ACS234
Maths and Data Modelling

Tutorial 8
Wednesday 1pm online

https://github.com/ineskris/ACS234/blob/master/Tutorial8/Tutorial8.ipynb

Done in Lecture (week 9/10)

- High Order ODE
- Euler’s, Mid Point, Heun’s Method

ODE solvers

<table>
<tr><th></th><th>1st-order ODE system</th><th>nth-order ODE</th></tr>
<tr><td>Formula</td><td>$\frac{dy}{dt} = f(t, y) \qquad y(t_0) = y_0$</td><td>$\frac{dy^n}{dt} = f(t, y(t), y'(t), \ldots, y^{n-1}(t), u(t))$</td></tr>
<tr><td>Example</td><td>$\frac{dy}{dt} = y \qquad y(0) = 1 \qquad \boxed{y(t) = \exp(t)}$</td><td>$\frac{dy^2}{dt} = -\frac{dy}{dt} + 6y \qquad \boxed{y(t) = A\exp(2t) + B\exp(-3t)}$</td></tr>
<tr><td>Euler</td><td>$y_{i+1} = y_i + hf(t_i, y_i) \qquad t_{i+1} = t_i + h$</td><td rowspan="3">$x_1(t) = y(t)$
$x_2(t) = y'(t)$
$\cdots$
$x_n(t) = y^{n-1}(t)$

$\dot{x}_1(t) = x_2(t)$
$\dot{x}_2(t) = x_3(t)$
$\cdots$
$\dot{x}_{n-1}(t) = x_n(t)$
$\dot{x}_n(t) = f(t, x_1(t), \ldots, x_n(t))$</td></tr>
<tr><td>Mid Point</td><td>$y_{i+\frac{1}{2}} = y_i + \frac{h}{2}f(t_i, y_i)$
$t_{i+\frac{1}{2}} = t_i + \frac{h}{2}$
$y_{i+1} = y_i + hf(t_{i+\frac{1}{2}}, y_{i+\frac{1}{2}})$</td></tr>
<tr><td>Heun</td><td>$y^p_{i+1} = y_i + hf(t_i, y_i) \qquad t_{i+1} = t_i + h$
$y_{i+1} = y_i + \frac{h}{2}[f(t_i, y_i) + f(t_{i+1}, y^p_{i+1})]$</td></tr>
</table>

Exercice 1

Compute the first two steps of the Mid Point and the Heun method with these 1st order ODE system with the given step size h and the initial condition. Compare the results with the Euler method obtained in Tutorial 6 Ex 1.

a) $h = 0.1$

$$\frac{dy}{dt} = 3y \qquad y(0) = 2$$

b) $h = 0.5$

$$\frac{dy}{dt} = t^2 + 1 \qquad y(1) = 4$$

Exercice 2

Solve the ODE below using the Euler Method.

$$y''(t) = -y'(t) + sin(ty) \qquad y(0) = 1 \qquad y'(0) = 2$$