**DEPARTMENT OF MATHEMATICS**
**IIT GUWAHATI**

| **MA 473** | **Computational Finance** | **Lab – 13** | **Date: 05.11.2024** |
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1. Consider the following Black-Scholes diffusion equation:

$$\begin{cases} dX(t) = \mu X dt + \sigma X dW(t), \\ X(0) = X_0. \end{cases}$$

The values of the parameters are $\mu = 0.75,\ \sigma = 0.30$ and $X_0 = 307$ and $t \in (0, 1)$.

(a) Obtain the exact solution of the above SDE.

(b) Solve the above SDE by

  (i) Euler-Maruyama Scheme.

  (ii) First-order Milstein Scheme.

(c) Plot the order of convergence in a loglog plot ($N$ vs. the mean error).

2. Consider the following Langevin SDE:

$$\begin{cases} dX(t) = -\mu X(t) dt + \sigma dW(t), \\ X(0) = X_0. \end{cases}$$

The values of the parameters are $\mu = 10, \sigma = 1$ and $X_0 = 0$, and $t \in (0, 4)$.

(a) Solve the above SDE by

  (i) Euler-Maruyama Scheme.

  (ii) First-order Milstein Scheme.

(b) Plot the order of convergence in a loglog plot ($N$ vs. the mean error).

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If we consider processes $X$ satisfying a SDE of the form

$$dX(t) = a(X(t))dt + b(X(t))dW(t),$$

with $X(0)$ fixed. Then

First-order Milstein Scheme:

$$\hat{X}(i+1) = \hat{X}(i) + a(\hat{X}(i))h + b(\hat{X}(i))\sqrt{h}Z_{i+1} + \frac{1}{2}b'(\hat{X}(i))b(\hat{X}(i))h(Z_{i+1}^2 - 1),$$

with the functions $a,\ b$ and their derivatives all evaluated at $\hat{X}(ih)$.

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