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VEHICLE ROUTING PROBLEM

DETERMINISTIC METHODS IN OPERATIONS RESEARCH

ESI6314

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1.Introduction:

1.1 Problem Statement:

A company must deliver products to its customers from its distribution center located at coordinates (0; 0) in the plane. The coordinates (x; y) of the customers are given, along with the demand of the customer.

The company could buy up to 20 trucks that can be used for delivery, each with capacity 80 and cost $1000. Each truck can visit each customer no more than once. Each truck will leave from the distribution center for delivery and then get back to the distribution center with maximum one delivery tour.

1. Find a best feasible solution and the corresponding objective value.

2. The VRP with Time Windows (VRPTW) is an extension of the VRP in which each customer i is associated with a time interval [ai; bi], called a time window.

With the time window data, find a best feasible solution and the corresponding objective value.

1.2 Structure of Report

In the first section, we mainly talk about the introduction and concept of vehicle routing problem followed by a general literature review of Vehicle Routing Problem (VRP). Next, problem data and input variables are defined, and we examine the given Vehicle Routing Problem (VRP). We will show different feasible solutions and choose the best solution which minimizes the total cost of travel. In the next section, we conclude the results and mention future scope. At the end, we mention python code we used and references for the project. *Google OR-Tools* in Python programming has been used to solve the problems.

2. Literature review

Vehicle Routing Problem:

The main aim of Vehicle routing problem is to optimize the routes of a fleet of homogeneous vehicles so as to serve all customer demand with minimum overall cost for the company. Here, vehicles should take routes such that a) the time and money of operation is minimized; b) each customer visited only once and by one vehicle only; c) Vehicle route should start at depot/warehouse and ends at the depot/warehouse only. Since Clarke and Wright (1964) proposed the first heuristic for the approximate solution of VRP, numerous studies have been devoted to find the exact or approximate solutions to different variants of VRP. The most common studied variant of VRP is Capacitated Vehicle Routing Problem (CVRP) and vehicle routing problems with time windows (VRPTW), in which the loading capacity of vehicles is considered as a constraint. Also, time windows are added. CVRP can be traced back to the study of Dantzig and Ramser (1959). VRP is studied with capacity constraints and with Time windows, such as by Madsen et al. (1995); Gendreau et al. (1999); Haghani and Jung (2005); Chen and Xu (2006); and Hong (2012), and VRP with Pickup and Delivery and Time Windows, as in Yang et al. (2004); Gendreau, et al. (2006); Cheung et al. (2008). VRP with Stochastic Travel Time (Lambert et al., 1993), VRP with Stochastic Demand and Travel Time (Cook and Russell, 1978), and VRP with Stochastic Travel Time and Service Time (Li et al., 2010).

The VRP definition states that Q vehicles initially located at the depot are going to deliver discrete quantities of goods to n customers. Determining the optimal route used by a group of vehicles when serving a group of users represents a VRP problem. The objective is to minimize the overall transportation cost. The solution of the classical VRP problem is a set of routes which all begin and end in the depot, and which satisfies the constraint that all the customers are served only once. The transportation cost can be improved by reducing the total travelled distance and by reducing the number of required vehicles. VRP is NP hard combinatorial optimization problem that can be solved only for small instances of the problem. Although the heuristic approach does not guarantee optimality, it yields best results in practice. Many vehicle routing problems involve scheduling visits to customers who are only available during specific time windows. These problems are known as vehicle routing problems with time windows (VRPTWs). If we add a time window to each customer, we get the vehicle routing problem with time windows. In addition to the capacity constraint, a vehicle now has to visit a customer within a certain time frame. The vehicle may arrive before the time window opens but the customer cannot be serviced until the time windows open. It is not allowed to arrive after the time window has closed. In the last twenty years the meta-heuristics has emerged as the most promising direction of research for the VRP family of problems.

When it comes to real-world problems which are very complex than the classical VRP. Therefore, in practice, the classical VRP problem is augmented by constraints, such as vehicle capacity or time interval in which each customer has to be served, revealing the Capacitated Vehicle Routing Problem (CVRP) and the Vehicle Routing Problem with Time Windows (VRPTW), respectively.

METHODOLOGY

There are different methods to solve CVRP and CVRPTW. They can be summarized in 3 main types:

1. **Vehicle flow formulations**—this uses integer variables associated with each arc that count the number of times that the edge is traversed by a vehicle. It is generally used for basic VRPs. This is good for cases where the solution cost can be expressed as the sum of any costs associated with the arcs. However it can't be used to handle many practical applications.
2. **Commodity flow formulations**—additional integer variables are associated with the arcs or edges which represent the flow of commodities along the paths travelled by the vehicles. This has only recently been used to find an exact solution.
3. **Set partitioning problem**—These have an exponential number of binary variables which are each associated with a different feasible route. The VRP is then instead formulated as a set partitioning problem which asks what the collection of routes with minimum cost that is satisfy the VRP constraints. This allows for very general route costs.

Here, we follow the Vehicle flow formulations used to find best solution to the problem. First, we define data, variables and constraints. We then create the mathematical modelling using the information and construct a programming structure which we can be able to run using solver tools. Once, the solution is obtained, we check for the feasibility and check for optimality.

We seek to minimize the total cost by minimizing the total Euclidean distance and the number of trucks used. Two nodes with same coordinates signify the same customer split in to two nodes. The demands of the duplicate nodes add up to the demand of the original node.

The integer programming optimization can be written as follows:

Minimize

$$\sum_{r=1}^{p}\sum_{i=0}^{n}\sum_{j=0,i\neq j}^{n} c_{ij}\, x_{rij}, \qquad (1)$$

Subject To

$$\sum_{r=1}^{p}\sum_{i=0,i\neq j}^{n} x_{rij} = 1, \qquad \forall j \in \{1,\dots,n\}, \qquad (2)$$

$$\sum_{j=1}^{n} x_{r0j} = 1, \qquad \forall r \in \{1,\dots,p\}, \qquad (3)$$

$$\sum_{i=0,i\neq j}^{n} x_{rij} = \sum_{i=0}^{n} x_{rji}, \qquad \forall j \in \{0,\dots,n\},\ r \in \{1,\dots,p\}, \qquad (4)$$

$$\sum_{i=0}^{n}\sum_{j=1,i\neq j}^{n} d_j\, x_{rij} \leqq Q, \qquad \forall r \in \{1,\dots,p\}, \qquad (5)$$

$$\sum_{r=1}^{p}\sum_{i\in S}\sum_{j\in S,i\neq j} x_{rij} \leqq |S|-1, \qquad \forall S \subseteq \{1,\dots,n\}, \qquad (6)$$

$$x_{rij} \in \{0,1\}, \qquad \forall r \in \{1,\dots,p\},\ i,j \in \{0,\dots,n\}, i \neq j. \qquad (7)$$

The binary decision variable x_{rij} is defined to indicate if the vehicle r, r ∈ {1, 2, . . ., p} traverses an arc (i, j) in an optimal solution. The objective function (1) minimizes the total travel cost. The model constraints (2) ensure that each customer is visited by exactly one vehicle. The constraints (3) and (4) guarantee that each vehicle can leave the depot only once, and the number of the vehicles arriving at every customer and entering the depot is equal to the number of the vehicles leaving. In the constraints (5) the capacity constraints are stated, making sure that the sum of the demands of the customers visited in a route is less than or equal to the capacity of the vehicle performing the service. The sub-tour elimination constraints (6) ensure that the solution contains no cycles disconnected from the depot. The remaining obligatory constraints (7) specify the definition domains of the variables.

3. CAPACITATED VEHICLE ROUTING PROBLEM

3.1 MATHEMATICAL MODELLING

Data:

Q, Number of truck available = 20
Lmax, capacity of each truck = 80
Cp, Cost of each truck= $1000
Cd, Cost of each Euclidean distance travelled = $1
Rij, routes travelled by truck
Dij, Distance travelled by truck

Hij, location of Distribution center, which is origin, (0,0)
Cij, Customer Coordinates are given along with respective their demand. Tc, Total cost involved in Distribution.

Variables:

T, the Total number of trucks purchased

Rij, The travel routes of Trucks

Constraints:

Capacity of each truck, C<= 80

Total number of trucks, Q<= 20

Each customer must be visited only once, ΣCij = 1 where i and j are x, y coordinates provided

Each truck can visit the distribution center only once, ΣR(0,0) = 1

Objective:

To minimize the total cost associated with vehicle routing distance and fixed operational cost associated truck.

Min ΣTc

Min [Cp x T + Dij x Cd]

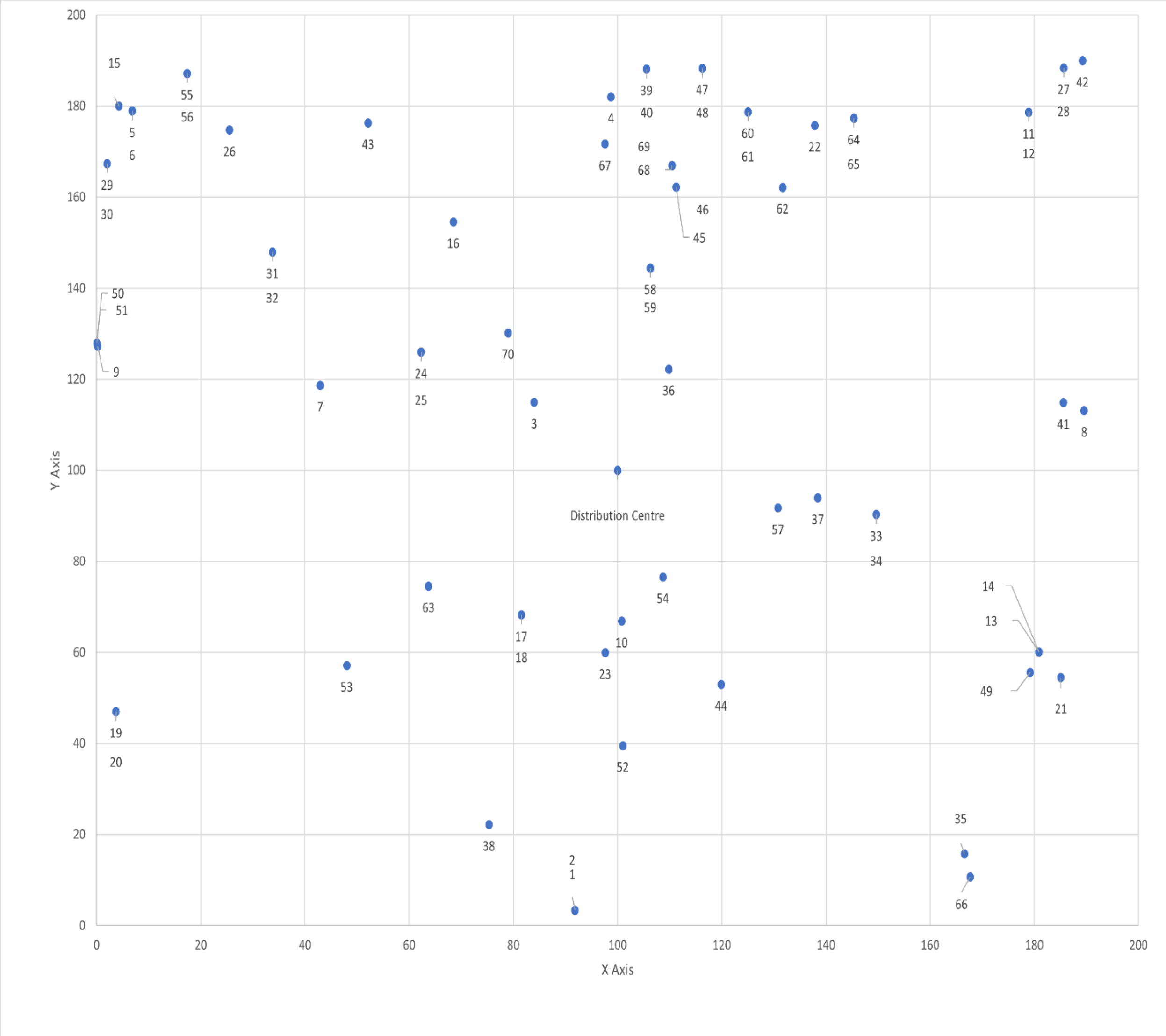

Figure 1: Locations of customers and the distribution center

we analyzed the truck requirement based on overall customer demand and capacity of each truck. This will give us a rough estimated number of trucks which company should in order to successfully satisfy all customer demand.

Theoretical number of trucks **=Total Demand of Trucks/ Capacity of Each required Truck**

=1223/80
= 15.28 ~ 16 Trucks.

Since trucks can only be an integer value, theoretical estimation of the total number of trucks is 16 and any greater number of vehicles should give us a feasible solution when we solve the problem with the given constraints.

Since VRP is a NP hard problem, when we solve it manually or using MS excel, will provide us a feasible solution but may not be an optimal one. It will take lot of time and efforts to find all the possible iterations and find optimal solution. So, to reduce the time spent, we used Python programming to construct the model and solve the model using available software tools and packages which gives us a feasible yet very close to optimal one. We have mentioned the sample code at the end of this report and provided links of actual code we implemented.

3.2. RESULTS

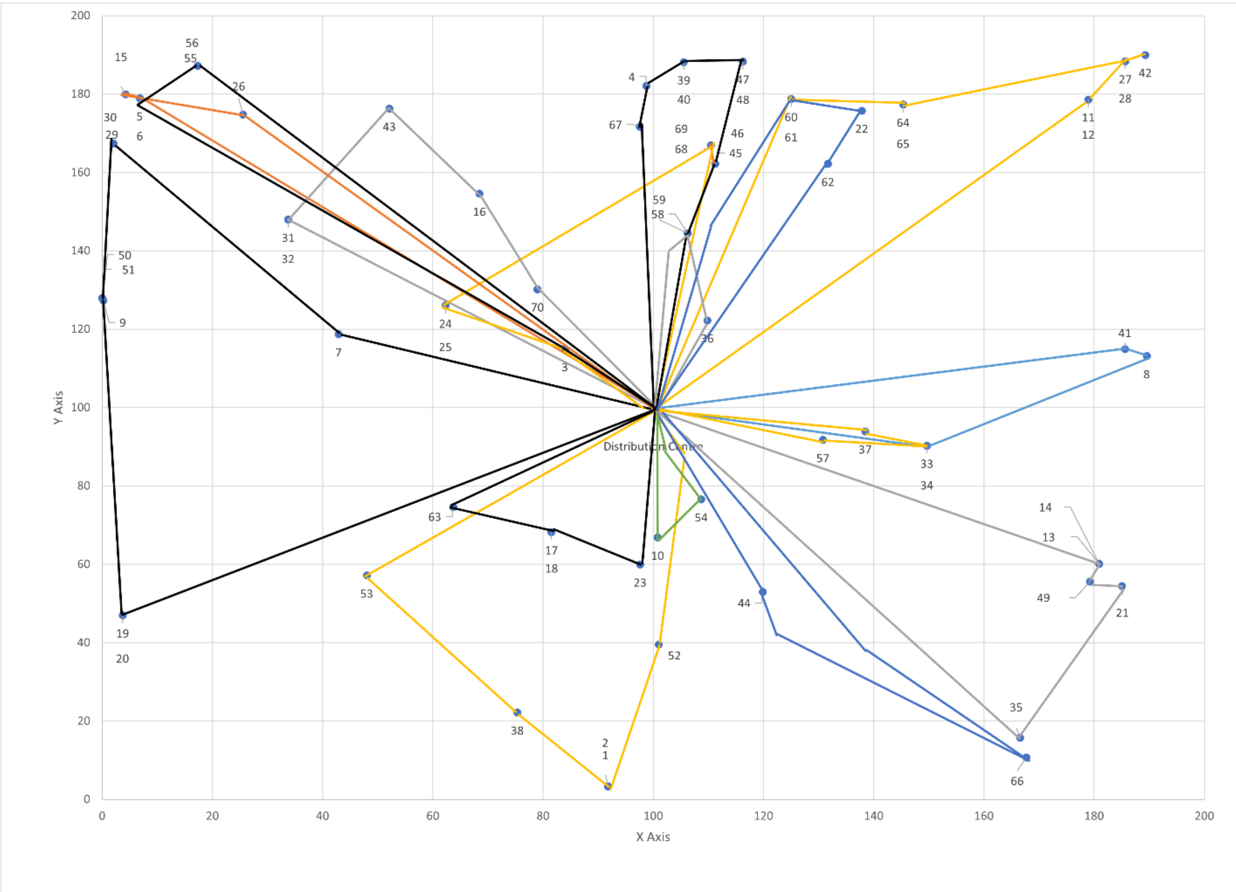

Figure 2: Feasible routes of the 16 trucks

- The minimum number of trucks required is feasible to be 16. Total distance travelled by all the trucks has a feasible value of 3140m.
- Cost of renting trucks: 16 × 1000 = $16,000
- Cost of travel: 3140 × 1 = $3140
- Total Cost = $19,140

3.3. Feasibility

- The cumulative load delivered by all the trucks is equal to the total demand placed by the customers. That is 305750lbs.
- Each customer is visited at least once.
- Load capacity of each truck does not exceed 20,000lbs.
- Number of trucks used do not exceed 20.

4. Capacitated Vehicle Routing Problem with Time Windows

4.1. Problem and Approach

Like the first problem, we seek to minimize the total cost by minimizing the total Euclidean distance and the number of trucks used. Two nodes with same coordinates signify the same customer split in to two nodes. The demands of the duplicate nodes add up to the demand of the original node. Additionally, the customers must be visited during a specified time window and accommodate the service times at each location.
The integer programming optimization can be written as follows:

Minimize

$$\sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} \qquad (1)$$

Subject to

$$\sum_{k\in K}\sum_{j\in\Delta^+(i)} x_{ijk} = 1 \quad \forall i \in V, \tag{2}$$

$$\sum_{j\in\Delta^+(0)} x_{0jk} = 1 \quad \forall k \in K, \tag{3}$$

$$\sum_{i\in\Delta^-(n+1)} x_{i,n+1,k} = 1 \quad \forall k \in K, \tag{4}$$

$$\sum_{i\in\Delta^-(j)} x_{ijk} - \sum_{i\in\Delta^+(k)} x_{jik} = 0 \quad \forall k \in K, j \in V \tag{5}$$

$$x_{ijk}(w_{ik} + s_i + t_{ij} - w_{jk}) \leq 0 \quad \forall k \in K, (i,j) \in A, \tag{6}$$

$$e_i \sum_{j\in\Delta^+(i)} x_{ijk} \leq w_{ik} \quad \forall k \in K, i \in V, \tag{7}$$

$$l_i \sum_{j\in\Delta^+(i)} x_{ijk} \geq w_{ik} \quad \forall k \in K, i \in V, \tag{8}$$

$$w_{0k} \geq E \quad \forall k \in K, \tag{9}$$

$$w_{n+1,k} \leq L \quad \forall k \in K, \tag{10}$$

$$\sum_{i\in V} q_i \sum_{j\in\Delta^+(i)} x_{ijk} \leq Q \quad \forall k \in K, \tag{11}$$

The binary decision variable x_{ijk} is defined to indicate if the vehicle k traverses an arc (i, j) in an optimal solution. Objective function (1) minimizes the total cost of all the routes in the solution. Constraint (2) ensures that each customer is assigned to exactly one route. Constraints (3), (4) and (5) guarantee that we have only one outcoming edge from the start depot vertex, only one incoming edge to the end depot vertex and each customer has the same number of incoming and outcoming edges. Inequality (6) prohibits service to start earlier than the earliest possible arrive time from the previous customer considering service and travel times. Inequalities (7), (8), (9), (10) provide feasibility with respect to the given time windows. Finally, constraint (11) prohibits overloaded routes with the total demand greater than vehicle capacity Q.

4.2. Results

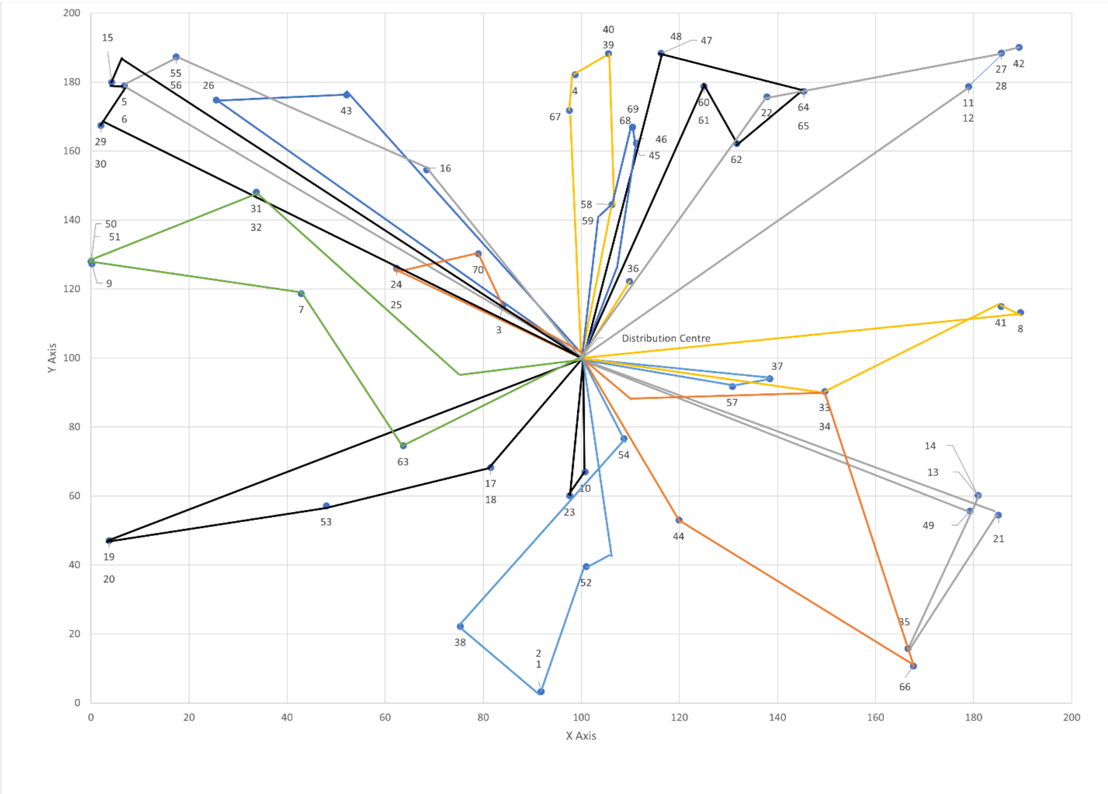

Figure 3: Feasible routes for the 17 trucks

- The minimum number of trucks required is evaluated to be 17. Total distance travelled by all the trucks has a feasible value of 3281m. Total time taken for all routes is 18834min.
- Cost of renting trucks: 17 × 1000 = $17,000
- Cost of travel: 3281 × 1 = $3281
- Total Cost = $20,281

4.3. Feasibility

- The cumulative load delivered by all the trucks is equal to the total demand placed by the customers. That is 305750lbs.
- Each customer is visited at least once.
- Load capacity of each truck does not exceed 20,000lbs.
- Number of trucks used do not exceed 20.
- The customers are visited within their respective time windows.

CONCLUSION AND FUTURE SCOPE

In conclusion, we determine that it is optimal to use 16 vehicles along the routes provided in the first part of the problem while satisfying all the constraints. The total Euclidean distance travelled by all 16 vehicles is 3314, the total cost associated with it stands at $19314.

For the second part of the problem, the optimal number of vehicles to be used is 17 vehicles, and the total Euclidean distance of all routes is 3799, the total customer demand of 1223 is satisfied with customer time windows and service time constraints. Total time of all routes travelled by the vehicles is minimized at 19154 units and the total cost associated is $20,799.

Even though the above-mentioned vehicle routes are feasible, and we state they are optimal solutions, we feel there is always a scope for improvement. We can improve the search of shortest path and reduce overall time of travel by improving search the algorithms in programming to suit the real-world needs, which requires extensive knowledge of operations research and excellent programming skills.

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