**DEPARTMENT OF MATHEMATICS**
**IIT GUWAHATI**

| **MA 473** | **Computational Finance** | **Lab – IV** | **Date: 20.08.2024** |
|---|---|---|---|

1. Consider the following Black-Scholes PDE for European call:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2(S)S^2\dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad 0 \le S < \infty,\ t \le T, \\ V(S,t) = V_T(S), \quad 0 \le S < \infty. \end{cases}$$

With the following transformation

$$\begin{cases} \xi = \dfrac{S}{S+q}, \quad q > 0, \\ \tau = T - t, \\ V(S,T) = (S+q)\,\overline{V}(\xi,\tau), \end{cases}$$

the above European call option problem becomes the following parabolic PDE:

$$\begin{cases} \dfrac{\partial \overline{V}}{\partial \tau} = \dfrac{1}{2}\overline{\sigma}^2(\xi)\xi^2\,(1-\xi)^2\,\dfrac{\partial^2 \overline{V}}{\partial \xi^2} + (r-\delta)\xi(1-\xi)\dfrac{\partial \overline{V}}{\partial \xi} - [r(1-\xi)+\delta\xi]\overline{V},\ 0 \le \xi \le 1,\ 0 \le \tau \le T, \\ \overline{V}(\xi,0) = \max(2\xi-1,0),\ 0 \le \xi \le 1, \\ \overline{V}(0,\tau) = \overline{V}(0,0)e^{-r\tau}, \quad \overline{V}(1,\tau) = \overline{V}(1,0)e^{-\delta\tau}, \quad 0 \le \tau \le T. \end{cases}$$

Solve the transformed PDE by the following schemes:

(i) Forward-Euler for time & central difference for space (FTCS) scheme.

(ii) Backward-Euler for time & central difference for space (BTCS) scheme.

(iii) Crank-Nicolson finite difference scheme.

The values of the parameters are $T = 1,\ r = 0.04,\ \sigma(S) = \dfrac{S}{4},\ \delta = 0.1.$

**Continued on the next page**

2. Consider the following European put option problem:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2(S)S^2\dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad 0 \le S < \infty,\ t \le T, \\ V(S,t) = V_T(S), \quad 0 \le S < \infty. \end{cases}$$

With the following transformation

$$\begin{cases} \xi = \dfrac{S}{S+q}, \quad q > 0, \\ \tau = T - t, \\ V(S,T) = (S+q)\,\overline{V}(\xi,\tau), \end{cases}$$

the above European put option problem becomes the following parabolic PDE:

$$\begin{cases} \dfrac{\partial \overline{V}}{\partial \tau} = \dfrac{1}{2}\overline{\sigma}^2(\xi)\xi^2\,(1-\xi)^2\,\dfrac{\partial^2 \overline{V}}{\partial \xi^2} + (r-\delta)\xi(1-\xi)\dfrac{\partial \overline{V}}{\partial \xi} - [r(1-\xi)+\delta\xi]\overline{V},\ 0 \le \xi \le 1,\ 0 \le \tau \le T, \\ \overline{V}(\xi,0) = \max(1-2\xi,0),\ 0 \le \xi \le 1, \\ \overline{V}(0,\tau) = \overline{V}(0,0)e^{-r\tau}, \quad \overline{V}(1,\tau) = \overline{V}(1,0)e^{-\delta\tau}, \quad 0 \le \tau \le T. \end{cases}$$

Solve the transformed PDE by the following schemes:

(i) Forward-Euler for time & central difference for space (FTCS) scheme.

(ii) Backward-Euler for time & central difference for space (BTCS) scheme.

(iii) Crank-Nicolson finite difference scheme.

The values of the parameters are $T = 1,\ r = 0.04,\ \sigma(S) = \dfrac{S}{4},\ \delta = 0.1.$

---

**Note :** $\overline{\sigma}(\xi) = \sigma(S(\xi)) = \sigma\left(\dfrac{q\xi}{1-\xi}\right).$

The output files should contain the following for above problems:

a) Plot the numerical solutions at the different times $t = 0,\ 0.25,\ 0.5,\ 0.75$ and $1$.

b) Draw the surface plot of the numerical solutions.

---