ACS234
Maths and Data Modelling

Find in this sheet a series of exercises to work during the holidays and which take up all the topics mentioned during the 3 previous tutorials.

Interpolation

Exercice 1 - Find the function of degree 3 that goes though these 3 points using :

a) Polynomial interpolation
b) Lagrange interpolation
c) Newton interpolation

x	-1	0	1	2
f(x)	-1	1	0	0

Exercice 2 - We have the data below representing the water kinematic viscosity v (in $m^2\ s^{-1}$) as a function of temperature T (in °C) :

T	15	16	17	18	19	20	21	22	23	24	25	26	27	28
v	1.14	1.11	1.08	1.06	1.03	1.01	0.983	0.960	0.938	0.917	0.896	0.876	0.857	0.839

1. What is the kinematic viscosity when T = 26.5 °C ?
2. For which temperature do we have v = 0.9 $m^2\ s^{-1}$?

Exercice 3

a) Find the Lagrange polynomial going though the first three points in red.
b) Find the Lagrange polynomial going though the first four points in red and blue.
c) Using both functions, interpolate the value of the f(3). Is it close to the real value in green ?

x	0.0	1.0	2.0	3.0	4.0
f(x)	0.0	2.0	36.0	252.0	1040.0

Exercice 1

$$f(x) = \frac{2}{3}x^3 + \frac{-3}{2}x^2 + \frac{-1}{6}x + 1$$

Exercice 2

1) You can choose to interpolate with 2,3 etc points using any methods. Only 2 points seems enough to approximate the function (see figure ex2.png), we have :

26	27
0.876	0.857

$$f(x) = -0.019x + 1.37$$

$$f(26.5) \approx 0.867$$

2) $f(x) \approx 0.9 \qquad x = \frac{(0.9 - 1.37)}{-0.019} \approx 24.73$

Exercice 3

a) $f_1(x) = a_0x^2 + a_1x \qquad a_0 = 16 \qquad a_1 = -14$

b) $f_2(x) = a_0x^3 + a_1x^2 + a_2x \qquad a_0 = 35 \qquad a_1 = -89 \qquad a_2 = 56$ (see figure ex3.png),

c) $f_1(3) = 102$

$f_2(3) = 312$

Simple Linear Regression

Exercice 1 - We have the weight of father and son given below. Calculate the coefficients of the linear model as well as the coefficient of determination R^2 . Give an interpretation.

Father	65	63	67	64
Son	68	66	68	65

Exercice 2 - We have the following linear model $y = a_0 + a_1 x + e$ and we know :
a) the regression line goes though $(x_1, y_1) = (2,2.5)$
b) $\bar{x} = 3.0$ and $\bar{y} = 5.0$
Find $\hat{a}_0$ and $\hat{a}_1$.

Exercice 3 - Can you write a Matlab (or Python) code to solve the 2 exercices above ?

Exercice 1

$$father = \hat{\beta} \times son$$

$$\hat{\beta} \approx 1.031$$

$$R^2 \approx 0.379$$

The proportionality between the weight of a father and his son is not obvious with a very low coefficient R^2

Exercice 2

We have the following equations :

$$\begin{aligned} 2.5 &= a_0 + a_1 \times 2 \\ 5 &= a_0 + a_1 \times 3 \end{aligned} \qquad \begin{aligned} \hat{a}_0 &= -2.5 \\ \hat{a}_1 &= 2.5 \end{aligned}$$