Tutorial 3

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PH 421: Photonics
Due: 12:30 Tuesday, August 29, 2023

Problem 3.1. Refractive index and relative permittivity
Using a transverse plane wave ansatz for the electric field of light propagating in a non-magnetic, dielectric medium, derive the relation between the refractive index and relative permittivity.

Problem 3.2. Gradients and wave-vectors
In class, we wrote down the Maxwell's equation in the specific case of a plane wave. We said that we could replace $\nabla \to \imath\mathbf{k}$ and $\partial/\partial t \to -\imath\omega$ in the Maxwell's equations, if our plane is of the form $\Psi = \mathbf{A}e^{\imath(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, where $\mathbf{A}$ is a constant vector. Using this form of Ψ, derive $\nabla \times \Psi$ in terms of the wave-vector components k_x, k_y, k_z and $\partial\Psi/\partial t$ in terms of ω. Note: you may use the identity $\nabla \times (f\mathbf{A}) = f\nabla \times \mathbf{A} + \nabla f \times \mathbf{A}$.

Problem 3.3. Full permutation symmetry
Convince yourself of the validity of full permutation symmetry for second order nonlinear susceptibility tensor using the following two methods:
(a) By looking at the quantum mechanical expression for $\chi^{(2)}$, and
(b) By deriving the electromagnetic energy density in a nonlinear medium.

Problem 3.4. Contracted d-tensor under the condition of Kleinman symmetry
Please derive the contracted d-tensor under the condition of Kleinman symmetry. How many distinct elements do you find?

Problem 3.5. Contracted d-tensor under the condition of Kleinman and C_4 symmetry
Please derive the contracted d-tensor under the condition of Kleinman symmetry and C_4 symmetry about the $z-$axis. How many distinct elements do you find?

Problem 3.6. Effective 'd' for a second order nonlinear experimental configuration with quartz
The nonlinear susceptibility (in contracted form) for quartz is known to be:

$$\begin{bmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We are performing a sum frequency generation, where the fundamental fields have amplitudes $\mathcal{E}(\omega_1)$ and $\mathcal{E}(\omega_2)$. Both of these fundamental waves are propagating at an angle θ with respect to the optic axis of the crystal (here z axis). The plane through the optic axis and the incident rays make an angle ϕ with respect to the x axis. We have set our polarizers such that the field $\mathcal{E}(\omega_1)$ lies in this plane, whereas $\mathcal{E}(\omega_2)$ is perpendicular to this plane.
(a) We would like to find out the nonlinear polarization at the sum frequency due to the interaction between these two incident (fundamental) rays. Please provide expressions for these.
(b) Using this expression, please calculate the d_{eff} for our experimental configuration.