DEPARTMENT OF MATHEMATICS
IIT GUWAHATI

MA 473 **Computational Finance** **Lab – VIII** **Date: 24.09.2024**

Consider the following Black-Scholes PDE for European call:

$$
\begin{cases}
\dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta) S \dfrac{\partial V}{\partial S} - rV = 0, \quad (0,\infty)\times(0,T],\ T>0 \\
V(S,t) = 0, \quad \text{for } S = 0, \\
V(S,t) = S - Ke^{-r(T-t)}, \quad \text{for } S \to \infty \\
\text{with the terminal condition } V(S,T) = \max(S-K, 0).
\end{cases}
$$

With the following transformation

$$
\begin{cases}
S = Ke^x, \quad t = T - \dfrac{2\tau}{\sigma^2}, \quad q := \dfrac{2r}{\sigma^2}, \quad q_\delta := \dfrac{2(r-\delta)}{\sigma^2}, \\
V(s,t) = V\left(Ke^x,\, T - \dfrac{2\tau}{\sigma^2}\right) =: v(x,\tau), \text{ and} \\
v(x,\tau) =: K \exp\left\{-\dfrac{1}{2}(q_\delta - 1)x - \left[\dfrac{1}{4}(q_\delta - 1)^2 + q\right]\tau\right\} y(x,\tau),
\end{cases}
$$

the above Black-Scholes PDE becomes the following 1-D heat conduction parabolic PDE:

$$
\begin{cases}
\dfrac{\partial y}{\partial \tau} = \dfrac{\partial^2 y}{\partial x^2},\ x \in \mathbb{R},\ \tau \geq 0, \\
y(x,0) = \max\left\{\exp(\tfrac{x}{2}(q_\delta + 1)) - \exp(\tfrac{x}{2}(q_\delta - 1)), 0\right\},\ x \in \mathbb{R}, \\
y(x,\tau) = 0,\ \text{for } x \to -\infty, \\
y(x,\tau) = \exp\left(\tfrac{1}{2}(q_\delta + 1)x + \tfrac{1}{4}(q_\delta + 1)^2\tau\right) \ \text{for } x \to \infty.
\end{cases}
$$

Solve the transformed PDE by the following finite element methods (FEMs):

(i) Piecewise-linear basis functions with the trapezoidal rule for the numerical quadratures and the Crank-Nicolson scheme (in time).

(ii) Piecewise-linear basis functions with the Simpson's rule for the numerical quadratures and the Crank-Nicolson scheme (in time).

The values of the parameters are $T = 1$, $K = 10$, $r = 0.06$, $\sigma = 0.3$ and $\delta = 0$.

The output files should contain the following for above problem:

a) Plot the numerical solution at the final time level.

b) Draw the surface plot of the numerical solution.

c) Use the double-mesh principle to calculate the L^2-norm errors. Use the log-log plot to show the order of convergence.
