

Brent's Algorithm

Brent's Algorithm combines the best features of the Newton-Raphson and Bisection techniques. Similar to the Bisection technique, Brent's algorithm assumes the root can be bracketed in an interval $[a, b]$. Due to its rapid convergence rates, Brent's algorithm first attempts to find the root using the Newton-Raphson scheme. If the estimate of the root falls outside the interval $[a, b]$, then Bisection is used to narrow the interval and the process is repeated. Eventually, Bisection will narrow the interval sufficiently such that the Newton-Raphson scheme will converge.

Algorithm

Given a function $F(x)$ for which the root is to be found:

- a) Pick $[a, b]$
- b) If $F(a) \cdot F(b) > 0 \Rightarrow$ Stop, the function does not cross the axis within $[a, b]$
- c) If $F(a) \cdot F(b) < 0$ Then

$$x_0 = (a+b)/2$$

$$\delta = -F(x_0)/F'(x_0)$$

$$\text{while } |\delta/x_0| > \varepsilon$$

$$x = x_0 + \delta$$

$$\text{if } x \notin [a, b] \quad x = (a+b)/2$$

$$\text{if } F(a) \cdot F(x) > 0 \text{ then } a = x$$

$$\text{if } F(a) \cdot F(x) < 0 \text{ then } b = x$$

$$x_0 = x$$

$$\delta = -F(x_0)/F'(x_0)$$

continue