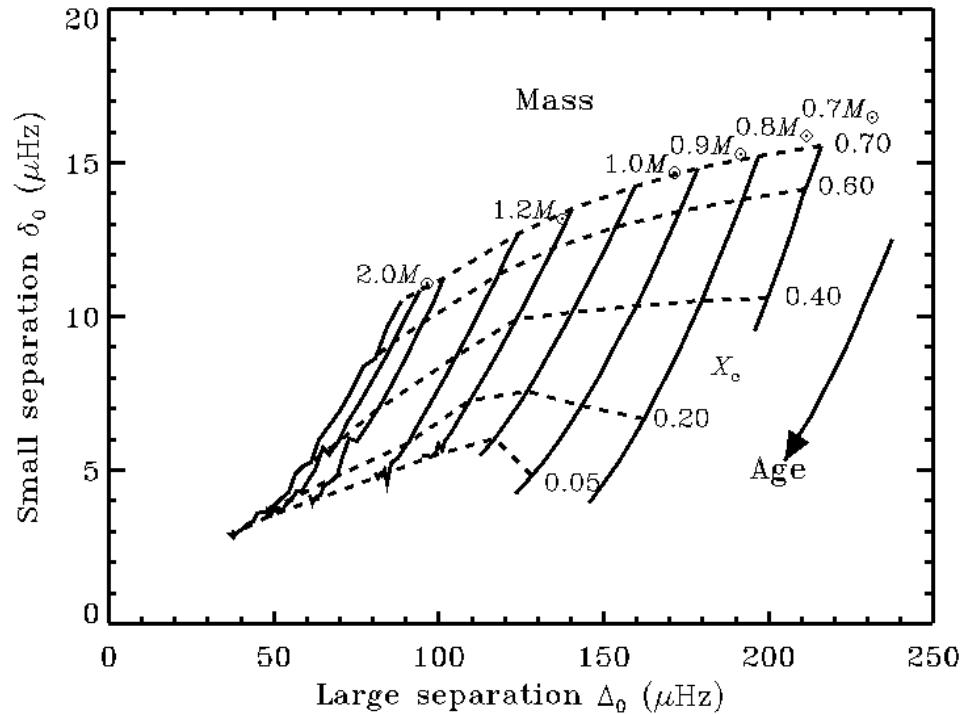


α Cen A (Bedding et al. 2004; ApJ 614, 380)

Hertzsprung-Russell Diagram



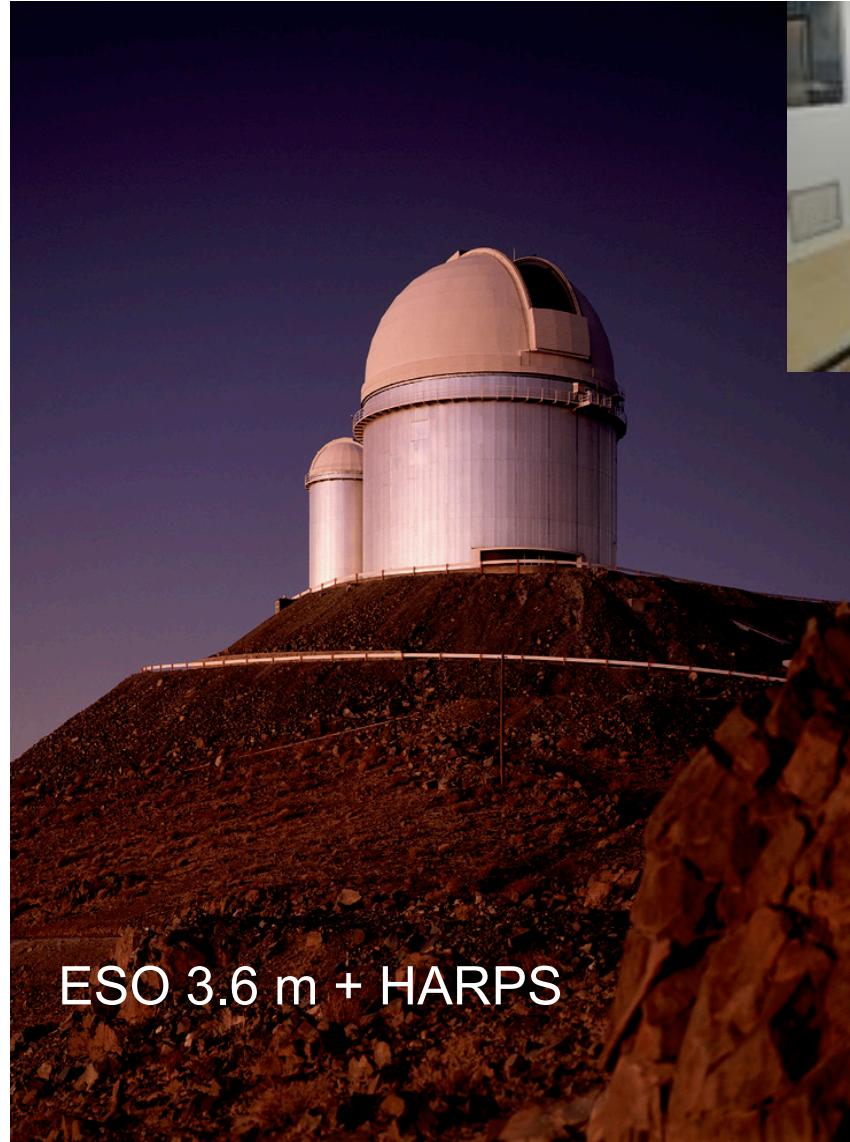
$$A \square \frac{1}{2\pi\omega_o} \left[\frac{c(R)}{R} - \int_0^R \frac{1}{2} \frac{dc}{dr} dr \right]$$



$$\nu_o \square \left(2 \int_0^R \frac{dr}{c} \right)^{-1}$$

The observational breakthrough

Very stable spectrographs, motivated by search for exo-planets



ESO 3.6 m + HARPS

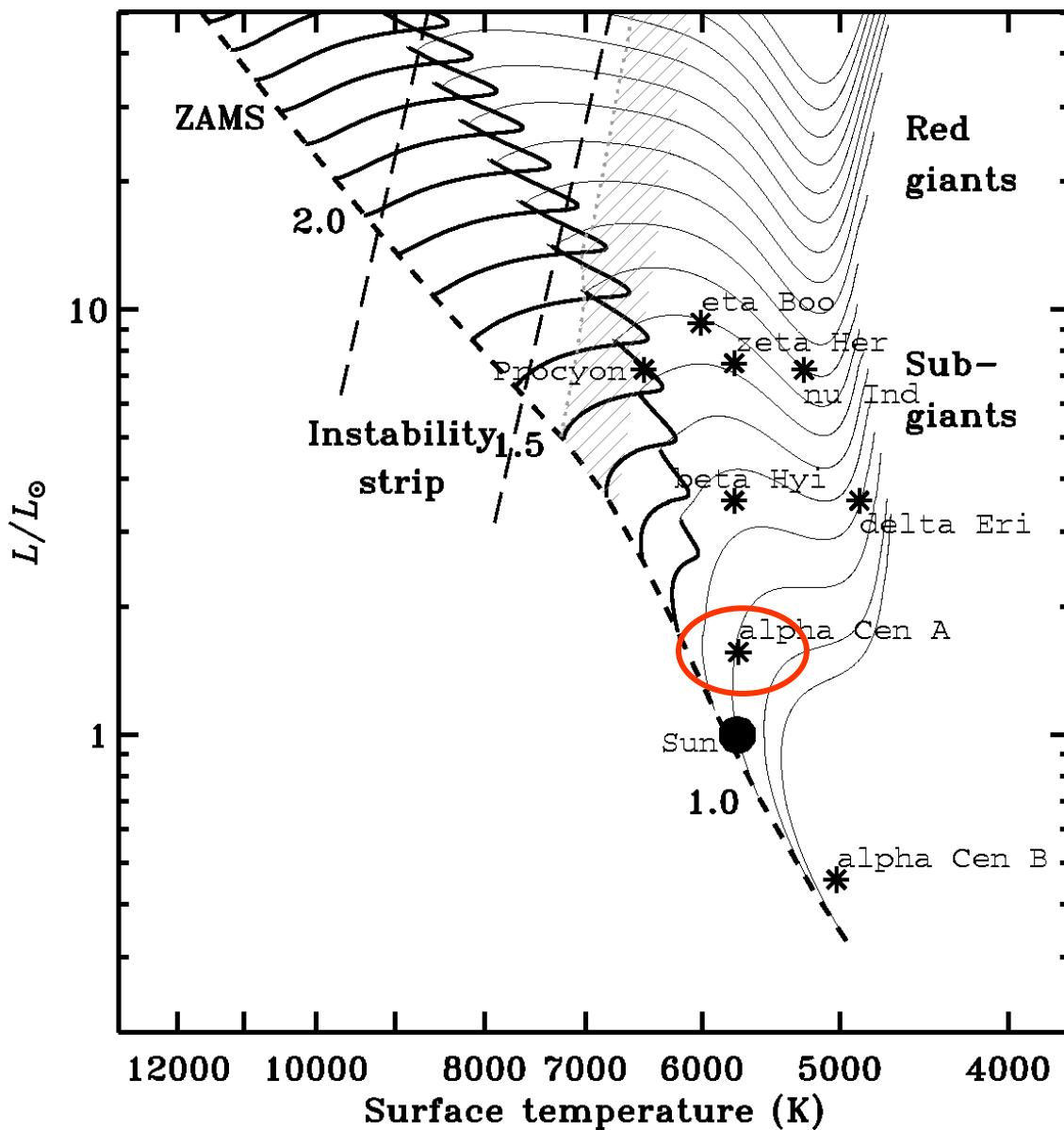


© Anglo-Australian Observatory

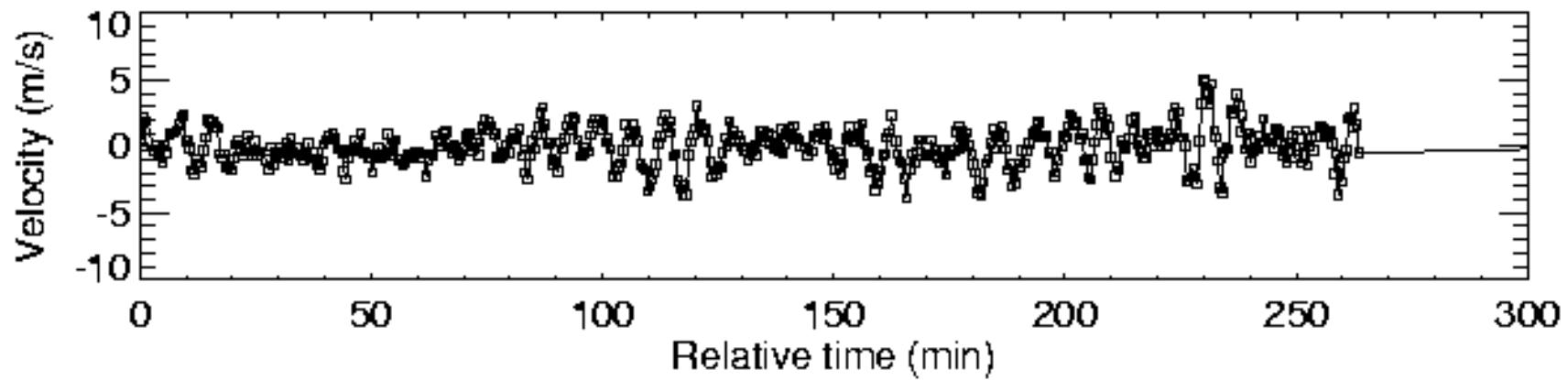


+ HARPS clone?

α Centauri A

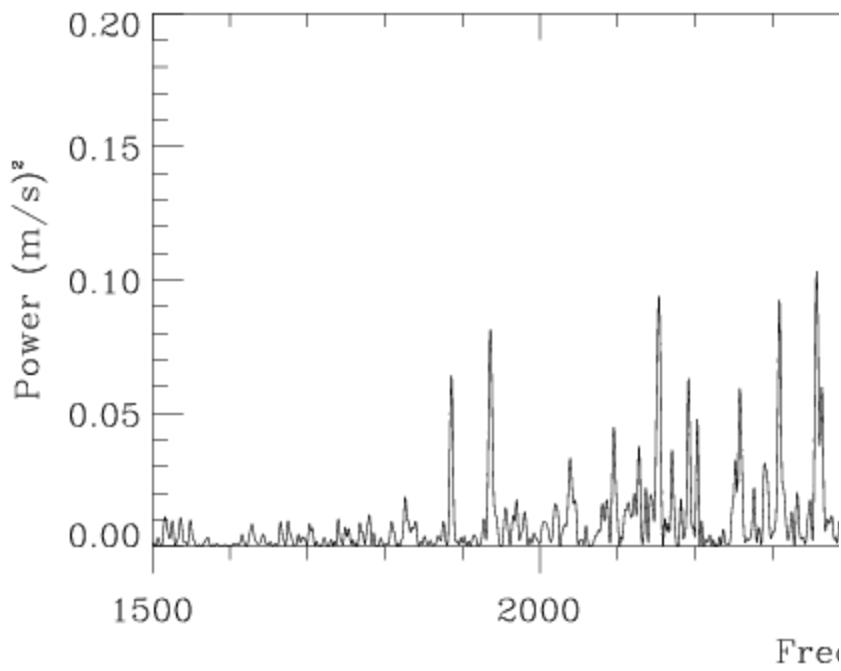


α Centauri A



Observations with UVES on VLT
(Butler et al, 2004; ApJ 600, L75)

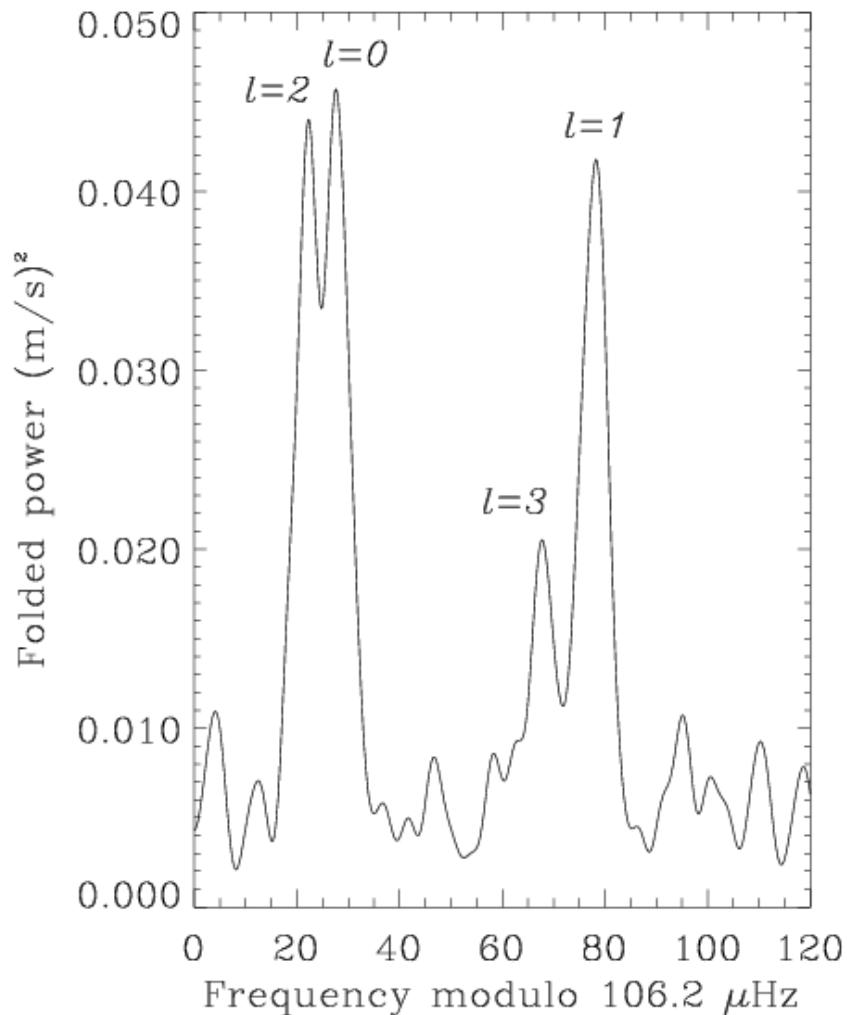
α Centauri A



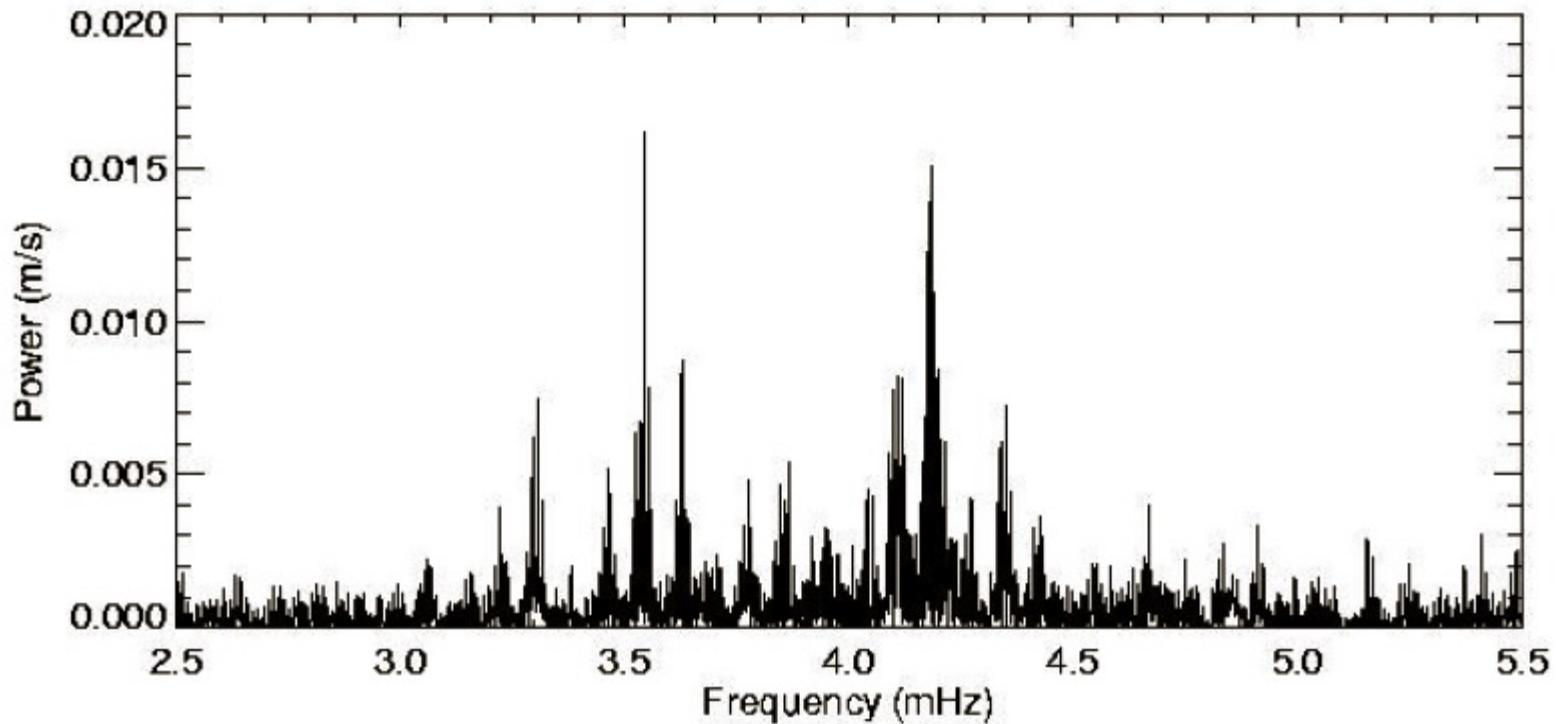
VLT(UVES) and AAT(UCLES)

optimally combined

Bedding et al. (2004; ApJ 614, 380)



α Centauri B



UVES (VLT) and UCLES (AAT)

Kjeldsen et al. (2005; submitted to ApJ)

Classical observables

	A	B	
M/M_{\odot}	1.105 ± 0.007	0.934 ± 0.006	(a)
T_{eff}	$(5830 \pm 30) \text{ K}$	$(5255 \pm 50) \text{ K}$	(b)
L/L_{\odot}	1.556 ± 0.011	0.504 ± 0.008	(b)
R/R_{\odot}	1.224 ± 0.003	0.863 ± 0.005	(c)
$Z_{\text{S}}/X_{\text{S}}$	0.037 ± 0.004	0.037 ± 0.004	(b)

(a) Pourbaix et al. (2002; Astron. Astrophys. 386, 280)

(b) Pijpers (2003; Astron. Astrophys. 400, 241)

(c) Kervella et al. (2003; Astron. Astrophys. 404 1087)

Fitting the α Cen system

Observable quantities for the system

$$\{y_i\} = \{M_A, T_{\text{eff},A}, L_A, (Z_s/X_s)_A, M_B, T_{\text{eff},B}, L_B, (Z_s/X_s)_B, [\text{osc. var.}]\} .$$

Model parameters:

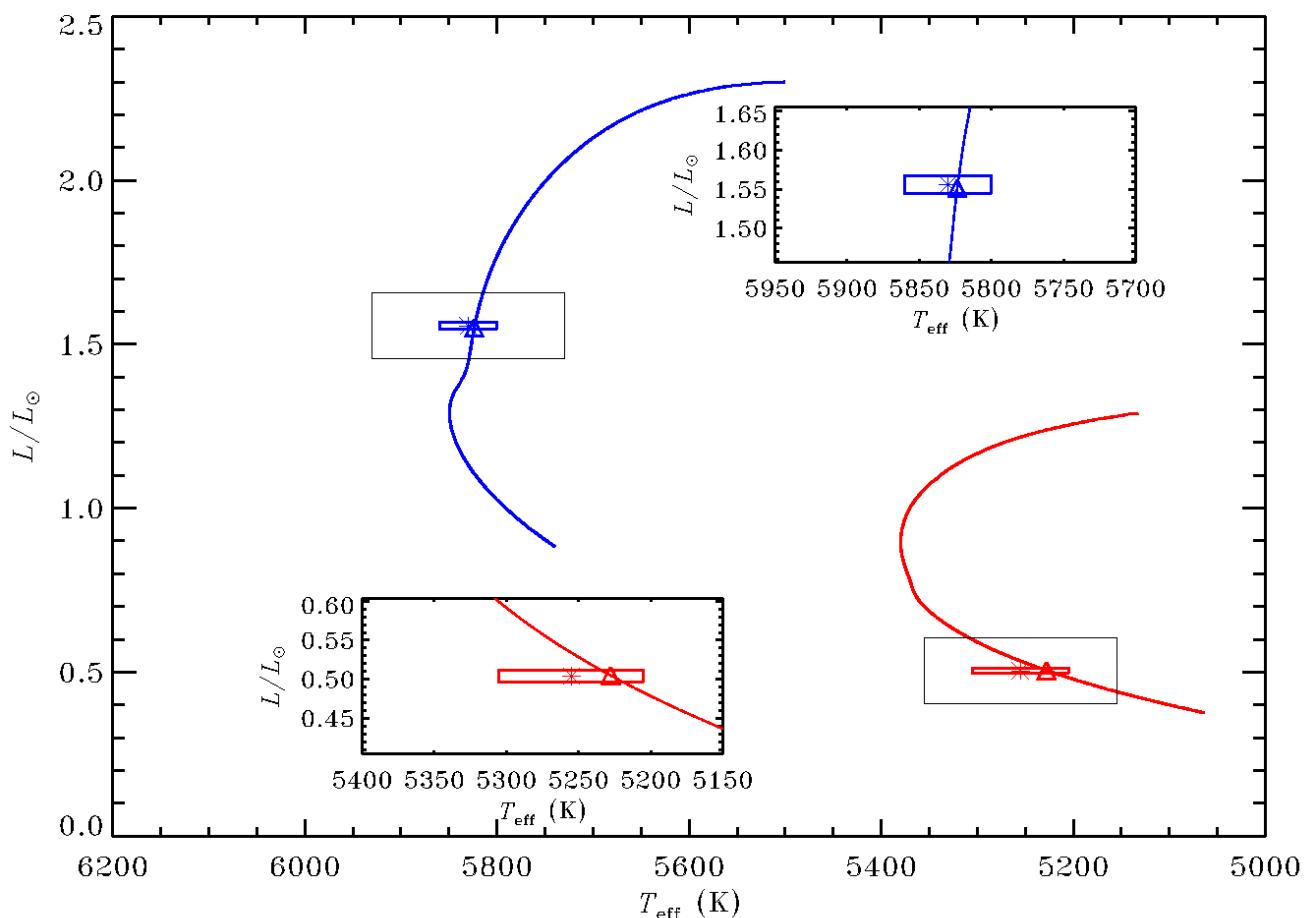
$$\{a_j\} = \{M_A^{(\text{mod})}, M_B^{(\text{mod})}, X_0, Z_0, \alpha_A, \alpha_B, \tau, [\psi_{\text{surf}}]\} .$$

Fit using Marquardt method, with centred differences, using an 8-processor Linux cluster.

Choice of oscillation variables, from Bedding et al. fits to Butler et al. observations:

$$\bar{\nu}_{21,0}, \overline{\Delta\nu_{21,0}}, \overline{\delta\nu_{21,0}/\Delta\nu_{21,0}}, \overline{\delta\nu_{21,1}/\Delta\nu_{21,1}}$$

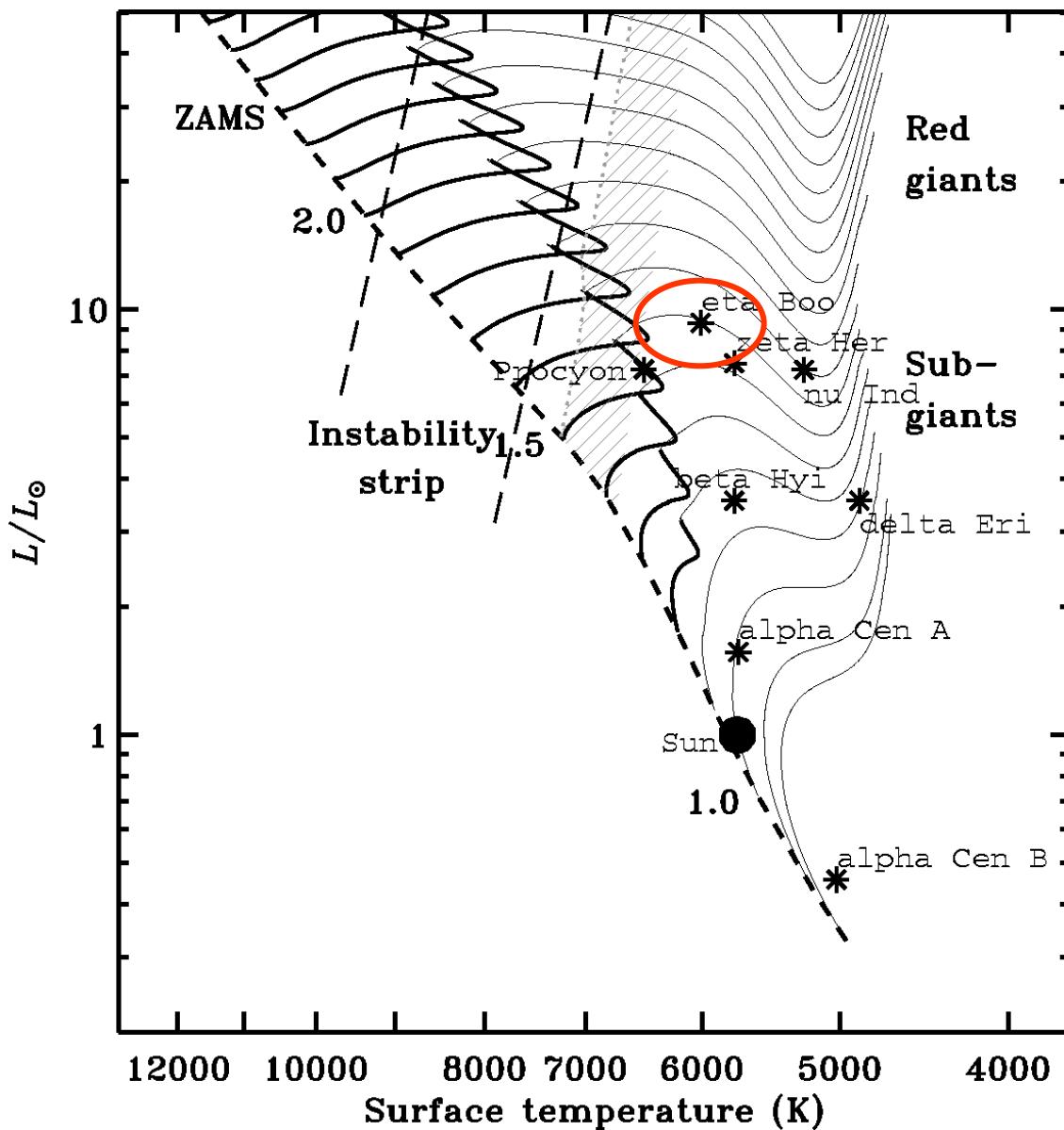
α Centauri system



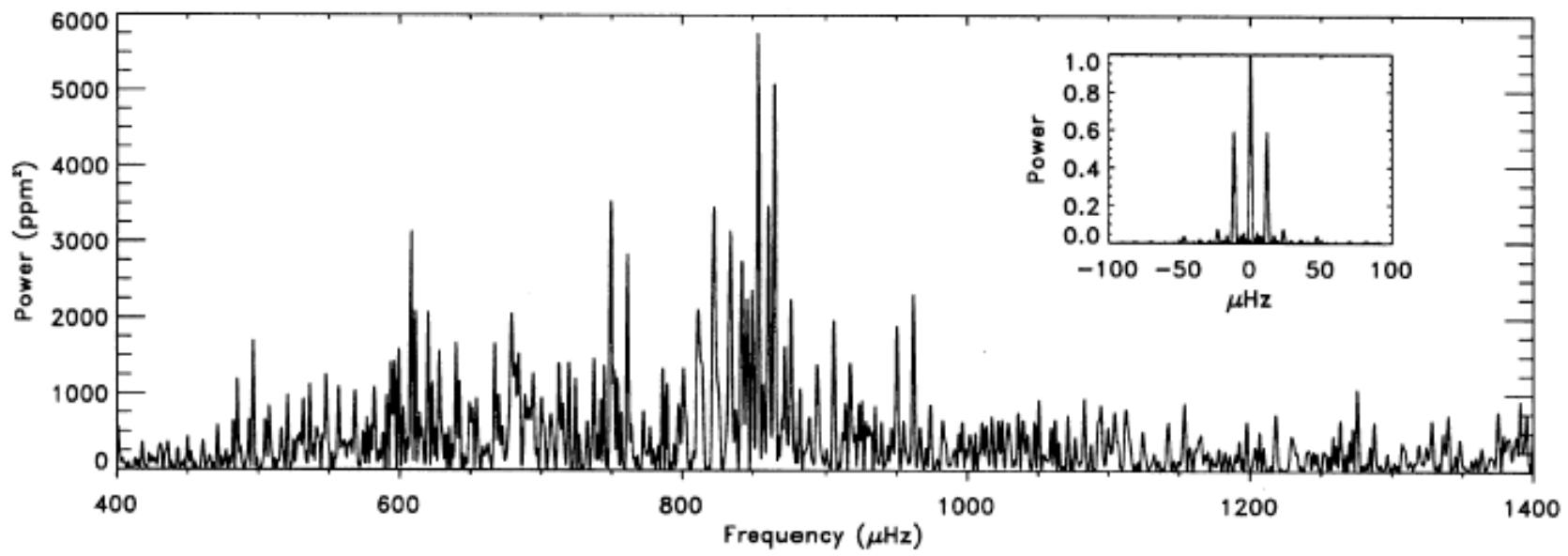
$M_A: 1.11111 M_\odot$
 $M_B: 0.92828 M_\odot$
 $X_0: 0.71045$
 $Z_0: 0.02870$
Age: 6.9848 Gyr

OPAL EOS, OPAL96 opacity, He, Z settling
(Teixeira et al.)

η Bootis



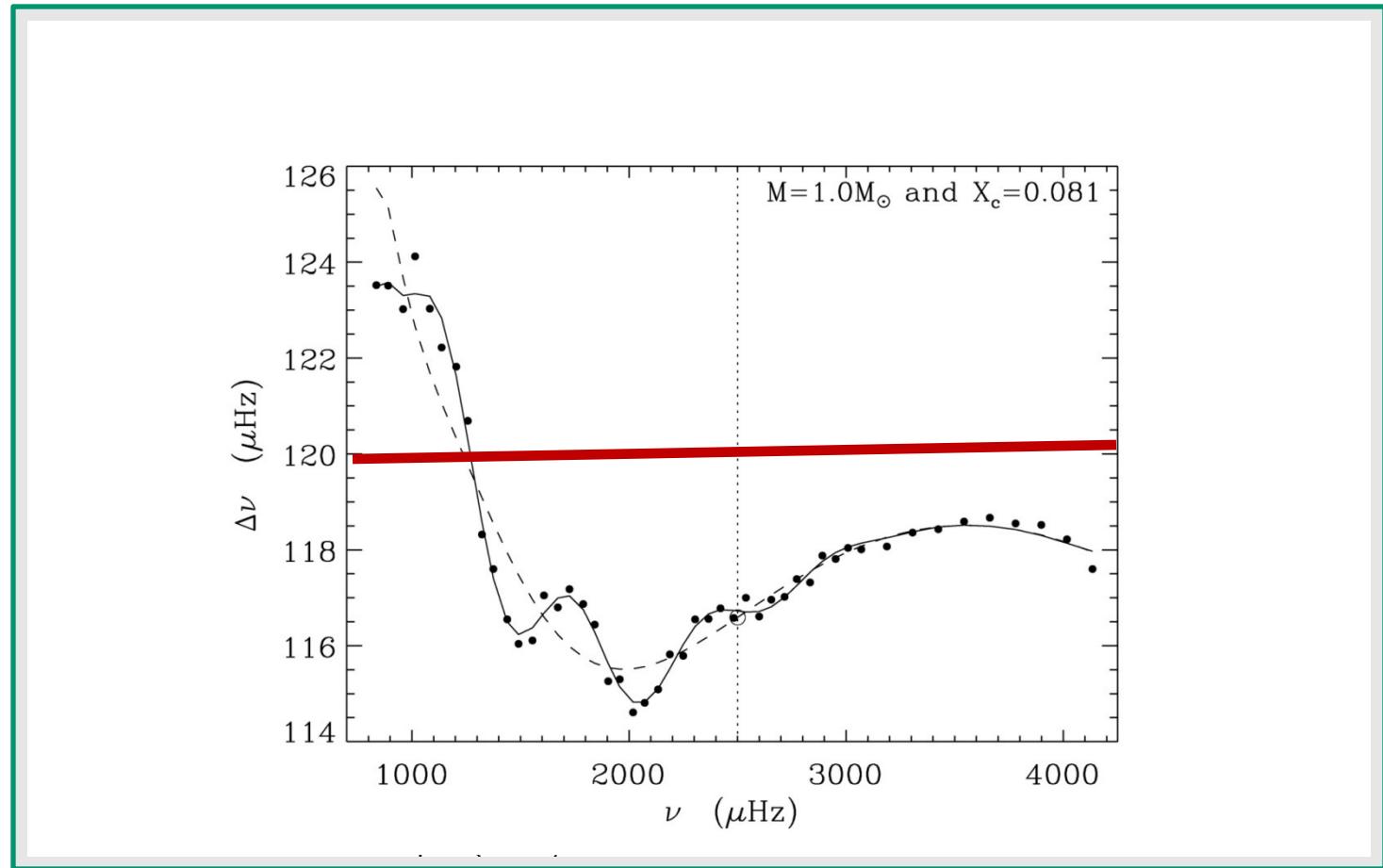
Observed power spectrum



Kjeldsen et al. (1995; AJ 109, 1313)

Some Physical Applications: Pulsating stars

STELLAR OSCILLATIONS: SOLAR TYPE STARS PROBING THE EXTERNAL LAYERS



Sharp features also affect the large (and small) separations

Lopes et al. (1994)

$$\Delta\nu_{l,n} = \nu_{l,n} - \nu_{l,n-1} \propto \frac{\nu}{n + \sqrt{l(l+1)}/2 + \beta(\nu)}$$

STELLAR OSCILLATIONS: SOLAR TYPE STARS PROBING THE EXTERNAL LAYERS

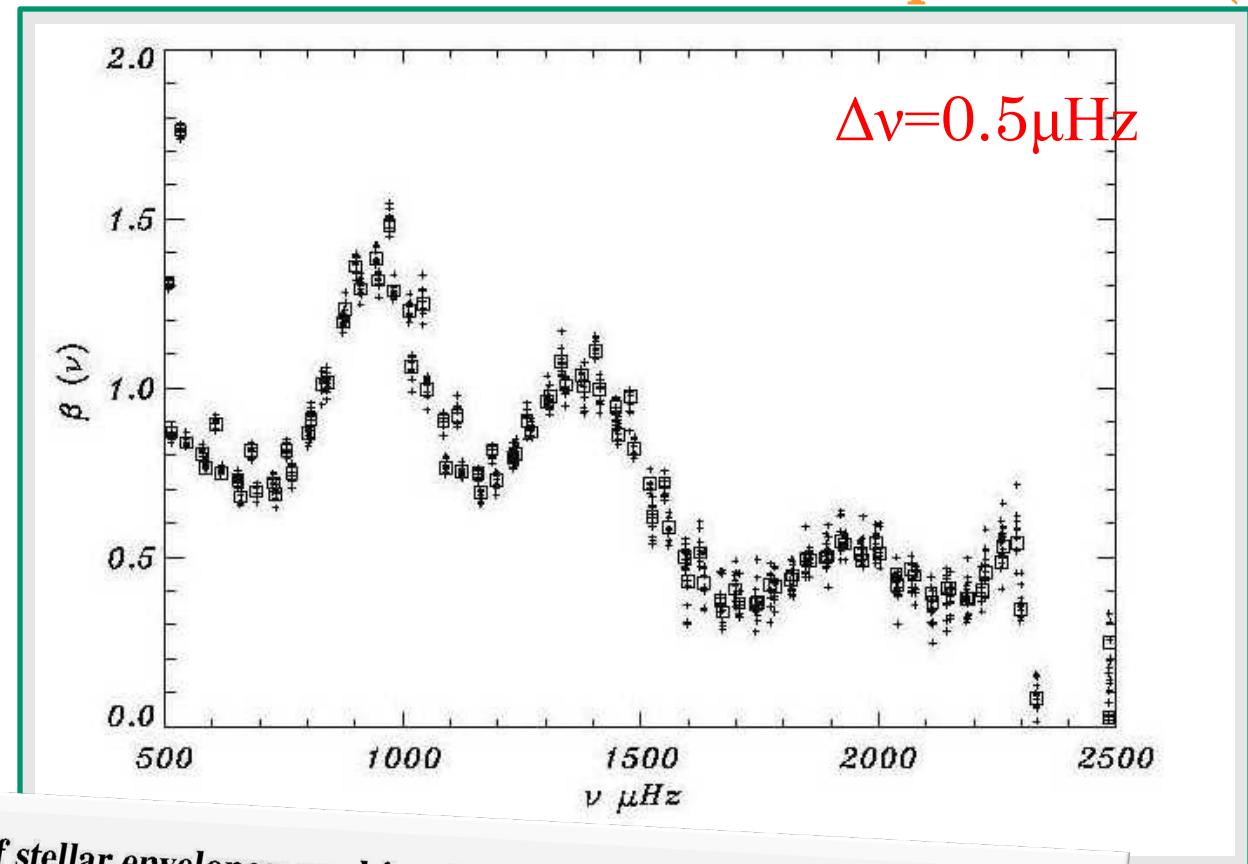
Lopes et al. (19)

COROT (COviction ROTation
and planetary Transits)
Space mission.
Launched 27 December 2006.

One of the first theoretical
studies in seismology to show the
power of COROT data to infer
properties about the interior of a
Star other than the Sun.

Example: Seismic parameter to
determine the content of helium
 Y and the mixing length
parameter α of convection on
solar type stars.

$$v(l,n) = v(l,n) + \Delta v$$



*Seismology of stellar envelopes: probing the outer layers of a star through
the scattering of acoustic waves*

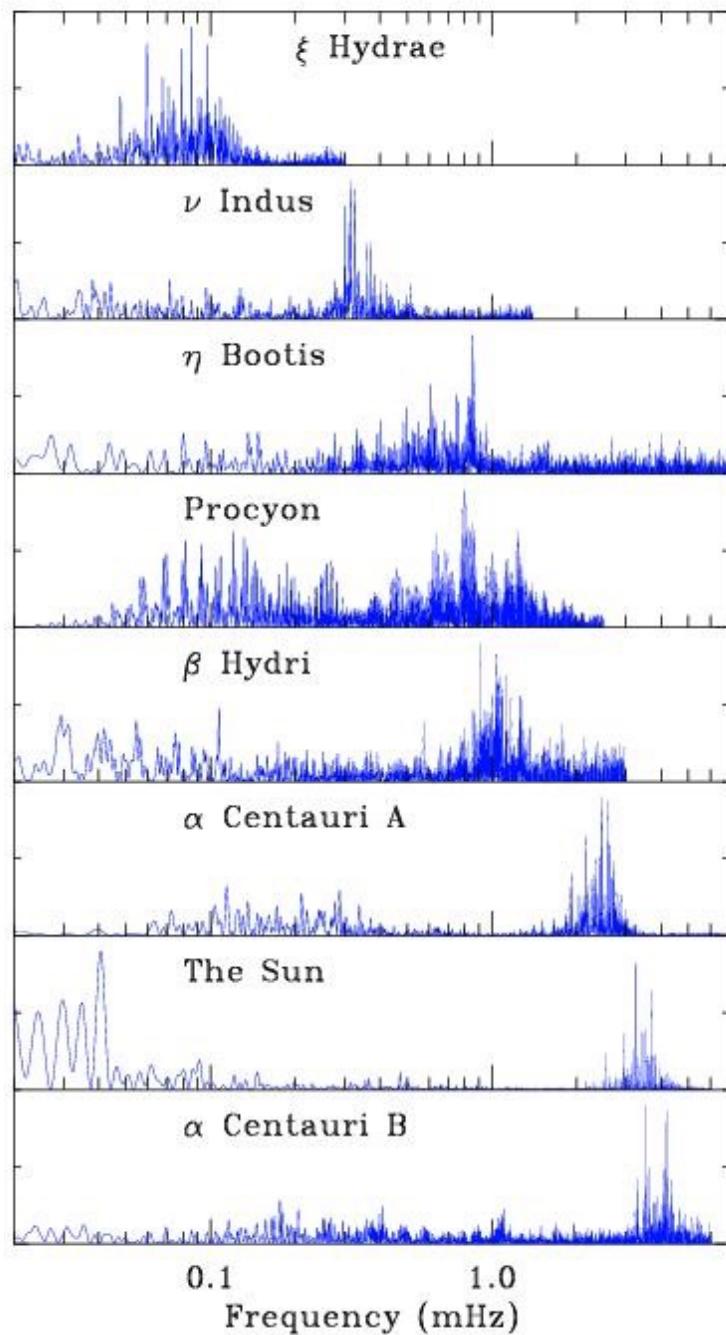
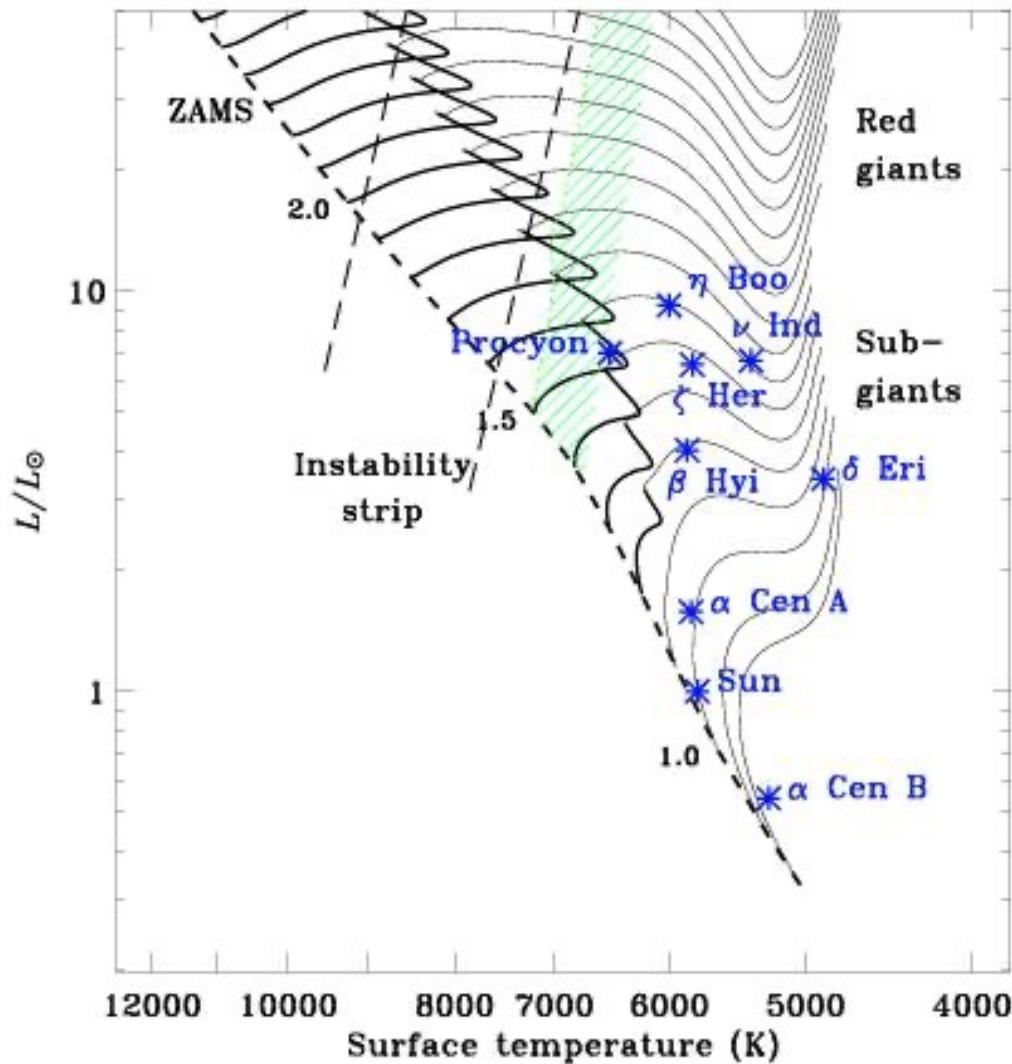
Ilídio P. Lopes¹★ and Douglas Gough^{1,2}★

¹Institute of Astronomy, Madingley Road, Cambridge CB3 0HA

²Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW

Stellar oscillations: *solar type* oscillations

Linear adiabatic oscillation



STELLAR OSCILLATIONS: SOLAR TYPE OSCILLATIONS

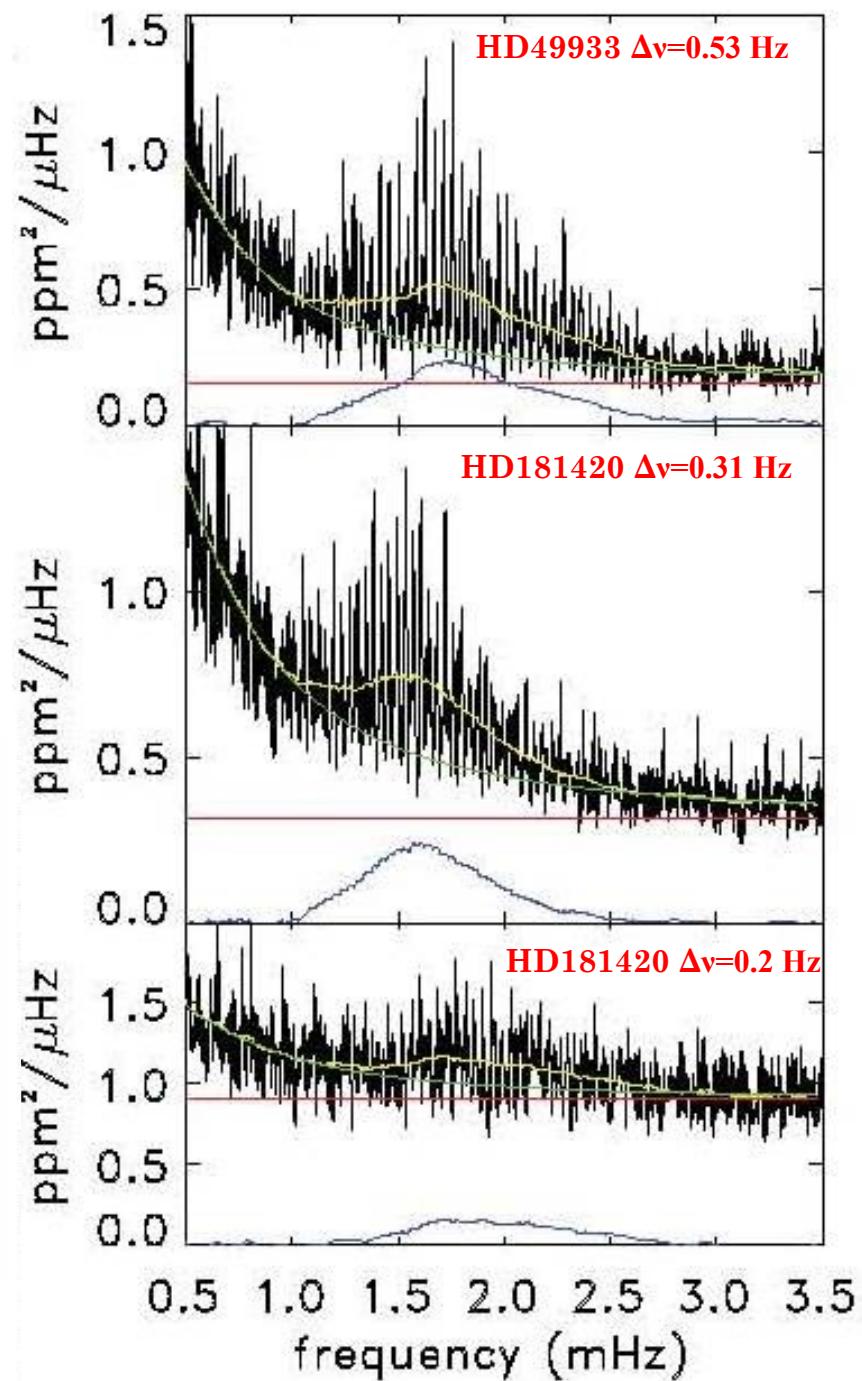
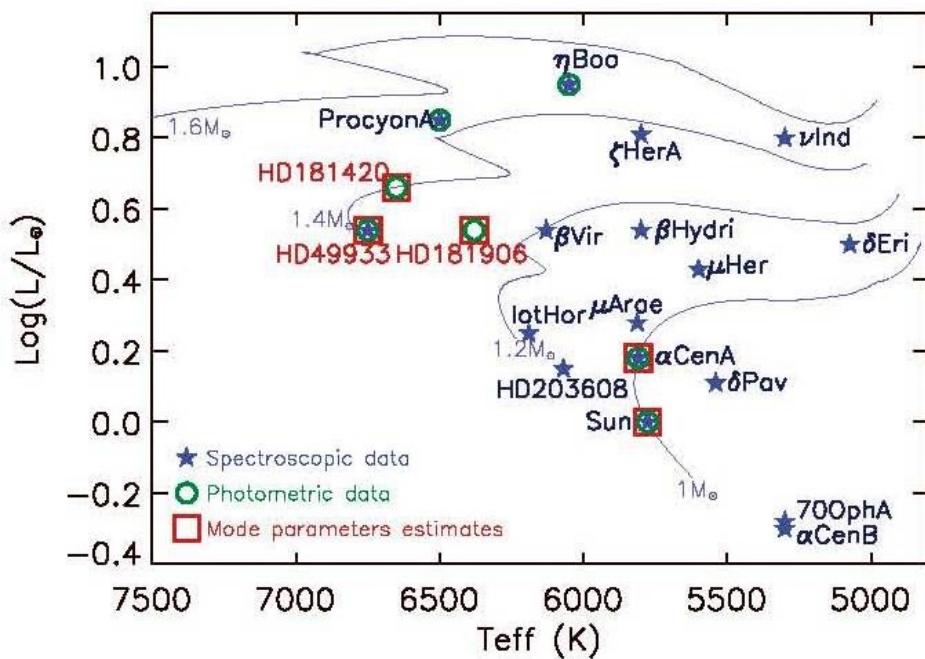
- COROT MISSION

(E. Michel et al.; Science 322:558,2008)

3 hotter F Stars

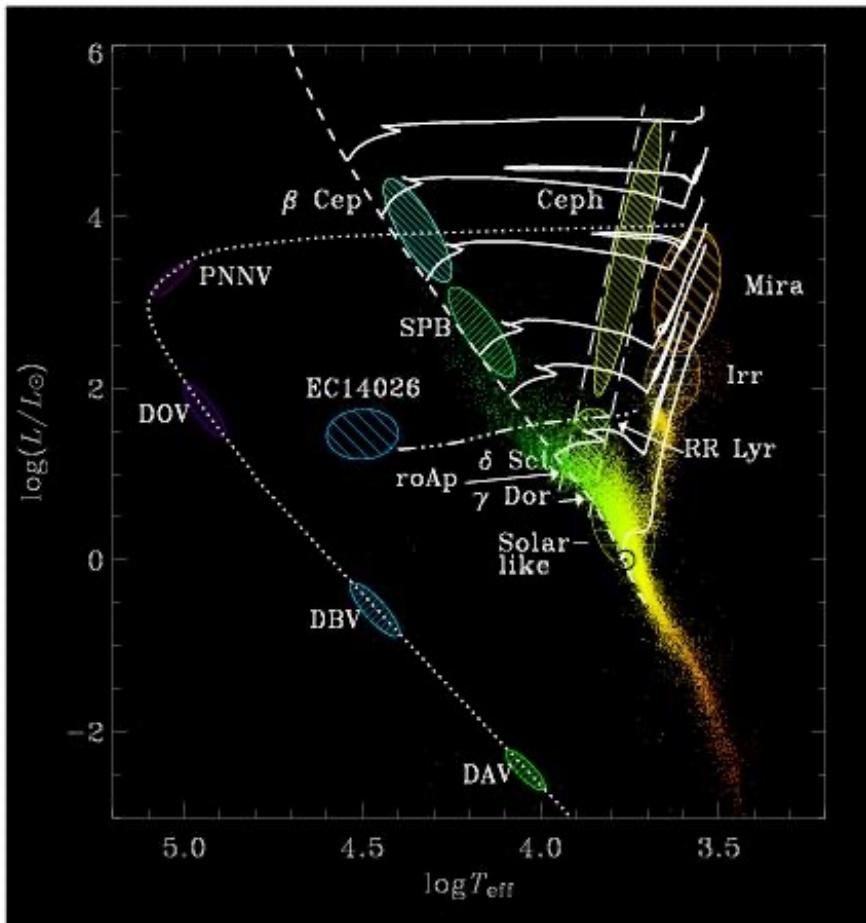
Equally spaced in frequency,

these are p-modes

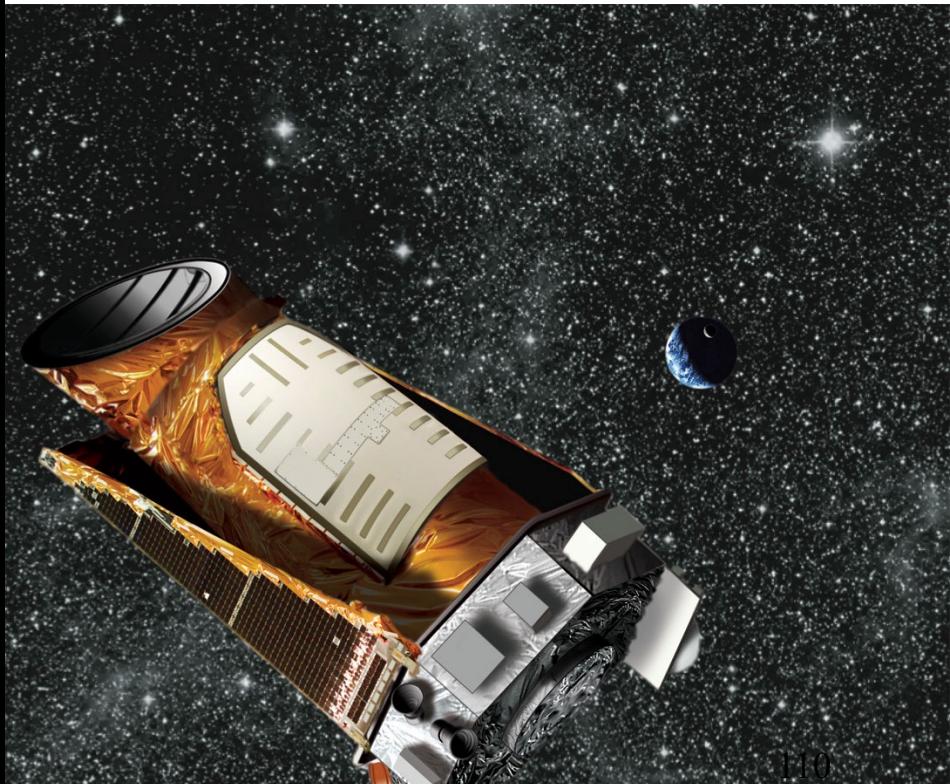


Asteroseismology

- Oscillating stars in the



- KEPLER MISSION (6/03/2009)
-



Lecture 4

END