Quasi-random number generator

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Pseudo-random vs. quasi-random

Pseudo-random number

- computer-generated number
- appears to be random
- generated by an entirely deterministic process

Quasi-random number

- low-discrepancy number
- taking previous draws into account

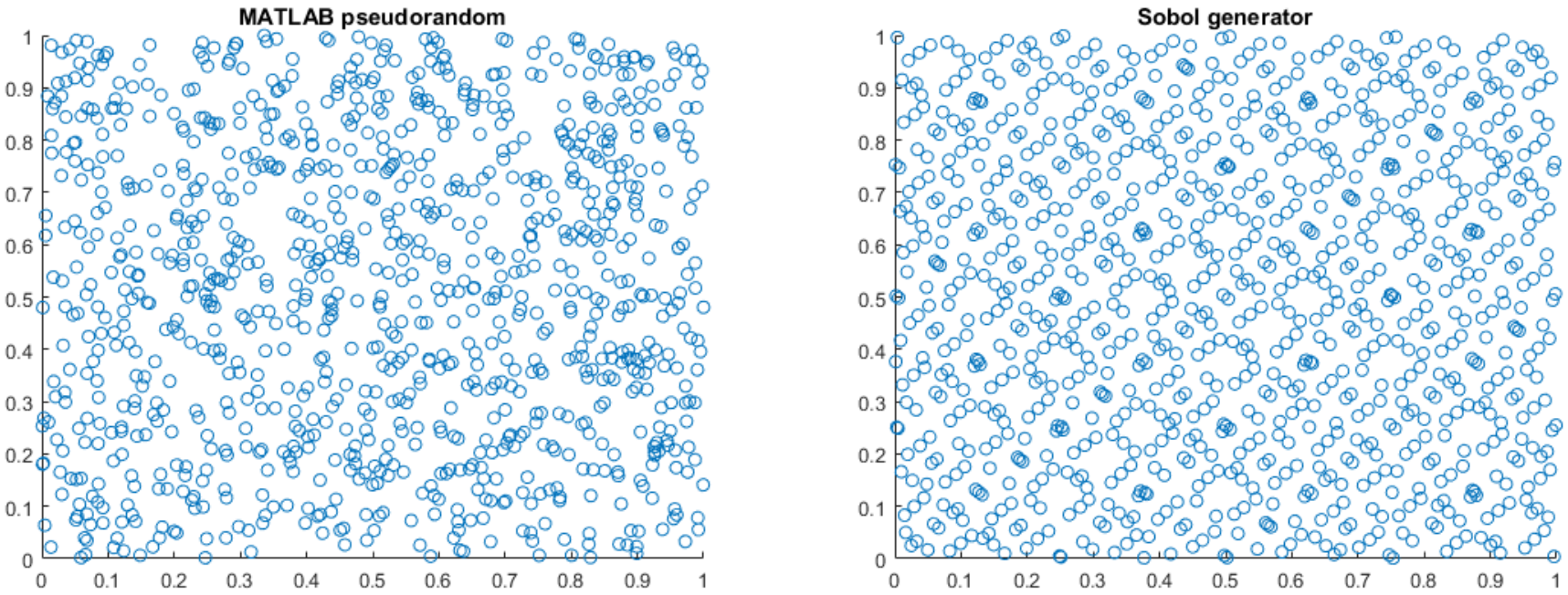


Figure: The comparison between pseudo- (left) and quasi-random numbers.

Usage

- useful in computational problems
- popular for financial Monte Carlo calculations
- asymptotic convergence is faster than when using pseudo-random numbers

Sobol' numbers

Algorithm 1 Generates the d-dimensional vector x_n in the Sobol sequence.

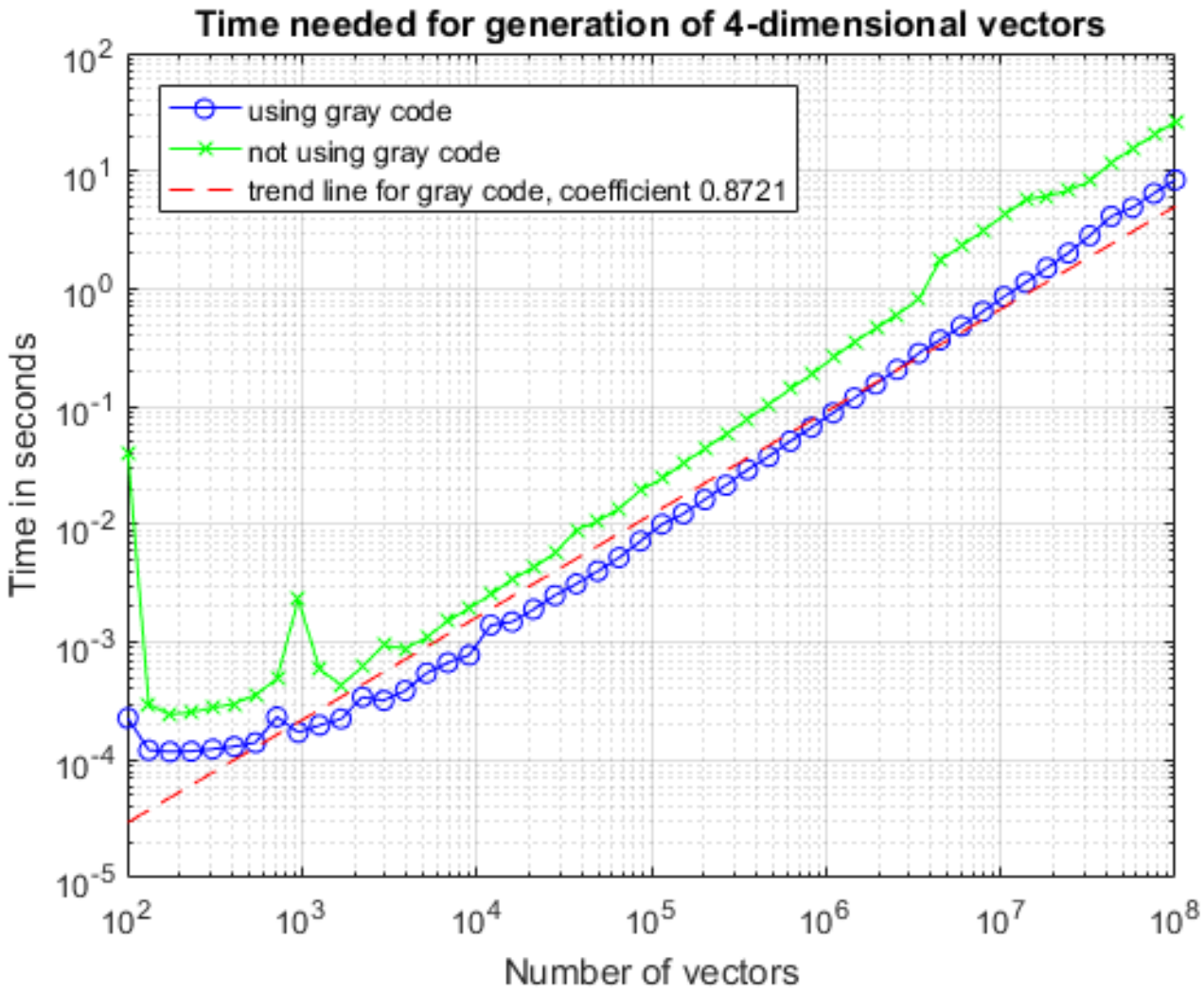
1: $\gamma(n) = n$ or $\gamma(n) = G(n)$
2: **for** $k = 1, \ldots, d$ **do**
3: $\quad p_k(z) = a_{k0} z^{g_k} + a_{k1} z^{g_k - 1} + \ldots + a_{k(g_k - 1)} z + a_{k g_k}$
4: $\quad$ calculate the direction integers v_{kl} using a_{kj} and binary addition
5: $\quad$ calculate x_{nk} based on which bits in $\gamma(n)$ are set
6: **end for**

Gray code

- any unique representation of n can be used for $\gamma(n)$
- $G(n)$ switches only one single bit for every increment in n
- this means that a single XOR operation has to be carried out for each dimension:

$$x_{nk} = x_{(n-1)k} \oplus v_{kj}$$

Gray code vs. not Gray code



Monte Carlo integration

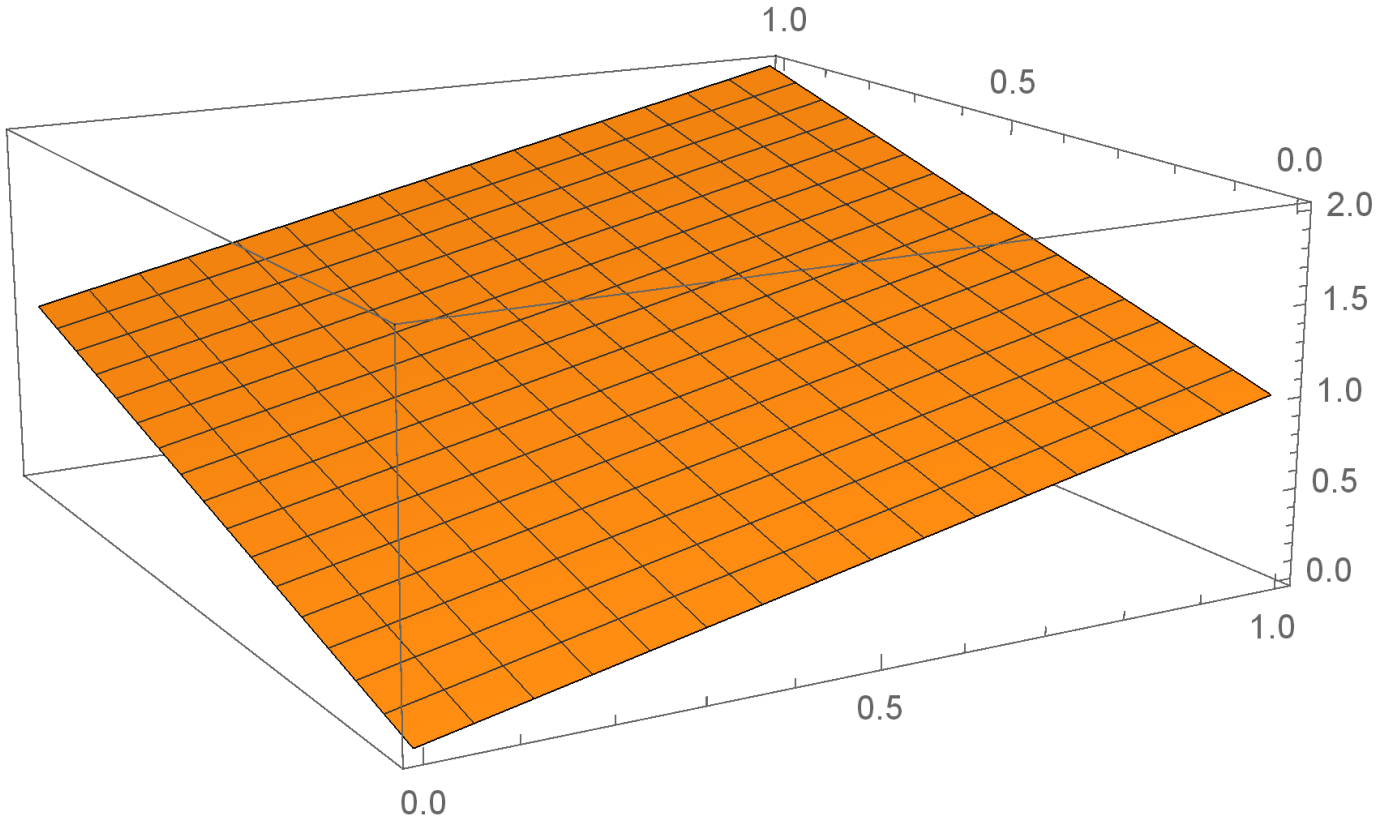
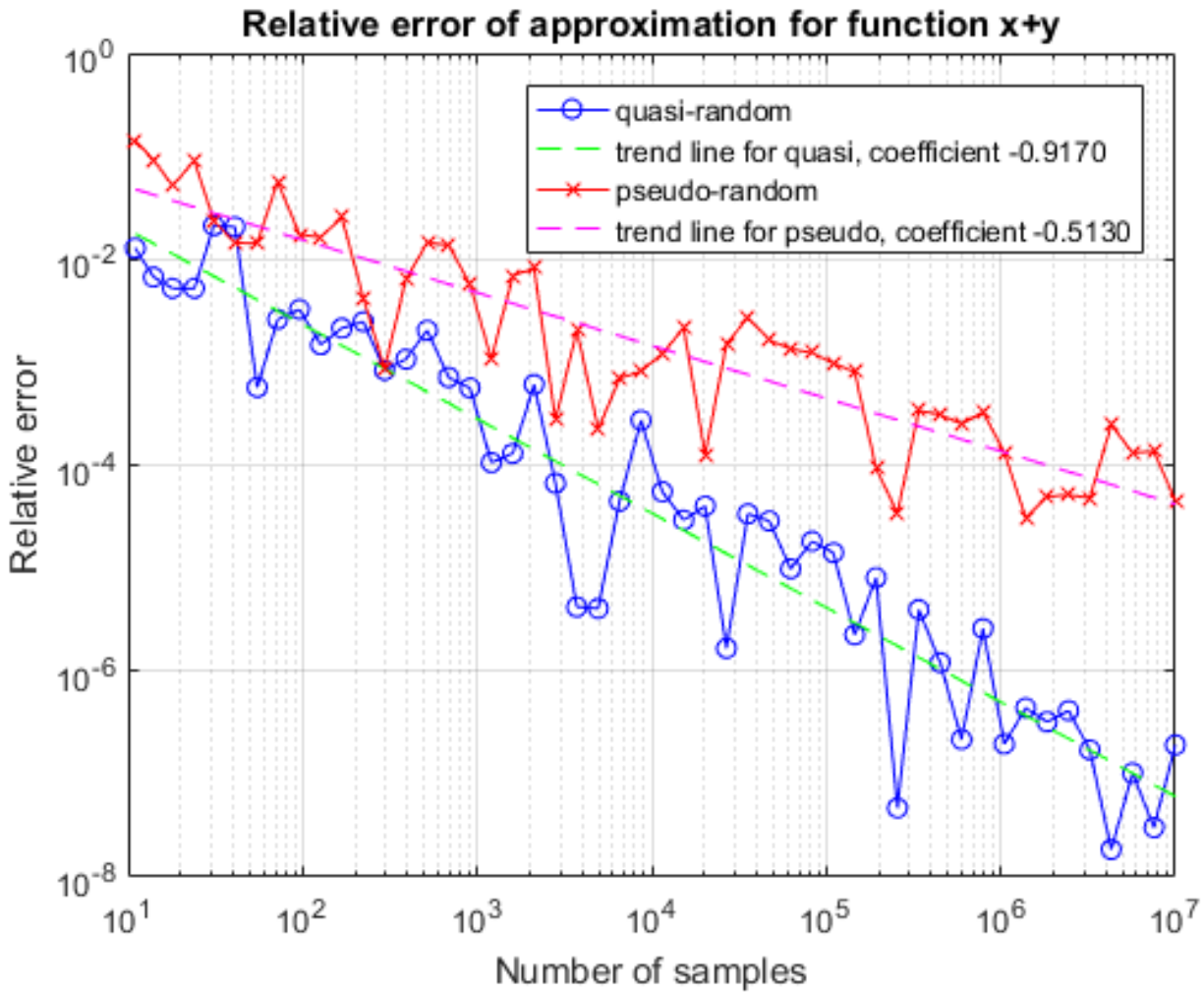


Figure: Plot of function $x + y$.


Relative error of approximation for function x+y
quasi-random
trend line for quasi, coefficient -0.9170
pseudo-random
trend line for pseudo, coefficient -0.5130
Relative error
Number of samples
10^0
10^-2
10^-4
10^-6
10^-8
10^1
10^2
10^3
10^4
10^5
10^6
10^7

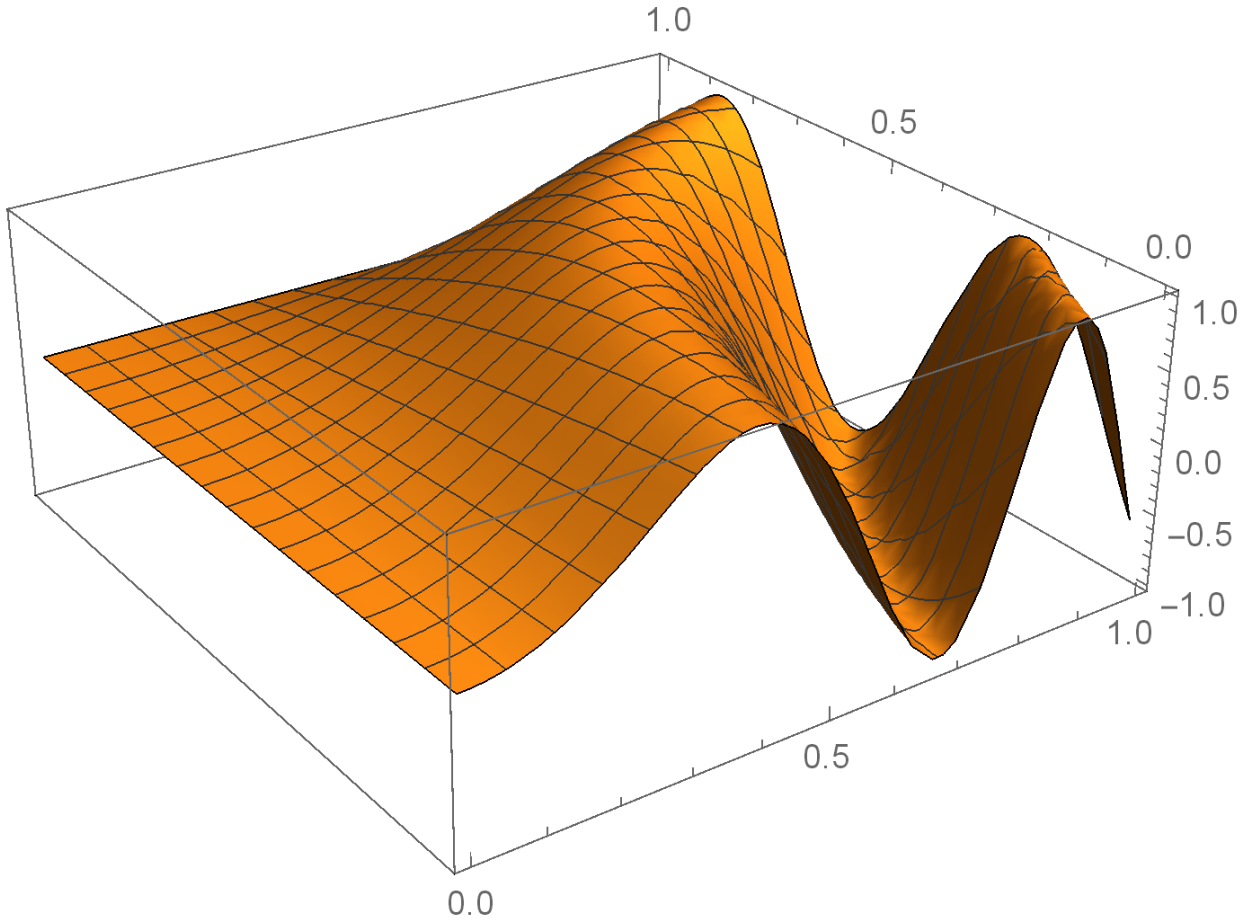
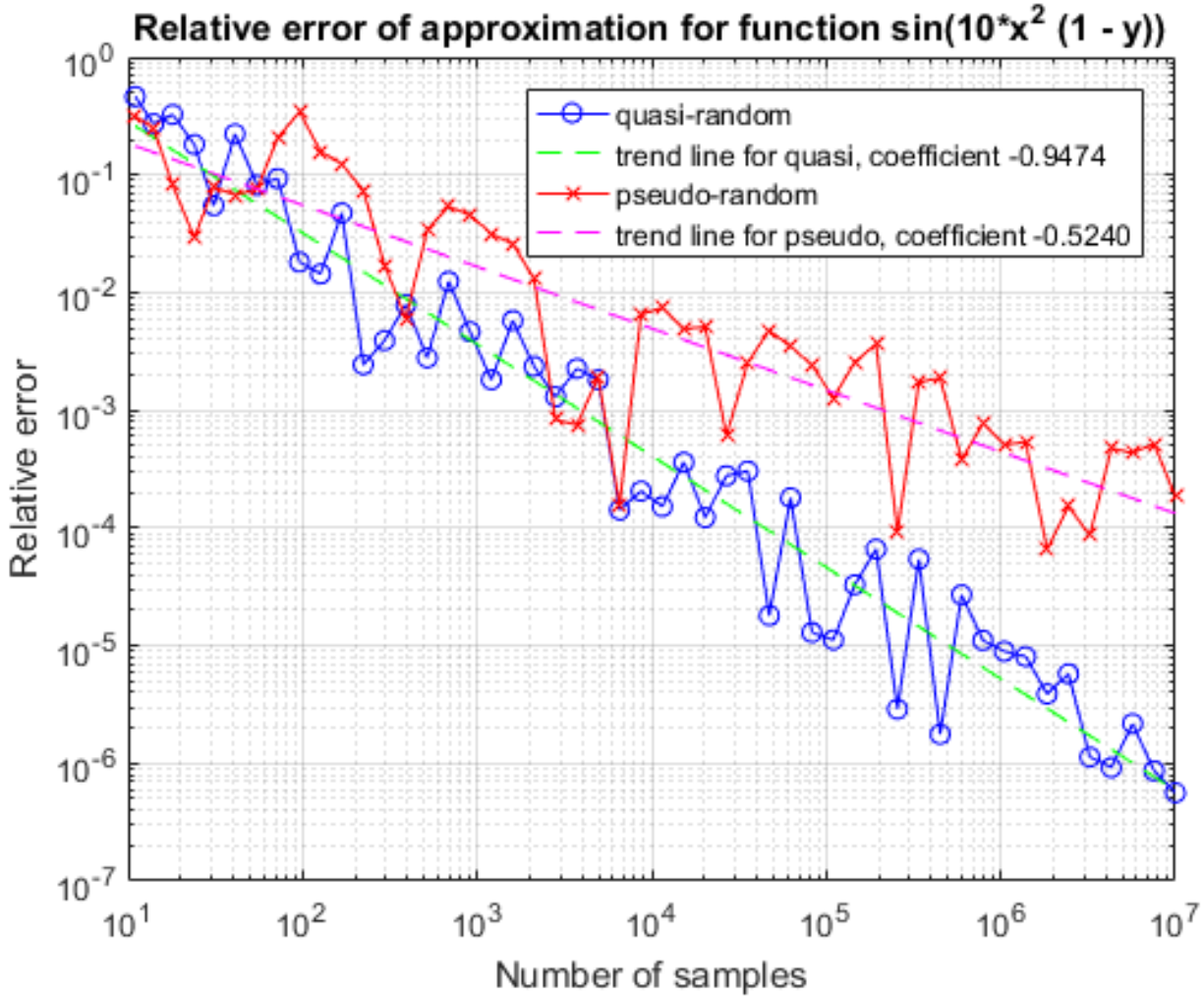


Figure: Plot of function $\sin(10x^2(1-y))$.


Relative error of approximation for function sin(10*x^2 (1 - y))
quasi-random
trend line for quasi, coefficient -0.9474
pseudo-random
trend line for pseudo, coefficient -0.5240
Relative error
Number of samples
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^1
10^2
10^3
10^4
10^5
10^6
10^7