Tutorial 5

Prof. Anshuman Kumar
PH 421: Photonics
Due: 12:30 Thursday, October 5, 2023

Problem 5.1. Sum frequency generation without undepleted pump approximation
Write a python code for calculating the intensities of all three waves as a function of position, for the sum frequency generation process in a lossless nonlinear crystal. Run your code for various initial conditions and interpret the results based on what we have learnt in class.

Problem 5.2. Spectrum of an optical cavity
Suppose that we have a cavity of length L, composed of two perfectly reflecting mirrors with a linear medium of refractive index n occupying the entire space within the cavity.
(a) Calculate the spacing between two neighbouring modes $\delta\nu$ of this cavity. This is called the free spectral range of the cavity.
(b) Suppose the mirrors are only partially reflecting with Fresnel reflection coefficient r (real). We input a light beam from the left end and observe the output on the right. Please provide an expression for the transmittance (intensity ratio of output to input) in terms of the wave-vector, cavity length and mirror reflectance.
(c) Plot the transmission spectrum above for different values of r (choose some reasonable numbers for the other parameters). What is the linewidth of each cavity mode? The ratio of the free spectral range from part (a) to the linewidth is often called the cavity finesse $\mathcal{F}$.
(d) In part (a), we assumed that the refractive index does not depend on the frequency. If the refractive index is a function of frequency, that is, $n = n(\nu)$, please find the correct expression for the free spectral range.

Problem 5.3. Derivation of gain threshold in a phase matched configuration of OPO
(a) Assuming perfect phase matching for the forward wave and poor phase match for the backward wave, calculate the gain threshold of a doubly resonant OPO in the small gain approximation. You are given the values of the cavity length L and reflectances at signal and idler frequencies.
(b) *Optional:* Repeat part (a) for a small phase mismatch Δk in the forward direction and poor phase match for the backward direction.

Problem 5.4. Gain bandwidth for an OPO
In a particular OPO, we use a pump at the telecom wavelength of 1.55 µm. For a particular orientation of the nonlinear crystal inside the cavity, perfect phase matching is achieved at a signal wavelength of about 2.782 µm. The refractive index difference at the signal and the idler frequencies is about 0.01. Moreover the nonlinear crystal is such that the refractive index dispersion $dn/d\nu \sim 10^{-15}$ for both these frequencies. The length of the cavity is about 15 cm and the cavity finesse is about 100. You may assume that the nonlinear crystal occupies the the entire inside of the cavity.
(a) Estimate the free spectral range of the cavity.
(b) Estimate the linewidth associated with each mode.
(c) Estimate the gain bandwidth.
(d) How many modes can fit into the gain bandwidth?

Problem 5.5. Isofrequency surfaces of a uniaxial medium
(a) Find the equation of isofrequency surfaces for a uniaxial medium.
(b) Find the direction of the electric fields for each point of the isofrequency surface that you derived.