DEPARTMENT OF MATHEMATICS
IIT GUWAHATI

MA 473 **Computational Finance** **Lab – V** **Date: 27.08.2024**

Consider the following Black-Scholes PDE for European call:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad (0,\infty)\times(0,T],\ T>0 \\ V(S,t) = 0, \quad \text{for } S = 0, \\ V(S,t) = S - Ke^{-r(T-t)}, \quad \text{for } S \to \infty \\ \text{with the terminal condition } V(S,T) = \max(S-K, 0). \end{cases}$$

With the following transformation

$$\begin{cases} S = Ke^x, \quad t = T - \dfrac{2\tau}{\sigma^2}, \quad q := \dfrac{2r}{\sigma^2}, \quad q_\delta := \dfrac{2(r-\delta)}{\sigma^2}, \\ V(s,t) = V\left(Ke^x,\, T - \dfrac{2\tau}{\sigma^2}\right) =: v(x,\tau), \text{ and} \\ v(x,\tau) =: K\exp\left\{-\dfrac{1}{2}(q_\delta - 1)x - \left[\dfrac{1}{4}(q_\delta - 1)^2 + q\right]\tau\right\} y(x,\tau) \end{cases}$$

the above Black-Scholes PDE becomes the following 1-D heat conduction parabolic PDE:

$$\begin{cases} \dfrac{\partial y}{\partial \tau} = \dfrac{\partial^2 y}{\partial x^2},\ x\in\mathbb{R},\ \tau \geq 0, \\ y(x,0) = y_0(x) = \max\left\{\exp(\tfrac{x}{2}(q_\delta+1)) - \exp(\tfrac{x}{2}(q_\delta-1)), 0\right\},\ x\in\mathbb{R}, \\ y(x,\tau) = 0,\ \text{for}\, x\to-\infty, \\ y(x,\tau) = \exp\left(\tfrac{1}{2}(q_\delta+1)x + \tfrac{1}{4}(q_\delta+1)^2\tau\right)\ \text{for}\, x\to\infty. \end{cases}$$

Solve the transformed PDE using the method of vertical lines (MOL), and address the resulting system of first-order ODEs in time using the following schemes: **explicit Euler, implicit Euler, and second-order Runge-Kutta methods (with an appropriately chosen step size).**

The values of the parameters are $\sigma = 0.3$, $K = 10$, $r = 0.06$, and $T = 1$. The exact solution of the transformed equation is given by

$$y(x,\tau) = \frac{1}{2\sqrt{\pi\,\tau}}\int_{-\infty}^{\infty} y_0(s)e^{-(x-s)^2/4\tau}ds$$

The output files should contain the following for above problem:

a) Plot the numerical solutions at the final time level.

b) Draw the surface plot of the numerical solutions.

c) Use the exact solution to calculate the error. Use the loglog plot to show the order of convergence, that is, $\Delta\, x$ vs. *Max. abs. error.*
