ACS234
Maths and Data Modelling

Tutorial 7
Wednesday 1pm online

https://github.com/ineskris/ACS234/blob/master/Tutorial7/Tutorial7.ipynb

Done in Lecture (week 8/9)

- ODEs systems
- Euler's Method

Ordinary Differential Equations

	1st-order ODE system	nth-order ODE
Formula	$\frac{dy}{dt} = f(t, y)$	$\frac{dy^n}{dt} = f(t, y(t), y'(t), \ldots, y^{n-1}(t))$
Example	$\frac{dy}{dt} = y \quad y(0) = 1 \quad \boxed{y(t) = \exp(t)}$	$\frac{dy^2}{dt} = -\frac{dy}{dt} + 6y \quad \boxed{y(t) = A\exp(2t) + B\exp(-3t)}$

Euler's Method

Taylor series : Soit f indéfiniment dérivable en un point a : $f(a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad y(t_0 + h) \approx y(t_0) + hy'(t_0)$

Euler's method : $\frac{dy}{dt} = f(t, y) \quad y_1 \approx y_0 + hf(t_0, y_0) \quad t_1 = t_0 + h \quad y(t_1) = y_1 \quad y(t_0) = y_0$

$$\frac{dy}{dt} = f(t, y) \qquad y(t_0) = y_0$$

$$y_{i+1} = y_i + hf(t_i, y_i)$$

Exercice 1

Use Euler's method to integrate these 1st order ODE system with the given step size h, number of steps and the initial condition. Compute for each step the exact analytical solution too.

a) $h = 0.1 \qquad n_{step} = 2$

$\frac{dy}{dt} = 3y \qquad y(0) = 2$

b) $h = 0.5 \qquad n_{step} = 3$

$\frac{dy}{dt} = t^2 + 1 \qquad y(1) = 4$

c) $h = 1 \qquad n_{step} = 2$

$\frac{dy}{dt} = ln(t) \qquad y(1) = 0$

Exercice 2

Find the step size h in question 1c) to have an absolute error inferior to 0.2 for the approximate y_2.

Draw the solution with both approximate using the Tangent line.

Exercice 1 - solution

Exercice 2 - solution