

# ACS234

# Maths and Data Modelling

**Tutorial 6**  
**Wednesday 1pm online**

<https://github.com/ineskris/ACS234/tree/master/Tutorial6>

## Done in Lecture (week 6/7)

- General Linear Regression
- Non Linear Regression

# General Linear Regression

$$\nearrow Y = X\hat{a} + e \quad \hat{a} = (X'X)^{-1}X'Y$$

Linear means the response Y is a linear function with the unknown parameter a.

Linear Regression with Nonlinear Terms, one example :  $y = a_0 + a_1x + a_2e^{-x} = Xa$       $X = \begin{pmatrix} 1 & x_0 & e^{-x_0} \\ 1 & x_1 & e^{-x_1} \\ 1 & x_2 & e^{-x_2} \\ 1 & x_3 & e^{-x_3} \end{pmatrix}$

## Exercise 1

Based on each dataset, estimate the parameters of the **general linear regression model**. Calculate the MSE error.

a)  $f_1(x) = a_0x + a_1\sin(x)$

x	0	3	10	20
y	0	3.28	8.91	21.8

b)  $f_2(x) = a_0 + a_1 \log(x)$

x	1	3	10	20
y	10	8.9	7.7	7

c)  $f(x) = a_1\sin(2\pi x) + a_2\cos(2\pi x)$

x	0	1
y	-3	-3

For questions b and c, don't you see a simpler way to estimate the parameters?

## Exercise 2

Use a **polynomial regression model of degree 2** to estimate the parameters from ex1a). What is the MSE error ?

# Non Linear Regression



the response Y is a non linear function with the unknown parameter a

$$\hat{a} = (X'X)^{-1}X'Y$$

Non Linear Regression with Nonlinear Terms, one example  $y = a_0 + e^{-a_1x}$

We want to minimise the difference between the response y and the estimated y induced by the model.

## Exercise 2

Estimate the parameter  $\eta$  of the Weibull Distribution given below using the data points and Matlab or Python.  
Write the function to minimise.

$$f(x) = \eta \times x^{\eta-1} \exp^{-(x^\eta)}$$

x	0.1	0.6	1.4	4
y	1.17	0.35	0.15	0.03

Tips :

In Matlab, you can use :

- lsqnonlin
- fminsearch

Equivalent in Python :

```
from scipy.optimize import curve_fit
```