DEPARTMENT OF MATHEMATICS
IIT GUWAHATI

MA 473 **Computational Finance** **Lab – XII** **Date: 29.10.2024**

1. The PDE modelling the Asian option for the European arithmetic average strike call (for $S > 0, A > 0, 0 \leq t \leq T$) is given by

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + S\frac{\partial V}{\partial A} - rV = 0. \tag{1}$$

with the condition

$$V(T, S, A) = (S_T - A)^+,$$

which corresponds to the payoff of the Asian option with an arithmetic average strike, where A represents the arithmetic average of the underlying asset price over a certain period up to time t. Specifically, it is defined as:

$$A = \frac{1}{t}\int_0^t S(\tau)\, d\tau,$$

where $S(\tau)$ is the asset price at time τ.

Boundary Conditions:

- For $S \to 0$: $V(t, 0, A) = 0$.
- For $S \to S_{\max}$: Choose an appropriate boundary condition reflecting the option behavior.
- Similarly, set boundary conditions as $A \to 0$ and $A \to A_{\max}$.

(a) Solve (1) without using any transformation by the following schemes:

(i) Forward-Euler for time & central difference for space (FTCS) scheme.

(ii) Backward-Euler for time & central difference for space (BTCS) scheme.

(iii) Central-Time and Central Space (CTCS) scheme.

2. By using the transformation $V(S, A, t) = \widetilde{V}(S, R, t) = S \cdot H(R, t)$, with $R = \dfrac{A}{S}$, transform (1) into the following form:

$$\begin{cases} \dfrac{\partial H}{\partial t} + \dfrac{1}{2}\sigma^2 R^2 \dfrac{\partial^2 H}{\partial R^2} + (1 - rR)\dfrac{\partial H}{\partial R} = 0, \\ H = 0, \quad \text{for } R \to \infty, \\ \dfrac{\partial H}{\partial t} + \dfrac{\partial H}{\partial R} = 0, \quad \text{for } R = 0, \\ H\left(R_T, T\right) = \left(1 - \dfrac{R_T}{T}\right)^+. \end{cases} \tag{2}$$

a) Solve the above transformed PDE (2) by the following schemes:

(i) Forward-Euler for time & central difference for space (FTCS) scheme.

(ii) Backward-Euler for time & central difference for space (BTCS) scheme.
(iii) Central-Time and Central Space (CTCS) scheme.

- Plot the solution surfaces for $T = 0.2, r = 0.05, \sigma = 0.25$ at different time levels.
