

Calcolo di limiti di successioni

Giustificare tutti i passaggi mediante la teoria studiata.

1. Usando i teoremi algebrici e di confronto, calcolare

$$\lim_{n \rightarrow +\infty} a_n$$

ove

$$(a) \quad a_n = \frac{n^3 + 2n}{1 - 5n + 3n^3};$$

$$(b) \quad a_n = \frac{2 - n}{1 + n^2};$$

$$(c) \quad a_n = \frac{n - n^5 + 6}{n^2 + 8};$$

$$(d) \quad a_n = \frac{\cos(\log n + n!)}{n^2};$$

$$(e) \quad a_n = n - \sqrt{n} \cos n^2;$$

$$(f) \quad a_n = \frac{\sin n + \sqrt{n}}{n};$$

$$(g) \quad a_n = \frac{2^n - n + n!}{n^3};$$

$$(h) \quad a_n = \frac{3^n + (-2)^n}{5^n + (-2)^n};$$

$$(i) \quad a_n = \frac{6^n + (-1)^n}{3^n + \sin n};$$

$$(j) \quad a_n = \frac{n \sin n^2 + n^2 \sin n}{(n+1)\sqrt{n^3}};$$

$$(k) \quad a_n = n - \frac{n^2 + 1}{n + 2};$$

$$(l) \quad a_n = \frac{\sqrt[3]{n} - \sqrt{n}}{2 + \sin n^2};$$

$$(m) \quad a_n = \frac{\cos n + \sin n}{n^2};$$

$$(n) \quad a_n = \frac{\sqrt[4]{n} - \sqrt[5]{n}}{\sqrt[6]{n} - \sqrt[7]{n}};$$

$$(o) \ a_n = \frac{\sqrt[4]{n} - n\sqrt[5]{n}}{\sqrt[6]{n} - n\sqrt[7]{n}};$$

$$(p) \ a_n = \frac{\log n + \sqrt[3]{n}}{n^n + \cos n + 2^n}.$$

Soluzioni: (a) $1/3$; (b) 0 ; (c) $-\infty$; (d) 0 ; (e) $+\infty$; (f) 0 ; (g) $+\infty$; (h) 0 ; (i) $+\infty$; (j) 0 ; (k) 2 ; (l) $-\infty$; (m) 0 ; (n) $+\infty$; (o) $+\infty$; (p) 0 .

2. Usando i teoremi algebrici, i criteri del rapporto, della radice, del rapporto \rightarrow radice, calcolare

$$\lim_{n \rightarrow +\infty} a_n$$

ove

$$(a) \ a_n = \frac{2^n}{n^3};$$

$$(b) \ a_n = \frac{3^n}{n!};$$

$$(c) \ a_n = \frac{n! \cdot 3^n}{n^n};$$

$$(d) \ a_n = \frac{n! \cdot 2^n}{n^n};$$

$$(e) \ a_n = \sqrt[n]{\frac{n!}{n^3}};$$

$$(f) \ a_n = \frac{1}{2^n} \left(1 + \frac{1}{n}\right)^{n^2};$$

$$(g) \ a_n = \frac{1}{4^n} \left(1 + \frac{1}{n}\right)^{n^2};$$

$$(h) \ a_n = \frac{2^{n^2}}{n^n};$$

$$(i) \ a_n = \sqrt[n]{n!};$$

$$(j) \ a_n = \frac{\sqrt[n]{n!}}{n};$$

$$(k) \ a_n = \sqrt[n]{n^2 + 2n + 6};$$

$$(l) \ a_n = \sqrt[n]{3^n + n^2 + 6n + 3};$$

$$(m) \ a_n = \sqrt[n]{n^3 + n \log n};$$

$$(n) \ a_n = \sqrt[n]{n^n + \log n};$$

$$(o) \ a_n = \sqrt[n]{2^n + n^3};$$

$$(p) \ a_n = \frac{n! \cdot 3^n}{n^{2n}}.$$

Soluzioni: (a) $+\infty$; (b) 0; (c) $+\infty$; (d) 0; (e) $+\infty$; (f) $+\infty$; (g) 0; (h) $+\infty$; (i) $+\infty$; (j) $1/e$; (k) 1; (l) 3; (m) 1; (n) $+\infty$; (o) 2; (p) 0.
