Tutorial 4

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PH 421: Photonics
Due: 12:30 Thursday, September 14, 2023

Problem 4.1. Time averages with complex envelopes
Consider two real fields $\mathbf{A}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$. Suppose they are represented in terms of the respective complex envelopes as $\mathbf{A}(\mathbf{r},t) = 2\Re\{\tilde{\mathbf{A}}(\mathbf{r})e^{-\imath\omega t}\}$ and $\mathbf{B}(\mathbf{r},t) = 2\Re\{\tilde{\mathbf{B}}(\mathbf{r})e^{-\imath\omega t}\}$. Please express the time average of the vector product, $\langle \mathbf{A}(\mathbf{r},t) \times \mathbf{B}(\mathbf{r},t)\rangle$ in terms of the respective complex envelopes. Note that the average is taken over timescales much longer than the time period of the wave.

Problem 4.2. Sum frequency generation in the un-depleted pump approximation
(a) Please express the intensity of the sum frequency wave in terms of the intensities of the two fundamental waves. You may assume that d_{eff}, refractive indices n_1, n_2, n_3, length of the nonlinear medium L and a phase mismatch of Δk. The beams may be assumed to be collinear.
(b) Is full permutation symmetry sufficient for having the same d_{eff} on the right side of the three coupled equations for the evolution of the respective electric fields at the three frequencies?

Problem 4.3. Phase matching in sum frequency generation in isotropic crystals
(a) For the case of collinear beams, please provide the condition for phase matching in terms of the refractive indices at the respective frequencies.
(b) Please convince yourself that achieving phase matching for a nonlinear material with normal dispersion of the refractive index is not possible.
(c) Please discuss the ways of achieving phase matching and their merits and demerits.

Problem 4.4. Manley-Rowe relations for sum frequency generation
(a) Please derive the Manley Rowe relations for the case of sum frequency generation.
(b) Provide an interpretation of the Manley-Rowe relations.
(c) Our coupled equations for the fields for the case of sum frequency generation are classical. However Manley-Rowe relations seem to be consistent with a quantum mechanical picture. How do you reconcile this?

Problem 4.5. Second harmonic generation in the depleted pump regime
Suppose we use normalized electric fields u_i such that

$$u_i e^{\imath\phi_i} = \left(\frac{2c\epsilon_0 n_i}{I_{\text{tot}}}\right)^{1/2} \mathcal{E}_i.$$

The beams are assumed to be collinear and the medium is lossless.
(a) Please derive the differential equation for u_2^2, that is, $d(u_2^2)/dz$. You may normalize the z coordinate for convenience of notation.
(b) How do we chose the sign of the right hand side of the equation in part (a).