

DEPARTMENT OF MATHEMATICS
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Consider the following Black-Scholes PDE for European call:

$$\begin{cases} \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \delta)S \frac{\partial V}{\partial S} - rV = 0, & (0, \infty) \times (0, T], \quad T > 0 \\ V(S, t) = 0, & \text{for } S = 0, \\ V(S, t) = S - Ke^{-r(T-t)}, & \text{for } S \rightarrow \infty \\ \text{with the terminal condition } V(S, T) = \max(S - K, 0). \end{cases}$$

With the following transformation

$$\begin{cases} S = Ke^x, \quad t = T - \frac{2\tau}{\sigma^2}, \quad q := \frac{2r}{\sigma^2}, \quad q_\delta := \frac{2(r - \delta)}{\sigma^2}, \\ V(s, t) = V\left(Ke^x, T - \frac{2\tau}{\sigma^2}\right) =: v(x, \tau), \text{ and} \\ v(x, \tau) =: K \exp\left\{-\frac{1}{2}(q_\delta - 1)x - \left[\frac{1}{4}(q_\delta - 1)^2 + q\right]\tau\right\} y(x, \tau) \end{cases}$$

the above Black-Scholes PDE becomes the following 1-D heat conduction parabolic PDE:

$$\begin{cases} \frac{\partial y}{\partial \tau} = \frac{\partial^2 y}{\partial x^2}, \quad x \in \mathbb{R}, \quad \tau \geq 0, \\ y(x, 0) = y_0(x) = \max\left\{\exp\left(\frac{x}{2}(q_\delta + 1)\right) - \exp\left(\frac{x}{2}(q_\delta - 1)\right), 0\right\}, \quad x \in \mathbb{R}, \\ y(x, \tau) = 0, \quad \text{for } x \rightarrow -\infty, \\ y(x, \tau) = \exp\left(\frac{1}{2}(q_\delta + 1)x + \frac{1}{4}(q_\delta + 1)^2\tau\right) \quad \text{for } x \rightarrow \infty. \end{cases}$$

Solve the transformed PDE using the method of horizontal lines (MOL), and address the resulting system of second-order ODEs in space variable using the following schemes: **central differences, and second-order Runge-Kutta methods (with an appropriately chosen step size)**.

The values of the parameters are $\sigma = 0.3$, $K = 10$, $r = 0.06$, and $T = 1$. The exact solution of the transformed equation is given by

$$y(x, \tau) = \frac{1}{2\sqrt{\pi\tau}} \int_{-\infty}^{\infty} y_0(s) e^{-(x-s)^2/4\tau} ds$$

The output files should contain the following for above problem:

- a) Plot the numerical solutions at the final time level.
 - b) Draw the surface plot of the numerical solutions.
 - c) Use the exact solution to calculate the error. Use the loglog plot to show the order of convergence, that is, Δx vs. *Max. abs. error*.
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