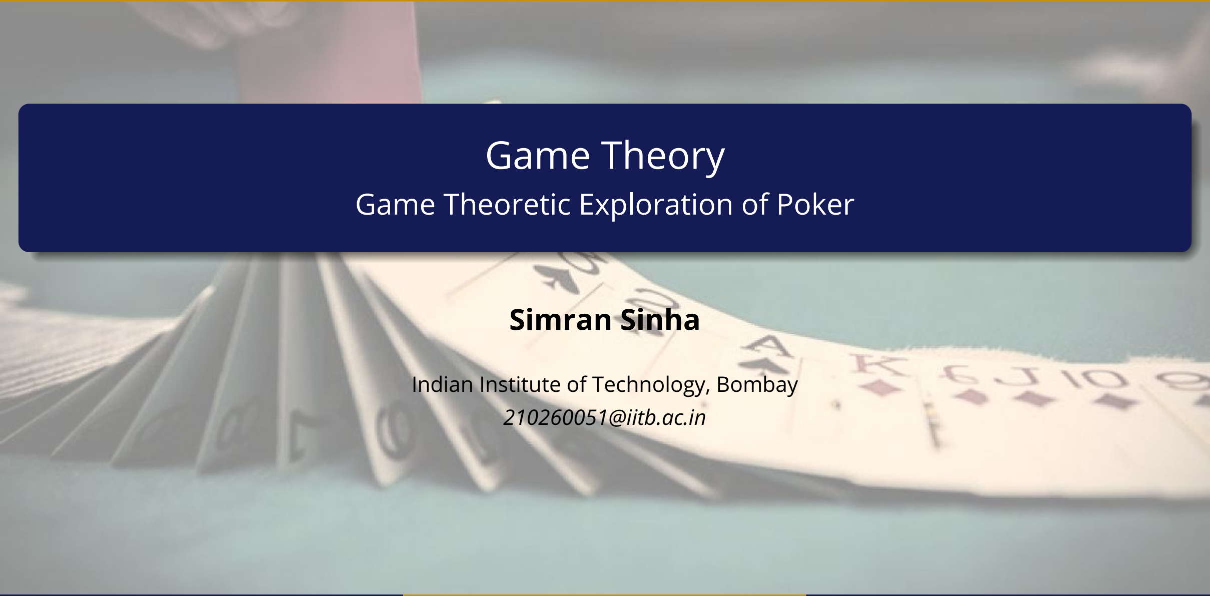


Game Theory

Game Theoretic Exploration of Poker

Simran Sinha

Indian Institute of Technology, Bombay

210260051@iitb.ac.in

Table of Contents

Motivation

- Poker is a popular strategic card game and has been a subject of fascination for mathematicians and computer scientists for decades
- Poker is a complex game involving incomplete information, random chance, and strategic decision-making
- It is a game of skill, probability, and decision making that can be used to study various aspects of human behavior and game theory
- By studying and developing poker-playing agents, we can improve our understanding of complex decision-making, risk-taking, and strategic thinking in humans and machines
- Additionally, there is a need to better understand the game-theoretic aspects of poker variants in order to develop winning strategies

Introduction

- Poker is a popular card game played worldwide for entertainment and as a competitive sport.
- It is a game of skill and strategy, where the player with the best hand wins.
- The game involves a deck of 52 cards, and each player is dealt with a set of cards from the deck.
- Players make bets on the strength of their cards, and the objective is to have the highest-ranking hand at the end of the game.
- There are several variations of poker, each with its own set of rules and gameplay.
- Some of the most popular versions of poker include Texas Hold'em, Omaha, Seven Card Stud, and Razz.

Kuhn Poker

Description of Kuhn Poker game in conventional poker terms as follows:

- Each player antes 1
- Each player is dealt one of the three cards, and the third is put aside unseen
- Player one can check or bet 1
  - If player one checks then player two can check or bet 1
    - If player two checks there is a showdown for the pot of 2 (i.e. the higher card wins 1 from the other player)
    - If player two bets then player one can fold or call
    - If player one folds then player two takes the pot of 3 (i.e. winning 1 from player 1)
    - If player one calls there is a showdown for the pot of 4 (i.e. the higher card wins 2 from the other player)
  - If player one bets then player two can fold or call
    - If player two folds then player one takes the pot of 3 (i.e. winning 1 from player 2)
    - If player two calls there is a showdown for the pot of 4 (i.e. the higher card wins 2 from the other player)

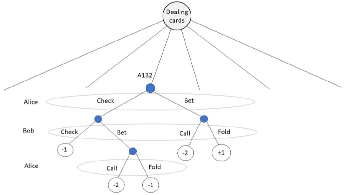
Nash Equilibria of Kuhn Poker

- Both Alice and Bob have a strictly dominant strategy of setting the parameters for calling with the 1 card to 0, which means they should always fold when they have the 1 card
- Similarly, the parameters for folding with card 3 are also strictly dominant because when a player has this card, they know they will always have the highest card, so they will never fold a bet
- Alice and Bob will both never bet with the 2, because when Alice has the 2, Bob has either the 1 or the 3. Bob will always fold with the 1 and always call with the 3, so Alice will never win anything when she bets with the 2
- Bob will always bet with the 3 when he gets the opportunity, this is when Alice does not bet.
- The remaining five parameters require a more complex analysis to determine the optimal strategy since they are not strictly dominated.

Contd.

- Total utility for Alice: $U_A(p,q) = \frac{1}{6}[p1(1-3q2) + p2(3q1-1) + p3(q2-q1) - q1]$
- If $1-3q2=0$ and $q2-q1=0$, Alice's strategy **p** does not influence her utility, that is, $q1=q2=\frac{1}{3}$ is the Nash equilibrium strategy for Bob.
- Alice's utility can be rewritten as $U_A(p,q) = \frac{1}{6}[q1(3p2-p3-1) + q2(p3-3p1) + (p1-p2)]$
- If $3p2-p3-1=0$ and $p3-3p1=0$, Bob's strategy **q** does not influence his utility. This is the case for the family of solutions $p1=\frac{p3}{3}$, $p2=\frac{p3}{3}+\frac{1}{3}$ and $0 \leq p3 \leq 1$, which is the family of Nash equilibrium solutions for Alice.

Figure: Partial game tree for A1B2 with the utility of Alice



Adding Blinds to Kuhn Poker

- Bob and Alice have to put in blinds of X and Y respectively; where $X < Y$
- Bob has to decide if he wants to put in the same amount as Alice (Y in total). If he does so, the game continues the same as before, only with Y as bet size
- If Bob folds, he loses his blind X to Alice
- Two new strategy parameters are introduced: q3 and q4
  - q3 is the probability that Bob calls Y while having the card with value 1
  - q4 is the probability that Bob calls Y while having the card with value 2
- If Bob has the card with value 3, he always calls, so we don't have to consider this
- For every different hand, we have to add the probabilities of Bob calling or folding the hand in the first place

Contd.

Utility for Alice:

$$U_A(p,q) = \frac{1}{6}Y[p1(-3q2q4 + 2q4 - 1) + p2(3q1q3 - 1)$$
$$+p3(-q1q3 + q2q4) + 2(q3 - 1) + q1q3] + \frac{1}{6}X[3 - 2q4 - q3]$$

if q3=q4=1, i.e., Bob will always call big blind Y , so this leaves us with the original game, only with bet size Y.

$$U_A(p,q) = \frac{1}{6}Y[p1(1 - 3q2) + p2(3q1 - 1) + p3(-q1 + q2)]$$

If q3 = q4 = 0, only dependent on strategy parameters of Alice.

$$U_A(\mathbf{p},(q1,q2,0,0)) = \frac{1}{6}Y[-(p1 + p2) - 2] + \frac{1}{2}X$$

Contd.

Alice wants to maximize her utility, p1=p2=0; $U_A = -\frac{1}{3}Y + \frac{1}{2}X$
If $Y > \frac{3}{2}X$ this strategy gives Alice a negative and thus Bob a positive utility
Bob can guarantee himself a utility of $\frac{1}{18}Y$ by playing $q = (\frac{1}{3}, \frac{1}{3}, 1, 1)$
But playing $q = (q1, q2, 0, 0)$ guarantees a utility of $\frac{1}{3}Y - \frac{1}{2}X$
If $\frac{1}{18}Y = \frac{1}{13}Y - \frac{1}{2}$, then $X = \frac{5}{9}Y$ and utilities of both strategies are equal
If $X \leq \frac{5}{9}Y$, Bob would prefer playing $q3 = q4 = 0$ over $q3 = q4 = 1$
If $X \geq \frac{5}{9}Y$, Bob would prefer playing $q3 = q4 = 1$ over $q3 = q4 = 0$.
Playing $q = (q1, q2, 0, 0)$ guarantees a positive utility if $\frac{2}{3}Y > X$ but is not a Nash equilibrium.

Evolutionary Game Theory

- Evolutionary Game Theory is a mathematical framework for studying the evolution of strategies in populations of interacting individuals.
- In the context of poker, evolutionary game theory can help us understand how different strategies evolve and become dominant over time
- One of the key ideas in evolutionary game theory is the concept of fitness, which measures the success of a strategy in a given environment
- By modeling the interactions between players and analyzing the evolution of different strategies, we can gain insights into how poker strategies have evolved over time

Replicator Dynamics

- Replicator Dynamics is a central concept in Evolutionary Game Theory that describes how the proportion of different strategies in a population can change over time.
- In the context of Poker, Replicator Dynamics can be used to model how the frequency of different strategies (e.g., aggressive vs. passive play) changes over time as players interact with each other.
- Basic equation for Replicator Dynamics:

$$\Delta p_i = p_i(f_i - \bar{f})$$

where:
Δp_i is the change in the frequency of strategy i
p_i is the current frequency of strategy i
f_i is the payoff of strategy i
$\bar{f}$ is the average payoff in the population

Contd.

- This equation describes how the frequency of each strategy changes over time, based on the payoffs associated with each strategy and the average payoff in the population.
- In the context of Poker, we can use Replicator Dynamics to study how different strategies (e.g., aggressive vs. passive play) evolve over time, and how the frequency of these strategies changes as players interact with each other.

Heuristic Payoff Table

- Heuristic Payoff Table is a tool used to analyze strategies in evolutionary game theory in poker
- It maps out the payoffs for different strategies in a game, and is used to determine the most successful strategies for players
- The table is typically constructed by analyzing a large number of hands played by different players, and recording the results
- The payoff table is usually represented as a matrix, where each cell represents the payoff for a particular strategy combination

Simplex Analysis

- Mathematical tool used to analyze the Heuristic Payoff Table and determine the most successful strategies for players
- The Simplex algorithm is used to identify the best response for each player, and to find the Nash Equilibrium, which is the combination of strategies that is stable against any deviation by either player
- Based on linear programming, which involves finding the maximum or minimum value of a linear objective function subject to linear constraints
- In the context of evolutionary game theory in poker, the objective function is the payoff function, and the constraints are the probabilities of each player playing a particular strategy
- The Simplex algorithm is used to iteratively improve the probability distribution until the Nash Equilibrium is reached, at which point no player has an incentive to deviate from their strategy

Conclusion

While our study has shown promising results in applying game theory to poker gameplay, there are several limitations that should be acknowledged.

- The sample size of the study was relatively small, which may limit the generalizability of the findings
- The study did not take into account the psychological factors that can influence poker gameplay

Thankyou!