ACS234
Maths and Data Modelling

Tutorial 9
Wednesday 1pm online

- High Order ODE
- RK4 method

ODE

	1st-order ODE system	nth-order ODE
Formula	$\frac{dy}{dt} = f(t, y)$ $\quad y(t_0) = y_0$	$\frac{dy^n}{dt} = f(t, y(t), y'(t), \ldots, y^{n-1}(t), u(t))$
Example	$\frac{dy}{dt} = y$ $\quad y(0) = 1$ $\quad \boxed{y(t) = \exp(t)}$	$\frac{dy^2}{dt} = -\frac{dy}{dt} + 6y$ $\quad \boxed{y(t) = A\exp(2t) + B\exp(-3t)}$
Euler	$y_{i+1} = y_i + hf(t_i, y_i)$ $\quad t_{i+1} = t_i + h$	$x_1(t) = y(t)$ $x_2(t) = y'(t)$ … $x_n(t) = y^{n-1}(t)$ $\dot{x}_1(t) = x_2(t)$ $\dot{x}_2(t) = x_3(t)$ … $\dot{x}_{n-1}(t) = x_n(t)$ $\dot{x}_n(t) = f(t, x_1(t), \ldots, x_n(t))$
Mid Point	$y_{i+\frac{1}{2}} = y_i + \frac{h}{2}f(t_i, y_i)$ $t_{i+\frac{1}{2}} = t_i + \frac{h}{2}$ $y_{i+1} = y_i + hf(t_{i+\frac{1}{2}}, y_{i+\frac{1}{2}})$	
Heun	$y^p_{i+1} = y_i + hf(t_i, y_i)$ $\quad t_{i+1} = t_i + h$ $y_{i+1} = y_i + \frac{h}{2}[f(t_i, y_i) + f(t_{i+1}, y^p_{i+1})]$	
RK4	$y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ $k_1 = f(t_i, y_i)$ $k_2 = f(t_i + \frac{h}{2}, y_i + \frac{hk_1}{2})$ $k_3 = f(t_i + \frac{h}{2}, y_i + \frac{hk_2}{2})$ $k_4 = f(t_i + h, y_i + hk_3)$	

Method :
—> state-space model (simultaneous equations of n 1st-order ODEs)
--> apply a numerical method (e.g. Euler's, Mid-point, Heun's, RK4) to each of the n 1st-order ODEs
>> in each step, the values of the state variables are updated simultaneously in each step of calculation
>> when all state variables are updated, move to the next step of calculation

Exercice 1

Compute the first two steps of the RK4 with thE 1st order ODE system below with the given step size h and the initial condition. Compare with the results obtained in Tutorial 8 Exercice 1a.

$$h = 0.1$$

$$\frac{dy}{dt} = 3y \qquad y(0) = 2$$

Exercice 2

Use classical RK4 method to solve the second-order below (cf hand out) :

$$y''(t) = -y'(t) + sin(ty) \qquad y(0) = 1 \qquad y'(0) = 2$$