Tutorial 7

Prof. Anshuman Kumar
PH 421: Photonics
Due: 12:30 Thursday, October 26, 2023

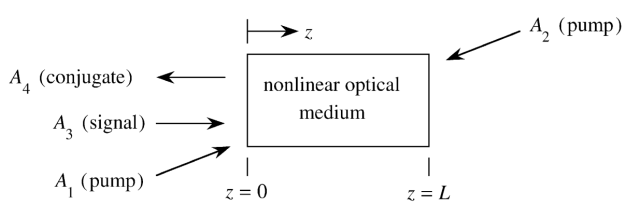
Problem 7.1. Symmetry of third order susceptibility in isotropic media
(a) List out the possible non-zero components of the third order optical susceptibility for an isotropic medium. Please provide justification for components which are zero and equal.
(b) Prove that $\chi_{1111} = \chi_{1212} + \chi_{1122} + \chi_{1221}$.

Problem 7.2. Polarization rotation in an isotropic nonlinear medium
(a) Consider a plane perfectly polarized (as opposed to partial or unpolarized) light wave traveling along the z axis, with a representation:

$$\mathbf{E} = (\mathcal{E}_x\hat{\mathbf{x}} + \mathcal{E}_y\hat{\mathbf{y}})e^{\imath(kz-\omega t)} + c.c. \tag{1}$$

The state of polarization can be characterizaed by the quantity $\eta = \mathcal{E}_y/\mathcal{E}_x = \tan(\theta/2)e^{\imath\delta}$. Write down η for various standard polarization states: left and right circular, linear polarization along x, y and 45° and elliptical polarization.
(b) Suppose we express the electric field above in the circular basis. What is the value of $I_{\text{rcp}}/I_{\text{lcp}}$ in terms of η?
(c) Consider a plane light wave of arbitrary elliptical polarization propagating in an isotropic medium with $\chi^{(3)}_{ijkl}$, where i, j, k, l are Cartesian indices. Express the difference in the refractive index between RCP and LCP light in terms of η, linear refractive index, the nonlinear susceptibility tensor components and the input intensity.
(d) Derive an expression for the rotation of the polarization ellipse after propagating a distance L in this medium.

Problem 7.3. Degenerate four wave mixing
Suppose the electric fields are given by $E_i(\mathbf{r}, t) = A_i e^{\imath(\mathbf{k}_i\cdot\mathbf{r}-\omega t)} + \text{c.c.}$. For simplicity, you can assume the



fields are scalars.
(a) For the scheme shown in the figure, write down the nonlinear polarization corresponding to the Fourier component $e^{\imath(k_i z'-\omega t)}$ for the pump waves and $e^{\imath(k_i z'-\omega t)}$ for the other two weaker waves in terms of the complex amplitudes A_i.
(b) Write down the evolution equations of the pump amplitudes and shown that the pump beams only pick up phases with no intensity changes.
(c) Assuming the pump intensities are exactly equal, write down the evolution equations of the signal and the backward moving wave and solve them given the initial conditions $A_4(L)$ and $A_3(0)$.
(d) Assuming no injected backward beam $A_4(L)$, write down the expression for the backward wave at the entrance of the crystal $z = 0$.