

Question 1

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$$\int P \, dx$$

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$$\int P \, dx = 2 \int \frac{dx}{x}$$

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$$y = -\frac{\cos x}{x^2} + \frac{c}{x^2}$$

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$$y = 1 \text{ at } x = 1$$

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$$y = 1 \text{ at } x = 1 \quad \Longrightarrow \quad c = \frac{5}{4} \quad \Longrightarrow \quad y = \frac{x}{2} \ln x - \frac{x}{4} + \frac{5}{4x}$$

Question 3

$$\frac{dT}{dt} = -k(T - \sin t)$$

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$$e^{kt} \frac{dT}{dt} + kTe^{kt} = ke^{kt} \sin t$$

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$$\Longrightarrow \quad e^{kt} T = k \int e^{kt} \sin t dt$$

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$$\Longrightarrow \quad e^{kt} T = k \int e^{kt} \sin t dt = k I_1 + c$$

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$$I_1 = \int e^{kt} \sin t \, dt$$

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$$I_2 = \int e^{kt} \cos t \, dt$$

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$$I_2 = \int e^{kt} \cos t \, dt = \frac{e^{kt}}{k} \cos t + \frac{1}{k} \int e^{kt} \sin t \, dt$$

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$$\begin{aligned} I_2 &= \int e^{kt} \cos t \, dt = \frac{e^{kt}}{k} \cos t + \frac{1}{k} \int e^{kt} \sin t \, dt \\ &= \frac{e^{kt} \cos t}{k} + \frac{1}{k} I_1 \end{aligned}$$

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$$I_1 = \frac{e^{kt}}{k} \sin t - \frac{1}{k} \left(\frac{e^{kt} \cos t}{k} + \frac{1}{k} I_1 \right)$$

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$$I_1 = \frac{e^{kt}}{k} \sin t - \frac{1}{k} \left(\frac{e^{kt} \cos t}{k} + \frac{1}{k} I_1 \right)$$

$$\left(1 + \frac{1}{k^2} \right) I_1 = \frac{1}{k^2} \left(k e^{kt} \sin t - e^{kt} \cos t \right)$$

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$$\left(1 + \frac{1}{k^2} \right) I_1 = \frac{1}{k^2} \left(k e^{kt} \sin t - e^{kt} \cos t \right)$$

$$\implies I_1 = \frac{1}{1 + k^2} \left(k e^{kt} \sin t - e^{kt} \cos t \right)$$

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$$e^{kt}T = kI_1 + c$$

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$$e^{kt}T = kI_1 + c = \frac{k}{1+k^2} \left(ke^{kt} \sin t - e^{kt} \cos t \right) + c$$

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$$e^{kt}T = kI_1 + c = \frac{k}{1+k^2} \left(ke^{kt} \sin t - e^{kt} \cos t \right) + c$$

$$\implies T = ce^{-kt} + \frac{k}{1+k^2} (k \sin t - \cos t)$$

Question 4

$$(s - t)(s + t - u)$$

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$$(s - t)(s + t - u) = s^2 - su - t^2 + tu$$

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Take

$$s = \frac{dy}{dx} \quad u = x \quad t = y$$

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$$s = \frac{dy}{dx} \quad u = x \quad t = y$$

$$0 = \left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} - y^2 + xy$$

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$$s = \frac{dy}{dx} \quad u = x \quad t = y$$

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$$\frac{dy}{dx} - y = 0$$

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$$\frac{dy}{dx} + y - x = 0$$

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$$\frac{dy}{dx} + y - x = 0 \quad \implies \quad \frac{dy}{dx} + y = x$$

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$$ye^x = \int xe^x \, dx = xe^x - \int e^x \, dx = xe^x - e^x + B$$

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$$y = x - 1 + Be^{-x}$$