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$$y(x) = e^{5x}\left(A\cos x + B\sin x\right)$$

Question 2

$$\frac{d^2y}{dx^2} + 4y = 0$$

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$$\Longrightarrow \quad \frac{d^4y}{dx^4} - 16y = 0$$

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$$\begin{aligned}
\frac{d^2y}{dx^2} + 4y = 0 \qquad &\Longrightarrow \qquad \frac{d^3y}{dx^3} + 4\frac{dy}{dx} = 0 \qquad \Longrightarrow \qquad \frac{d^4y}{dx^4} + 4\frac{d^2y}{dx^2} = 0 \\
&\Longrightarrow \qquad \frac{d^4y}{dx^4} - 16y = 0 \qquad \Longrightarrow \qquad \frac{d^5y}{dx^5} - 16\frac{dy}{dx} = 0 \\
&\Longrightarrow \qquad \frac{d^6y}{dx^6} - 16\frac{d^2y}{dx^2} = 0
\end{aligned}$$

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&\Longrightarrow \qquad \frac{d^6y}{dx^6} - 16\frac{d^2y}{dx^2} = 0 \qquad \Longrightarrow \qquad \frac{d^6y}{dx^6} + 64y = 0
\end{aligned}$$

Question 3 (a)

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$$

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Solutions are

$$y = e^{-x} \qquad y = xe^{-x} \qquad y = x^2e^{-x}$$

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Solutions are

$$y = e^{-x} \qquad y = xe^{-x} \qquad y = x^2e^{-x}$$

General solution

$$y = e^{-x}\left(A + Bx + Cx^2\right)$$

Question 3 (b)

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Auxiliary equation

$$(\lambda - 1)^4 = 0$$

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Auxiliary equation

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$$\implies \qquad \lambda^4 - 4\lambda^3 + 6\lambda^2 - 4\lambda + 1 = 0$$

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Auxiliary equation

$$(\lambda - 1)^4 = 0$$

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So a suitable differential equation is

$$\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$$