Abstract Algebra

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Previous year question of CSIR-NET maths exam.

Syllabus: Permutations and combinations, Pigeon-hole principle, inclusion-exclusion principle, dearrangement

QUESTION WITH ONE CORRECT ANSWER

1. The number of 4 digit numbers with no two digits common is

   A. 4536 B. 3024 C. 5040 D. 4823

2. The number of words that can be formed by permuting the letters of 'MATHEMATICS' is

   A. 5040 B. 4989600 C. 11! D. 8!

3. Let $M = \{(a_1, a_2, a_3) : a_i \in \{1, 2, 3, 4\}, a_1 + a_2 + a_3 = 6\}$, Then the number of elements in M is

   A. 8 B. 9 C. 10 D. 12

4. In a group of 265 persons, 200 like singing, 110 like dancing and 55 like painting. If 60 persons like both singing and dancing, 30 like both singing and painting and 10 like all three activities, then the number of persons who like *only* dancing and painting is

   A. 10 B. 20 C. 30 D. 40

5. The number of surjective maps from a set of 4 elements to a set of 3 elements is

   A. 36 B. 64 C. 69 D. 81

6. An ice cream shop sells ice cream in five possible falvours: Vanilla, Chocolate, Strawberry, Mango and Pineapple. How many combinations of three scoop cones are possible? [Note: The repeatation of flavours allowed but the order in which the flavours are chosen does not matter.]

   A. 10 B. 20 C. 35 D. 243

7. We are given a class consisting of 4 boys and 4 girls. A committee that consists of a President, a Vice-President and a Secretary is to be chosen among the 8 students of the class. Let a denote the number of ways of choosing the committee in such a way that the committee has at least one boy and at least one girl. Let b deonote the number of ways of choosing the committee in such a way that the number of girls is greater than or equal to that of the boys. Then

   ◯ $a = 288$ ◯ $b = 168$ ◯ $a = 144$ ◯ $b = 192$

Syllabus: Fundamental theorem of arithmetic, divisibility in $\mathbb{Z}$, congruences, Chinease remainder theorem, Euler's ϕ function, Primitive roots

8. The number of positive divisors of 50000 is

   A. 20 B. 30 C. 40 D. 50

9. The number of multiplies of 10^{44} that divide 10^{55} is

   A. 11 B. 12 C. 121 D. 144

10. The unit digit of 2^{100} is

    A. 2 B. 4 C. 6 D. 8

11. The last digit of 38^{2011} is

    A. 6 B. 2 C. 4 D. 8

12. The last two digit of 7^{81} are

    A. 07 B. 17 C. 37 D. 47

13. For any integers a, b let $N_{a,b}$ denote the number of positive integers $x < 1000$ satisfying $x = a(\text{mod } 27)$ and $x = a(\text{mod } 37)$. Then

    A. there exist a, b such that $N_{a,b} = 0$

    B. for all a, b, $N_{a,b} = 1$

    C. for all a, b , $N_{a,b} > 1$

    D. there exist a, b such that $N_{a,b} = 1$, and there exist a, b such that $N_{a,b} = 2$

14. The number of elements in the set $\{m : 1 \leq m \leq 1000, m$ and 1000 are relatively prime$\}$ is

    A. 100 B. 250 C. 300 D. 400

15. If n is a positive integer such that the sum of all positive integers a satisfy $1 \leq a \leq n$ and $\gcd(a, n) = 1$ is equal to $240n$, then the number of summands, namely, $\phi(n)$ is

    A. 120 B. 124 C. 240 D. 480

16. For a positive integer m, let $\phi(m)$ denote the number of integers k such that $1 \leq k \leq m$ and $\gcd(k, m) = 1$. Then which of the followong statements are necessarily true?

    ◯ $\phi(n)$ divides n for every positive integer n.

    ◯ n divides $\phi(a^n - 1)$ for all positive integers a and n.

    ◯ n divides $\phi(a^n - 1)$ for all positive integers a and n such that $\gcd(a, n) = 1$

    ◯ a divides $\phi(a^n - 1)$ for all positive integers a and n such that $\gcd(a, n) = 1$

17. For positive integers m and n let $F_n = 2^{2^n} + 1$ and $G_m = 2^{2^n} - 1$. Which of the following statements are true?

    ◯ F_n divides G_m whenever $m > n$

    ◯ $\gcd(F_n, G_m) = 1$ whenever $m = n$

    ◯ $\gcd(F_n, G_m) = 1$ whenever $m \neq n$

    ◯ G_m divides F_n whenever $m < n$

Syllabus: Groups, subgroups, normal subgroup, quotient groups, homomorphism, cyclic groups, Cayley theorem, class equation, Sylow theorem

18. Let $G = \mathbb{Z}_{10} \times \mathbb{Z}_{15}$, then

    ◯ G contain exactly one element of order 2

    ◯ G contain exactly 5 element of order 3

    ◯ G contain exactly 24 element of order 5

    ◯ G contain exactly 24 element of order 10

19. Consider a group G. Let $Z(G)$ be its centre, i.e., $Z(G) = \{g \in G : gh = hg \text{ for all } h \in G\}$. For $n \in \mathbb{N}$, the set of positive integers, define

$$J_n = \{(g_1, \cdots, g_n) \in Z(G) \times \cdots \times Z(G) : g_1 \cdots g_n = e\}$$

    As a subset of the direct product group $G \times \cdots \times G$, J_n is

    A. not necessarily a subgroup

B. a subgroup but not necessarily a normal subgroup

C. a normal subgroup

D. isomorphic to the direct product $Z(G) \times \cdots \times Z(G)$ ($(n-1)$ times)

20. Let G be a group of order 77. then the center of G is isomorphic to

A. $\mathbb{Z}_1$ B. $\mathbb{Z}_7$ C. $\mathbb{Z}_{11}$ D. $\mathbb{Z}_{77}$

21. How many normal subgroups does a non-abelian group G of order 21 have other than the identity subgroup $\{e\}$ and G?

A. 0 B. 1 C. 3 D. 7

22. Let G be a nonabelian group. Then, its order can be

○ 25 ○ 55 ○ 125 ○ 55

23. Let G be a group of order 45. Then

○ G has an element of order 9

○ G has a subgroup of order 9

○ G has a normal subgroup of order 9

○ G has an normal subgroup of order 5

24. The total number of non-isomophic groups of order 122 is

A. 2 B. 1 C. 61 D. 4

25. For any group G of order 36 and any subgroup H of G order 4,

○ $H \subset Z(G)$

○ $H = Z(G)$

○ H is normal in G

○ H is an abelian group.

26. Let $H = \{e, (12)(34)\}$ and $K = \{e, (12)(34), (13)(24), (14)(23)\}$ be subgroup of S_4, where e denotes the identify element of S_4. Then

○ H and K are normal subgroup of S_4

○ H is normal in K and K are normal in A_4

○ H is normal in A_4 but not in S_4

○ H is normal in S_4 but H is not

27. Which of the following numbers can be orders of permutations σ of 11 symbols such that σ does not fix any symbol?

○ 18 ○ 30 ○ 15 ○ 28

28. Let $\sigma = (1\ 2)(3\ 4\ 5)$ and $\tau = (1\ 2\ 3\ 4\ 5\ 6)$ be permutation in S_6, the group of permutations on six symbols. Which of the following statements are true?

○ The subgroup $\langle\sigma\rangle$ and $\langle\tau\rangle$ are isomorphic to each other.

○ $\langle\sigma\rangle$ and $\langle\tau\rangle$ are conjugate in S_6.

○ $\langle\sigma\rangle \cap \langle\tau\rangle$ is trivial group.

- ◯ $\langle\sigma\rangle$ and $\langle\tau\rangle$ commute.

29. Let S_n denote the symmetric group on n symbols. The group $S_3 \oplus \mathbb{Z}/2\mathbb{Z}$ is isomorphic to which of the following groups?
   - ◯ $\mathbb{Z}/12\mathbb{Z}$
   - ◯ $\mathbb{Z}/6\mathbb{Z} \oplus \mathbb{Z}/6\mathbb{Z}$
   - ◯ A_4, the alternating group of order 12
   - ◯ D_6, the dihedral group of order 12

30. Given the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$ the matrix A is defined to be the one whose $i-$th column is the $\sigma(i)-$th column of the identity matrix I. Which of the following is correct?

   A. $A = A^{-2}$ B. $A = A^{-4}$ C. $A = A^{-5}$ D. $A = A^{-1}$

31. Let G denote the group $S_4 \times S_3$. Then
   - ◯ a $2-$Sylow subgroup of G is normal
   - ◯ a $3-$Sylow subgroup of G is normal
   - ◯ G has a non-trivial normal subgroup
   - ◯ G has a normal subgroup of order 72

32. For a positive integer $n \geq 4$ and a prime number $p \leq n$, let $U_{p,n}$ denote the union of all $p-$Sylow subgroups of the alternating group A_n on letters n letters. Also let $K_{p,n}$ denote the subgroup of A_n generated by $U_{p,n}$, and let $\mid K_{p,n} \mid$ denote the order of $K_{p,n}$. Then

   ◯ $K_{2,4} = 12$ ◯ $K_{2,4} = 4$ ◯ $K_{2,5} = 60$ ◯ $K_{3,5} = 30$

33. Determine which of the following cannot be the class equation of a group

   ◯ $10 = 1+1+1+2+5$ ◯ $4 = 1+1+2$ ◯ $8 = 1+1+3+3$ ◯ $6 = 1+2+3$

34. The number of group homomorphism from the symmetric group S_3 to $\mathbb{Z}/6\mathbb{Z}$ is

   A. 1 B. 2 C. 3 D. 6

35. Consider the group $G = \mathbb{Q}/\mathbb{Z}$ where $\mathbb{Q}$ and $\mathbb{Z}$ are the groups of rational numbers and integers respectively. Let n be a positive integers. Then is there is a cyclic subgroup of order n?

   A. not necessarily

   B. yes, a unique one

   C. yes, but not necessarily a unique one

   D. never

36. Let p be a prime number. The order of a $p-$Sylow subgroup of the group $GL_{50}(\mathbb{F}_p)$ of invertible 50×50 matrices from the finite field $\mathbb{F}_p$, equals

   A. p^{50} B. p^{125} C. p^{1250} D. p^{1225}

37. For which of the following primes p, does the polynomial $x^4 + x + 6$ have a root of multiplicity > 1 over a field of characteristic p?

   A. 2 B. 3 C. 5 D. 7

38. In the group of all invertible 4×4 matrices with entries in the field of 3 elements, any 3−Sylow subgroup has cardinality

A. 3 B. 81 C. 243 D. 729

39. For a matrix A as given below, which of them satisfy $A^6 = I$?

A. $\begin{pmatrix} \cos\frac{pi}{4} & \sin\frac{pi}{4} & 0 \\ -\sin\frac{pi}{4} & \cos\frac{pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{pi}{3} & \sin\frac{pi}{3} \\ 0 & -\sin\frac{pi}{3} & \cos\frac{pi}{3} \end{pmatrix}$

C. $\begin{pmatrix} \cos\frac{pi}{6} & 0 & \sin\frac{pi}{6} \\ 0 & 1 & 0 \\ -\sin\frac{pi}{6} & 0 & \cos\frac{pi}{6} \end{pmatrix}$

D. $\begin{pmatrix} \cos\frac{pi}{2} & \sin\frac{pi}{2} & 0 \\ -\sin\frac{pi}{2} & \cos\frac{pi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

40. Suppose $(F, +, \cdot)$ is the field with 9 elements. Let $G = (F, +)$ and $H = (F\ \{0\}, \cdot)$ denote the underlying additive and multiplicative groups respectively. Then

- ◯ $G \cong (\mathbb{Z}/3\mathbb{Z}) \times (\mathbb{Z}/3\mathbb{Z})$
- ◯ $G \cong (\mathbb{Z}/9\mathbb{Z})$
- ◯ $H \cong (\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z})$
- ◯ $G \cong (\mathbb{Z}/3\mathbb{Z}) \times (\mathbb{Z}/3\mathbb{Z})$ and $H \cong (\mathbb{Z}/8\mathbb{Z})$

41. Consider the multiplicative group G of all the (complex) 2^n−th roots of unity where $n = 0, 1, 2, \cdots$.Then

- ◯ Every proper subgroup of G is finite.
- ◯ G has a finite set of generators
- ◯ G is cyclic
- ◯ Every finite subgroup of G is cyclic

Syllabus: Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.Polynomial rings and irreducibility criteria

42. The number of non-trivial ring homomorphism form $\mathbb{Z}_{12}$ to $\mathbb{Z}_{28}$ is

A. 1 B. 3 C. 4 D. 7

43. Let $\mathcal{R}$ be non-zero commutative ring with unity $1_{\mathcal{R}}$. Dfine the caracteristic of $\mathcal{R}$ to be the of $1_{\mathcal{R}}$ in $(R, +)$ if it is finite and to be the order of $1_{\mathcal{R}}$ in $(R, +)$ is infinite. We denote the caracteristic of R by . In the following, let $\mathcal{R}$ and S be non-zero commutative rings with unity. Then

- ◯ char($\mathcal{R}$) is always a prime number.
- ◯ If S is a quotient ring of char($\mathcal{R}$), then either char(S) divides the char($\mathcal{R}$), or char(S)= 0.
- ◯ If S is a subring of $\mathcal{R}$ containig $1_{\mathcal{R}}$ then char(S)= char($\mathcal{R}$).
- ◯ If char($\mathcal{R}$) is a prime number, then char($\mathcal{R}$) is a field.

44. Let $\mathcal{R}$ be the ring obtained by taking the quotient of $(\mathbb{Z}/6\mathbb{Z})[X]$ by the principal ideal $(2X+4)$. Then
   - ◯ $\mathcal{R}$ has infinitely many elements.
   - ◯ $\mathcal{R}$ is a field.
   - ◯ 5 is a unit in $\mathcal{R}$.
   - ◯ 4 is a unit in $\mathcal{R}$.

45. In which of the following fields, the polynomial

$$x^3 - 312312x + 123123$$

is irreducible in $\mathbb{F}[x]$?
   - A. the field $\mathbb{F}_3$ with 3 elements
   - B. the field $\mathbb{F}_7$ with 7 elements
   - C. the field $\mathbb{F}_{13}$ with 13 elements
   - D. the filed $\mathbb{Q}$ of rational numbers

46. Let $f(x) = x^3 + 2x^2 + 1$ and $g(x) = 2x^2 + x + 2$. Then over $\mathbb{Z}_3$
   - A. $f(x)$ and $g(x)$ are irreducible
   - B. $f(x)$ is irreducible, but $g(x)$ is not
   - C. $g(x)$ is irreducible, but $f(x)$ is not
   - D. neither $f(x)$ nor $g(x)$ is irreducible

47. Let I_1 be the ideal generated by $x^4 + 3x^2 + 2$ and I_2 be the ideal generated by $x^3 + 1$ in $\mathbb{Q}[x]$. If $F_1 = \mathbb{Q}[x]/I_1$ and $F_2 = \mathbb{Q}[x]/I_2$, then
   - A. F_1 and F_2 are fields
   - B. F_1 is a field, but F_2 is not a fields
   - C. F_1 is not a field while F_2 is a field
   - D. Neither F_1 nor F_2 is a field

48. Let $\langle p(x)\rangle$ denote the ideal generated by the polynomial $p(x)$ in $\mathbb{Q}[x]$. If $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^3 - x^2 + x - 1$, then
   - ◯ $\langle f(x)\rangle + \langle g(x)\rangle = \langle x^3 + x\rangle$
   - ◯ $\langle f(x)\rangle + \langle g(x)\rangle = \langle f(x)g(x)\rangle$
   - ◯ $\langle f(x)\rangle + \langle g(x)\rangle = \langle x^2 + 1\rangle$
   - ◯ $\langle f(x)\rangle + \langle g(x)\rangle = \langle x^4 - 1\rangle$

49. Let $\mathbb{R}[x]$ be the polynomial ring over $\mathbb{R}$ in one variable. Let $I \subset \mathbb{R}[x]$ be an ideal. Then
   - ◯ I is a maximal ideal if and only if I is a non-zero prime ideal
   - ◯ I is a maximal ideal if and only if the quotien ring $\mathbb{R}[x]/I$ is isomorphic to $\mathbb{R}$
   - ◯ I is a maximal ideal if and only if $I = (f(x))$, where $f(x)$ is a non-constant irrudicible polynomial over $\mathbb{R}$
   - ◯ I is a maximal ideal if and only if there exists a nonconstant polynomial $f(x) \in I$ of degree ≤ 2

Syllabus: Fields, finite fields, field extensions, Galois Theory

50. Let F be a field of 8 elements and

$$A = \{x \in F : x^7 = 1 \text{ and } x^k \neq 1 \text{ for all natural numbers } k < 7\}.$$

Then the number of element in A is

A. 1 B. 2 C. 3 D. 6

51. Let F and F' be two finite fields of order q and q' respectively. Then:
   - ◯ F' contains a subfields isomorphic to F if and only if $q \leq q'$.
   - ◯ F' contains a subfields isomorphic to F if and only if q divides q'.
   - ◯ If the g.c.d of q and q' is not 1, then both are isomorphic to subfields of some finite field L.
   - ◯ Both F and F' are quotient rings of the ring $\mathbb{Z}[X]$.

52. Let ω be a complex number such that $\omega^3 = 1$ and $\omega = 1$. Suppose L is the field $\mathbb{Q}(\sqrt[3]{2}, \omega)$ generated by $\sqrt[3]{2}$ and ω over the filed $\mathbb{Q}$ of rational numbers. Then the number of subfields K of L such that $\mathbb{Q} \subsetneq K \subsetneq L$ is

A. 1 B. 2 C. 3 D. 4

53. The degree of the extension $\mathbb{Q}(\sqrt{2} + \sqrt[3]{2})$ over the field $\mathbb{Q}(\sqrt{2})$ is

A. 1 B. 2 C. 3 D. 6

54. Let I_1 be the ideal generated by x^2+1 and I_2 be the ideal generated by $x^3 - x^2 + x - 1$ in $\mathbb{Q}[x]$. If $R_1 = \mathbb{Q}[x]/I_1$ and $R_2 = \mathbb{Q}[x]/I_2$, then
   - ◯ R_1 and R_2 are fields
   - ◯ R_1 is a field and R_2 is not a field
   - ◯ R_1 is an integral domain and R_2 is not an integral domain
   - ◯ R_1 and R_2 are not integral domain

55. Let $R = \mathbb{Q}[x]/I$, where I is the ideal generated by $1 + x^2$. Let y to be the coset of x in R. Then
   - ◯ $y^2 + 1$ is irreducible over R
   - ◯ $y^2 + y + 1$ is irreducible over R
   - ◯ $y^2 - y + 1$ is irreducible over R
   - ◯ $y^3 + y^2 + y + 1$ is irreducible over R

56. Let $f(x) = x^3 + 2x^2 + x - 1$. Determine in which of the following cases f is irreducible over the field k.
   - ◯ $k = \mathbb{Q}$, the field of rational numbers.
   - ◯ $k = \mathbb{R}$, the field of real numbers.
   - ◯ $k = \mathbb{F}_2$, the finite field of 2 elements.
   - ◯ $k = \mathbb{F}_3$, the finite field of 3 elements.

57. Which of the following is true?
   - ◯ $\sin 7$ is algebraic over $\mathbb{Q}$
   - ◯ $\cos \pi/17$ is algebraic over $\mathbb{Q}$
   - ◯ $\sin^{-1} 1$ is algebraic over $\mathbb{Q}$
   - ◯ $\sqrt{2} + \sqrt{\pi}$ is algebraic over $\mathbb{Q}$

58. Let $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^3 + 1$. Then in $\mathbb{Q}[x]$,

- ◯ $\gcd(f(x), g(x)) = x + 1$
- ◯ $\gcd(f(x), g(x)) = x^2 - 1$
- ◯ $\operatorname{lcm}(f(x), g(x)) = x^5 + x^3 + x^2 + x + 1$
- ◯ $\operatorname{lcm}(f(x), g(x)) = x^5 + x^4 + x^3 + x^2 + x + 1$

59. For a positive integer n, let $f_n(x) = x^{n-1} + x^{n-2} + \cdots + x + 1$. Then
    - ◯ $f_n(x)$ is an irreducible polynomial in $\mathbb{Q}[x]$ for every positive integers n.
    - ◯ $f_p(x)$ is an irreducible polynomial in $\mathbb{Q}[x]$ for every prime number p.
    - ◯ $f_{p^e}(x)$ is an irreducible polynomial in $\mathbb{Q}[x]$ for every prime number p and every positive integer e.
    - ◯ $f_p(x^{p^{e-1}})$ is an irreducible polynomial in $\mathbb{Q}[x]$ for every prime number p and every positive integer e.

60. Consider the ring $R = \mathbb{Z}[-\sqrt{-5}] = \{a + b\sqrt{-5}\}$ and the element $\alpha = 3 + \sqrt{-5}$ of R. Then
    - ◯ α is a prime.
    - ◯ α is irreducible.
    - ◯ R is not a unique factorization domain.
    - ◯ R is not an integral domain.

61. Consider the polynomial $f(x) = x^4 - x^3 + 14x^2 + 5xz + 16$. Also for a prime number p, let $\mathbb{F}_p$ denote the field with p elements. Which of the following are always true?
    - ◯ Considering f as a polynomial with coefficients in $\mathbb{F}_3$, it has no roots in $\mathbb{F}_3$.
    - ◯ Considering f as a polynomial with coefficients in $\mathbb{F}_3$, it has a product of two irreducible factors of degree 2 over $\mathbb{F}_3$.
    - ◯ Considering f as a polynomial with coefficients in $\mathbb{F}_7$, it has a irreducible factors of degree 3 over $\mathbb{F}_7$.
    - ◯ f is a product of two polynomial of degree 2 over $\mathbb{Z}$

62. For a positive integer m, let a_m denote the number of disjoint prime ideals of the ring $\frac{\mathbb{Q}[x]}{\langle x^m - 1\rangle}$. Then

    ◯ $\alpha_4 = 2$ ◯ $\alpha_4 = 3$ ◯ $\alpha_5 = 2$ ◯ $\alpha_5 = 3$

63. Which of the following integral domains are Euclidean domains?
    - ◯ $\mathbb{Z}[\sqrt{-3}] = \{a + b\sqrt{-3} : a, b \in \mathbb{Z}\}$
    - ◯ $\mathbb{Z}[x]$
    - ◯ $\mathbb{R}[x^2, x^3] = \{f(x) = \sum_{i=0}^{n} a_i x^i \in \mathbb{R}[x] : a_1 = 0\}$
    - ◯ $\left(\frac{\mathbb{Z}[x]}{(2,x)}\right)[y]$ where x, y are independent variables and $(2, x)$ is the ideal generated by 2 and x

64. Let $\mathbb{Z}[i]$ denote the ring of Gaussian integers. For which of the following values of n is the quotient ring $\mathbb{Z}[i]/n\mathbb{Z}[i]$ an integral domain?

    ◯ 2 ◯ 13 ◯ 19 ◯ 7

65. For which of the following values of n, does the finite field $\mathbb{F}_{5^n}$ with 5^n elements contain a non-trivial 93^{rd} root of unity?

    ◯ 92 ◯ 30 ◯ 15 ◯ 6

66. Let G be the Galois group of the splitting field of $x^5 - 2$ over $\mathbb{Q}$. Then, which of the following statements are true?

- ◯ G is cyclic
- ◯ G is non-abelian
- ◯ the order of G is 20
- ◯ G has an element of order 4

67. Which of the following is/are true?
    - ◯ Given any positive integer n, there exists a field extension of $\mathbb{Q}$ of degree n.
    - ◯ Given a positive integer n, there exist fields F and K such that $F \subset K$ and K is Galois over F with $[K : F] = n$.
    - ◯ Let K be a Galois exension of $\mathbb{Q}$ with $[K : \mathbb{Q}] = 4$. Then there is a field L such that $\mathbb{Q} \subset L \subset K, [L : \mathbb{Q}] = 2$ and L is a Galois extension of $\mathbb{Q}$.
    - ◯ There is algebraic extension K of $\mathbb{Q}$ such that $[K : \mathbb{Q}]$ is not finite.

68. Let G denote the group of all the automorphisms of the field $F_{3^{100}}$ that consist of 3^{100} elements. Then the number of distinct subgroup of G is equal to

    A. 4 B. 3 C. 100 D. 9

69. Let p, q be distinct primes. Then
    - A. $\mathbb{Z}/p^2q\mathbb{Z}$ has exactly 3 distinct ideals
    - B. $\mathbb{Z}/p^2q\mathbb{Z}$ has exactly 3 distinct ideals
    - C. $\mathbb{Z}/p^2q\mathbb{Z}$ has exactly 2 distinct ideals
    - D. $\mathbb{Z}/p^2q\mathbb{Z}$ has a unique maximal ideals

70. Let $f(x) = x^4 + 3x^3 - 9x^2 + 7x + 27$ and let p be a prime. Let $f_p(x)$ denote the corresponding polynomial with coefficients in $\mathbb{Z}/p\mathbb{Z}$. Then
    - ◯ $f_2(x)$ is irreducible over $\mathbb{Z}/2\mathbb{Z}$
    - ◯ $f(x)$ is irreducible over $\mathbb{Q}$
    - ◯ $f_3(x)$ is irreducible over $\mathbb{Z}/3\mathbb{Z}$
    - ◯ $f(x)$ is irreducible over $\mathbb{Z}$

71. Pick the correct statements:
    - ◯ $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are isomorphic as $\mathbb{Q}-$vector spaces
    - ◯ $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are isomorphic as fields
    - ◯ $Gal_{\mathbb{Q}}(\mathbb{Q}(\sqrt{2})/\mathbb{Q}) \cong Gal_{\mathbb{Q}}(\mathbb{Q}(i)/\mathbb{Q})$
    - ◯ $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are both Galois extensions of $\mathbb{Q}$