Inflation Expectations, Wages and On-the-Job Search

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Inês Gonçalves Raposo †

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Abstract

In this paper, I design and implement a survey of United States workers to study the causal effect of higher inflation expectations on workers' job search decisions. I use hypothetical scenarios to decompose and quantify the impact of inflation expectations into direct and indirect effects: Direct effects are those caused by changes in inflation expectations, keeping other expectations constant. Indirect effects are caused by spill-overs from inflation expectations to expectations about the real economy. Through a within-subject design, I identify each of these effects at the individual and aggregate levels. I find that, on average, the direct effects of inflation expectations are positive and statistically significant. On average, workers associate higher inflation with higher unemployment. This produces an indirect effect that mutes average intentions to search. Workers' responses to higher expected inflation are heterogeneous with respect to age, gender and job tenure.

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Keywords: *inflation expectations*, *unemployment*, *job search*, *survey design*

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†Università Commerciale Luigi Bocconi, Milan (ines.goncalves@phd.unibocconi.it).

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1 Introduction

The 2021-2022 inflationary episode saw the highest inflation rates in decades across many advanced economies. In countries such as the United States, the United Kingdom, and throughout the euro area, the surge in inflation occurred against a backdrop of tight labor markets. This situation raised concerns that workers, in attempting to catch up with unexpected price increases and to mitigate further expected real income losses, would exacerbate price pressures. Workers' demands could materialize either through direct requests for pay raises or by increasing their job search efforts to find higher-paying positions. By 2024, while inflation rates have cooled, real wages have not yet returned to pre-inflation levels.

How do inflation expectations affect employed workers' search behavior? In this paper, I design and implement a survey of employed workers in the United States. I first elicit workers' one-year ahead macroeconomic expectations, their plans to search for a higher-paid job in the near future, and their demographic and job-related characteristics. Then, I induce changes in their macroeconomic expectations by presenting hypothetical scenarios in which the Central Bank announces forecasts for inflation and unemployment rates. I focus on short-term inflation expectations, given their importance for price and wage setting (Weber et al., 2023) and their predominant role in the recent inflation episode (Hajdini, 2023; Werning, 2022). In particular, I assess how each announcement translates into a change in expectations. Workers adjust their inflation expectations towards the information provided in Central Bank announcements, and adjust their planned behavior in line with these posteriors.

I separately identify the effects of higher inflation expectations on planned search behavior into direct and indirect effects. The direct effect represents the impact of higher inflation expectations when labor market expectations remain constant. In practice, however, changes in inflation expectations can spill over into unemployment expectations. The indirect effect, therefore, arises from updated labor market expectations in response to inflation expectations. In canonical job search models, workers optimally decide on their search effort by equating the marginal cost with the expected marginal benefit. This expected benefit includes the value from switching jobs, weighted by the probability of receiving an offer. When the labor market is slack, the probability of finding a job for a given search effort is lower, reducing incentives to search. Depending on how workers perceive the co-movements between inflation and unemployment, these spillovers could either offset or amplify the direct effects of inflation expectations.

My survey design accounts for these potential dynamics. Decomposing and quantifying changes in behavior into direct and indirect effects provides a more comprehensive understanding of the mechanisms through which inflation expectations influence behavior. On average, a 1 percentage point increase in inflation expectations raises intentions to search for a higher-paying job by 1.2 percentage points. However, workers tend to associate higher inflation with higher unemployment, which dampens average intentions to search. This "supply-side" view of inflation provides a potential reason for why real wages have not grown as much as initially anticipated at the onset of the inflationary surge.

My within-subject design enables me to go beyond average effects and to characterize the cross-sectional distribution of individual-level effects. First, it allows me to examine the distribution of the effects of Central Bank announcements on inflation and labor market expectations, providing direct evidence on heterogeneous effects of Central Bank communication on household expectations. Second, it captures the distribution of the effects of these expectations on behavior across heterogeneous workers. This distribution is relevant, as shifts in the composition of job seekers play a significant role in aggregate wage cyclicality (Gertler et al., 2020; Grigsby et al., 2021; Bauer and Lochner, 2020; Black and Figueiredo, 2022).

To validate respondents' intentions to search, I re-contact survey participants three months after the initial survey to assess their actual behavior. Reported intentions to search effectively predict whether employed workers pursued a higher-paying job, confirming that *ex-ante* planned behavior aligns closely with *ex-post* realized behavior.

Related literature. This paper contributes to a broad literature on the formation of macroeconomic expectations and its impact on household behavior (see Candia et al. (2020) or Fuster and Zafar (2022) for a review). In particular, this paper is most closely related to a nascent strand of the literature that leverages survey data to study the impact of inflation expectations on labor market expectations and actions. Several studies show that households associate higher expected inflation to expected losses in income (Hajdini et al., 2022; Stantcheva, 2024; Baek and Yaremko, 2024). These studies show how, given higher inflation expectations, workers choose whether to engage in costly wage bargaining (Guerreiro et al., 2024), adjust reservation wages (Baek and Yaremko, 2024) or intensify on-the-job search (Pilossoph and Ryngaert, 2024). Georgarakos et al. (2024) focus on the impact of inflation uncertainty, and find that reducing inflation uncertainty increases planned search intensity. These papers present complementary methodologies and datasets, offering a thorough understanding of worker behavior in an inflationary environment. This paper adds to this body of evidence by documenting the individual-level heterogeneity of behavioral responses of workers employed in a variety of occupations, industries and states, and by explicitly accounting for the role of unemployment expectations.

Secondly, this paper contributes to a growing literature on the links between business cycle dynamics and labor market transitions. Job-to-job transitions matter for reallocation, wage growth and productivity growth. Several papers explore the theoretical links between business cycle fluctuations and job ladder dynamics that take place through on-the-job search (Faccini and Melosi, 2023; Moscarini and Postel-Vinay, 2022; Trigari, 2009; Krause and Lubik, 2007). Empirically, there is contrasting evidence of countercyclical search among the employed (Bransch et al., 2024; Ahn and Shao, 2021; Elsby et al., 2015). Some studies highlight that different motives for searching on-the-job can generate offsetting cyclical properties, depending on whether employed workers search for a job to avoid unemployment or to climb the job ladder (Fujita, 2010; Simmons, 2023). Other studies use survey design to recover perceived returns or costs of on-the-job search (Adams-Prassl et al., 2023; Miano, 2023). I contribute to this literature by providing empirical evidence on on-the-job search given workers' different perceived stages of the business cycle, accounting for both the effects of perceived labor market slack and inflation.

By measuring the spill-overs of inflation expectations to unemployment expectations and its effects on behavior, this work is related to a strand of literature that examines the joint formation of macroeconomic expectations through mental models of the macroeconomy and perceived sources of fluctuations. This literature draws on both cross-sectional and time series data on expectations (Bhandari et al., 2019; Hou, 2020; Jain et al., 2022; Ferreira and Pica, 2024) as well as experimental methods (Andre et al., 2022; Binetti et al., 2024).

Methodologically, masuring the causal effect of *one* macroeconomic expectation is challenging - macroeconomic variables, as well as their expectation counterparts, are correlated and likely to jointly impact behavior. Depending on the underlying economic shocks, the magnitude and sign of the correlation between macroeconomic variables may differ not only across individuals, but across time and within-individuals, limiting the use of observational data on choices and expectations. This simultaneity also poses challenges under experimental settings - Fuster and Zafar (2022) or Coibion et al. (2023) discuss how an "exclusion restriction" problem arises when an information treatment is used to instrument for changes in expectations and behavior is regressed on this instrument.

To address these challenges, I build on the literature on survey measurement of expectations (see Manski (2018); Bachmann et al. (2022) for a review). I elicit subjective choice probabilities under incomplete scenarios, following the stated preference literature (Manski, 1999) and estimate treatment effects based on conditional expectations as in Giustinelli and Shapiro (2024),Wiswall and Zafar (2021), Ameriks et al. (2020) or Arcidiacono et al. (2020). Hypothetical scenarios, beyond having a causal interpretation, allow for specification of environments that may not be observable in practice (Armantier et al., 2022). I specify scenarios where the Central Bank makes different announcements of economic forecasts with no policy commitment, akin to what is usually referred to as "Delphic" forward guidance (Campbell et al., 2012). The within-individual comparison of how expectations update in response to different types of Central Bank announcements contributes to the empirical literature studying how Central Banks can use communication to affect agents' expectations (Coibion et al., 2022, 2020; Haldane and McMahon, 2018; Binder, 2017).

Structure The rest of the paper is structured as follows: Section 2 explains the analytical framework underlying the survey design. Section 3 describes the empirical strategy. Section 4 presents the survey results. Section 6 complements and validates the results. Section 7 conducts further robustness exercises. Finally, Section 8 concludes.

2 Conceptual framework

This section explains how I measure the individual subjective effect of an increase in inflation expectations on behavior. Let Y denote a binary outcome: to search or not for a higher-paid job (denoted by "search" or S_i).

At time t, individual i forms expectations about macroeconomic variables (in particular, inflation π and unemployment u) at $t+j$. These j periods ahead expectations are used to formulate a subjective probability of an outcome Y at time $t+k$, with $k \leq j$. Let the probability of individual i taking action Y be a linear function of individual fixed and time-varying characteristics (α_{i1}, v_{it}), as well as of macroeconomic expectations:

$$P_{it}(Y_{it+k} = 1) = \alpha_{i1} + \beta_{i1}\pi^e_{it+j,t} + \beta_{i2}u^e_{it+j,t} + v_{it} \tag{1}$$

Where $P_{it}(Y_{it+k} = 1)$ is the subjective probability that individual i searches for a higher-paid job in period $t+k$, $\pi^e_{it+j,t}$ is the expected inflation at $t+j$ evaluated at time t, $u^e_{it+j,t}$ is expected unemployment at time $t+j$ evaluated at time t.

As mentioned previously, while realized inflation and unemployment rates co-move, this is also the case for their expectation counterparts. In reduced form, this can be expressed as:

$$u^e_{it+j,t} = \alpha_{i2} + \gamma_i \pi^e_{it+j,t} + \varepsilon_{it} \tag{2}$$

A positive shock to inflation expectations may lead to a contemporaneous revision in unemployment expectations, which is captured by γ_i. As indicated by the subscript i, the sign of this revision can be heterogeneous across individuals.

These individual-level regressions are unobserved. In this paper, I design hypothetical scenarios that generate controlled variation in inflation expectations and assess how planned behavior responds. In doing so, I assume that unemployment expectations are a sufficiently good instrument for individuals' expectations about the real economy and labor markets, such that no "spill-overs" from higher inflation expectations to other macroeconomic variables would have a first-order effect on search behaviors.[1]

Stated preference methods allow the researcher to generate controlled exogenous variation and measure responses that could not be identified from variation in actual choices (or revealed preferences) alone.[2] The elicitation of choice probabilities, a particular type of stated preference, allows respondents to express uncertainty about factors that can affect decision-making.[3] In this paper, I elicit choice probabilities conditional on different hypothetical scenarios. Conceptually,

[1]This assumption is supported both by my data collection and other existing surveys. In particular, other macroeconomic expectations have no predictive power over individuals' realized search efforts, once unemployment and inflation expectations are controlled for. Note that this assumption is made with regards to search behaviors only.

[2]These encompass methods such as conditional probability elicitations (Giustinelli and Shapiro (2024)), vignettes or strategic surveys (Ameriks et al., 2020; Armantier et al., 2022).

[3]For a discussion see, for example, Blass et al. (2010)

hypothetical scenarios correspond to a function assigning an environment to each member of the population (Manski, 1999). In practice, I partially specify the environment by fixing a state of the world such that the Central Bank communicates their predictions. This is akin to what is described in the Central Bank communication literature as Delphic forward guidance (Campbell et al., 2012), whereby the Central Bank announces their expected future economic conditions without making any binding promises on future policy. Individuals' baseline expectations are made conditional on a state of the world that *differs* from the one I describe in the scenarios. The within-subject design controls for unobserved heterogeneity and the effects of inflation expectations on planned behavior can be point identified at the individual level.

For the remainder of this paper, let $j = 12$ and $k = 3$.[4] Changes in planned behaviour caused by changes in inflation expectations can be decomposed into a direct effect - if labor market conditions and unemployment expectations were unchanged - and an indirect effect - driven by revisions in unemployment expectations. I can manipulate individuals' inflation expectations and identify both effects solely using the elicitation (i.e. without resorting to regression estimation). Let subscripts 0 and 1 be used to denote prior and posteriors, respectively. I can retrieve the subjective treatment effect for individual i of a 1 percentage point increase in her inflation expectations by eliciting the following objects:

1. Priors:
   - Inflation expectations $\mathbb{E}_{it}(\pi_{t+12}) \equiv \pi^e_{i0}$
   - Unemployment expectations $\mathbb{E}_{it}(u_{t+12}|\pi^e_i = \pi^e_{i0}) \equiv u^e_{i0}$
   - Probability of searching for a higher-paid job given inflation and unemployment expectations $P_{it}(Y_{it+3} = 1|\pi^e_{i0}, u^e_{i0}) \equiv S_i$
2. Expectations under hypothetical scenario:
   - Inflation expectations: π^e_{i1}
   - Change in inflation expectations under the scenario: $\pi^e_{i1} - \pi^e_{i0} \equiv \Delta$
   - Expected unemployment rate conditional on π^e_{i1}: $\mathbb{E}_{it}(u_{t+12}|\pi^e_{i1}) = u^e_{i1}$
3. Planned behavior under hypothetical scenario:
   - Probability to search conditional on higher expected inflation, keeping expected unemployment constant: $P_{it}(Y_{it+3} = 1|\pi^e_{i1}, u^e_{i0}) \equiv S^D_i$
   - Probability to search conditional on unemployment expectations u^e_1: $P_{it}(Y_{it+3} = 1|\pi^e_{i0}, u^e_{i1}) \equiv S^I_i$

[4] Empirical work by Glick et al. (2022) shows that while one-year ahead inflation expectations impact wage inflation, long-term expectations play no role. Furthermore, Armantier et al. (2022) find evidence that long-term expectations are irresponsive to persistent inflation shocks

Given these objects, we can retrieve β_{i1}, β_{i2} and γ_i from Equations 1 and 2 as:

$$\beta_{i1} \equiv \frac{S_i^D - S_i}{\pi_{i1}^e - \pi_{i0}^e} \qquad \beta_{i2} \equiv \frac{S_i^I - S_i}{u_{i1}^e - u_{i0}^e} \qquad \gamma_i \equiv \frac{u_{i1}^e - u_{i0}^e}{\pi_{i1}^e - \pi_{i0}^e}$$

The total individual treatment effect can therefore be estimated as the sum of the direct effect of inflation expectations and the indirect effect of unemployment expectations that changed in response to higher inflation expectations ($\beta_{1i} + \gamma_i \beta_{2i}$). This treatment effect is subjective, in the sense that it is evaluated by the individual. It is also *ex-ante* , as it is evaluated prior to the treatment.[5]

[5] Some papers, such as Arcidiacono et al. (2020) and Giustinelli and Shapiro (2024) have used the term subjective ex-ante treatment effect. Others (for example Ameriks et al. (2020)) have referred to it simply as treatment effects.

3 Research design

The survey is composed of 6 blocks. Participants are first asked about their economic expectations. Secondly, they are asked to report the percent chances of taking different actions given their current expectations. Then, respondents are shown hypothetical scenarios. The hypothetical scenarios describe a situation where a source discloses information about the expected evolution of an aggregate variable - namely, price inflation and the national unemployment rate. To isolate the variation of interest, the scenario pins down other macroeconomic variables that could otherwise be expected co-move with the main variable. Table 1 maps the theoretical objects described in Section 2 into the different survey blocks.

Table 1: Mapping: Elicitation and analytical framework

Survey Block	Environment specification	Elicited objects
Baseline expectations	n.a.	π^e_{i0}, u^e_{i0}
Baseline behavior	n.a.	S_i
Scenario 1.	Fed expects 7 percent inflation	$\tilde{u}^e_{i1}$
Scenario 2.	Fed expects 7 percent inflation, unemployment expectations unchanged (u^e_{i0})	π^e_{i1}, S^D_i
Scenario 3.	Fed expects unemployment rate $\tilde{u}^e_{i1}$, expected inflation unchanged (π^e_{i0})	u^e_{i1}, S^I_i
Controls	n.a.	See Table 13

The exact phrasing of the scenarios is specified below:

Scenario 1. Higher expected inflation

Suppose that, after an unexpected shock, the Fed announces that it expects the inflation rate to be 7 percent 12 months from now. In your opinion, 12 months from now, what will be the national unemployment rate?

Scenario 2. Higher expected inflation, *ceteris paribus*

> *Suppose that, after an unexpected shock, the Fed announces that it expects the inflation rate to be 7 percent 12 months from now.*
>
> *The job market will be just as you first thought. The expected national unemployment rate is* `[respondent's baseline expected unemployment]` *percent. It will be as easy to find or lose a job as it was before the shock.*

Scenario 3. Higher expected unemployment, *ceteris paribus*

> *Suppose that, due to an unexpected shock, the Fed announces that it expects the national unemployment rate to be* `[respondent's expected unemployment rate when inflation rate is 7 percent]` *percent 12 months from now.*
>
> *Predictions for the inflation rate are exactly in line with your first thoughts: Prices are expected to increase by* `[respondent's baseline inflation expectation]` *percent over the next 12 months.*

After each scenario is described, I re-elicit respondents' expectations over the object described under that scenario. For example, in Scenario 1:

> *How do you expect prices to evolve over the next 12 months following the Fed's announcement? By how much?*

By explicitly eliciting individuals' expectations under the hypothetical scenario, I make sure that the changes in behavior that I identify correspond to the changes in expectations that would happen in that scenario.[6]

To illustrate, consider Scenario 2, which specifies an expected inflation π^S and keeps labor market conditions constant. Recall that in order to identify the direct effect (β_{i1}), the difference in current and hypothetical planned behaviors must be divided by the difference in inflation expectations:

$$\beta_{i1} = \frac{S_i^D - S_i}{\pi_{i1}^e - \pi_0^e}$$

If the individual's expected inflation under the scenario is $\pi_{i1}^e \neq \pi^S$ but it is assumed to be π^S, then the recovered effect would be, instead:

$$\tilde{\beta}_{i1} = \beta_{i1} + \frac{\pi_{i1}^e - \pi^S}{(\pi^S - \pi_{i0}^e)(\pi_{i1}^e - \pi_{i0}^e)} \left(S_i^D - S_i\right)$$

[6]As a robustness measure, I re-estimate these effects taking into account the scenarios and not the re-elicitations. Results do not change significantly.

Each scenario block concludes with the re-elicitation of planned behavior:

> *In this scenario, what is the percent chance that you search for a higher-paid job to replace your current job over the next 3 months?*

Finally, the last part of the survey elicits demographic and other control variables about respondents' economic and professional characteristics that could explain heterogeneous responses to macroeconomic shocks. These include age, gender, race, education, liquidity constraints, home ownership status, commuting status to work, time and reason for last pay raise, area and state of residence, inclusion of cost of living (COLA) adjustments in pay, labor union membership, type of work contract, tenure at current job and occupation.

4 Survey Results

4.1 Sample description

The survey was administered on-line on the 24^{th}, 25^{th} and 30^{th} of August 2023. Participants were recruited in Prolific through convenience sampling. In total, 722 participants started the survey and 682 individual responses were collected, which makes for an attrition rate lower than 5%. The median completion time was 9 minutes. The survey sample spans across the United States, with responses from 46 states. It also broadly captures the U.S professional fabric, covering responses from individuals employed in 18 out of 20 sectors included in the North American Industry Classification System (NACE), and in all 23 major occupational groups of the Standard Occupational Classification System (SOC). Table 1 compares the demographic composition of my survey sample to that of the employed population in the Current Population Survey (CPS). The two are broadly aligned, with my survey sample being more educated, less female and undersampling young individuals. A more detailed description of the sample, including occupation, area of residence, home ownership status is available in the Data Appendix.

Table 2: Sample composition

	Survey	CPS
High-School Degree or Less	12,6	33,8
Some College Education	24,7	25,7
College Degree or More	62,7	40,5
Age 18-34	29	33,5
Age 35-49	42,5	32,1
Age 50-65	24	27,4
Over 65	4,4	6,9
Female	41,5	46,8
White	77,4	77
Part-time	15,8	16,7
Northeast	16,7	17,4
Midwest	23,2	21,3
South	27,6	37,5
West	32,4	23,8
Management, professional, related	60,4	42,9
Service occupations	10,3	16,2
Sales and office	19,5	19,2
Farming, fishing, and forestry	0,3	0,6
Construction, and maintenance	3,9	8,4
Production, transportation, and material moving	5,6	12,7

Average shares for Current Population Survey computed using 2022 monthly data on employed respondents only. Survey weights used.

4.2 Average treatment effects

Table 14 shows average treatment effects (ATE). The average magnitude and statistical significance of the effects of inflation expectations on search behavior are muted when variation is not isolated from its spill-overs through labor market expectations.

Table 3: Average treatment effects

	ATE: Inflation expectations		
	Direct (β_1)	Indirect ($\beta_2\gamma$)	Total ($\beta_1 + \beta_2\gamma$)
Search	1.014***	-0.250	0.764
	(0.294)	(0.533)	(0.632)

4.3 Individual treatment effects

There is rich heterogeneity behind the average treatment effects that I identify. In the following subsections, I characterize the cross-sectional distribution of each individual treatment effects (ITE) of interest. These correspond to the direct and indirect effects. These effects are identified by comparing baseline and re-elicited behaviors under the Scenarios 2 and 3 described in Section 3.

Table 4 summarizes the distribution of individual treatment effects under the direct and indirect scenarios, as well as inflation expectations and planned search behavior before and after the scenarios were presented.

Table 4: Individual Treatment Effects - Search

Direct scenario					
	π_0^e	π_1^e	p_0(search)	p_1(search)	β_{i1}
Mean	4.46	5.88	25.24	28.94	0.76
SD	4.48	4.07	31.52	32.15	5.11
p10	0	0	0	0	-2
p25	1	4	0	1	0
p50	4	7	10	16	0
p75	7	8	40	50	1.67
p90	10	10	80	85	5
Indirect scenario					
	u_0^e	u_1^e	p_0(search)	p_1(search)	$\beta_{i2}\gamma_i$
Mean	8.98	11.18	25.24	26.99	-0.14
SD	9.61	10.38	31.52	30.11	9.29
p10	3	4	0	0	-3
p25	4	5	0	1	0
p50	5	8	10	15	0
p75	10	13	40	45	1
p90	19	22	80	76	4.4

Table 5 shows the share of negative, zero and positive individual treatment effects by scenario. Overall, the share of individuals revising their intentions to search upward is very similar in both scenarios.

Table 5: Share of search ITE, by sign

	Sign of ITE		
	Negative	Zero	Positive
Direct Scenario	0.19	0.43	0.39
Indirect Scenario	0.24	0.40	0.36

Heterogeneity On average, inflation expectations directly increase workers' plans to search for higher-paid jobs. However, the indirect effect of updated unemployment expectations mitigates these revisions. Behind these findings there is wide heterogeneity.

To study heterogeneity in the magnitude and the sign of the response to the scenarios, I run OLS and ordered probit regressions reported in Table 6. The first column studies how workers' baseline probability to search on-the-job varies with respondents' characteristics; The two following columns assess heterogeneity in the magnitude of responses to direct and indirect scenarios; The last two columns study the sign (negative, null or positive) of the response to direct and indirect scenarios.

Responses to the direct scenario - recall, that induces *ceteris paribus* variation in inflation expectations - are heterogeneous across gender and education. Responses to the indirect scenario, that focuses on the *labor market*, are heterogeneous for different levels of job tenure.

The regressions are estimated with respect to a baseline category with the following characteristics: white man aged between 30 and 40 years old, college-educated, working full-time in the same city that he lives in; who has had a pay raise over the past year; employed at his current job for more than 6 years; owns a home with an outstanding mortgage; with a pay contract without cost of living adjustments and not a member of a labor union; employed in a management occupation in the South region of the United States.

Direct effects of higher inflation expectations are lower for individuals between 50 and 59 years of age. Female and non-college educated workers are more likely to reduce the percent chance of search for a higher-paid job. Indirect effects through unemployment expectations are heterogeneous across job tenures. In particular, workers with lower job tenures would increase their intentions to search for a higher-paid job more than workers who have been in their job for longer than 6 years.[7]

[7] Note that given the large mass of zero ITEs, coefficient estimates should be interpreted with caution. Relative differences with respect to the baseline category are the focal point of this heterogeneity analysis.

Table 6: Heterogeneity in ITE - on-the-job search

	Probability to search	ITE Size - OLS Direct (β_{i1})	ITE Size - OLS Indirect ($\beta_{i2}\gamma_i$)	ITE Sign - Oprobit Direct (β_{i1})	ITE Sign - Oprobit Indirect ($\beta_{i2}\gamma_i$)
Under 30 years old	-3.84	0.19	-3.21**	0.33	-0.03
	(5.26)	(0.61)	(1.56)	(0.21)	(0.22)
Between 40 and 49 years old	-0.87	-0.33	-1.00	-0.10	0.12
	(3.97)	(0.45)	(1.24)	(0.15)	(0.17)
Between 50 and 59 years	-4.13	-0.94**	-0.13	-0.23	0.09
	(4.24)	(0.48)	(1.33)	(0.16)	(0.18)
Over 60 years old	-9.96*	-0.83	-1.04	-0.13	0.05
	(5.16)	(0.58)	(1.66)	(0.20)	(0.23)
Female	0.96	-0.07	-1.06	-0.24**	-0.14
	(3.16)	(0.36)	(0.99)	(0.12)	(0.13)
No college degree	-3.99	-0.75*	-1.13	-0.14	-0.10
	(3.40)	(0.39)	(1.07)	(0.13)	(0.15)
Renter	8.24**	-0.07	-1.11	-0.06	-0.14
	(3.51)	(0.40)	(1.07)	(0.13)	(0.15)
Never had a pay raise	13.16**	0.47	-2.09	-0.15	-0.19
	(5.94)	(0.67)	(1.89)	(0.23)	(0.27)
Tenure lower than 1 year	4.03	0.33	4.32**	0.19	0.47*
	(6.40)	(0.72)	(1.91)	(0.25)	(0.27)
Tenure between 1 and 2 years	14.54***	-0.64	4.28**	-0.19	0.40*
	(5.32)	(0.60)	(1.70)	(0.20)	(0.24)
Tenure between 2 and 6 years	8.07**	-0.71*	2.37**	-0.22	0.28*
	(3.54)	(0.40)	(1.10)	(0.13)	(0.15)
Constant	23.41***	2.31***	-0.79		
	(6.10)	(0.69)	(1.92)		
Other controls	Y	Y	Y	Y	Y
Observations	462	454	365	462	371

"Other controls" include dummies for race, home ownership status, commuter status, residence, liquidity constraints, part-time contracts, cost of living adjustment clauses, labor union membership, occupation and region.

5 Follow-up survey: contrasting planned and realized behavior

The causal effects estimated in the previous sections are evaluated *ex-ante* by respondents. These subjective causal effects are informative to the extent that planned behavior predicts actual behavior. To test whether that is the case, I re-contacted survey participants 3 months after the original survey was fielded. The data collection took place between 29$^{\text{th}}$ November and December 1$^{\text{st}}$. Out of 682 respondents, 500 completed the follow-up study. The follow-up study was designed to contrast reported probabilities to search for a higher-paid job with actual self-reported behavior. In this sense, respondents are asked the following questions:

Have you searched for a higher-paid job in the last 3 months?

Additionally, I elicit labor market outcomes:

Have you received a pay raise in the last 3 months?

Have you changed jobs in the last 3 months?

When the survey was originally fielded, the average percent chance of search on-the-job over the next 3 months was 25 percent. Three months later, 38 percent of respondents who completed the follow-up survey reported to have searched for a higher-paid job. The average search intention among respondents who ended up searching for a higher-paid job was 46 percent, compared to 12 percent among workers who did not search.
Table 7 contrasts realized and planned behavior in more detail. Subjective search probabilities predict ex-post search behavior. [8] In both cases, there is a positive correlation between planned and realized behavior.

Table 7: Ex-post behavior and ex-ante beliefs

	Searched
P(search)	0.78***
	(0.059)
Constant	0.50
	(0.121)
Controls	Y
Observations	497
R-squared	0.301

[8] Note that uncertainty and the materialization of shocks may lead to divergences between *ex-ante* assessments and *ex-post* choices.

6 Validating direct effects of inflation expectations

6.1 A simpler scenario: The effects of housing inflation expectations

In this section, I focus on the particular role of shelter inflation expectations in affecting individuals' job search behavior. I elicit expectations of *local* rental prices.[9] While inflationary pressures have eased since the peak in June 2022 (9.1%), shelter inflation has been taking longer to cool off (Kmetz et al., 2023). On average, housing services account for one-third of the overall personal expenditures in the United States (Hazell et al., 2022). These facts illustrate how housing may be particularly salient to survey respondents, and how a scenario around that object may be realistic and easy to understand. Moreover, with regards to the simultaneity issue described in the introduction, a shock to local housing prices is more likely to affect agents' inflation expectations, without spilling-over to expectations about the labor market. In support of the former, Dhamija et al. (2023) find that households overweight house price expectations when thinking about their inflation expectations. With regards to the latter, Kuchler and Zafar (2019) document extrapolation from local home price changes to formation of national inflation expectations but zero effects on unemployment expectations.[10]

I elicit one-year ahead rent inflation expectations r^e_{i0} and r^e_{i1} based on the scenario transcribed below:

Higher expected rent inflation, *ceteris paribus*

> *Suppose the following:*
> *The news report that average home rents in the city where you live are expected to <u>increase by 9 percent</u> over the next 12 months. This expected increase in rents is not caused by changes in jobs, wages or other prices.*

Let S_i denote the same baseline search as previously defined and S^R_i define search under the direct scenario.

> *In this scenario, by what percentage do you think home rents in your area will increase over the next year?*

On average, I find that a 1 pp increase in rent inflation expectations leads to a 0.294 pp increase in intentions to search for higher-paid jobs, and this effect is significant at a 95% confidence level.

[9] My focus on rent inflation expectations, instead of house price expectations, is in line with the BLS methodology to measure all shelter component of housing services.

[10] In my survey, the cross-correlations between inflation expectations, rent inflation expectations and unemployment expectations are of 0.43, 0.32 and 0.21,respectively.

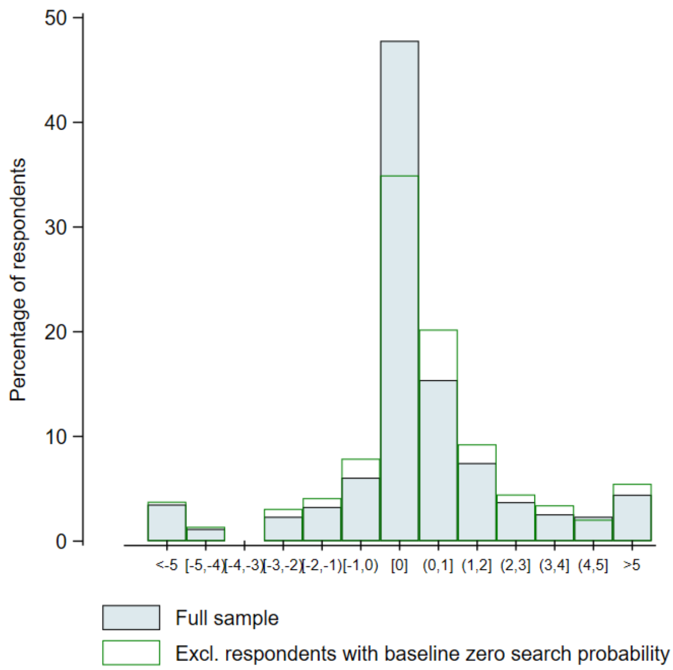


Figure 1: Distribution of ITEs to Rent Scenario

Table 8 shows how rent expectations change in response to a scenario where local news informs that rents are expected to increase 9% over the next year. As can be seen, rent expectations after the scenario become significantly less dispersed and concentrated around 9%. The 10^{th} percentile increases from 0 to 6 percent and the 90^{th} percentile decreases from 20 to 14% expected rent inflation. On average, the percent chance of searching for a higher-paid job increases from 23.7 to 26.11%, with median intentions increasing from 8.50 to 14%. On average, ITE are small and feature a significant mass around 0. As can be seen from Figure 1, this correspond to a large extent to respondents with baseline 0 percent chances of searching for a higher-paid job. On average, a 1 pp increase in rent inflation expectations leads to a 0.29 pp increase in intentions to search for a higher-paid job.

Table 8: Individual Treatment Effects - Rent scenario

	r_0^e	r_1^e	p_0(search)	p_1(search)	ITE
Mean	7.84	10.25	23.71	26.11	0.29
SD	7.82	5.11	31.51	31.23	3.46
p10	0	6	0	0	-1
p25	2	9	0	1	0
p50	5.5	9	8.5	14	0
p75	12	10	39	40	0.6
p90	20	14	76	81	2.79

Table 9 shows nearly 48% of respondents do not change intentions to search following the rent shock, 35% increase intentions to search and 16.8% decrease search intentions.

Table 9: Share ITE responses, by sign

	Sign of ITE		
	Negative	Zero	Positive
Search	0.17	0.48	0.35

Heterogeneity Table 10 shows heterogeneity of individual treatment effects along observable characteristics. The baseline category is defined as previously. The first column is, as before, a regression of baseline search behavior on observable characteristics.

Consistent with the general inflation scenario (Columns 3 and 5 of Table ??), ITEs are lower for female and non-college educated individuals. However, some new results emerge compared to the direct effects estimated before. We see that renters have higher baseline search intentions, but also higher ITEs to the rent scenario; Additionally, there are heterogeneous ITEs with respect to age, with lower estimated ITEs for all age categories compared to the baseline of 30 to 40 years old. Lastly, individuals who never had a pay raise have a higher estimated ITE compared to individuals who had a pay raise sometime over the past year.

Table 10: Heterogeneity in ITE - on-the-job search - rent scenario

		ITE	
	Search	Size OLS	Sign - Oprobit
Under 30 years old	-3.98	-1.16**	-0.20
	(5.22)	(0.53)	(0.20)
Between 40 and 50 years	1.60	-0.65	-0.50***
	(4.34)	(0.44)	(0.17)
Between 50 and 60 years	-1.40	0.14	-0.41**
	(4.92)	(0.50)	(0.19)
Over 60 years old	-12.47**	-0.65	-0.58***
	(5.69)	(0.58)	(0.22)
Female	2.05	-0.66*	-0.27**
	(3.46)	(0.35)	(0.13)
No college degree	-1.67	-0.52	-0.29**
	(3.62)	(0.36)	(0.14)
Renter	9.91***	0.75**	0.17
	(3.76)	(0.38)	(0.15)
Never had a pay raise	4.86	0.38	0.57**
	(6.72)	(0.70)	(0.26)
Tenure lower than 1 years	4.39	0.69	-0.07
	(7.12)	(0.72)	(0.27)
Tenure between 1 and 2 years	19.69***	0.29	-0.06
	(5.92)	(0.60)	(0.23)
Tenure between 2 and 6 years	13.25***	0.02	0.09
	(3.85)	(0.39)	(0.15)
Constant	20.69***	-0.46	
	(6.82)	(0.69)	
Other controls	Y	Y	Y
Observations	388	380	388

7 Further robustness, extensions and discussion

7.1 Zero individual treatment effects

As highlighted in Section 4, a sizeable share of respondents does not change their planned behavior in the scenarios. To respondents who, for one of the scenarios, did not change their intentions to search, the survey asks why. Respondents can select all options that apply from the following: a) Scenario is not different enough b) Chances of *finding* a higher-paid job would not be affected by the scenario c) Doesn't know how to look for a higher-paid job d) Doesn't have time to look for a higher-paid job e) Satisfied with my current job or f) The scenario was difficult to understand. In providing this list of options, I account for three main reasons behind zero-effects:

1. Scenario complexity

2. Expected returns of search relatively low

3. Expected costs of search are relatively high

Table 17 shows, for each scenario, the share of zero individual treatment effects by individuals with baseline zero or hundred percent chances of searching for a job.

Table 11: Zero individual treatment effects, by scenario

	Scenario		
	Inflation	Unemployment	Rent
Total Search ITE	*500*	*418*	*581*
of which: Zeros	214	163	230
of which: P(Search) = 0	49.5%	55.2%	49.6%
of which: P(Search) = 100	15.0%	10.4%	14.3%

Across the scenarios, the most common answer was that the scenarios were not different enough to change respondents' planned behavior. The second most frequent answer was that respondents are satisfied with their current job. Third, same chances of finding a higher-paid job - while this is correlated with the first reason, it specifically ties with the expected returns from job search under the two scenarios, rather than potential income effects. Very few respondents mention that they do not know or cannot look for a higher-paid job, and only 3 respondents select difficulty in understanding scenarios as an option for zero changes in planned behavior.

Overall, this sub-section suggests that null ITEs may be more reflective of respondents' actual job constraints or non-pecuniary benefits, than of scenario complexity or confusion.

7.2 Consistency with other empirical evidence

Table 12: Comparison with other surveys' expectations

	Median one-year ahead inflation expectation (Aug 23)	Mean one-year ahead inflation expectation (Aug 23)	U.S. CPI YoY percent change (Aug 24, BLS)
Survey of Consumer Expectations (NYFed)	4.9%		2.5% (all items)
Michigan Survey of Consumers	3.5%	5.6%	3.2% (core)
This paper	4%	4.7%	

Table 12 compares the median and mean one-year ahead inflation expectations of my survey to those in long-running surveys established in the measurement of inflation expectations, the Survey of Consumer Expectations of the New York Fed and the Michigan Survey of Consumers. The last column of the table shows realized inflation one year later.

My elicitations correlate positively with similar measures in more extensive and widely accepted surveys. The longitudinal nature of these surveys can be used to understand how planned job seeking behavior correlates with realized job-to-job transitions. Figures 2 and 3 illustrate how realized inflation and job-to-job transitions correlate with self-reported survey measures of on-the-job search behavior - namely, those elicited in the Labor Market Survey, the in-depth quadrimestral module in the SCE focused on job search. Figure 2 shows that job-to-job transitions co-move positively with realized inflation. Figure 3 shows, especially since 2016, a co-movement between the self-reported percent chances of switching jobs and the actual job-to-job transitions.

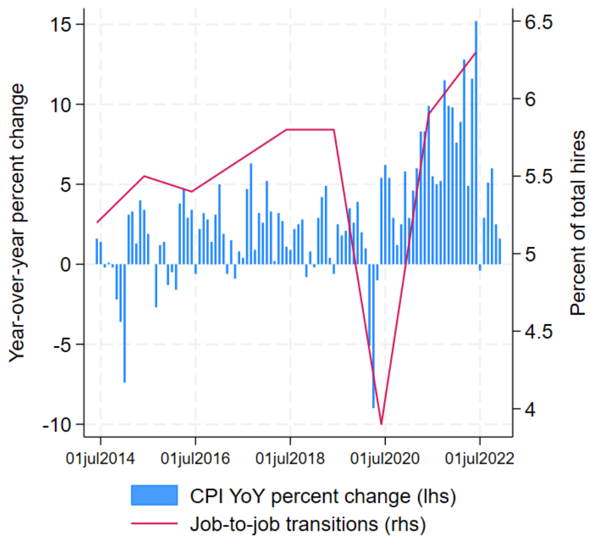
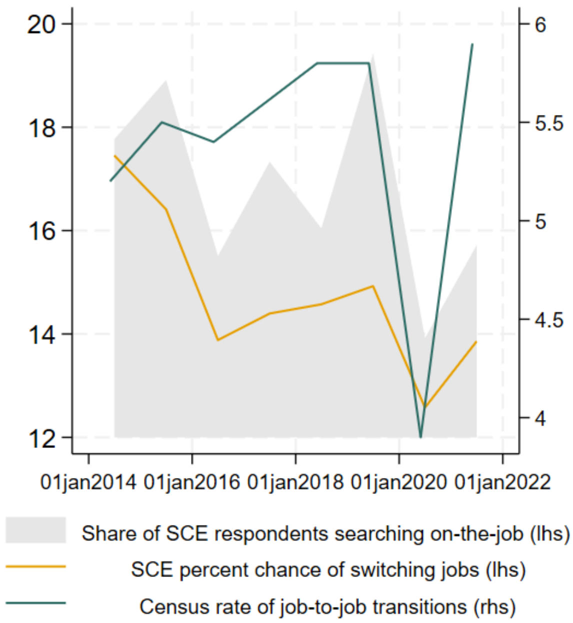


Figure 2: Inflation and job-to-job transitions - realizations

Source: Federal Reserve Bank of Cleveland (CPI YoY percent change) and US Census Bureau (Rate of job-to-job hires, non seasonally adjusted).

Figure 3: Survey measures of job search and realized job-to-job transitions



Source: Labor Market Survey, Federal Reserve Bank of New York (on-the-job search indicators and percent chance of working for a different employer in 4 months) and US Census Bureau (Rate of job-to-job hires, non seasonally adjusted).

Lastly, Figure 9 shows individual revisions in their unemployment expectations in response to higher inflation expectations. This corresponds to how I defined γ_i in Section 2. Around two thirds of respondents revise their unemployment expectations up in response to a 1 percentage point increase in inflation expectations. Most individuals (41% of respondents) revise their unemployment expectations upwards by at most 1 percentage point. For 6% of respondents the magnitude of revisions is between 1 and 2 percentage points, while 8% of respondents revise by more than 2 percentage points. Almost a fifth of respondents (19%) does not revise their unemployment expectations at all. On average, these findings are consistent with findings there is heterogeneity in belief updating even when provided with the same information about macroeconomic variables (Andre et al., 2022), and that on average individuals hold a recessionary view of inflation.

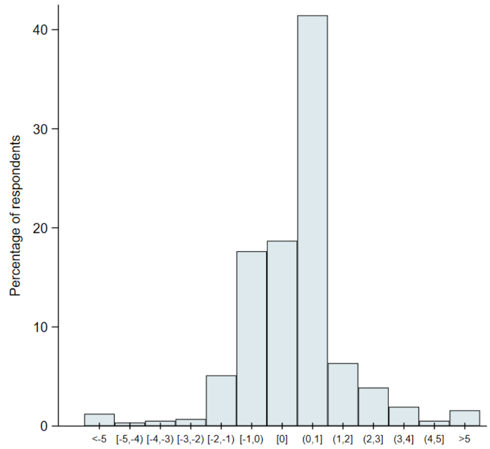


Figure 4: Changes in unemployment expectations (pp) given a 1 pp increase in inflation expectations

7.3 Extensions: other variables

The Data Appendix extends the analysis to workers' propensity to ask employers for a pay raise. Workers are more likely to search on-the-job than to ask for a nominal pay raise. Only a small share of workers would ask directly for a pay raise to their employer. This is compatible with workers facing some degree of nominal wage rigidity, documented in the literature. While the model I present in the next section assumes a specific form of wage rigidity (Calvo) that keeps the analysis tractable, I will comment on how inflation expectations change the wage bargaining outcome. The expected *frequency* of wage renegotiations, however, will be kept fixed by assumption.

7.4 Consistency with macroeconomic models

A recent literature has focused on effects of monetary policy on labor market flows, and how these flows can result in inflation pressures. To the extent that monetary policy affects households' expectations (Binder et al., 2022), these effects can be related to the ones of this paper.Most studies addressing the effect of monetary policy in frictional labor markets use modelling choices where transmission channels operate through labor demand. For example, when firms use wages to compete over workers, on-the-job search co-moves positively with inflation (Moscarini and Postel-Vinay, 2022). A monetary tightening increases propensity to search on-the-job and resulting job-to-job transitions (Faccini and Melosi, 2023). Graves et al. (2023) explicitly focuses on the role of labor supply in a partial setting with sticky wages, studying the participation margin. In this stylized model, a monetary contraction produces a substitution (reduces the job finding rate and hence, returns to search) and an income effect (increases the marginal utility of consumption) on workers' decision to participate in the labor market. Cantore et al. (2022) consider a two-agent model with hand-to-mouth and saver households, but no on-the-job search. An increase in interest rates increases debt repayments and generates an income effect in labor supply. In particular, it generates an increase in working hours at the bottom of the income distribution, though on average hours and labor earnings decline. Unconstrained households reduce consumption because of intertemporal substitution, as well as higher returns to savings.

In ongoing work, I study the choice of optimal search intensity through the lens of a New Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) model. The purpose of the model is to provide insight into how inflation expectations theoretically affect on-the-job search, mapping with the direct and indirect effects elicited in the survey. The model features search and matching frictions, as well as nominal price and wage rigidities. The model follows closely the works of Gertler et al. (2020) and Gertler et al. (2008). Workers search on-the-job to improve match quality, but do not directly negotiate their pay when joining a new job, joining the firms' existing payscale. Wages are negotiated in nominal terms and there are wage rigidities. Match surplus is decreasing with expected inflation across all match qualities. I describe the model framework in the Model Appendix, but leave a full-fledged theoretical exposition of the results for future work.

7.5 Experimenter demand effects

Recent evidence that demand effects in online experiments is quantitatively small (see Fuster and Zafar (2023); De Quidt et al. (2018); Clifford et al. (2021), and in particular Roth et al. (2022) for evidence on macroeconomic information). I follow best practices to minimize concerns of experimenter demand effects (see, e.g. De Quidt et al. (2019); Falk and Zimmermann (2013)). In particular, the purpose of the survey and instructions are neutrally framed. Respondents are not primed in any direction of updating and are informed that their decisions are anonymous.

7.6 External validity and reverse causality

The elicitation of stated choices or behavior may prompt concerns regarding the unbiasedness or external validity of the estimated effects. Evidence (for example, Fuster et al. (2021); Fuster and Zafar (2022)) suggests that if scenarios are realistic and relevant for individuals, stated choices are meaningful and retrieve similar preference estimates to actual choices. With regards to inflation expectations in particular, Coibion et al. (2023) find that transitory shocks in inflation expectations lead to persistent effects on spending. The authors argue that a potential mechanism is revision of planned behavior that is followed through even after the shock has worn out.

A final issue for discussion is that, while I explore the link from inflation expectations to unemployment expectations, it could be argued that the inverse direction also affects behavior. I focus on the causal chain from inflation expectations to unemployment expectations to study responses to inflationary shocks. Additionally, there is empirical evidence that while news about inflation move both inflation and unemployment expectations, news about real economic variables do not generate this co-movement (Hou, 2020).

8 Conclusion

The recent inflationary period has revived interest in measuring the role of inflation expectations for nominal wage growth. Studies that take labor market frictions into account show how wage pressures can materialize, of which on-the-job search is an important mechanism. My survey sheds light on how increases in inflation expectations may have heterogeneous effects on individuals' search behavior. These behaviors are important as they could materialize into an effect on aggregate wages. Existing experimental studies contributing to measurement of this response do not take into account that individuals may hold heterogeneous views of what causes higher inflation - depending on how individuals observe or interpret shocks, some may expect unemployment and inflation to co-move positively, and others negatively. By doing that, my work allows for a more complete description, not only by identifying which individuals' responses to expected inflation are muted or exacerbated, but also why.

I find that inflationary pressures may per se increase search for higher-paid jobs. This effect, however, is counteracted by how average individuals think about the real economy in an inflationary environment. There is rich heterogeneity behind these average effects.

The findings of this paper raise interesting policy implications with respect to Central Bank communication and its use for expectations' management. In particular, this paper highlights how managing expectations of one macroeconomic variable may spill-over to how individuals view the broader economic reality.

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A Data Appendix

A.1 Detailed sample composition

Table 13: Detailed sample composition

	N	Percent
Under 30 years old	92	13.49
Between 30 and 39 years old	231	33.87
Between 40 and 49 years old	165	24.19
Between 50 and 59 years old	122	17.89
Over 60 years old	72	10.56
Male	399	58.50
White	528	77.42
College educated	428	62.76
Liquidity constrained	173	25.37
Rents	244	35.78
Owns with outstanding mortgage	248	36.36
Owns without outstanding mortgage	139	20.38
Works and lives in the same place	513	75.22
Last pay raise one year ago or less	422	61.88
Last pay raise more than 1 year ago	186	27.27
Never had a pay raise	74	10.85
Lives in a city	200	29.33
Lives in a town	69	10.12
Lives in a suburb	309	45.31
Lives in a rural area	104	15.25
Pay includes COLA	197	28.89
Labor union member	60	8.80
Part-time worker	108	15.84
Job tenure lower than 1 year	70	10.26
Job tenure between 1 and 2 years	69	10.12
Job tenure between 2 and 6 years	231	33.87
Job tenure higher than 6 years	312	45.75
Occupation		
Management	108	15.84
Business and Financial Operations	64	9.38
Computer, Math and Engineering	105	15.40
Science and Education	55	8.06
Arts, Design and Media	17	2.49
Healthcare	56	8.21
Sales and Related	50	7.33
Office and Administrative Support	75	11.00
Production	14	2.05
Transportation and Material Moving	22	3.23
Other	116	17.01

Northeast Region	126	18.48
Midwest Region	168	24.63
South Region	264	38.71
West Region	124	18.18
Observations	682	100

A.2 Aggregating individual treatment effects

The previous sub-sections identify average treatment effects and unpack the underlying heterogeneity at the individual level. Each of the effects is based on two observations per individual, which effectively identify how behavior would change if expectations changed from one point to the other. This change, however, may not be the same for other points of the individual's inflation expectations distribution. To evaluate this, one could elicit behavior conditional on different inflation expectations, keeping unemployment constant. An alternative approach that minimizes the burden on respondents is to interpret these effects as *local*. Under minimal assumptions, I can aggregate responses to identify a curve between expectations and planned behavior. To illustrate, consider Figure 5, that plots survey responses for four hypothetical individuals (A, B, C and D). For each individual, the survey elicitation recovers a linear relationship between two points, corresponding to prior and posterior inflation expectations, respectively.

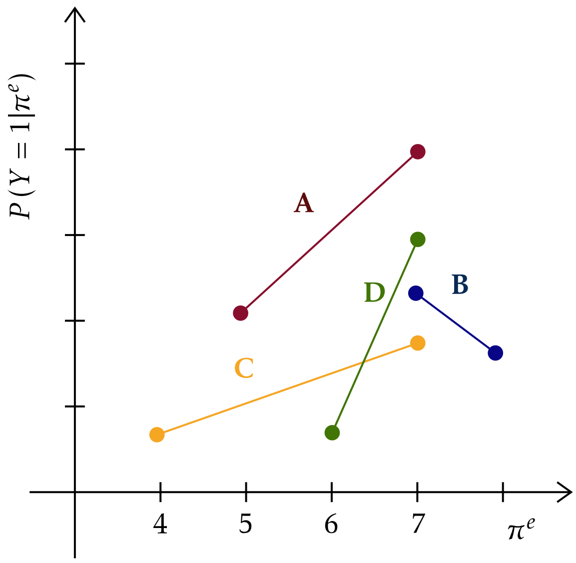


Figure 5: Illustration of hypothetical individual responses

The slope of each line is β_{i1}, the subjective direct effect of inflation expectations on behavior (job search or wage bargaining). A first assumption required is that this effect holds *locally*, this is, for every point of the inflation expectation distribution between the two elicited points[11].

For each integer value of inflation expectations j, let $\mathcal{J}$ be the set of respondents such that $j \in [\pi_i^{min}, \pi_i^{max}]$ and $j+1 \in [\pi_i^{min}, \pi_i^{max}]$. Let N_j be the number of respondents in that set. Then, for each $\pi^e \in [j, j+1]$, we can estimate an average effect $\beta_1(j)$, with standard deviation $\sigma(j)$:

[11] Let π_0^e and π_1^e denote the prior and posterior inflation expectations, $\pi^e \in [\min\{\pi_0^e, \pi_1^e\}, \max\{\pi_0^e, \pi_1^e\}] \equiv [\pi_i^{min}, \pi_i^{max}]$

$$\beta_1(j) = \frac{1}{N_j}\sum_{i\in\mathcal{J}}\beta_{i1}, \quad \sigma(j) = \sqrt{\frac{1}{N_j - 1}\sum_{i\in\mathcal{J}}(\beta_{i1} - \beta_1(j))^2}$$

As an example, in Figure 5, this aggregation method would imply using individual's C response to identify the effect between $\pi^e = [4,5)$; individuals A and C for $\pi^e = [5,6)$; individuals A, C and D for $\pi^e = [6,7)$; and individual B for $\pi^e = [7,8)$.

Figure 6 plots the aggregate relationship between planned behavior and inflation expectations, *ceteris paribus*. In each plot, I estimate $\beta_1(j)$ and plot 95% confidence bands. In line with previous findings, there is a positive relationship between inflation expectations and both intentions to search, as well as to bargain for a higher nominal wages.

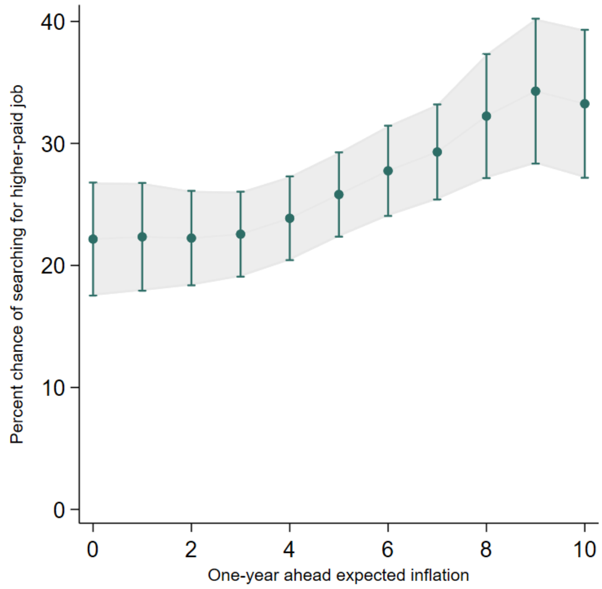


Figure 6: Average treatment effects

A.3 Inflation expectations and wage bargaining

The elicitation is done to the same individuals on the same date and session, and follows the same flow as described in the main text. The elicited objects are:

- Probability of asking current employer for a raise given inflation and unemployment expectations $P_{it}(Y_{it+3} = 1|\pi^e_{i0}, u^e_{i0}) \equiv W_i$
- Probability to ask for a pay raise conditional on higher expected inflation, keeping expected unemployment constant: $P_{it}(Y_{it+3} = 1|\pi^e_{i1}, u^e_{i0}) \equiv W^D_i$
- Probability to ask for a pay raise conditional on unemployment expectations u^e_1: $P_{it}(Y_{it+3} = 1|\pi^e_{i0}, u^e_{i1}) \equiv W^I_i$

The elicitation question is:

What is the percent chance that you will ask your employer for a pay increase over the next 3 months?

Table 14: Average treatment effects

	ATE: Inflation expectations		
	Direct (β_1)	Indirect ($\beta_2\gamma$)	Total ($\beta_1 + \beta_2\gamma$)
Wage bargaining	1.259***	0.139	1.399*
	(0.399)	(0.552)	(0.711)

Table 15 presents similar summary statistics for the reported percent chances of asking for a pay raise following the direct and indirect scenarios. As in the previous sub-section, a scenario where the Fed reports an expected 7% inflation rate shifts the median respondents' expectations under that scenario towards that value. In terms of wage setting behavior, the median percent chance of asking for a pay raise increases from 5 to 8%. The 90$^{\text{th}}$ percentile also increases from 55 to 58%. On average, the percent chance of asking for a pay raise increases from 17 to 18.4%. The ITE is small on average, with significant heterogeneity.

The indirect scenario shows the effects of a change in unemployment expectations equivalent to that caused by a 1 percentage point increase in inflation expectations. An equivalent change in unemployment expectations generates a small increase on average intentions to ask for a pay raise, from 17.04 to 17.29%. While the median intention increases from 5 to 8 percent, the right-tail of the distribution decreases from 55 to 50%. The extremes of the distribution of ITE are of slightly larger magnitudes than under the inflation expectations scenario.

Table 15: Individual Treatment Effects - wage bargaining

Direct scenario					
	π_0^e	π_1^e	W_i	W_i^D	β_{i1}
Mean	4.46	5.88	17.04	18.41	0.96
SD	4.48	4.07	25.68	25.58	7.26
p10	0	0	0	0	-3.12
p25	1	4	0	0	0
p50	4	7	5	8	0
p75	7	8	25	25	1.97
p90	10	10	55	58	6.67
Indirect scenario					
	u_0^e	u_1^e	W_i	W_i^I	$\beta_{i2}\gamma_i$
Mean	8.98	11.18	17.04	17.29	0.10
SD	9.61	10.38	25.68	23.58	9.93
p10	3	4	0	0	-3.71
p25	4	5	0	0	-0.14
p50	5	8	5	8	0
p75	10	13	25	25	0.83
p90	19	22	55	50	5

Table 16 shows that in response to a scenario where the Fed announces 7% inflation, 48% of respondents increase intentions to ask for a higher-paid job to their current employer (9 pp higher than search behavior). A scenario focusing instead on unemployment rates increases wage bargaining intention of 37% of respondents, with a larger share of null or negative treatment effects.

Table 16: Share of wage bargaining ITE, by sign

	Sign of ITE		
	Negative	Zero	Positive
Direct Scenario	0.20	0.31	0.48
Indirect Scenario	0.26	0.38	0.36

Participants also reported in the follow-up study actual wage bargaining behavior which can be contrasted to elicited behavior.

Have you asked your employer for a pay raise in the last 3 months?

The average 18.5 percent chance of asking employers for a pay raise, which aligns well with the fact that 17 percent of respondents reported having asked for a pay raise 3 months later. Figure 7 contrasts realized and planned behavior in more detail.

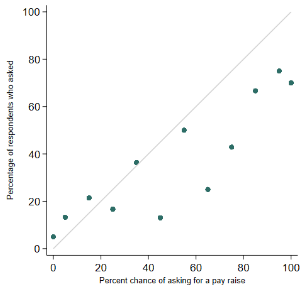


Figure 7: Wage bargaining: planned and realized behavior

Table 17: Zero individual treatment effects, by scenario

	Scenario	
	Inflation	Unemployment
Wage bargaining behavior:		
Zeros	187	157
of which: P(Raise) = 0	80.7%	75.8%
of which: P(Raise) = 100	2.7%	2.5%

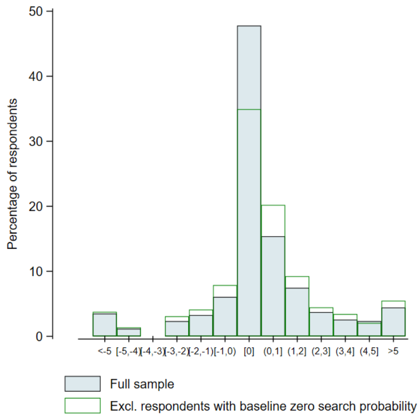
I elicit the main reasons for not asking for a pay raise for individuals with baseline zero percent chances of doing so. The most frequently mentioned reason by survey respondents is that their employer will not accept giving a pay raise (selected by 38% of respondents). A fifth of respondents mention their pay already automatically adjusts for changes in cost of living as the main reason for not asking for a pay raise. For 16% respondents, overall economic conditions are not favourable to asking; Finally, 17% respondents had already recently asked for a pay raise.

Table 18: Reasons for not asking for a pay raise

	Share	N
My employer will not accept it	38%	93
My pay automatically adjusts to changes in the cost of living	20%	49
I already asked for a pay increase recently	17%	41
The economy	16%	39
My pay is negotiated in collective bargaining	10%	23
Current work contract ending soon	2%	5
My partner recently had a pay increase	1%	2

A.4 Additional tables and figures

Rent scenario - individual treatment effects



Search

Figure 8: Individual effects of higher rent expectations on search

Joint unemployment and inflation expectations

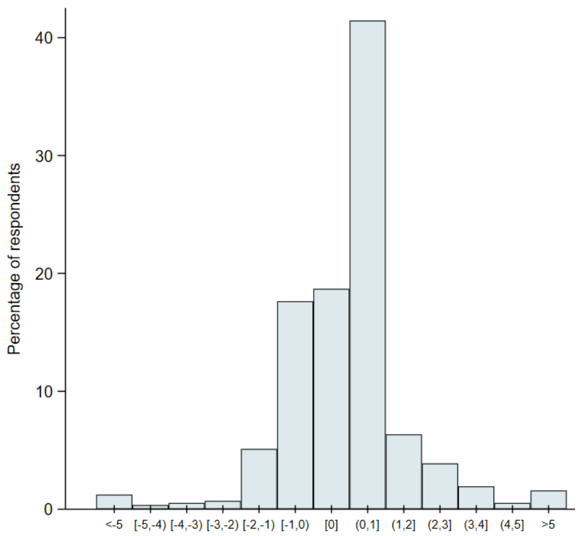


Figure 9: Individual percentage point changes in unemployment expectations in response to a 1 pp increase in inflation expectations

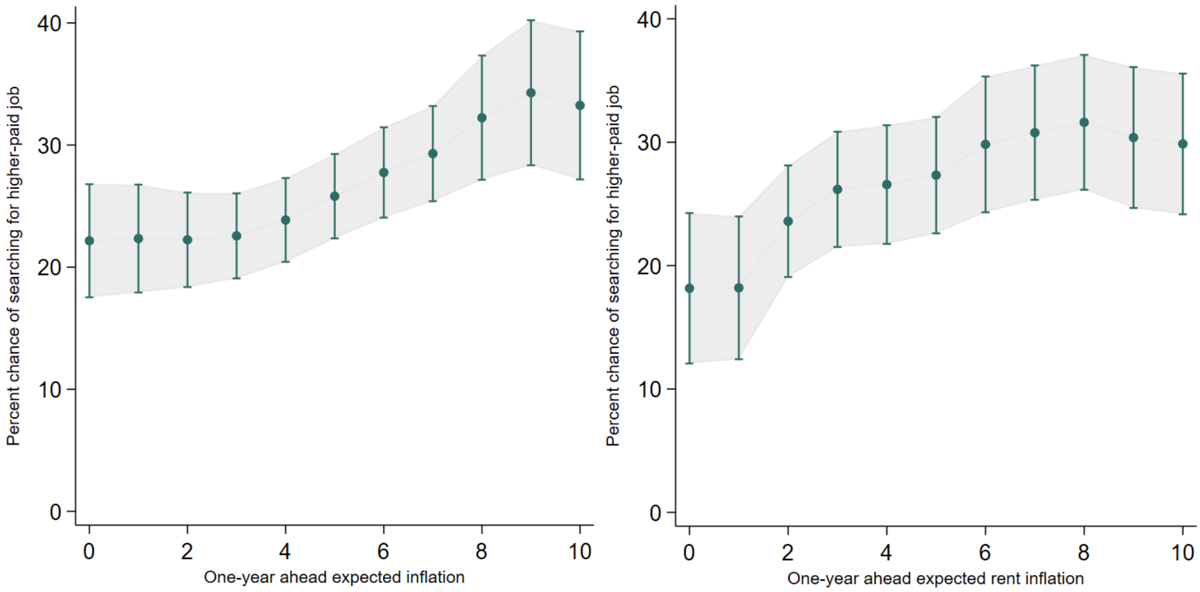


Figure 10: Search behavior and inflation expectations - general and rent inflation

B Model appendix

This appendix studies the choice of optimal search intensity through the lens of a New Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) model. The purpose of the model is to understand the theoretical mechanisms through which inflation expectations may affect on-the-job search, and to map these with the direct and indirect effects elicited in the survey. The model features search and matching frictions, as well as nominal price and wage rigidities. The model follows closely the works of Gertler et al. (2020) and Gertler et al. (2008). The former introduces idiosyncratic match quality and on-the-job search in a search and matching model with staggered wage bargaining. The latter estimates a New Keynesian DSGE with search and matching frictions, but does not consider on-the-job search. I first present the model extensively, but focus my analysis on the labour market block, which the main differentiating element and object of interest.

B.1 Model description

Environment

There are three types of agents in the economy: households, wholesale firms and retailers. Households consume, save and supply labour to wholesale firms. Wholesale firms hire labour through a search and matching process with idiosyncratic match quality $k \in \{g, b\}$ and produce intermediate goods sold at relative price x_t. Retailers operate under monopolistic competition, aggregate intermediate goods into a single final good, and sell this good to households. Retailers choose the price of the final good optimally, given Calvo nominal price rigidities. Furthermore, a monetary authority sets interest rates based on a Taylor rule and a government finances unemployment benefits through lump-sum taxes.

Timing

Consider the following intra-period timing of events: at the beginning of the period, shocks materialize. A share ρ of the workforce exogenously separates and can only search in the next period. Wholesale firms post vacancies, workers and firms meet, and matches form. Production takes place by firms with the workforce size and composition reflecting the outcomes from the search and matching process. Firms and workers bargain on the wage with a probability λ. Retailers produce and the household consumes and saves.

Households

There is a representative household with a continuum of family members of measure 1 and perfect consumption insurance (Merz (1995); Andolfatto (1996)). Within the household, a measure g_t is employed in good matches, b_t is employed in bad matches, and 1 - g_t - b_t collect unemployment benefits. The family pools all income and benefits before deciding how much to consume and to save. Households do not earn utility from leisure.

Let B_{t-1} be the beginning of period bonds, r_t the one-period nominal interest rate, p_t the nominal price level, Π_t lump-sum profits, T_t government transfers. There is a number l_t

of employed workers that is determined through a search and matching process, and $1-l_t$ unemployed family members. The value of the representative household is Ω_t:

$$\Omega_t = \max_{c_t, B_t} \{\log(c_t) + \beta \mathbb{E}_t \Omega_{t+1}\}$$

$$\text{s.t.} \quad c_t + \frac{B_t}{r_t p_t} + c(\zeta_t) = w_t g_t + \phi w_t b_t + (1 - g_t - b_t) u_B + \Pi_t + T_t + \frac{B_{t-1}}{p_t} \tag{c.1}$$

$$g_t = \delta_t^g g_{t-1} + \xi f_t s_t \tag{c.2}$$

$$b_t = \delta_t^b b_{t-1} + \xi \bar{\varphi}_t f_t s_t \tag{c.3}$$

With $\bar{\varphi}_t$ is the quality mix of bad to good hires in the economy and the total cost of search is given by $c(\zeta_t) = \frac{\zeta_0}{1+\eta_\zeta} \left(\zeta_g^{1+\eta_\zeta} g_{t-1} + \bar{\zeta}_{bt}^{1+\eta_\zeta} b_{t-1} \right)$, with $\bar{\zeta}$ the average search intensity of workers employed in bad matches. Given period employment l_t, the representative household chooses consumption c_t and savings B_t subject to a budget constraint (c.1) and to an employment accumulation equation for good and bad matches (c.2 and c.3, respectively).

Vacancies, match quality and hires

The search and matching process is analogous to Gertler et al. (2020), with the important difference that new hires start working within the period. Match quality is idiosyncratic, with good matches occurring with probability ξ and bad matches with probability 1-ξ. Bad matches produce a fraction ϕ of the output produced by good matches. At the beginning of period t, the existing labour force in efficiency units of firm i is:

$$l_{it-1} = g_{it-1} + \phi b_{it-1} = (1+\phi) g_{it-1} \tag{3}$$

Firms post vacancies, workers search, and they match randomly. Workers in bad matches search on-the-job with intensity ζ_{bt} to improve match quality.Workers in good matches face probability ζ_g to search on-the-job for any other match, allowing for relocation shocks. I normalize the search intensity of unemployed workers to 1. After meeting and randomly drawing match quality, firms and workers decide whether or not to actually form a match (i.e. a hire).
Let $g_{t-1} = \int g_{it-1} di$ and $b_{t-1} = \int b_{it-1} di$ denote the total of good and bad matches in the economy in the beginning of period t. Then, the number of unemployed workers in the beginning of the period is:

$$u_{t-1} = 1 - l_{t-1} \tag{4}$$

The total efficiency units of search in the labour market at a given time t is defined as:

$$s_t = u_{t-1} + (1-\rho)(\zeta_g g_{t-1} + \zeta_{bt} b_{t-1}) \tag{5}$$

Where $(1-\rho)$ is the probability that the match is not destroyed, and employed workers in good and bad matches are weighted by their respective search intensities. The matching process in the economy is described as:

$$m_t = \sigma_m s_t^{\sigma} v_t^{1-\sigma} \tag{6}$$

Where σ denotes elasticity of matches to units of search effort, σ_m the match efficiency and v_t the total number of vacancies posted.

The job-finding probability per unit of search intensity in the economy is:

$$f_t = \frac{m_t}{s_t} \tag{7}$$

The probabilities of finding a good and bad match are, respectively:

$$f_t^g = \xi f_t \qquad\qquad f_t^b = (1-\xi) f_t \tag{8}$$

Similarly, a firm that posts a vacancy will meet workers with probability q_t^m:

$$q_t^m = \frac{m_t}{v_t} \tag{9}$$

The probabilities of a posted vacancy resulting in bad and a good hire are, respectively:

$$q_t^b = (1-\xi)\left(1-(1-\rho)\frac{\zeta_{bt} b_{t-1}}{s_t}\right) q_t^m \qquad\qquad q_t^g = \xi q_t^m \tag{10}$$

These job-filling probabilities account for the fact that hires are conditional on realized match quality.[12] When firms post vacancies, the expected hires in efficiency units are:

$$q_t = q_t^g + \phi q_t^b \tag{11}$$

The number of hires in the economy in a given period are equal to $q_t v_t$.

Firms

Each period, wholesale firms produce output using only labor as an input. The ratio of bad-to-good matches in firm i, $\varphi_{it} \equiv b_{it}/g_{it}$, describes the quality mix of the firm's total labor force. The stock of labor in efficiency units is:

$$l_{it} = (1 + \phi \varphi_{it}) g_{it} \tag{12}$$

The number of good and bad matches in the firm evolves through retentions - workers who do not separate from the firm into unemployment or into another match - and new hires. The laws of motion for good and bad matches in firm i are:

[12] While all good matches lead to hires, bad matches only lead to hires if the worker is either unemployed or employed in a good match and forced to search, which occurs with probability ζ_g.

$$g_{it} = (1-\rho)(1-\zeta_g f_t)g_{it-1} + q_t^g v_{it} \equiv \delta_t^g g_{it-1} + q_t^g v_{it} \tag{13}$$
$$b_{it} = (1-\rho)(1-\zeta_{bt} f_t^g)b_{it-1} + q_t^b v_{it} \equiv \delta_t^b b_{it-1} + q_t^b v_{it} \tag{14}$$

Matches separate exogenously with probability ρ. Retention probabilities in good and bad matches are denoted by $\delta_t^k, k \in \{g,b\}$. Their convex combination yields the firm's retention probability of a unit of labour in efficiency units:

$$\delta_{it} = \frac{\delta_t^g + \phi\varphi_{it-1}\delta_t^b}{1+\phi\varphi_{it-1}} \tag{15}$$

The firm's hiring rate is measured with respect to its existing workforce at the beginning of time t, l_{it-1}:

$$\varkappa_{it} = \frac{q_t v_{it}}{l_{it-1}} \tag{16}$$

The employment stock in firm i at the end of period t is:

$$l_{it} = (\delta_{it} + \varkappa_{it})l_{it-1} \tag{17}$$

Let x_t be the relative price of intermediate goods, w_{it} the nominal wage per efficiency unit of labour at firm i, a_t the productivity per efficiency unit of labour ($a_t = \frac{1-\zeta}{l_t}$),[13] $\Lambda_{t,t+1} \equiv \beta\lambda_{t+1}/\lambda_t = \beta u'(c_{t+1})/u'(c_t)$ the firm's discount rate. Firms use their employment stock l_{it} to produce output $y_{it} = a_t l_{it}$. They add labour by choosing the hiring rate for labour in efficiency units, $\varkappa_{it}$. Hiring activities involve costs that are assumed to be quadratic in the hiring rate ($\varkappa_{it}$) and linear in the existing stock of employment at the time of hiring (l_{it-1}).

The firm value F is homogeneous in the stock of labour, due to the assumption of constant returns. The firm decides the hiring rate to maximize expected discounted profits subject to the laws of motion for labor (l_{it}) and quality mix (φ_{it}), taking as given the expected path of wages w_t. The value of each firm is:

$$F_t(l_{it-1},\varphi_{it-1},w_{it}) = \max_{\varkappa_{it},l_{it}} \left\{ x_t a_t l_{it} - \frac{\kappa}{2}\varkappa_{it}^2 l_{it-1} - \frac{w_{it}}{p_t} l_{it} + E_t\Lambda_{t,t+1}F_{t+1}(l_{it},\varphi_{it},w_{it+1}) \right\} \tag{18}$$

subject to:

$$\varphi_{it} = \frac{\delta_t^b\varphi_{it-1} + q_t^b v_{it}/g_{it-1}}{\delta_t^g + q_t^g v_{it}/g_{it-1}} = \frac{\delta_t^b \frac{\varphi_{it-1}}{1+\phi\varphi_{it-1}} + \frac{\bar{\varphi}_t^h}{1+\phi\bar{\varphi}_t^h}\varkappa_{it}}{\delta_t^g \frac{1}{1+\phi\varphi_{it-1}} + \frac{1}{1+\phi\bar{\varphi}_t^h}\varkappa_{it}}$$
$$l_{it} = (\delta_{it} + \varkappa_{it})l_{it-1}$$

[13] The productivity per efficiency unit of labour is independent of the firm and *its logarithm* is assumed to follow an autoregressive process with persistence parameter ρ_a and standard deviation σ_a

The firm's hiring decision yields:

$$\kappa \varkappa_{it} l_{it-1} = x_t a_t l_{it-1} - \frac{w_{it}}{p_t} l_{it-1} + E_t \Lambda_{t,t+1} \frac{\partial F_{t+1}}{\partial \varkappa_{it}}$$

Now, note that $F_{t+1} \equiv F_{t+1}(l_{it}(\varkappa_{it}), \varphi_{it}(\varkappa_{it}), w_{it+1})$. Therefore:

$$\frac{\partial F_{t+1}}{\partial \varkappa_{it}} = \frac{\partial F_{t+1}}{\partial l_{it}} \frac{\partial l_{it}}{\partial \varkappa_{it}} + \frac{\partial F_{t+1}}{\partial \varphi_{it}} \frac{\partial \varphi_{it}}{\partial \varkappa_{it}} = \frac{\partial F_{t+1}}{\partial l_{it}} l_{it-1} + \frac{\partial F_{t+1}}{\partial \varphi_{it}} \frac{\partial \varphi_{it}}{\partial \varkappa_{it}}$$

Using that:

$$\frac{\partial F_t(l_{it-1}, \varphi_{it-1}, w_{it})}{\partial l_{it-1}} = \frac{\kappa}{2} \varkappa_{it}^2 + \kappa \delta_{it} \varkappa_{it}$$

We have:

$$\kappa \varkappa_{it} l_{it-1} = x_t a_t l_{it-1} - \frac{w_{it}}{p_t} l_{it-1} + E_t \Lambda_{t,t+1} \left[\frac{\kappa}{2} \varkappa_{it+1}^2 + \kappa \delta_{it+1} \varkappa_{it+1} \right] l_{it-1} + E_t \Lambda_{t,t+1} \frac{\partial F_{t+1}}{\partial \varphi_{it}} \frac{\partial \varphi_{it}}{\partial \varkappa_{it}}$$

Where the second term reflects the composition effect of hiring in the future value of the firm. I follow the assumption in Gertler et al. (2020) that firm retention of good and bad matches is identical in the steady-state. As a result, the steady-state search intensity of workers in good matches will be a fraction ξ of that of workers in bad matches i.e. $\xi \zeta_{bt} = \zeta_g$. Under that assumption, it can be shown that $\frac{\partial F_{t+1}}{\partial \varphi_{it}} = 0$.[14] This simplifying assumption lends considerable tractability to the model and allows me to focus on the link between inflation expectations, search intensity and hiring that are not driven by quality composition adjustment motives of the firm.

Given the assumption of constant returns to scale, the firm's hiring decision can be expressed as:

$$\kappa \varkappa_{it}(\varphi_{it}, w_{it}) = x_t a_t - \frac{w_{it}}{p_t} + E_t \Lambda_{t,t+1} \left[\frac{\kappa}{2} \varkappa_{it+1}^2 + \kappa \delta_{it+1} \varkappa_{it+1} \right]$$

In order to define the wage bargaining process, let the value of the firm of hiring one additional efficiency unit of labour at time t *after* workers have already joined the firm at time t. This is, define this value net of current adjustment costs and taking hiring and worker composition as given, $J_t(\varphi_{it}, w_{it})$. We have that:

$$J_t(\varphi_{it}, w_{it}) \equiv \frac{\partial (F_t(l_{it-1}, \varphi_{it-1}, w_{it}) + \frac{\kappa}{2} \varkappa_{it}^2 l_{it-1})}{\partial l_{it}} = x_t a_t - \frac{w_{it}}{p_t} + E_t \Lambda_{t,t+1} \frac{\partial F_{t+1}}{\partial l_{it}}$$

Where the last term is simply:

[14] Crucially, it holds that $\delta + \varkappa = 1$ in the steady state. See Appendix of Gertler et al. (2020) for further details

$$\frac{\partial F_{t+1}}{\partial l_{it}} = \frac{\partial F_{t+1}}{\partial l_{it}} + \frac{\partial F_{t+1}}{\partial l_{it+1}}\frac{\partial l_{it+1}}{\partial l_{it}} = -\frac{\kappa}{2}\varkappa_{it+1}^2 + (\delta_{it+1} + \varkappa_{it+1})J_{t+1}(\varphi_{it+1}, w_{it+1})$$

This value, which corresponds to the firm's surplus, will consist of average expected profits per efficiency unit of labour:

$$J_t(\varphi_{it}, w_{it}) = x_t a_t - \frac{w_{it}}{p_t} - E_t\Lambda_{t,t+1}\frac{\kappa}{2}\varkappa_{it+1}^2 + E_t\Lambda_{t,t+1}(\delta_{it+1} + \varkappa_{it+1})J_{t+1}(\varphi_{it+1}, w_{it+1})$$

Workers

The value functions for unemployed workers, workers in good matches and workers in bad matches are defined in units of efficient labour. Workers in bad matches search with variable search intensity. The cost of search intensity for each match quality $k \in \{g, b\}$ is:

$$c(\zeta_{kt}) = \frac{\zeta_0}{1+\eta_\zeta}{\zeta_{kt}}^{1+\eta_\zeta} \tag{19}$$

Where η_ζ is the search cost elasticity and ζ_0 is a scale parameter. Let the average value of being employed in a match of quality k at the end of period t be:

$$\bar{V}_t^k = \int V_t^k(w_{it}^k, \varphi_{it})dG_t(w, \varphi) \tag{20}$$

In the beginning of period t, workers in bad matches optimally choose search intensity ζ_{bt}, taking as given the end of period wages and composition. Workers in bad matches may upgrade to a good match, with average value $\bar{V}_t^g$, or continue in their current match, with value $V_t^b(\varphi_{it}, w_{it})$. Workers in good matches do not choose their search effort, they simply observe the shock ζ_g in the beginning of the period. The value of search for each match quality k is:[15]

$$S_t^b = \max_{\zeta_{bt}}\left\{\zeta_{bt}f_t^g\bar{V}_t^g + (1-\zeta_{bt}f_t^g)V_t^b - c(\zeta_{bt})\right\} \tag{21}$$

$$S_t^g = (1-\zeta_g f_t)V_t^g + \zeta_g(f_t^g\bar{V}_t^g + f_t^b\bar{V}_t^b) - c(\zeta_g) \tag{22}$$

Workers in bad matches will choose, for each given (φ_{it}, w_{it}), the search intensity that equalizes the marginal benefit and cost of one additional search unit:

$$\zeta_0\zeta_{bt}^{\eta_\zeta} = f_t^g\left(\bar{V}_t^g - V_t^b\right) \tag{23}$$

[15] Recall that workers and firms match, but not all matches lead to hires. I express the values of search, employment and unemployment in terms of the average unconditional values of good and bad matches and refer to Gertler et al. (2020, 2008); Gertler and Trigari (2009) for first-order equivalence results between values of matches conditional and unconditional on being hired.

Consider the value of being employed in a bad and a good match at time t. These values are defined after hires take place and are net of current search costs. The value of a worker hired in a bad match employed in firm i is:

$$V_t^b \equiv V_t^b(\varphi_{it}, w_{it}^b) = \left\{ \frac{w_{bt}}{p_t} + E_t \left\{ \Lambda_{t,t+1} \left[(1-\rho) S_{t+1}^b + \rho U_{t+1} \right] \right\} \right\} \tag{24}$$

The value of a worker hired in a good match will be:

$$V_t^g \equiv V_t^g(\varphi_{it}, w_{it}^g) = \left\{ \frac{w_{gt}}{p_t} + E_t \left\{ \Lambda_{t,t+1} \left[(1-\rho) S_{t+1}^g + \rho U_{t+1} \right] \right\} \right\} \tag{25}$$

The value of an unemployed worker is:

$$U_t = u_B + E_t \left[\Lambda_{t,t+1} \left[f_{t+1}^g \bar{V}_{t+1}^g + f_{t+1}^b \bar{V}_{t+1}^b + (1 - f_{t+1}) U_{t+1} \right] \right] \tag{26}$$

I now define the match surplus, which will be essential to characterize the contract wage. The match surplus for a workers in a good and bad matches will be, respectively:

$$H_t \equiv H_t(\varphi_{it}, w_{it}) = V_t^g - U_t \tag{27}$$

$$H_t^b \equiv H_t^b(\varphi_{it}, w_{it}) = V_t^b - U_t \tag{28}$$

Wage-setting

There is staggered Nash wage bargaining, where wage contracts are negotiated in *nominal* terms. Define $\pi_t = p_t/p_{t-1}$. Let w denote the nominal wage per unit of labour quality. As such, w will be the nominal wage of a good match, and ϕw the nominal wage of a bad match. For simplicity, I follow Gertler et al. (2020) and assume that wage bargaining is led by workers in good matches, and workers in bad matches simply reap the quality-adjusted bargained nominal wage.

Every period, there is a probability $1-\lambda$ to renegotiate the wage. In other words, each period there is a share λ of firms that cannot renegotiate the contract wage. Firms who cannot renegotiate wages will index wages to past inflation according to the following rule:

$$w_{it} = \pi_t^\gamma w_{it-1}, \quad \gamma \in (0,1) \tag{29}$$

Let w_{it}^* denote the nominal wage per unity of labour quality chosen if wage renegotiations are possible at time t. Let H_t and J_t denote the match surplus for workers employed in good matches and for firms, as defined in the previous sub-section. The optimally renegotiated wage w_{it}^* is chosen to maximize the Nash product of one unit of labour quality to a firm and a worker in a good match:

$$\max_{w_{it}^*} \quad H_t^\eta(w_{it}) J_t^{1-\eta}(w_{it}) \tag{30}$$

subject to:

$$w_{it+j} = \begin{cases} w_{it+j-1}\pi_{t+j-1}^{\gamma}, \text{ with probability } \lambda \\ w_{it+j-1}^{*}, \text{ with probability } 1-\lambda \end{cases}, \quad \text{for every } t+j, j \geq 1.$$

The expected duration of an optimally chosen contract wage will be $\frac{1}{1-\lambda}$, where λ is a measure of nominal wage stickiness in the economy. The first order condition is:

$$\chi_t^* J_t^* = (1-\chi_t^*) H_t^* \tag{31}$$

$$\chi_t^* = \frac{\eta}{\eta + (1-\eta)\mu_t^*/\epsilon_t^*} \tag{32}$$

Where $\epsilon_t^* = p_t \partial H_t^* / \partial w_{it}^*$ and $\mu_t(\varphi_{it}, w_{it}) = -p_t \partial J_t(\varphi_{it}, w_{it}^*)/\partial w_{it}^*$ will be the effects of a rise in the real wage on the worker and firm surpluses, respectively. I express ϵ_t and μ_t recursively as:

$$\epsilon_t = 1 + (1-\rho)\lambda E_t \Lambda_{t,t+1}(1-\zeta_g f_{t+1})\pi_t^{\gamma}\frac{p_t}{p_{t+1}}\epsilon_{t+1} + \mathcal{O}_1 \tag{33}$$

$$\mu_t(w_{it}^*) = 1 + \lambda E_t \Lambda_{t,t+1}\pi_t^{\gamma}\frac{p_t}{p_{t+1}}\mu_{t+1}(\pi_t^{\gamma} w_{it}^*) + \mathcal{O}_2 \tag{34}$$

Where $\mathcal{O}_1$ and $\mathcal{O}_2$ are composition terms that are equal to zero up to a first order in the steady-state. Average nominal wages per unit of labour quality are defined as:

$$w_t = \int_{w,\varphi} w_{it} dG_t(\varphi, w)$$

To a first order approximation, the evolution of average nominal wages per unit of labour quality is equal to a linear combination of the target nominal wage contract and last period's nominal wages partially adjusted to inflation:

$$w_t = (1-\lambda)w_t^* + \lambda \int_{w,\varphi} (\pi_t^{\gamma} w_{it-1}) dG_{t-1}(\varphi, w)$$

Retailers and price-setting

There is a measure one of monopolistic competitive retailers indexed by j. These retailers repackage goods produced by wholesalers into intermediate goods, and price them at nominal p_{jt}. Intermediate goods y_{jt} are purchased by a competitive final goods sector, that aggregates them into a final good y_t, taking the final price p_t as given.

$$y_t = \left[\int_0^1 y_{jt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{35}$$

where ε_p denotes the elasticity of substitution between different varieties. The demand facing retailer j will be:

$$y_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon_p} y_t, \text{ with } p_t = \left(\int_0^1 p_{jt}^{1-\varepsilon_p} dj\right)^{\frac{1}{1-\varepsilon_p}}$$

Retailers are price-setters in this economy. They purchase intermediate goods at relative price x_t and choose p^*_{jt} to maximize expected discounted future profits. There are Calvo-style nominal price rigidities: every period, retailers have a probability $1-\psi$ of choosing p^*_{jt}. Formalizing the optimization problem of retailer j:

$$\begin{aligned} \max_{p_{jt}} \quad & \Pi_t = \left(\frac{p_{jt}}{p_t} - x_t\right) y_{jt} + \psi E_t \Lambda_{t,t+1} \Pi_{t+1} \\ s.t. \quad & y_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon_p} y_t \end{aligned}$$

As can be seen from the first order condition, the optimal price p^*_{jt} will be a function of $p_t x_t$, the nominal marginal cost faced by the retailers. This nominal marginal cost is simply the relative nominal price of the wholesale good.

From the hiring equation of wholesale firms, we can express the real marginal cost faced by retailers (x_t) as a function of real wages and of current and expected future discounted hiring costs.[16]

$$x_t = \frac{1}{a_t}\left[\kappa \varkappa_{it}(\varphi_{it}, w_{it}) + \frac{w_{it}}{p_t} - E_t \Lambda_{t,t+1}\left[\frac{\kappa}{2}\varkappa_{it+1}^2 + \kappa \delta_{it+1}\varkappa_{it+1}\right]\right]$$

In this environment, the aggregate price level will be:

$$p_t = (1-\psi)p_t^* + \psi p_{t-1} \tag{36}$$

Monetary policy There is a monetary authority that sets rates based on the following Taylor rule:

$$r_t = r\,(\pi_t)^{\phi}\, e^{\varepsilon_t^m} \tag{37}$$

Where ε_t^m is a monetary policy shock.

Government The government finances unemployment benefits through lump-sum transfers:

$$T_t + (1 - g_t - b_t)u_B = 0 \tag{38}$$

Resource constraint Aggregate output is equal to the total of resources allocated towards consumption, vacancy posting costs and search costs:

$$y_t = c_t + \frac{\kappa}{2}\int_0^1 \varkappa_t^2 l_{it-1} di + \frac{\zeta_0}{1+\eta_\zeta}(1-\rho)\left({\zeta_n}^{1+\eta_\zeta} g_t + \zeta_{bt}^{1+\eta_\zeta} b_t\right) \tag{39}$$

[16]See **?** for discussion on how search and matching frictions change the nature of the real marginal cost through use of labor.

The role of prices How do current prices p_t affect optimal search? From the optimal hiring condition, we can observe that an increase in p_t reduces unit labour costs $\frac{w_{it}}{p_t}$ and increases $\varkappa_{it}$. As a result, firms post more vacancies, f_t increases, and the probability of finding a good match $f_t^g = \xi f_t$ also increases. To assess how prices affect the flow value of improving match quality i.e. $\bar{V}_t^g - V_t^b$, note that we can write it as:

$$\bar{V}_t^g - V_t^b = \int \frac{w_{it}^g}{p_t} dG_t(w,\varphi) - \frac{w_{it}^b}{p_t} + E_t\left[\Lambda_{t,t+1}\left[(1-\rho)\bar{S}_{t+1}^g - S_{t+1}^b\right]\right] \tag{40}$$

$$\bar{S}_{t+1}^g = \bar{V}_{t+1}^g + f_t^b \bar{V}_{t+1}^b - c(\zeta_g) \tag{41}$$

$$S_{t+1}^b = \max_{\zeta_{bt+1}}\left\{\zeta_{bt+1} f_{t+1}^g \bar{V}_{t+1}^g + (1-\zeta_{bt+1} f_{t+1}^g) V_{t+1}^b - c(\zeta_{bt+1})\right\} \tag{42}$$

From (21) we see that higher p_t erodes the difference in flow gains from upgrading match quality $\int \frac{w_{it}^g}{p_t} dG_t(w,\varphi) - \frac{w_{it}^b}{p_t}$

B.2 Expected inflation, hiring and search intensity

To deepen the intuition about the mechanisms through which expected inflation affects on-the-job search, I derive log-linear equations that describe search and hiring dynamics. A more detailed account of these results is available in the following sections. Let $\hat{y}_t$ denote the log-deviation of any variable y_t at time t from its steady state value $\bar{y}$.

The loglinearized surplus for a worker in a good match is:

$$\begin{aligned}
\hat{\hat{H}}_t = {} & \frac{\bar{w}^r}{\bar{H}}(\hat{w}_t^{*r} + \lambda\beta\delta\epsilon E_t(\hat{w}_t^{*r} + \gamma\hat{\pi}_t - \hat{\pi}_{t+1} - \hat{w}_{t+1}^{*r}) + \\
& \beta(1-\rho-\delta)\frac{\bar{H}^a}{\bar{H}} E_t(\hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^a) - f\beta\frac{\bar{H}^a}{\bar{H}} E_t(\hat{f}_{t+1} + \hat{H}_{t+1}^a + \hat{\Lambda}_{t,t+1}) + \\
& + \beta(\delta - 1 - \rho)(1 - \frac{\bar{H}^a}{\bar{H}}) E_t\hat{f}_{t+1} + \beta\delta E_t(\hat{\Lambda}_{t,t+1} + \hat{H}_{t+1})
\end{aligned}$$

Where $\epsilon = \partial H/\partial w$ evaluated at the steady state, w^r denotes *real* wages, H^a denotes the surplus in *all* matches (good and bad) and f_{t+1} denotes the probability of finding *any* match. Focusing on the first term, we can observe that surplus of workers in good matches *decreases* with expected inflation.

Similarly, the surplus of workers employed in bad matches is:

$$\begin{aligned}
\hat{H}_t^b = {} & \frac{\phi\bar{w}^r}{H^b}(\hat{w}_t^{*r} + \beta\lambda\delta\epsilon E_t(\gamma\hat{\pi}_t + \hat{w}_t^{*r} - \hat{\pi}_{t+1} - \hat{w}_{t+1}^{*r})) + \chi_\Lambda E_t\hat{\Lambda}_{t,t+1} + \chi_f E_t\hat{f}_{t+1} \\
& + \delta\beta E_t\hat{H}_{t+1}^b(w_{t+1}^*) + \beta(1-\rho-\delta)\frac{\bar{H}}{H^b} E_t\hat{\hat{H}}_{t+1} - \beta f\frac{\bar{H}^a}{H^b} E_t(\hat{\hat{H}}_{t+1}^a)
\end{aligned}$$

Where χ_Λ and χ_f are functions of the model's primitives and are reported extensively in the next subsections. As expected, expected inflation decreases the surplus of workers in bad matches as well.

The loglinearized optimal search intensity for a worker in a bad match is:

$$\eta\hat{\zeta}_{bt}(w_t^*) = \frac{(\bar{H}\hat{\bar{H}}_t - H^b\hat{H}_t^b(w_t^*))}{\bar{H} - H^b} + \hat{f}_t^g$$

Where $\bar{H} - H^b$ is the steady-state difference in the surplus from a good and a bad match and $\hat{f}_t^g$ denotes log-deviations from the steady-state probability of finding a good match. Search intensity of workers in bad matches will be increasing in the surplus of good matches relative to bad matches, as well as in the probability of finding a good match. Recall that:

$$\hat{f}_t^g = \hat{f}_t = \hat{m}_t - \hat{s}_t$$

The loglinearized hiring rate is:

$$\hat{\varkappa}_t(w_t^*) = \frac{xa}{\kappa\bar{\varkappa}}(\hat{x}_t + \hat{a}_t) - \frac{\bar{w}^r}{\kappa\bar{\varkappa}}\hat{w}_t^r + \beta\lambda E_t(\hat{\varkappa}_{t+1}(\pi_t^\gamma w_t^*) - \hat{\varkappa}_{t+1}(w_{t+1}^*)) + \frac{\beta}{2}(1+\delta)E_t\hat{\Lambda}_{t,t+1} +$$
$$\beta\delta\lambda E_t(\hat{\delta}_{t+1}(\pi_t^\gamma w_t^*) - \hat{\delta}_{t+1}(w_{t+1}^*)) + \beta\delta E_t(\hat{\delta}_{t+1}(w_{t+1}^*)) + \beta E_t(\hat{\varkappa}_{t+1}(w_{t+1}^*))$$

Deviations from steady-state hires are driven by expected future changes in the firm's labor force, through hires and retentions. These, in turn, can be expressed recursively, so that:

$$\hat{\varkappa}_t(w_t^*) = \frac{xa}{\kappa\bar{\varkappa}}(\hat{x}_t + \hat{a}_t) - \frac{\bar{w}^r}{\kappa\bar{\varkappa}}\hat{w}_t^r - \frac{\bar{w}}{\kappa\bar{\varkappa}}\mu E_t(\gamma\hat{\pi}_t + \hat{w}_t^{*r} - \hat{\pi}_{t+1} - \hat{w}_{t+1}^{*r}) +$$
$$+ \frac{1-\rho-\delta}{\delta\eta}\frac{\phi\bar{w}^{*r}}{\bar{H} - H^b}\epsilon E_t(\gamma\hat{\pi}_t + \hat{w}_t^{*r} - \hat{\pi}_{t+1} - \hat{w}_{t+1}^{*r})$$
$$+ \frac{\beta}{2}(1+\delta)E_t\hat{\Lambda}_{t,t+1} + \beta\delta E_t(\hat{\delta}_{t+1}(w_{t+1}^*)) + \beta E_t(\hat{\varkappa}_{t+1}(w_{t+1}^*))$$

On the one hand, higher expected future inflation increases current period hires ($\frac{\bar{w}\mu}{\kappa\varkappa}$). As can be seen by the fourth term of the equation above, this positive effect is however tapered by the changes in expected retention of matches, which depend on η (the cost of search elasticity).

Discounting and expected inflation An increase in expected inflation in $t+1$ decreases the marginal utility of consumption at t. Recall that the discount rate of firms in the model is $\Lambda_{t,t+1} \equiv \beta u'(c_{t+1})/u'(c_t)$, with $u'(c_t) = r_t\beta E_t\left(\frac{\lambda_{t+1}}{\pi_{t+1}}\right)$. Consequently, an increase in expected inflation will *increase* the discount rate of firms and workers use in the labor market to make search and hiring decisions. Firms discount more heavily the hiring costs they will have to pay on their next period labor force - this in turn, affects hiring decisions in the present period. Workers will discount more heavily future possible upgrades in match quality, even though inflation *erodes* the difference in flow values from good and bad matches.
In this appendix, I follow GHT20 closely and show that, under the assumption of equal retention of good and bad quality matches in the steady-state, there is no composition effect in nominal wage setting. The proof follows the structure of the Appendix of GHT20, and is established by using a set of auxiliary results, which I also prove. The appendix is structured as follows: first, I present the set of auxiliary results which allow me to prove the lack of composition effect; then, I present two results that allow me to conveniently express the average surplus as

approximately the surplus in a firm with an average wage and average composition; Finally, I derive the loglinearized equations of the labor market block of the model. Note that, while my model and results build from GHT20, it differs from the authors' model in important ways, namely *nominal* wage rigidity, model timing and the existence of within-period hires. As such, it is not straightforward that their results carry over to my setting, which justifies the need for derivations.

B.3 The average contract wage

To derive the average contract wage through the Nash Bargaining first order condition, I need the average firm surplus and the average workers' surplus in good matches in renegotiating firms. While the wage is negotiated by workers in good matches, the contract wage will depend on average retention rates, which hinge on search intensity of workers in bad matches. This, in turn, depends on the surplus of the workers employed in these bad matches.

Take a firm i renegotiating wages. With probability λ, next period's nominal wage w_{it+1} will be equal to this period's wage adjusted for inflation $\pi_t^\gamma w_{it}^*$. With probability $1-\lambda$, it will be equal to next period's target nominal wage w_{it+1}^*. The Nash bargaining condition (recall, w^* is nominal) will be:

$$\begin{aligned}\chi_t^* J_t^*(\varphi_{it}, w_{it}^*) &= (1-\chi_t^*) H_t^*(\varphi_{it}, w_{it}^*) \\ \chi_t^* &= \frac{\eta}{\eta + (1-\eta)\mu_t^*/\epsilon_t^*}\end{aligned}$$

With:

$$\begin{aligned}\epsilon_t(\varphi_{it}, w_t^*(\varphi_{it})) &= p_t \frac{\partial H_t(\varphi_{it}, w_{it}^*(\varphi_{it}))}{\partial w_{it}^*(\varphi_{it})} \\ \mu_t(\varphi_{it}, w_t^*(\varphi_{it})) &= -p_t \frac{\partial J_t(\varphi_{it}, w_{it}^*(\varphi_{it}))}{\partial w_{it}^*(\varphi_{it}))}\end{aligned}$$

In particular, we can show that these can be written recursively as:

$$\begin{aligned}\epsilon_t &= 1 + (1-\rho)\lambda E_t \Lambda_{t,t+1}(1-\zeta_g f_{t+1})\pi_t^\gamma \frac{p_t}{p_{t+1}} \epsilon_{t+1} + \mathcal{O}_1 \\ \mu_t(w_t^*) &= 1 + \lambda E_t \Lambda_{t,t+1}[\delta_{it+1} + \varkappa_{it+1}]\pi_t^\gamma \frac{p_t}{p_{t+1}} \mu_{t+1}(\pi_t^\gamma w_t^*) + \mathcal{O}_2\end{aligned}$$

Where $\mathcal{O}_1$ and $\mathcal{O}_2$ are composition terms which will be zero in the steady-state.

The goal is to solve for the **average** contract wage w_t^*. In order to solve for the contract wage, we need the log-linearized expressions for the average firm and worker surplus. I drop the subscript i to refer to a firm with average wage and average composition. First, the log-linearized Nash bargaining condition for a renegotiating firm with average composition is:

$$\hat{J}_t(\varphi_t, w_t^*) + (1-\chi)^{-1}\hat{\chi}_t(w_t^*) = \hat{H}_t(\varphi_t, w_t^*)$$

B.3.1 Average worker surplus in good matches

$$\begin{aligned}
&H_t(\varphi_t, w_t^*) = \frac{w_t^*}{p_t} - u_B + \\
&E_t\Lambda_{t,t+1}\Big[(1-\rho)(1-\zeta_g f_{t+1})\Big[\lambda H_{t+1}(\varphi_{it+1}, \pi_t^\gamma w_t^*) + (1-\lambda)H_{t+1}(\varphi_{it+1}, w_{it+1}^*)\Big]\Big] + \\
&+ E_t\Lambda_{t,t+1}\Big[(1-\rho)\zeta_g f_{t+1}\bar{H}_{t+1}^a - f_{t+1}\bar{H}_{t+1}^a\Big]
\end{aligned}$$

Which can be re-written as:

$$\begin{aligned}
&H_t(\varphi_t, w_t^*) = \frac{w_t^*}{p_t} - u_B \\
&+ \lambda E_t\Lambda_{t,t+1}\Big[(1-\rho)(1-\zeta_g f_{t+1})\Big]\Big[H_{t+1}(\varphi_{it+1}, \pi_t^\gamma w_t^*) - H_{t+1}(\varphi_{it+1}, w_{it+1}^*)\Big] \\
&+ E_t\Lambda_{t,t+1}[(1-\rho)(1-\zeta_g f_{t+1})]H_{t+1}(\varphi_{it+1}, w_{it+1}^*) \\
&+ E_t\Lambda_{t,t+1}\Big[(1-\rho)\zeta_g f_{t+1}\bar{H}_{t+1}^a - f_{t+1}\bar{H}_{t+1}^a\Big]
\end{aligned}$$

Using the definition of retention rate $\delta_{t+1}^g = (1-\rho)(1-\zeta_g f_t)$, log-linearizing and using w^r to express the *real* wage:

$$\begin{aligned}
&\bar{H}\hat{H}_t = \bar{w}^r\hat{w}_t^r + \\
&\lambda\beta\bar{\delta}^g\bar{H}E_t(\hat{H}_{t+1}(\pi_t^\gamma w_{it}^*) - \hat{H}_{t+1}(w_{t+1}^*)) + \\
&+ \beta\bar{\delta}^g\bar{H}E_t(\hat{\Lambda}_{t,t+1} + \hat{\delta}_{t+1}^g + \hat{H}_{t+1}) + \\
&+ E_t\beta\bar{H}^a(1-\rho-\delta^g - f)(\hat{H}_{t+1}^a + \hat{\Lambda}_{t,t+1} + \hat{f}_{t+1})
\end{aligned}$$

Using $\epsilon_t = 1 + \lambda E_t\Lambda_{t,t+1}\delta_{t+1}^g\pi_t^\gamma \frac{p_t}{p_{t+1}}\epsilon_{t+1}$:

$$E_t[H_{t+1}(\pi_t^\gamma w_t^*) - H_{t+1}(w_{t+1}^*)] = E_t\epsilon_{t+1}\left(\pi_t^\gamma \frac{w_t^{*r}}{\pi_{t+1}} - w_{t+1}^{*r}\right)$$

Log-linearizing around the steady state, defining the steady-state values of the surplus in good matches $\bar{H}$, the steady-state real wage $\bar{w}^r$ and defining $\epsilon = \frac{1}{1-\delta^g\lambda\beta}$:

$$E_t[\hat{H}_{t+1}(\pi_t^\gamma w_t^*) - \hat{H}_{t+1}(w_{t+1}^*)] = \frac{\bar{w}^r}{\bar{H}}\epsilon E_t(\hat{w}_t^{*r} + \gamma\hat{\pi}_t - \hat{\pi}_{t+1} - \hat{w}_{t+1}^{*r})$$

Substituting into the main expression, using $\delta^g\hat{\delta}^g = -(1-\rho)\zeta_g f\hat{f} = (\delta_t^g - (1-\rho))\hat{f}$, rearranging and using the surplus approximation $\hat{H}_{t+1}(\varphi_{it+1}, w_{it+1}^*(\varphi_{it+1})) = \hat{H}_{t+1}(\varphi_{t+1}, w_{t+1}^*)$:

$$\begin{aligned}
&\hat{H}_t(\varphi_t, w_t^*) = \frac{\bar{w}^r}{\bar{H}}\left(\hat{w}_t^r + \lambda\beta\delta^g\epsilon E_t(\hat{w}_t^{*r} + \gamma\hat{\pi}_t - \hat{\pi}_{t+1} - \hat{w}_{t+1}^{*r})\right) + \\
&\beta(1-\rho-\delta^g)\frac{\bar{H}^a}{\bar{H}}E_t(\hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^a) - f\beta\frac{\bar{H}^a}{\bar{H}}E_t(\hat{f}_{t+1} + \hat{H}_{t+1}^a + \hat{\Lambda}_{t,t+1}) + \\
&+ \beta(\delta^g - 1 - \rho)(1 - \frac{\bar{H}^a}{\bar{H}})E_t\hat{f}_{t+1} + \beta\delta^g E_t(\hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}(\varphi_{t+1}, w_{t+1}^*))
\end{aligned}$$

B.3.2 Average firm surplus in good matches

$$\begin{aligned}
&J_t(\varphi_t, w_t^*) = x_t a_t - \frac{w_t^*}{p_t} - \lambda E_t\Lambda_{t,t+1}\frac{\kappa}{2}\varkappa_{it+1}^2(\varphi_{it+1}, \pi_t^\gamma w_t^*) + \\
&- (1-\lambda)E_t\Lambda_{t,t+1}\frac{\kappa}{2}\varkappa_{it+1}^2(\varphi_{it+1}, w_{it+1}^*) + \\
&+ \lambda E_t\Lambda_{t,t+1}(\delta_{it+1} + \varkappa_{it+1}) \times J_{t+1}(\varphi_{it+1}, \pi_t^\gamma w_t^*) \\
&+ (1-\lambda)E_t\Lambda_{t,t+1}(\delta_{it+1} + \varkappa_{it+1}) \times J_{t+1}(\varphi_{it+1}, w_{it+1}^*)
\end{aligned}$$

Which can be combined with the hiring condition to write:

$$
\begin{aligned}
&J_t(\varphi_t, w_t^*) = x_t a_t - \frac{w_t^*}{p_t} + \lambda E_t \Lambda_{t,t+1} \frac{\kappa}{2} \varkappa_{it+1}^2(\varphi_{it+1}, \pi_t^\gamma w_{it}^*) + \\
&+ (1-\lambda) E_t \Lambda_{t,t+1} \frac{\kappa}{2} \varkappa_{it+1}^2(\varphi_{it+1}, w_{it+1}^*) + \\
&+ \lambda E_t \Lambda_{t,t+1} \Big(\delta_{it+1}(\varphi_{it+1}, \pi_t^\gamma w_t^*) \times J_{t+1}(\varphi_{it+1}, \pi_t^\gamma w_t^*) - \delta_{it+1}(\varphi_{it+1}, w_{it+1}^*) \times J_{t+1}(\varphi_{it+1}, w_{it+1}^*) \Big) \\
&+ E_t \Lambda_{t,t+1} \left(\delta_{it+1}(\varphi_{it+1}, w_{it+1}^*) \times J_{t+1}(\varphi_{it+1}, w_{it+1}^*) \right)
\end{aligned}
$$

Loglinearizing:

$$
\begin{aligned}
&\hat{J}_t = xa(\hat{x}_t + \hat{a}_t) - w^r \hat{w}_t^r + \beta \left(\frac{\varkappa}{2} + \delta \right) \hat{\Lambda}_{t,t+1} + \beta \varkappa \hat{\varkappa}_{it+1}(w_{it+1}^*) + \beta \lambda \varkappa E_t \left(\hat{\varkappa}_{it+1}(\pi_t^\gamma w_t^*) - \hat{\varkappa}_{it+1}(w_{it+1}^*) \right) + \\
&+ \beta \lambda \delta E_t \left(\hat{\delta}_{it+1}(\pi_t^\gamma w_t^*) - \hat{\delta}_{it+1}(w_{it+1}^*) \right) + \beta \lambda \delta E_t \left(\hat{J}_{t+1}(\pi_t^\gamma w_t^*) - \hat{J}_{t+1}(w_{it+1}^*) \right) + \\
&+ \beta \delta \left(\hat{\delta}_{it+1}(w_{it+1}^*) + \hat{J}_{t+1}(w_{it+1}^*) \right)
\end{aligned}
$$

Using $\varkappa + \delta = 1$:

$$
\begin{aligned}
&\hat{J}_t(\varphi_t, w_t^*) = xa(\hat{x}_t + \hat{a}_t) - w^r \hat{w}_t^r + \beta \left(\frac{1+\delta}{2} \right) E_t \hat{\Lambda}_{t,t+1} + \beta \lambda E_t \left(\hat{\varkappa}_{it+1}(\pi_t^\gamma w_t^*) - \hat{\varkappa}_{it+1}(w_{t+1}^*) \right) + \\
&+ \beta \lambda \delta E_t \left(\hat{\delta}_{it+1}(\pi_t^\gamma w_t^*) - \hat{\delta}_{it+1}(w_{it+1}^*) \right) + \beta \delta E_t \hat{\delta}_{it+1}(w_{t+1}^*) + \beta E_t \hat{J}_{t+1}(\varphi_{t+1}, w_{t+1}^*)
\end{aligned}
$$

B.3.3 Average hiring rate at renegotiating firms

Considering that the composition term will be zero up to a first order, we can express the average hiring rate at a renegotiating firm as:

$$
\kappa \varkappa_t(\varphi_t, w_t^*) = x_t a_t - \frac{w_t^*}{p_t} + E_t \Lambda_{t,t+1} \left[\frac{\kappa}{2} \varkappa_{t+1}^2 + \kappa \delta_{t+1} \varkappa_{t+1} \right]
$$

Loglinearizing:

$$
\hat{\varkappa}_t(\varphi_t, w_t^*) = \frac{xa}{\kappa \bar{\varkappa}}(\hat{x}_t + \hat{a}_t) - \frac{\bar{w}^r}{\kappa \bar{\varkappa}} \hat{w}_t^r + \beta(\varkappa + \delta) E_t \hat{\varkappa}_{t+1} + \beta(\frac{\varkappa}{2} + \delta) E_t \hat{\Lambda}_{t,t+1} + \beta \delta E_t \hat{\delta}_{t+1}
$$

Which results in, given that $\varkappa + \delta = 1$ in the steady state:

$$
\hat{\varkappa}_t(\varphi_t, w_t^*) = \frac{xa}{\kappa \bar{\varkappa}}(\hat{x}_t + \hat{a}_t) - \frac{\bar{w}^r}{\kappa \bar{\varkappa}} \hat{w}_t^r + \beta E_t \hat{\varkappa}_{t+1} + \frac{\beta}{2}(1+\delta) E_t \hat{\Lambda}_{t,t+1} + \beta \delta E_t \hat{\delta}_{t+1}
$$

Considering nominal wage rigidity:

$$
\begin{aligned}
&\hat{\varkappa}_t(\varphi_t, w_t^*) = \frac{xa}{\kappa \bar{\varkappa}}(\hat{x}_t + \hat{a}_t) - \frac{\bar{w}^r}{\kappa \bar{\varkappa}} \hat{w}_t^r + \beta E_t(\lambda \hat{\varkappa}_{t+1}(\pi_t^\gamma w_t^*) + (1-\lambda) \hat{\varkappa}_{t+1}(w_{it+1}^*)) + \frac{\beta}{2}(1+\delta) E_t \hat{\Lambda}_{t,t+1} + \\
&\beta \delta E_t(\lambda \hat{\delta}_{t+1}(\pi_t^\gamma w_t^*) + (1-\lambda) \hat{\delta}_{t+1}(w_{it+1}^*))
\end{aligned}
$$

Rearranging:

$$
\begin{aligned}
&\hat{\varkappa}_t(\varphi_t, w_t^*) = \frac{xa}{\kappa \bar{\varkappa}}(\hat{x}_t + \hat{a}_t) - \frac{\bar{w}^r}{\kappa \bar{\varkappa}} \hat{w}_t^r + \beta \lambda E_t(\hat{\varkappa}_{t+1}(\pi_t^\gamma w_t^*) - \hat{\varkappa}_{t+1}(w_{it+1}^*)) + \frac{\beta}{2}(1+\delta) E_t \hat{\Lambda}_{t,t+1} + \\
&\beta \delta \lambda E_t(\hat{\delta}_{t+1}(\pi_t^\gamma w_t^*) - \hat{\delta}_{t+1}(w_{it+1}^*)) + \beta \delta E_t(\hat{\delta}_{t+1}(w_{it+1}^*)) + \beta E_t(\hat{\varkappa}_{t+1}(w_{it+1}^*))
\end{aligned}
$$

B.3.4 Average retention rate at renegotiating firms

Recall the definition of retention rate:

$$\delta_t = \frac{\delta_t^g + \phi\varphi_{t-1}\delta_t^b}{1+\phi\varphi_{t-1}} = (1-\rho)\frac{(1-\zeta_g f_t) + \phi\varphi_{t-1}(1-\zeta_{bt} f_t^g)}{1+\phi\varphi_{t-1}}$$

Loglinearizing around a steady-state where $\delta^g = \delta^b = \delta$ and $\zeta_g = \zeta_{bt}\xi$, and recalling that $\delta_t^g = (1-\rho)(1-\zeta_g f_t)$ and $\delta_t^b = (1-\rho)(1-\zeta_{bt}\xi f_t)$, we can show:

$$\delta\hat{\delta}_t^g = \delta(\hat{\delta_t^b}) = -(1-\rho)\zeta_g f(\hat{\zeta}_{bt} + \hat{f}_t)$$

Which simplifies deviations in average retention rate as:

$$\hat{\delta}_t = \frac{\delta - (1-\rho)}{\delta}(\hat{\zeta}_{bt} + \hat{f}_t)$$

Note that:

$$E_t(\hat{\delta}_{t+1}(\pi_t^\gamma w_t) - \hat{\delta}_{t+1}(w_{it+1}^*)) = \frac{\delta-(1-\rho)}{\delta}E_t\left(\hat{\zeta}_{bt+1}(\pi_t^\gamma w_t) - \hat{\zeta}_{bt+1}(w_{it+1}^*)\right)$$

B.3.5 Average search intensity at renegotiating firms

The optimal search intensity for workers in bad matches in renegotiating firms is:

$$\zeta_0\zeta_{bt}^{\eta_\zeta}(\varphi_t, w_t^*) = f_t^g\left(\bar{H}_t - H_t^b(\varphi_t, w_t^*)\right)$$

Log-linearizing and using $\hat{f}_t^g = \hat{f}_t$

$$\zeta_0\zeta_b^\eta\eta\hat{\zeta}_{bt} = f^g(\bar{H}\hat{\bar{H}}_t - H_t^b\hat{H}_t^b(w_t^*)) + f^g\hat{f}_t^g(\bar{H}_t - \bar{H}_t^b)$$

Which simplifies into:

$$\eta\hat{\zeta}_{bt}(w_t^*) = \frac{(\bar{H}\hat{\bar{H}}_t - H^b\hat{H}_t^b(w_t^*))}{\bar{H} - H^b} + \hat{f}_t^g$$

But then:

$$\eta E_t\left(\hat{\zeta}_{bt+1}(\pi_t^\gamma w_t^*) - \hat{\zeta}_{bt+1}(w_{it+1}^*)\right) = -\frac{H^b}{\bar{H} - H^b}E_t\left(\hat{H}_{t+1}^b(\pi_t^\gamma w_t^*) - \hat{H}_{t+1}^b(w_{it+1}^*)\right)$$

B.3.6 Average worker surplus in bad matches

$$H_t^b(\varphi_t, w_t^*) = \phi\frac{w_t^*}{p_t} - u_B \underbrace{-E_t\Lambda_{t,t+1}(1-\rho)c(\zeta_{bt+1})}_{(a)} + \underbrace{E_t\Lambda_{t,t+1}(1-\rho)(\zeta_{bt+1}f_{t+1}^g)\bar{H}_{t+1}}_{(b)} +$$

$$+\underbrace{E_t\Lambda_{t,t+1}(1-\rho)(1-\zeta_{bt+1}f_{t+1}^g)H_{t+1}^b(\varphi_{it+1}, w_{t+1})}_{(c)} - E_t\Lambda_{t,t+1}f_{t+1}\bar{H}_{t+1}^a$$

Now, note that $E_t\zeta_{bt+1} = \lambda\zeta_{bt+1}(\pi_t^\gamma w_t^*) + (1-\lambda)\zeta_{bt+1}(w_{it+1}^*)$. As such, we can re-write ***(a)*** as:

$$\begin{aligned}
&-E_t\Lambda_{t,t+1}(1-\rho)c(\zeta_{bt+1}) = \\
&-\lambda E_t\Lambda_{t,t+1}(1-\rho)c(\zeta_{bt+1}(\pi_t^\gamma w_t^*)) - (1-\lambda)E_t\Lambda_{t,t+1}(1-\rho)c(\zeta_{bt+1}(w_{it+1}^*))
\end{aligned}$$

By a similar token, we can re-write ***(b)***:

$$\begin{aligned}
&E_t\Lambda_{t,t+1}(1-\rho)(\zeta_{bt+1}f_{t+1}^g)\bar{H}_{t+1} = \\
&\lambda E_t\Lambda_{t,t+1}(1-\rho)(\zeta_{bt+1}(\pi_t^\gamma w_t^*)f_{t+1}^g)\bar{H}_{t+1} + \\
&(1-\lambda)E_t\Lambda_{t,t+1}(1-\rho)(\zeta_{bt+1}(w_{it+1}^*)f_{t+1}^g)\bar{H}_{t+1}
\end{aligned}$$

Likewise, for ***(c)***:

$$\begin{aligned}
&E_t\Lambda_{t,t+1}(1-\rho)(1-\zeta_{bt+1}f_{t+1}^g)H_{t+1}^b(w_{t+1}) = \\
&\lambda E_t\Lambda_{t,t+1}(1-\rho)(1-\zeta_{bt+1}(\pi_t^\gamma w_t^*)f_{t+1}^g)H_{t+1}^b(\pi_t^\gamma w_t^*) \\
&(1-\lambda)E_t\Lambda_{t,t+1}(1-\rho)(1-\zeta_{bt+1}(w_{it+1}^*)f_{t+1}^g)H_{t+1}^b(w_{t+1}^*)
\end{aligned}$$

Which we can re-write as:

$$\begin{aligned}
&E_t\Lambda_{t,t+1}(1-\rho)(1-\zeta_{bt+1}f_{t+1}^g)H_{t+1}^b(w_{t+1}) = \\
&= \lambda E_t\Lambda_{t,t+1}(1-\rho)(H_{t+1}^b(\pi_t^\gamma w_t^*) - H_{t+1}^b(w_{it+1}^*)) \\
&- \lambda E_t\Lambda_{t,t+1}(1-\rho)f_{t+1}^g\left(\zeta_{bt+1}(\pi_t^\gamma w_t^*)H_{t+1}^b(\pi_t^\gamma w_t^*) - \zeta_{bt+1}(w_{it+1}^*)H_{t+1}^b(w_{it+1}^*)\right) \\
&+ E_t\Lambda_{t,t+1}(1-\rho)(1-\zeta_{bt+1}(w_{it+1}^*)f_{t+1}^g)H_{t+1}^b(w_{it+1}^*)
\end{aligned}$$

Loglinearizing and collecting terms:

$$\begin{aligned}
&\hat{H}_t^b(\varphi_t, w_t^*) = \frac{\phi}{H^b}\bar{w}^r\hat{w}_t^{*r} - \beta(1-\rho)\zeta_0\frac{\zeta_b^{1+\eta_\zeta}}{H^b}E_t\left[\lambda\hat{\zeta}_{bt+1}(\pi_t^\gamma w_t^*) + (1-\lambda)\hat{\zeta}_{bt+1}(w_{it+1}^*)\right] - \\
&-\beta(1-\rho)\frac{c(\zeta_b)}{H^b}E_t\hat{\Lambda}_{t,t+1} + \beta(1-\rho-\delta)\frac{\bar{H}}{H^b}E_t\left[\hat{\Lambda}_{t,t+1} + \hat{\bar{H}}_{t+1}\right] \\
&-\beta f\frac{\bar{H}^a}{H^b}E_t\left(\hat{f}_{t+1} + \hat{\Lambda}_{t,t+1} + \hat{\bar{H}}_{t+1}^a\right) \\
&-\beta(1-\rho-\delta)\left(1-\frac{\bar{H}}{H^b}\right)E_t\left(\hat{f}_{t+1} + \lambda\hat{\zeta}_{bt+1}(\pi_t^\gamma w_t^*) + (1-\lambda)\hat{\zeta}_{bt+1}(w_{it+1}^*)\right) \\
&+\delta\beta\lambda E_t\left(\hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^b(\pi_t^\gamma w_t^*)\right) \\
&+\delta\beta(1-\lambda)E_t\left(\hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^b(w_{it+1}^*)\right)
\end{aligned}$$

Collecting terms, this can be further simplified as:

$$\begin{aligned}
&\hat{H}_t^b(\varphi_t, w_t^*) = \frac{\phi}{H^b}\bar{w}^r\hat{w}_t^{*r} + \chi_\Lambda E_t\hat{\Lambda}_{t,t+1} + \chi_f E_t\hat{f}_{t+1} \\
&+ \beta\lambda\delta E_t[\hat{H}_{t+1}^b(\pi_t^\gamma w_t^*) - \hat{H}_{t+1}^b(w_{t+1}^*)] \\
&+ \delta\beta E_t\hat{H}_{t+1}^b(w_{t+1}^*) + \beta(1-\rho-\delta)\frac{\bar{H}}{H^b}E_t\hat{\bar{H}}_{t+1} - \beta f\frac{\bar{H}^a}{H^b}E_t(\hat{\bar{H}}_{t+1}^a)
\end{aligned}$$

With $\chi_\Lambda = \left[-\beta(1-\rho)\frac{c(\zeta_b)}{H^b} + \beta(1-\rho-\delta)\frac{\bar{H}}{H^b} - \beta f\frac{\bar{H}^a}{H^b} + \delta\beta\right]$ and $\chi_f = \left[-\beta f\frac{\bar{H}^a}{H^b} - \beta(1-\rho-\delta)\left(1-\frac{\bar{H}}{H^b}\right)\right]$. Finally, note that:

$$E_t[\hat{H}^b_{t+1}(\pi_t^\gamma w_t^*) - \hat{H}^b_{t+1}(w^*_{t+1})] = \frac{\phi}{H^b}\bar{w}^r(1+\beta\lambda\delta+(\beta\lambda\delta)^2 ...)(\gamma\hat{\pi}_t + \hat{w}_t^{*r} - \hat{\pi}_{t+1} - \hat{w}^{*r}_{t+1})$$

Using $\epsilon = \frac{1}{1-\delta\lambda\beta}$:

$$E_t[\hat{H}^b_{t+1}(\pi_t^\gamma w_t^*) - \hat{H}^b_{t+1}(w^*_{t+1})] = \frac{\phi}{H^b}\bar{w}^r\epsilon(\gamma\hat{\pi}_t + \hat{w}_t^{*r} - \hat{\pi}_{t+1} - \hat{w}^{*r}_{t+1})$$

Substituting:

$$\begin{aligned}\hat{H}^b_t &= \frac{\phi}{H^b}\bar{w}^r E_t\left(\hat{w}_t^{*r} + \beta\lambda\delta\epsilon\left(\gamma\hat{\pi}_t + \hat{w}_t^{*r} - \hat{\pi}_{t+1} - \hat{w}^{*r}_{t+1}\right)\right) + \chi_\Lambda E_t\hat{\Lambda}_{t,t+1} + \chi_f E_t\hat{f}_{t+1}\\ &+ \delta\beta E_t\hat{H}^b_{t+1}(w^*_{t+1}) + \beta(1-\rho-\delta)\frac{\bar{H}}{H^b}E_t\hat{\bar{H}}_{t+1} - \beta f\frac{\bar{H}^a}{H^b}E_t(\hat{\bar{H}}^a_{t+1})\end{aligned}$$

B.4 Auxiliary steady-state results

B.4.1 Effect of composition on contract wage

A firm has composition φ_{it} and contract wage $w^*_{it}(\varphi_{it})$. Let the implicit function $\mathbf{F}_t(\varphi_{it}, w^*_{it}(\varphi_{it})) \equiv \eta J_t(\varphi_{it}, w^*_{it}(\varphi_{it})) - (1-\eta)H_t(\varphi_{it}, w^*_{it}(\varphi_{it}))$. By the surplus sharing rule, $\mathbf{F} = 0$. Applying the implicit function theorem we have that:

$$\frac{\partial w_t^*(\varphi_{it})}{\partial\varphi_{it}} = -\frac{\partial\mathbf{F_t}/\partial\varphi_{it}}{\partial\mathbf{F}_t/\partial w^*_{it}}$$

With

$$\begin{aligned}\frac{\partial\mathbf{F}_t}{\partial\varphi_{it}} &= \eta\frac{\partial J_t}{\partial\varphi_{it}} - (1-\eta)\frac{\partial H_t}{\partial\varphi_{it}}\\ \frac{\partial\mathbf{F}_t}{\partial w_t^*} &= \eta\frac{\partial J_t}{\partial w_t^*} - (1-\eta)\frac{\partial H_t}{\partial w_t^*}\end{aligned}$$

Evaluated at the steady state:

$$\frac{\partial w^*}{\partial\varphi} = -\frac{\eta\partial J/\partial\varphi - (1-\eta)\partial H/\partial\varphi}{\eta\partial J/\partial w - (1-\eta)\partial H/\partial w} \tag{S1}$$

B.4.2 Effect of composition and wages on worker surplus in good matches

Consider the following auxiliary expressions for average surplus in all matches, in good matches and in bad matches, respectively:

$$\begin{aligned}
\bar{H}_t &\equiv \xi \bar{H}_t^g + (1-\xi)\bar{H}_t^b \\
\bar{H}_t^g &\equiv \bar{V}_t^g - U_t \\
\bar{H}_t^b &\equiv \bar{V}_t^b - U_t
\end{aligned}$$

The value of a worker in a good match will be, net of search costs:

$$V_t^g \equiv V_t^g(\varphi_{it}, w_{it}^g) = \left\{ \frac{w_{gt}}{p_t} + E_t\left\{\Lambda_{t,t+1}\left[(1-\rho)S_{t+1}^g + \rho U_{t+1}\right]\right\}\right\}$$

Where, recall:

$$\begin{aligned}
S_t^b &= \max_{\zeta_{bt}}\left\{\zeta_{bt} f_t^g \bar{V}_t^g + (1-\zeta_{bt} f_t^g) V_t^b - c(\zeta_{bt})\right\} \\
S_t^g &= (1-\zeta_g f_t) V_t^g + \zeta_g (f_t^g \bar{V}_t^g + f_t^b \bar{V}_t^b) - c(\zeta_g)
\end{aligned}$$

The value of an unemployed worker is:

$$U_t = u_B + E_t\left[\Lambda_{t,t+1}\left[f_{t+1}^g \bar{V}_{t+1}^g + f_{t+1}^b \bar{V}_{t+1}^b + (1-f_{t+1})U_{t+1}\right]\right]$$

The surplus of workers in good matches **net of search costs** can be written as:

$$\begin{aligned}
H_t = {} & \frac{w_{gt}}{p_t} - u_B + E_t\Lambda_{t,t+1}\left((1-\rho)(1-\zeta_g f_{t+1})V_{t+1}^g\right) + \\
& + E_t\Lambda_{t,t+1}\left((1-\rho)\zeta_g(f_{t+1}^g \bar{V}_{t+1}^g + f_{t+1}^b \bar{V}_{t+1}^b) - f_{t+1}^g \bar{V}_{t+1}^g - f_{t+1}^b \bar{V}_{t+1}^b\right) + \\
& + E_t\Lambda_{t,t+1}(\rho - 1 + f_{t+1})U_{t+1}
\end{aligned}$$

Which reduces to:

$$\begin{aligned}
H_t(\varphi_{it}, w_{it}) = {} & \frac{w_{gt}}{p_t} - u_B + E_t\Lambda_{t,t+1}\left[(1-\rho)(1-\zeta_g f_{t+1})H_{t+1}^g\right] + \\
& + E_t\Lambda_{t,t+1}\left[(1-\rho)\zeta_g f_{t+1}\bar{H}_{t+1}^a - f_{t+1}\bar{H}_{t+1}^a\right]
\end{aligned}$$

Which, given the nominal wage rigidity, will be:

$$\begin{aligned}
H_t(\varphi_{it}, w_{it}) = {} & \frac{w_{gt}}{p_t} - u_B + \\
& E_t\Lambda_{t,t+1}\left[(1-\rho)(1-\zeta_g f_{t+1})\left(\lambda H_{t+1}(\varphi_{it+1}, \pi_t^\gamma w_t) + (1-\lambda)H_{t+1}(\varphi_{it+1}, w_{it+1}^*)\right)\right] + \\
& + E_t\Lambda_{t,t+1}\left[(1-\rho)\zeta_g f_{t+1}\bar{H}_{t+1}^a - f_{t+1}\bar{H}_{t+1}^a\right]
\end{aligned}$$

$$\frac{\partial H_t(\varphi_{it}, w_{it})}{\partial \varphi_{it}} = (1-\rho)(1-\zeta_g f_{t+1}) E_t \left\{ \Lambda_{t,t+1} \left(\lambda \frac{\partial H_{t+1}(\varphi_{t+1}, \pi_t^{\gamma} w_t)}{\partial \varphi_{t+1}} \right) \frac{d\varphi_{t+1}}{d\varphi_t} \right\} +$$
$$+ (1-\lambda) E_t \left\{ \Lambda_{t,t+1} \left(\frac{\partial H_{t+1}(\varphi_{t+1}, w_{t+1}^*(\varphi_{t+1}))}{\partial \varphi_{t+1}} + \frac{\partial H_{t+1}(\varphi_{t+1}, w_{t+1}^*(\varphi_{t+1}))}{\partial w_{t+1}^*} \frac{\partial w_{t+1}^*}{\partial \varphi_{t+1}} \right) \frac{d\varphi_{t+1}}{d\varphi_t} \right\}$$

Recall that retention of good matches is $\delta_t^g = (1-\rho)(1-\zeta_g f_t)$. So in the steady state, $\partial H_t / \partial \varphi_{it}$ will be:

$$\frac{\partial H}{\partial \varphi} \left(1 - \delta^g \beta \frac{d\varphi'}{d\varphi} \right) = \delta^g \beta (1-\lambda) \frac{\partial H}{\partial w} \frac{\partial w^*(\varphi)}{\partial \varphi} \frac{d\varphi'}{d\varphi} \tag{S2}$$

As for the effect of wages on worker surplus in good matches, we have:

$$\frac{\partial H}{\partial w} = \frac{1}{p_t} + E_t \Lambda_{t,t+1} \left[(1-\rho)(1-\zeta_g f_{t+1}) \lambda \left[\pi_t^{\gamma} \frac{\partial H_{t+1}(\varphi_{it+1}, \pi_t^{\gamma} w_{it})}{\partial (\pi^{\gamma} w_{it})} + \frac{\partial H_{t+1}(\varphi_{it+1}, \pi^{\gamma} w_{it})}{\partial \varphi_{it+1}} \frac{\partial \varphi_{it+1}}{\partial w_{it}} \right] \right] +$$
$$+ E_t \Lambda_{t,t+1} \left[(1-\rho)(1-\zeta_g f_{t+1})(1-\lambda) \left[\frac{\partial H_{t+1}(\varphi_{it+1}, w_{it+1}^*)}{\partial \varphi_{it+1}} + \frac{\partial H_{t+1}(\varphi_{it+1}, w_{it+1}^*)}{\partial w_{it+1}^*} \frac{\partial w_{it+1}^*}{\partial \varphi_{it+1}} \right] \frac{\partial \varphi_{it+1}}{\partial w_{it}} \right]$$

In the steady-state:

$$\frac{\partial H}{\partial w} \left(1 - \delta^g \beta \lambda \pi^{\gamma} - \delta^g \beta (1-\lambda) \frac{\partial w^*(\varphi)}{\partial \varphi} \frac{d\varphi'}{dw} \right) = 1/p + \delta^g \beta \frac{\partial H}{\partial \varphi} \frac{d\varphi'}{dw} \tag{S3}$$

B.4.3 Effect of composition and wages on firm surplus

The firm surplus is, for each given composition and wage:

$$J_t(\varphi_{it}, w_{it}) = x_t a_t - \frac{w_{it}}{p_t} - E_t \Lambda_{t,t+1} \frac{\kappa}{2} \varkappa_{it+1}^2 (\varphi_{it+1}, w_{it+1}) +$$
$$+ E_t \Lambda_{t,t+1} (\delta_{it+1}(\varphi_{it+1}, w_{it+1}) + \varkappa_{it+1}(\varphi_{it+1}, w_{it+1})) J_{t+1}(\varphi_{it+1}, w_{it+1})$$

With

$$J_{t+1}(\varphi_{it+1}, w_{it+1}) = \lambda J_{t+1}(\varphi_{it+1}, \pi_t^{\gamma} w_{it}) + (1-\lambda) J_{t+1}(\varphi_{it+1}, w_{it+1}^*)$$
$$\varkappa_{t+1}(\varphi_{it+1}, w_{it+1}) = \lambda \varkappa_{t+1}(\varphi_{it+1}, \pi_t^{\gamma} w_{it}) + (1-\lambda) \varkappa_{t+1}(\varphi_{it+1}, w_{it+1}^*)$$
$$\delta_{t+1}(\varphi_{it+1}, w_{it+1}) = \lambda \delta_{t+1}(\varphi_{it+1}, \pi_t^{\gamma} w_{it}) + (1-\lambda) \delta_{t+1}(\varphi_{it+1}, w_{it+1}^*)$$

We will use these auxiliary expressions:

$$\frac{\partial J_{t+1}(\varphi_{it+1}, w_{it+1})}{\partial \varphi_{it+1}} = \lambda \frac{\partial J_{t+1}(\varphi_{it+1}, \pi_t^{\gamma} w_{it})}{\partial \varphi_{it+1}} + (1-\lambda)\frac{\partial J_{t+1}(\varphi_{it+1}, w^*_{it+1}(\varphi_{it+1}))}{\partial \varphi_{it+1}} + \\ + (1-\lambda)\frac{\partial J_{t+1}(\varphi_{it+1}, w^*_{it+1}(\varphi_{it+1}))}{\partial w^*_{it+1}} \frac{\partial w^*_{it+1}}{\partial \varphi_{it+1}}$$

$$\frac{d\varkappa_{t+1}(\varphi_{it+1}, w_{it+1})}{d\varphi_{it+1}} = \lambda \frac{\partial \varkappa_{t+1}(\varphi_{it+1}, \pi_t^{\gamma} w_{it})}{\partial \varphi_{it+1}} + (1-\lambda)\frac{\partial \varkappa_{t+1}(\varphi_{it+1}, w^*_{it+1}(\varphi_{it+1}))}{\partial \varphi_{it+1}} + \\ + (1-\lambda)\frac{\partial \varkappa_{t+1}(\varphi_{it+1}, w^*_{it+1}(\varphi_{it+1}))}{\partial w^*_{it+1}} \frac{\partial w^*_{it+1}}{\partial \varphi_{it+1}}$$

We can write:

$$\frac{\partial J_t}{\partial \varphi_{it}} = -\kappa E_t \Lambda_{t,t+1}\left(\varkappa_{it+1}(\varphi_{it+1}, w_{it+1})\frac{d\varkappa_{it+1}(\varphi_{it+1}, w_{it+1})}{d\varphi_{it+1}} \frac{d\varphi_{it+1}}{d\varphi_{it}}\right) + \\ + E_t \Lambda_{t,t+1}\left(\frac{d\delta_{it+1}}{d\varphi_{it}} + \frac{d\varkappa_{it+1}}{d\varphi_{it}}\right) J_{t+1}(\varphi_{it+1}, w_{it+1}) + \\ + E_t \Lambda_{t,t+1}(\delta_{it+1}(\varphi_{it+1}, w_{it+1}) + \varkappa_{it+1}(\varphi_{it+1}, w_{it+1}))\frac{\partial J_{t+1}(\varphi_{it+1}, w_{it+1})}{\partial \varphi_{it+1}} \frac{d\varphi_{it+1}}{d\varphi_{it}}$$

Note that in the steady state $\delta + \varkappa = 1$.

$$\frac{\partial J}{\partial \varphi} = -\beta J \left(\frac{d\varkappa}{d\varphi}\right)\frac{d\varphi'}{d\varphi} + \beta\left(\frac{\partial J}{\partial \varphi} + (1-\lambda)\frac{\partial J}{\partial w}\frac{\partial w^*}{\partial \varphi}\right)\frac{d\varphi'}{d\varphi}$$

Which yields:

$$\frac{\partial J}{\partial \varphi}\left(1 - \beta \frac{d\varphi'}{d\varphi}\right) = -\beta J \frac{d\varkappa}{d\varphi}\frac{d\varphi'}{d\varphi} + \beta(1-\lambda)\frac{\partial J}{\partial w}\frac{\partial w^*}{\partial \varphi}\frac{d\varphi'}{d\varphi} \tag{S4}$$

To compute $\partial J_t/\partial w_t$, we will use the following expressions:

$$
\begin{aligned}
\frac{\partial J_{t+1}(\varphi_{it+1}, w_{it+1})}{\partial w_{it}} &= \left(\lambda \pi_t^{\gamma} \frac{\partial J_{t+1}(\varphi_{t+1}, \pi_t^{\gamma} w_{it})}{\partial \pi_t^{\gamma} w_{it}}\right) + \\
&+ \left(\lambda \frac{\partial J_{t+1}(\varphi_{t+1}, \pi_t^{\gamma} w_{it})}{\partial \varphi_{t+1}} + (1-\lambda)\frac{\partial J_{t+1}(\varphi_{t+1}, w_{t+1}^{*})}{\partial \varphi_{t+1}} + (1-\lambda)\frac{\partial J_{t+1}(\varphi_{t+1}, w_{t+1}^{*})}{\partial w_{t+1}^{*}} \frac{\partial w_{t+1}^{*}}{\partial \varphi_{t+1}}\right) \frac{d\varphi_{t+1}}{d w_{it}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\varkappa_{t+1}(\varphi_{it+1}, w_{it+1})}{d w_{it}} &= \left(\lambda \pi_t^{\gamma} \frac{\partial \varkappa_{t+1}(\varphi_{t+1}, \pi_t^{\gamma} w_{it})}{\partial \pi_t^{\gamma} w_{it}}\right) + \\
&+ \left(\lambda \frac{\partial \varkappa_{t+1}(\varphi_{t+1}, \pi_t^{\gamma} w_{it})}{\partial \varphi_{t+1}} + (1-\lambda)\frac{\partial \varkappa_{t+1}(\varphi_{t+1}, w_{t+1}^{*})}{\partial \varphi_{t+1}} + (1-\lambda)\frac{\partial \varkappa_{t+1}(\varphi_{t+1}, w_{t+1}^{*})}{\partial w_{t+1}^{*}} \frac{\partial w_{t+1}^{*}}{\partial \varphi_{t+1}}\right) \frac{d\varphi_{t+1}}{d w_{it}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\delta_{t+1}(\varphi_{it+1}, w_{it+1})}{d w_{it}} &= \left(\lambda \pi_t^{\gamma} \frac{\partial \delta_{t+1}(\varphi_{t+1}, \pi_t^{\gamma} w_{it})}{\partial \pi_t^{\gamma} w_{it}}\right) + \\
&+ \left(\lambda \frac{\partial \delta_{t+1}(\varphi_{t+1}, \pi_t^{\gamma} w_{it})}{\partial \varphi_{t+1}} + (1-\lambda)\frac{\partial \delta_{t+1}(\varphi_{t+1}, w_{t+1}^{*})}{\partial \varphi_{t+1}} + (1-\lambda)\frac{\partial \delta_{t+1}(\varphi_{t+1}, w_{t+1}^{*})}{\partial w_{t+1}^{*}} \frac{\partial w_{t+1}^{*}}{\partial \varphi_{t+1}}\right) \frac{d\varphi_{t+1}}{d w_{it}}
\end{aligned}
$$

We can express:

$$
\begin{aligned}
\frac{\partial J_t}{\partial w_t} &= -\frac{1}{p_t} - \kappa E_t \Lambda_{t,t+1} \varkappa_{it+1}(\varphi_{it+1}, w_{it+1}) \frac{d\varkappa_{it+1}(\varphi_{it+1}, w_{it+1})}{d w_{it}} + \\
&+ E_t \Lambda_{t,t+1} \left(\frac{d\delta_{it+1}}{d w_{it}} + \frac{d\varkappa_{it+1}}{d w_{it}}\right) J_{t+1}(\varphi_{it+1}, w_{it+1}) + \\
&+ E_t \Lambda_{t,t+1} \left(\delta_{it+1} + \varkappa_{it+1}\right) \frac{\partial J_{t+1}(\varphi_{it+1}, w_{it+1})}{\partial w_t}
\end{aligned}
$$

Evaluating at the steady-state yields:

$$
\begin{aligned}
\frac{\partial J}{\partial w} &= -\frac{1}{p} - \beta J \left(\lambda \pi^{\gamma} \frac{\partial \varkappa}{\partial w} + \left(\frac{\partial \varkappa}{\partial \varphi} + (1-\lambda)\frac{\partial \varkappa}{\partial w} \frac{\partial w^{*}}{\partial \varphi}\right) \frac{d\varphi'}{d w}\right) + \\
&+ \beta \left(\lambda \pi^{\gamma} \frac{\partial J}{\partial w}\right) + \beta \left(\frac{\partial J}{\partial \varphi} + (1-\lambda)\frac{\partial J}{\partial w} \frac{\partial w^{*}}{\partial \varphi}\right) \frac{d\varphi'}{d w}
\end{aligned}
$$

Which yields:

$$
\frac{\partial J}{\partial w}\left(1 - \beta\lambda\pi^{\gamma} - \beta(1-\lambda)\frac{\partial w^{*}}{\partial \varphi} \frac{d\varphi'}{d w}\right) = -\frac{1}{p} - \beta J \frac{d\varkappa}{d w} + \beta \frac{\partial J}{\partial \varphi} \frac{d\varphi'}{d w} \tag{S5}
$$

B.4.4 Effect of composition and wages on hiring rate

Under the assumption of equal retention of good and bad matches, we have, from the optimal hiring condition:

$$\varkappa_{it}(\varphi_{it}, w_{it}) = \frac{1}{\kappa}\left[x_t a_t - \frac{w_{it}}{p_t} + E_t\Lambda_{t,t+1}\left[\frac{\kappa}{2}\varkappa^2_{it+1} + \kappa\delta_{it+1}(\varphi_{it+1}, w_{it+1})\varkappa_{it+1}(\varphi_{it+1}, w_{it+1})\right]\right]$$

$$\begin{aligned}\frac{d\varkappa_{it}}{d\varphi_{it}} &= E_t\Lambda_{t,t+1}(\varkappa_{it+1} + \delta_{it+1})\frac{d\varkappa_{it+1}}{d\varphi_{it+1}}\frac{d\varphi_{it+1}}{d\varphi_{it}} + E_t\Lambda_{t,t+1}\varkappa_{it+1}(\varphi_{it+1}, w_{it+1})\frac{d\delta_{it+1}}{d\varphi_{it+1}}\frac{d\varphi_{it+1}}{d\varphi_{it}} = \\ &= E_t\Lambda_{t,t+1}\varkappa_{it+1}\left(\frac{d\varkappa_{it+1} + \delta_{it+1}}{d\varphi_{it}}\right)\frac{d\varphi_{it+1}}{d\varphi_{it}} + E_t\Lambda_{t,t+1}\delta_{it+1}\frac{d\varkappa_{it+1}}{d\varphi_{it+1}}\frac{d\varphi_{it+1}}{d\varphi_{it}}\end{aligned}$$

Evaluated at the steady state:

$$\frac{d\varkappa}{d\varphi}\left(1 - \beta\delta\frac{d\varphi'}{d\varphi}\right) = \beta\delta(1-\lambda)\frac{d\varkappa}{dw}\frac{\partial w^*}{\partial\varphi}\frac{d\varphi'}{d\varphi} \tag{S6}$$

As for the effect of wages:

$$\begin{aligned}\frac{d\varkappa_{it}}{dw_{it}} &= -\frac{1}{\kappa p_t} + E_t\Lambda_{t,t+1}(\varkappa_{it+1} + \delta_{it+1})\frac{\partial\varkappa_{it+1}}{\partial w_{it+1}}\frac{dw_{it+1}}{d\varphi_{it+1}}\frac{d\varphi_{it+1}}{dw_{it}} + E_t\Lambda_{t,t+1}\varkappa_{it+1}(\varphi_{it+1}, w_{it+1})\frac{\partial\delta_{it+1}}{\partial w_{it+1}}\frac{\partial w_{it+1}}{\partial\varphi_{it+1}}\frac{d\varphi_{it+1}}{dw_{it}} = \\ &= -\frac{1}{\kappa p_t} + E_t\Lambda_{t,t+1}\varkappa_{it+1}\left(\frac{\partial\varkappa_{it+1}}{\partial w_{it+1}} + \frac{\partial\delta_{it+1}}{\partial w_{it+1}}\right)\frac{\partial w_{it+1}}{\partial\varphi_{it+1}}\frac{d\varphi_{it+1}}{dw_{it}} + \delta_{it+1}\frac{\partial\varkappa_{it+1}}{\partial w_{it+1}}\frac{dw_{it+1}}{d\varphi_{it+1}}\frac{d\varphi_{it+1}}{dw_{it}}\end{aligned}$$

Evaluated at the steady state:

$$\frac{d\varkappa}{dw} = -\frac{1}{\kappa p} + \beta\delta\left[\pi^\gamma\lambda\frac{d\varkappa}{dw} + \frac{\partial\varkappa}{\partial\varphi}\frac{d\varphi'}{dw} + (1-\lambda)\frac{d\varkappa}{dw}\frac{\partial w^*}{\partial\varphi}\frac{d\varphi'}{dw}\right]$$

$$\frac{d\varkappa}{dw}\left(1 - \pi^\gamma\beta\delta\lambda - \beta\delta(1-\lambda)\frac{\partial w^*}{\partial\varphi}\frac{d\varphi'}{d\varphi}\right) = -\frac{1}{\kappa p} + \beta\delta\frac{d\varkappa}{d\varphi}\frac{d\varphi'}{dw} \tag{S7}$$

B.4.5 Effect of composition and wage on future composition

The law of motion for composition given φ_{it-1}, w_t is:

$$\varphi_{it} = \frac{\delta_t^b \varphi_{it-1} + q_t^b v_{it}/g_{it-1}}{\delta_t^g + q_t^g v_{it}/g_{it-1}} = \frac{\delta_t^b \frac{\varphi_{it-1}}{1+\phi\varphi_{it-1}} + \frac{\bar{\varphi}_t{}^h}{1+\phi\bar{\varphi}_t{}^h}\varkappa_{it}}{\delta_t^g \frac{1}{1+\phi\varphi_{it-1}} + \frac{1}{1+\phi\bar{\varphi}_t^h}\varkappa_{it}}$$

Therefore

$$\varphi_{it+1} = \frac{\delta_{t+1}^b \varphi_{it} + q_t^b v_{it+1}/g_{it}}{\delta_{t+1}^g + q_{t+1}^g v_{it+1}/g_{it}} = \frac{\delta_{t+1}^b \frac{\varphi_{it}}{1+\phi\varphi_{it}} + \frac{\bar{\varphi}_{t+1}{}^h}{1+\phi\bar{\varphi}_{t+1}{}^h}\varkappa_{it+1}}{\delta_{t+1}^g \frac{1}{1+\phi\varphi_{it}} + \frac{1}{1+\phi\bar{\varphi}_{t+1}^h}\varkappa_{it+1}}$$

$$\frac{d\varphi_{it+1}}{d\varphi_{it}} = \frac{\partial\varphi_{it+1}}{\partial\varphi_{it}} + \frac{\partial\varphi_{it+1}}{\partial\varkappa_{it+1}}\frac{\partial\varkappa_{it+1}}{\partial\varphi_{it}} + \frac{\partial\varphi_{it+1}}{\partial\delta_{t+1}^b}\frac{\partial\delta_{t+1}^b}{\partial\zeta_{bt+1}}\frac{\partial\zeta_{bt+1}}{\partial\varphi_{it}}$$

$$\frac{d\varphi_{it+1}}{dw_{it}} = \frac{\partial\varphi_{it+1}}{\partial w_{it}} + \frac{\partial\varphi_{it+1}}{\partial\varkappa_{it+1}}\frac{\partial\varkappa_{it+1}}{\partial w_{it}} + \frac{\partial\varphi_{it+1}}{\partial\delta_{t+1}^b}\frac{\partial\delta_{t+1}^b}{\partial\zeta_{bt+1}}\frac{\partial\zeta_{bt+1}}{\partial w_{it}}$$

From the definition of φ_{it}, we have:

$$\frac{\partial\varphi_{it}}{\partial\varphi_{it-1}} = \frac{\frac{\delta_t^b}{(1+\phi\varphi_{it-1})^2}\left(\frac{\delta_t^g}{1+\phi\varphi_{it-1}} + \frac{\varkappa_{it}}{1+\phi\bar{\varphi}_t{}^h}\right) + \frac{\phi\delta_t^g}{(1+\phi\varphi_{it-1})^2}\left(\delta_t^b\frac{\varphi_{it-1}}{1+\phi\varphi_{it-1}} + \frac{\bar{\varphi}_t{}^h}{1+\phi\bar{\varphi}_t{}^h}\varkappa_{it}\right)}{\left(\frac{\delta_t^g}{1+\phi\varphi_{it-1}} + \frac{\varkappa_{it}}{1+\phi\bar{\varphi}_t{}^h}\right)^2}$$

$$\frac{\partial\varphi_{it}}{\partial\varkappa_{it}} = \frac{(1+\phi\bar{\varphi}_{it}^h)(1+\phi\varphi_{it-1})}{[\delta_t^g(1+\phi\bar{\varphi}_{it}^h) + \varkappa_{it}(1+\phi\varphi_{it-1})]^2}\left(\bar{\varphi}_t^h\delta_t^g - \varphi_{it-1}\delta_t^b\right)$$

$$\frac{\partial\varphi_{it}}{\partial\delta_t^b} = \frac{\frac{\varphi_{it-1}}{1+\phi\varphi_{it-1}}}{\frac{\delta_t^g}{1+\phi\varphi_{it-1}} + \frac{\varkappa_{it}}{1+\phi\bar{\varphi}_t^h}}$$

As before, we have $\frac{\partial\delta_t^b}{\partial\zeta_{bt}} = -(1-\rho)f_t^g$. In addition, $\frac{\partial\varphi_{it+1}}{\partial w_{it}} = 0$. Although $\bar{\varphi}_t^h$ the ratio of bad to good matches among new hires is denoted with subscript t (i.e. end of period), these new hires are made from the initial pool of searchers s_t that depends on $g_{t-1}, b_{t-1}, u_{t-1}$. In the steady state evaluation, the ratio of good-to-bad workers among new hires is equal to the ratio of good-to-bad workers overall i.e. $\bar{\varphi}^h = \varphi$.

Evaluating the expressions at the steady-state yields:

$$\frac{\partial\varphi'}{\partial\varphi} = \frac{\frac{\delta^b}{(1+\phi\varphi)^2}\left(\frac{\delta^g}{1+\phi\varphi} + \frac{\varkappa}{1+\phi\varphi}\right) + \frac{\phi\delta^g}{(1+\phi\varphi)^2}\left(\frac{\delta^b\varphi}{1+\phi\varphi} + \frac{\varphi\varkappa}{1+\phi\varphi}\right)}{\left(\frac{\delta^g}{1+\phi\varphi} + \frac{\varkappa}{1+\phi\varphi}\right)^2} = \frac{\delta^b}{1+\phi\varphi}\frac{1}{(\delta^g+\varkappa)} + \frac{\delta^b+\varkappa}{(\delta^g+\varkappa)^2}\frac{\delta^g\phi\varphi}{1+\phi\varphi}$$

$$\frac{\partial\varphi'}{\partial\varkappa} = \frac{(1+\phi\varphi)^2}{[(\delta^g+\varkappa)(1+\phi\varphi)]^2}\left(\varphi\delta^g - \varphi\delta^b\right) = \frac{\varphi\delta^g - \varphi\delta^b}{(\delta^g+\varkappa)^2}$$

$$\frac{\partial\varphi'}{\partial\delta^b} = \frac{\frac{\varphi}{1+\phi\varphi}}{\frac{\delta^g}{1+\phi\varphi} + \frac{\varkappa}{1+\phi\varphi}} = \frac{\varphi}{\delta^g+\varkappa}$$

Putting it together yields:

$$\frac{d\varphi'}{d\varphi} = \frac{\delta^b}{1+\phi\varphi}\frac{1}{(\delta^g+\varkappa)} + \frac{\delta^b+\varkappa}{(\delta^g+\varkappa)^2}\frac{\delta^g\phi\varphi}{1+\phi\varphi} + \frac{\varphi\delta^g-\varphi\delta^b}{(\delta^g+\varkappa)^2}\frac{\partial\varkappa}{\partial\varphi} - (1-\rho)f^g\frac{\varphi}{\delta^g+\varkappa}\frac{\partial\zeta_b}{\partial\varphi}$$

Note that under the assumption that $\delta^g = \delta^b = \delta$, it will be that:

$$\frac{\partial\varphi'}{\partial\varphi} = \delta \qquad \frac{\partial\varphi'}{\partial\varkappa} = 0 \qquad \frac{\partial\varphi'}{\partial\delta^b} = \varphi$$

So under that assumption we have a simplified:

$$\frac{d\varphi'}{d\varphi} = \delta - (1-\rho)f^g\varphi\frac{\partial\zeta_b}{\partial\varphi} \tag{S8}$$

As for the effect of wages on future composition evaluated in the steady-state:

$$\frac{d\varphi'}{dw} = -(1-\rho)f^g\varphi\frac{\partial\zeta_b}{\partial w} \tag{S9}$$

B.4.6 Effect of composition and wages on search intensity

To get $\partial\zeta_b/\partial\varphi$ and $\partial\zeta_b/\partial w$ and complete the previous expressions we have, for any composition and wage, the search intensity can be re-written in terms of worker surplus:

$$\zeta_0\zeta_{bt}^{\eta_\zeta}(\varphi_{it}, w_{it}) = f_t^g\left(\bar{H}_t - H_t^b(\varphi_t, w_t)\right)$$

$$\frac{\partial\zeta_{bt}}{\partial\varphi_{it}} = -\frac{\zeta_{bt}^{1-\eta_\zeta}}{\eta_\zeta\zeta_0}f_t^g\frac{\partial H_t^b}{\partial\varphi_{it}}$$

Defining $\tau \equiv \frac{\zeta_b^{1-\eta_\zeta}}{\eta_\zeta\zeta_0}f^g$ we have, at the steady state:

$$\frac{\partial\zeta_b}{\partial\varphi} = -\tau\frac{\partial H^b}{\partial\varphi} \tag{S10}$$

Similarly, we have:

$$\frac{\partial\zeta_{bt}}{\partial w_{it}} = -\frac{\zeta_{bt}^{1-\eta_\zeta}}{\eta_\zeta\zeta_0}f_t^g\frac{\partial H_t^b(\varphi_{it}, w_{it})}{\partial w_{it}}$$

Evaluating at the steady state yields:

$$\frac{\partial\zeta_b}{\partial w} = -\tau\frac{\partial H^b}{\partial w} \tag{S11}$$

B.4.7 Effect of composition and wages on worker surplus in bad matches

The worker surplus in a bad match will be, similar to that in a good match:

$$H_t^b(\varphi_{it}, w_{it}) = \phi \frac{w_{it}}{p_t} - u_B - E_t\Lambda_{t,t+1}(1-\rho)c(\zeta_{bt+1}) + E_t\Lambda_{t,t+1}(1-\rho)(\zeta_{bt+1}f_{t+1}^g)\bar{H}_{t+1}^g +$$
$$+ E_t\Lambda_{t,t+1}(1-\rho)(1-\zeta_{bt+1}f_{t+1}^g)H_{t+1}^b(\varphi_{it+1}, w_{it+1}) - E_t\Lambda_{t,t+1}f_{t+1}\bar{H}_{t+1}$$

This surplus is maximized given the search decision of the worker at time t. However, note that $\zeta_{bt+1} = \zeta_{bt+1}(\varphi_{it+1}, w_{it+1})$.
Let:

$$\frac{\partial H_{t+1}^b}{\partial \varphi_{it+1}} = \lambda \frac{\partial H^b(\varphi_{it+1}, \pi_t^\gamma \varphi_{it})}{\partial \varphi_{it+1}} + (1-\lambda)\frac{\partial H^b(\varphi_{it+1}, w_{it+1}^*)}{\partial \varphi_{it+1}}$$

Therefore:

$$\frac{\partial H_t^b}{\partial \varphi_{it}} = (1-\rho)E_t\left[\Lambda_{t,t+1}(1-\zeta_{bt+1}f_{t+1}^g)\left(\frac{\partial H_{t+1}^b(\varphi_{it+1}, w_{it+1})}{\partial \varphi_{it+1}}\right)+\right.$$
$$\left.(1-\lambda)\left(\frac{\partial H_{t+1}^b(\varphi_{it+1}, w_{it+1}^*(\varphi_{it+1})}{\partial w_{it+1}^*}\frac{\partial w_{it+1}^*}{\partial \varphi_{it+1}}\right)\right]\frac{d\varphi_{it+1}}{d\varphi_{it}} + Z_\varphi$$

With:

$$Z_\varphi = E_t\Lambda_{t,t+1}(1-\rho)\left[f_{t+1}^g\left[\bar{H}_{t+1}^g - H_{t+1}^b\right] - \zeta_0\zeta_{bt+1}^{\eta_\zeta}\right]\frac{\partial \zeta_{bt+1}}{\partial \varphi_{it+1}}\frac{d\varphi_{it+1}}{d\varphi_{it}}$$

Substituting the optimal search intensity condition at $t+1$, we will simply have $Z_\varphi = 0$.
Let $\tau_2 \equiv \zeta_0(\zeta_b)^{\eta_\zeta}$. We have that:

$$\frac{\partial H^b}{\partial \varphi} = \beta(1-\rho)(1-\zeta_b f^g)\left(\frac{\partial H^b}{\partial \varphi} + \left((1-\lambda)\frac{\partial H^b}{\partial w}\frac{\partial w^*}{\partial \varphi}\right)\right)\frac{d\varphi'}{d\varphi}$$

Which simplifies to, given the definition of retention:

$$\frac{\partial H^b}{\partial \varphi}\left(1 - \beta\delta^b\frac{d\varphi'}{d\varphi}\right) = \beta\delta^b\left((1-\lambda)\frac{\partial H^b}{\partial w}\frac{\partial w^*}{\partial \varphi}\right)\frac{d\varphi'}{d\varphi} \tag{S12}$$

Finally, the effect of wages on the surplus of bad matches is:

$$\frac{\partial H_t^b}{\partial w_{it}} = \frac{\phi}{p_t} + (1-\rho)(1-\zeta_{bt+1} f_{t+1}^g)\lambda E_t \left[\Lambda_{t,t+1} \left[\pi_t^\gamma \frac{\partial H_{t+1}^b(\varphi_{it+1}, \pi_t^\gamma w_{it})}{\partial \pi_t^\gamma w_{it}} + \frac{\partial H_{t+1}^b}{\partial \varphi_{it+1}} \frac{d\varphi_{it+1}}{dw_{it}} \right] \right]$$
$$+ (1-\rho)(1-\zeta_{bt+1} f_{t+1}^g)(1-\lambda) E_t \left[\Lambda_{t,t+1} \left[\frac{\partial H_{t+1}^b(\varphi_{it+1}, w_{it}^*)}{\partial \varphi_{t+1}} + \frac{\partial H_{t+1}^b(\varphi_{it+1}, w_{it}^*)}{\partial w_{t+1}^*} \frac{dw_{t+1}^*}{d\varphi_{t+1}} \right] \frac{d\varphi_{it+1}}{dw_{it}} \right] + Z_w$$

With Z_w:

$$Z_w = \pi_t^\gamma \lambda E_t \Lambda_{t,t+1}(1-\rho) \left[\left[f_{t+1}^g \left[\bar{H}_{t+1}^g - H_{t+1}^b \right] \right] - \zeta_0 \zeta_{bt+1}^{\eta_\zeta} \right] \frac{\partial \zeta_{bt+1}(\varphi_{it+1}, \pi_t^\gamma w_{it})}{\partial \pi_t^\gamma w_{it}}$$

But if search is chosen optimally at $t+1$, $Z_w = 0$. Evaluating at the steady-state:

$$\frac{\partial H^b}{\partial w} \left(1 - \delta^b \lambda \beta \pi^\gamma - \delta^b (1-\lambda)\beta \frac{dw^*}{d\varphi} \frac{d\varphi'}{dw} \right) = \frac{\phi}{p} + \delta^b \beta \frac{\partial H^b}{\partial \varphi} \frac{d\varphi'}{dw} \tag{S13}$$

B.4.8 Effect of composition and wages on retention rate

The average retention rate in firm i is:

$$\delta_{it} = \frac{\delta_t^g + \phi \varphi_{it-1} \delta_t^b}{1 + \phi \varphi_{it-1}}$$
$$\delta_t^b = (1-\rho)(1 - \zeta_{bt}(\varphi_{it}, w_{it}) f_t^g)$$

The derivative of retention with respect to composition is:

$$\frac{d\delta_{it}}{d\varphi_{it}} = \frac{\partial \delta_{it}}{\partial \delta_t^b} \frac{\partial \delta_t^b}{\partial \zeta_{bt}} \frac{\partial \zeta_{bt}}{\partial \varphi_{it}}$$

With:

$$\frac{\partial \delta_{it}}{\partial \varphi_{it-1}} = \frac{\phi(\delta_t^b - \delta_t^g)}{(1+\phi\varphi_{it-1})^2} \qquad \frac{\partial \delta_{it}}{\partial \delta_t^b} = \frac{\phi \varphi_{it-1}}{1+\phi\varphi_{it-1}} \qquad \frac{\partial \delta_t^b}{\partial \zeta_{bt}} = -(1-\rho) f_t^g$$

Evaluating at the steady-state, this yields:

$$\frac{d\delta}{d\varphi} = -(1-\rho) f^g \frac{\phi\varphi}{1+\phi\varphi} \frac{\partial \zeta_b}{\partial \varphi} \tag{S14}$$

As for the effect of wages:

$$\frac{d\delta_t}{dw_t} = \frac{\partial \delta_t}{\partial \delta_t^b} \frac{\partial \delta_t^b}{\partial \zeta_{bt}} \frac{\partial \zeta_{bt}}{\partial w_t}$$

Evaluating at the steady-state:

$$\frac{d\delta}{dw} = -(1-\rho) f^g \frac{\phi\varphi}{1+\phi\varphi} \frac{\partial \zeta_b}{\partial w} \tag{S15}$$

B.5 Putting it all together:

$$\frac{\partial w^*}{\partial \varphi} = -\frac{\eta \partial J/\partial \varphi - (1-\eta)\partial H/\partial \varphi}{\eta \partial J/\partial w - (1-\eta)\partial H/\partial w} \tag{S1}$$

$$\frac{\partial H}{\partial \varphi}\left(1 - \delta^g \beta \frac{d\varphi'}{d\varphi}\right) = \delta^g \beta(1-\lambda)\frac{\partial H}{\partial w}\frac{\partial w^*(\varphi)}{\partial \varphi}\frac{d\varphi'}{d\varphi} \tag{S2}$$

$$\frac{\partial H}{\partial w}\left(1 - \delta^g \beta \lambda \pi^\gamma - \delta^g \beta(1-\lambda)\frac{\partial w^*(\varphi)}{\partial \varphi}\frac{d\varphi'}{dw}\right) = 1/p + \delta^g \beta \frac{\partial H}{\partial \varphi}\frac{d\varphi'}{dw} \tag{S3}$$

$$\frac{\partial J}{\partial \varphi}\left(1 - \beta \frac{d\varphi'}{d\varphi}\right) = -\beta J \frac{d\varkappa}{d\varphi}\frac{d\varphi'}{d\varphi} + \beta(1-\lambda)\frac{\partial J}{\partial w}\frac{\partial w^*}{\partial \varphi}\frac{d\varphi'}{d\varphi} \tag{S4}$$

$$\frac{\partial J}{\partial w}\left(1 - \beta\lambda\pi^\gamma - \beta(1-\lambda)\frac{\partial w^*}{\partial \varphi}\frac{d\varphi'}{dw}\right) = -\frac{1}{p} - \beta J \frac{d\varkappa}{dw} + \beta \frac{\partial J}{\partial \varphi}\frac{d\varphi'}{dw} \tag{S5}$$

$$\frac{d\varkappa}{d\varphi}\left(1 - \beta\delta \frac{d\varphi'}{d\varphi}\right) = \beta\delta(1-\lambda)\frac{d\varkappa}{dw}\frac{\partial w^*}{\partial \varphi}\frac{d\varphi'}{d\varphi} \tag{S6}$$

$$\frac{d\varkappa}{dw}\left(1 - \pi^\gamma \beta\delta\lambda - \beta\delta(1-\lambda)\frac{\partial w^*}{\partial \varphi}\frac{d\varphi'}{d\varphi}\right) = -\frac{1}{\kappa p} + \beta\delta \frac{d\varkappa}{d\varphi}\frac{d\varphi'}{dw} \tag{S7}$$

$$\frac{d\varphi'}{d\varphi} = \delta - (1-\rho) f^g \varphi \frac{\partial \zeta_b}{\partial \varphi} \tag{S8}$$

$$\frac{d\varphi'}{dw} = -(1-\rho) f^g \varphi \frac{\partial \zeta_b}{\partial w} \tag{S9}$$

$$\frac{\partial \zeta_b}{\partial \varphi} = -\tau \frac{\partial H^b}{\partial \varphi} \tag{S10}$$

$$\frac{\partial \zeta_b}{\partial w} = -\tau \frac{\partial H^b}{\partial w} \tag{S11}$$

$$\frac{\partial H^b}{\partial \varphi}\left(1 - \beta\delta^b \frac{d\varphi'}{d\varphi}\right) = \beta\delta^b \left((1-\lambda)\frac{\partial H^b}{\partial w}\frac{\partial w^*}{\partial \varphi}\right)\frac{d\varphi'}{d\varphi} \tag{S12}$$

$$\frac{\partial H^b}{\partial w}\left(1 - \delta^b \lambda\beta\pi^\gamma - \delta^b(1-\lambda)\beta \frac{dw^*}{d\varphi}\frac{d\varphi'}{dw}\right) = \frac{\phi}{p} + \delta^b \beta \frac{\partial H^b}{\partial \varphi}\frac{d\varphi'}{dw} \tag{S13}$$

$$\frac{d\delta}{d\varphi} = -(1-\rho) f^g \frac{\phi\varphi}{1+\phi\varphi}\frac{\partial \zeta_b}{\partial \varphi} \tag{S14}$$

$$\frac{d\delta}{dw} = -(1-\rho) f^g \frac{\phi\varphi}{1+\phi\varphi}\frac{\partial \zeta_b}{\partial w} \tag{S15}$$

Where I simplified the expressions taking $\delta^b = \delta^g = \delta$. In sum, we have a system of **15** equations and **15** unknowns:

$$\left\{\frac{\partial w^*}{\partial \varphi}, \frac{\partial J}{\partial \varphi}, \frac{\partial J}{\partial w}, \frac{\partial H}{\partial \varphi}, \frac{\partial H}{\partial w}, \frac{d\varphi'}{d\varphi}, \frac{d\varphi'}{dw}, \frac{\partial \zeta_b}{\partial w}, \frac{\partial \zeta_b}{\partial \varphi}, \frac{\partial H^b}{\partial \varphi}, \frac{\partial H^b}{\partial w}, \frac{d\varkappa}{dw}, \frac{d\varkappa}{d\varphi}, \frac{d\delta}{dw}, \frac{d\delta}{d\varphi}\right\}$$

We can see that $\partial w^*/\partial \varphi = 0$ solves the system for any given price level p (or, more simply, that $\partial w^{*r}/\partial \varphi = 0$, with $w^{*r} = w/p$).

$$
\begin{aligned}
&\frac{\partial w^*}{\partial \varphi} = \frac{\partial H}{\partial \varphi} = \frac{\partial \varkappa}{\partial \varphi} = \frac{\partial J}{\partial \varphi} = \frac{\partial H^b}{\partial \varphi} = \frac{\partial \zeta_b}{\partial \varphi} = \frac{d\delta}{d\varphi} = 0 \\
&\frac{\partial H}{\partial w} = \frac{1}{p(1-\delta^g \beta\lambda\pi^\gamma)} \\
&\frac{\partial J}{\partial w}(1-\beta\lambda\pi^\gamma) = -\frac{1}{p} - \beta J\lambda\pi^\gamma \frac{\partial \varkappa}{\partial w} \\
&\frac{d\varkappa}{dw} = -\frac{1}{\kappa p}(1-\pi^\gamma \beta\delta\lambda)^{-1} \\
&\frac{d\varphi'}{d\varphi} = \delta \\
&\frac{d\varphi'}{dw} = -(1-\rho)f^g \frac{\partial \zeta_b}{\partial w} \\
&\frac{\partial \zeta_b}{\partial w} = -\frac{\partial H^b}{\partial w} \\
&\frac{\partial H^b}{\partial w} = \frac{\phi}{p} * (1-\delta^b \lambda\beta\pi^\gamma)^{-1} \\
&\frac{d\delta}{dw} = (1-\rho)f^g \frac{\phi\varphi}{1+\phi\varphi}\frac{\partial H^b}{\partial w}
\end{aligned}
$$

B.6 Composition effects: Surplus approximations

See GHT Model Appendix C.2.6 for derivations. In short, we can approximate period-ahead surplus at renegotiating firms with period-ahead surplus of a firm with average composition and average wages. Crucial to obtaining this result is a first-order Taylor expansion around the average values for composition and wages, and the results that $\partial H/\partial\varphi = \partial w^*/\partial\varphi = 0$, which yield $\eta_{H_\varphi} = \eta_{w_\varphi} = 0$, which my model verifies. The firm surplus approximations, instead, hinge on $\partial J/\partial\varphi = \partial H^b/\partial\varphi = \partial w^*/\partial\varphi = 0$, also verified by my model. In turn, these yield $\eta_{J_\varphi} = \eta_{Hb_\varphi} = \eta_{w_\varphi} = 0$.
The approximations for surplus of workers in good matches in renegotiating firms:

$$
\begin{aligned}
&\hat{H}_{t+1}(\varphi_{it+1}, w^*_{it+1}(\varphi_{it+1})) = \hat{H}_{t+1}(\varphi_{t+1}, w^*_{t+1}) \\
&\hat{H}_{t+1}(\varphi_{it+1}, \pi^\gamma w^*_t) = \hat{H}_{t+1}(\varphi_{t+1}, w^*_{t+1}) + \eta_{H_w}(\gamma\hat{\pi}_t + \hat{w}^*_t - \hat{w}^*_{t+1})
\end{aligned}
$$

The approximations for firm surplus:

$$
\begin{aligned}
&\hat{J}_{t+1}(\varphi_{it+1}, w^*_{it+1}(\varphi_{it+1})) = \hat{J}_{t+1}(\varphi_{t+1}, w^*_{t+1}) \\
&\hat{J}_{t+1}(\varphi_{it+1}, \pi^\gamma w^*_t) = \hat{H}_{t+1}(\varphi_{t+1}, w^*_{t+1}) + \eta_{J_w}(\gamma\hat{\pi}_t + \hat{w}^*_t - \hat{w}^*_{t+1})
\end{aligned}
$$

The approximations for surplus of workers in bad matches:

$$
\begin{aligned}
&\hat{H}^b_{t+1}(\varphi_{it+1}, w^*_{it+1}(\varphi_{it+1})) = \hat{H}^b_{t+1}(\varphi_{t+1}, w^*_{t+1}) \\
&\hat{H}^b_{t+1}(\varphi_{it+1}, \pi^\gamma w^*_t) = \hat{H}^b_{t+1}(\varphi_{t+1}, w^*_{t+1}) + \eta_{H^b_w}(\gamma\hat{\pi}_t + \hat{w}^*_t - \hat{w}^*_{t+1})
\end{aligned}
$$

Finally,

$$
\hat{w}^*_{t+1}(\varphi_{it+1}) \approx \hat{w}^*_{t+1}(\varphi_{t+1})
$$

Because we also have that $\partial\varkappa/\partial\varphi = \partial\delta/\partial\varphi = 0$, similar approximations can be derived for log-deviations in hires and retentions.
Where η_{H_w} denotes the steady-state elasticity of surplus H with respect to w, and so on for η_{J_w} and $\eta_{H^b_w}$.