

**ACS234**

# **Maths and Data Modelling**

**Tutorial 7**

**Wednesday 1pm online**

**<https://github.com/ineskris/ACS234/blob/master/Tutorial7/Tutorial7.ipynb>**

## Done in Lecture (week 8/9)

- ODEs systems
- Euler's Method

# Ordinary Differential Equations

	1st-order ODE system	nth-order ODE
Formula	$\frac{dy}{dt} = f(t, y)$	$\frac{dy^n}{dt} = f(t, y(t), y'(t), \dots, y^{n-1}(t))$
Example	$\frac{dy}{dt} = y \quad y(0) = 1 \quad \boxed{y(t) = \exp(t)}$	$\frac{dy^2}{dt} = -\frac{dy}{dt} + 6y \quad \boxed{y(t) = A \exp(2t) + B \exp(-3t)}$

## Euler's Method

Taylor series : Let  $f$  be indefinitely differentiable at a point  $a$ :  $f(a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n \quad y(t_0 + h) \approx y(t_0) + hy'(t_0)$

Euler's method :  $\frac{dy}{dt} = f(t, y) \quad y_1 \approx y_0 + hf(t_0, y_0) \quad t_1 = t_0 + h \quad y(t_1) = y_1 \quad y(t_0) = y_0$

$$\frac{dy}{dt} = f(t, y) \quad y(t_0) = y_0$$

$$y_{i+1} = y_i + hf(t_i, y_i)$$

## Stability

Solution of an ODE is :

- Stable if solutions resulting from perturbations of initial value remain close to original solution
- Asymptotically stable if solutions resulting from perturbations converge back to original solution
- Unstable if solutions resulting from perturbations diverge away from original solution without bound

## Exercise 1

Use Euler's method to integrate these 1st order ODE system with the given step size  $h$ , number of steps and the initial condition. Compute for each step the exact analytical solution too.

a) $h = 0.1$	$n_{step} = 2$	b) $h = 0.5$	$n_{step} = 3$	c) $h = 1$	$n_{step} = 2$
$\frac{dy}{dt} = 3y$	$y(0) = 2$	$\frac{dy}{dt} = t^2 + 1$	$y(1) = 4$	$\frac{dy}{dt} = \ln(t)$	$y(1) = 0$

## Exercise 2

Find a step size  $h$  in question 1c) to have an absolute error inferior to 0.05 between the analytical solution and the approximate  $y_2$ .

## Exercise 3

$$\frac{dy}{dt} = -3y \quad y(0) = 2$$

For the 1st order ODE above, find the condition on the step size  $h$  for the difference equation to be stable. See Hand out part 5.