Previous year question of CSIR-NET maths exam.

Syllabus: Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, GreenŠs function.

Question with one correct answer

December 2014

1. Let $y:\mathbb{R}\to\mathbb{R}$ be differentiable and satisfy the ODE:

$$\frac{dy}{dx} = f(y), \quad x\in\mathbb{R}$$
$$y(0) = y(1) = 0$$

Where, $f:\mathbb{R}\to\mathbb{R}$ is a lipschitz condition function. Then

A. $y(x) = 0$ if and only if $x \in 0, 1$

B. y is bounded.

C. y is strictly increasing.

D. $\frac{dy}{dx}$ is unbounded.

2. For $\lambda\in\mathbb{R}$, consider the boundary value problem

$$\left.\begin{matrix} x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + \lambda y = 0, \quad x\in[1,2] \\ y(1) = y(2) = 0 \end{matrix}\right\} \quad -(P_2)$$

Which of the following statement is true?

A. There exists a $\lambda_0 \in \mathbb{R}$ such that (P_λ) has a nontrivial solution for any $\lambda > \lambda_0$

B. $\lambda\in\mathbb{R} : (P_\lambda)$ has a nontrivial solution is a dense subset of $\mathbb{R}$

C. For any continuous function $f:[1,2]\to\mathbb{R}$ with $f(x)\neq 0$ for some $x\in[1,2]$, there exists a solution u of (P_λ) for some $\lambda\in\mathbb{R}$ such that $\int_1^2 fu \neq 0$

D. There exists a $\lambda\in\mathbb{R}$ such that (P_λ) has two linearly independent solutions.

3. The system of ODE

$$\begin{aligned} \frac{dx}{dt} &= (1+x^2) && y,t\in\mathbb{R} \\ \frac{dy}{dt} &= -(1+x^2) && x,t\in\mathbb{R} \\ (x(0), y(0)) &= (a,b) \end{aligned}$$

has a solution:

- ◯ only if $(a,b) = (0,0)$
- ◯ For any $(a,b)\in\mathbb{R}\times\mathbb{R}$
- ◯ Such that $x^2(t) + y^2(t) = a^2 + b^2$ for all $t\in\mathbb{R}$

- ◯ Such that $x^2(t)+y^2(t)\to\infty$ as $t\to\infty$ if $a>0$ and $b>0$.

4. Let $y:\mathbb{R}\to\mathbb{R}$ be a solution of the ODE
$$\frac{d^2y}{dx^2}-y=e^{-x},\quad x\in\mathbb{R}$$
$$y(0)=\frac{dy}{dx}(0)=0$$
then
   - ◯ y attains its maximum on $\mathbb{R}$
   - ◯ y is bounded on $\mathbb{R}$
   - ◯ $\lim_{x\to\infty}e^{-x}y(x)=\frac{1}{4}$
   - ◯ $\lim_{x\to\infty}e^{x}y(x)=\frac{1}{4}$

5. Let P,Q be continuous real valued function defined on $[-1,1]$ and $u_i:[-1,1]\to\mathbb{R}, i=1,2$ be solutions of the ODE.
$$\frac{d^2u}{dx^2}+P(x)\frac{du}{dx}+Q(x)u=0,\quad x\in[-1,1]$$
satisfying $u_1\geq 0, u_2\leq 0$ and $u_1(0)=u_2(0)=0$. Let ;w denote the Wronskian of u_1 and u_2, then
   - ◯ u_1 and u_2 are linearly independent.
   - ◯ u_1 and u_2 are linearly independent.
   - ◯ $w(x)=0$ for all $x\in[-1,1]$
   - ◯ $w(x)\neq 0$ for some $x\in[-1,1]$

June 2015

6. The singular integral of the ODE
$$(xy'-y)^2=x^2(x^2-y^2)$$
is
   A. $y=x\sin x$
   B. $y=x\sin(x+\frac{\pi}{4})$
   C. $y=x$
   D. $y=x+\frac{\pi}{4}$

7. The initial value problem
$$y'=2\sqrt{y},\quad (0)=a$$
has
   A. a unique solution if $a<0$
   B. no solution if $a<0$
   C. infinitely many solutions if $a=0$.
   D. a unique solution if $a\geq 0$

8. Let P be a continuous function on $\mathbb{R}$ and W the wronskian linearly independent solutions y_1 and y_2 of the ODE.
$$\frac{d^2}{dy^2}+(1+x^2)\frac{dy}{dx}+P(x)y=0,\quad x\in\mathbb{R}.$$
Let $W(1)=a, W(2)=b$ and $W(3)=c$, then

- ◯ $a < 0$ and $b > 0$
- ◯ $a < b < c$ or $a > b > c$
- ◯ $\frac{a}{|a|} = \frac{b}{|b|} = \frac{c}{|c|}$
- ◯ $0 < a < b$ and $b > c > 0$

Ⓢ