Probability

Type: *Manually Countable outcomes.*

Q1. A bag contains 2 green, 3 blue and 6 black balls. If a ball is drawn at random what is the probability that it is :

1. A black ball. 2. Not a blue ball?

Solution:

1. **6/11** 2. (2 green + 6 black) /11 = **8/11**

Q2. Two coins one with head on both sides and other coin with head in one side and tail in other side is in a box. A coin is taken at random and found head in one side. What is the probability that other side is a head?

Solution:

After drawing a coin and seeing a 'Head' on one side, the other side can be 'Head' (if it is a faulty coin) and other side can be 'Tail' (if it is normal coin), of which 'Head' event is favorite.

P = (H)/(H, T) = **1/2**

Q3. What is the probability that a two digit number selected at random will be a multiple of '3' and not a multiple of '5'?

Solution:

P = [2 digit multiples of '3' which are not multiples of '5'/[Two digit numbers]
= [(12, 15, 18, ... 99) - (15, 30, 45, 60, 75, 90)]/[10 -> 99]
= 30 - 6/90
= **24/90**

Q4. If two dice are thrown simultaneously, what is the probability that the first die shows up 6 and the second die does not show up 6?

A. 1/36 B. 1/9 C. 5/36 D. None of these

Solution:

P = [(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)]/[(1, 1), (1, 2) -> (6, 6)] = **5/36**

Q5. If two dice are thrown simultaneously, what is the probability that one die shows up ‘2’ and the other shows up ‘5’?

A. 1/18 B. 5/36 C. 1/36 D. None of these

Solution:

P = [(2, 5), (5, 2)]/[(1, 1), (1, 2) -> (6, 6)] = 2/36 = **1/18**

Q6.What is the probability of getting same numbers when two dice are thrown
A. 1/6 B.1/3 C.1/4 D.1/2

Solution:
P = [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]/[(1, 1), (1, 2) -> (6, 6)] = 6/36 = **1/6**

Q7. Two dice are tossed. The probability that the total score is a prime number is:
A. 1/6 B. 5/12 C. ½ D. 7/9

P = [(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)]/(6*6)
= 15/36 = **5/12**

Q8. Out of a pack of 52 cards one is lost; from the remainder of the pack, two cards are drawn and found to be spades. Find the chance that the missing card is a spade.
A. 11/50 B. 11/49 C. 10/49 D. 12/50

Solution:
Total possible missing cards: 50 [Out of 52 cards in a set, 2 of the cards are found already. Remaining 50 can be missing.]
Favorite: 11 [Out of 13 spades, two spades already discovered. Remaining 11 could be missing is favorite.]
P = **11/50**

Type: *Countable outcomes(Use of P&C concepts to count)*

Q9. A locker at the Deautche Bank in Germany can be opened by dialing a fixed three-digit code (between 000 and 999). Don, the King of Drug Mafia in India, only knows that the number is three-digit number and has only one six. Using this information he tries to open the locker by dialing three digits at random. The probability that he succeeds in his mission is?
A. 1/900 B. 1/216 C. 1/243 D. 3/216

P = ['1' correct lock code]/Count(Three digit patterns with exactly one '6') = 1/Count ([6 _ _] OR [_ 6 _] OR [_ _ 6])

Where each of _ can be done in (9) ways.

P = 1/([6 (9) (9)] OR [(9) 6 (9)] OR [(9) (9) 6]) = 1/[9*9 + 9*9 + 9*9] = **1/243**

Q10. What is the probability of getting at least one six in a single throw of three unbiased dice?
A. 1/6 B. 125 / 216 C. 81 / 216 D. 91 / 216

P = [(6, 6, 6) OR (6, 6, _) OR (6, _, 6) OR (_, 6, 6) OR (6, _, _) OR (_, 6, _) OR (_, _, 6)]/(6*6*6)

where each _ can have 5 readings Viz 1, 2, 3, 4, 5.

P = [1 + 5 + 5 + 5 + 5*5 + 5*5 + 5*5]/216 = **91/216**

Q11. If four dice are thrown simultaneously, what is the probability that sum of the numbers is exactly 20?
A. 31/1296 B. 35/1296 C. 37/1296 D. None of these

Solution:
Out of total 1296 options, favorite combinations are

(6, 6, 6, 2) which can be arranged in 4 ways,
(6, 6, 5, 3) which can be arranged in 4!/2! = 12 ways
(6, 6, 4, 4) which can be arranged in 4!/2!2! = 6 ways
(5, 5, 6, 4) which can be arranged in 4!/2! = 12 ways
(5, 5, 5, 5) which can be arranged in 1 way.

Therefore total favorite cases are 35 ways.
P = **35/1296**

Q12. If 5 coins are tossed together, what is the probability of getting exactly 2 heads?
A. ¼ B. 1/16 C. 5/16 D. None of these

Solution:

Out of total of 32 options, favorite combination is HHTTT which can be arranged in 5!/3!2! = 10 ways.
P = 10/32 = **5/16**

Type: *Compound Events*
Q13. When two balls are drawn on succession(one after another) with replacement from a box consisting of 6 white and 8 black balls, find the probability that
1. Both are white 2. One is white and the other is black.
3. Two balls being of same color when drawn ball is not replaced.
4. One ball being white and other being black when drawn ball is not replaced.

Solution:

1. Both are white = First ball is white AND Second ball is white = (6/14)*(6/14)

2. One is white and other is black = [(First white AND second is black) AND (First black AND first is white)]

= (6/14)*(8/14) + (8/14)*(6/14)

3. Two balls being of same color = [(W, W) OR (B, B)] = (6/14)*(5/13) + (8/14)*(7/13)

4. One ball being white and other being black = [(W, B) OR (B, W)] = [(6/14)*(8/13) + (8/14)*(6/13)]

Q14. The probability that Suresh can solve the problem is 2/3 and Ramesh can solve it is ¾. If both of them attempt the problem, then what is the probability that the problem gets solved?
A. ½ B. 11/12 C. 2/3 D. None of these

Solution:

P(Problem gets solved) = P(Ramesh solves AND Suresh doesn't solve) OR
P(Ramesh doesn't solve AND Suresh solves) OR
P(Ramesh solves and Suresh solves)
P = (3/4)*(1/3)+(1/4)*(2/3)+(3/4)*(2/3) = **11/12**

Q15. Ramesh person draws a card from a pack of 52, shuffles it. He continues doing it till he draws a heart. What is the probability that he has to make 3 trials?
A. 274/1700 B. 123/1720 C. 247/1700 D. 234/1500

Solution:

P(Ramesh makes 3 trials)=P(Ramesh doesn't get a spade in first trial) AND
P(Ramesh doesn't get a spade in second trial) AND
P(Ramesh gets spade in third trial)

P = (39/52)*(38/51)*(13/50)

Q16. For the FIFA world cup, Paul the octopus has been predicting the winner of each match with amazing success. It is rumored that in a match between 2 teams A and B, Paul picks A with the same probability as A's chances of winning.Let's assume such rumors to be true and that in a match between Ghana and Bolivia, Ghana the stronger team has a probability of 2/3 of winning the game. What is the probability that Paul will correctly pick the winner of the Ghana-Bolivia game?

A. 4/9 B. 2/3 C. 1/9 D. 5/9

Solution:

P(Paul picking the right winner)
= Paul selects Bolivia AND Bolivia wins OR Paul selects Ghana AND Ghana wins
= 2/3 * 2/3 + 1/3 * 1/3 = **5/9**

Q17. a, b, c are chosen randomly and with replacement from the set {1, 2, 3, 4, 5}. Find the probability that (a*b+c)
even.

Solution:

We get a*b+c as even in following conditions:

a is odd AND b is odd AND c is odd OR
a is even AND b is even AND c is even OR
a is even AND b is odd AND c is even OR
a is odd AND b is even AND c is even

3/5 * 3/5 * 3/5 + 2/5 * 2/5 * 2/5 + 2/5 * 3/5 * 2/5 + 3/5 * 2/5 * 2/5 = 59/125

Type: Mutually Exclusive/Mutually Non Exclusive Events [Don't double count your favorite outcomes]

Q18. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

A. 1 / 2 B. 2 / 5 C. 8 / 15 D. 9 / 20

Solution:

P = Count[(3, 6, 9, 12, **15**, 18) + (5, 10, **15**, 20) - **15**]/Count(1 ->20) = **9/20**

Q19. When two cards are drawn simultaneously from a pack of cards, what is the probability that both are kings or both are blacks?

Solution:

P(A U B) = P(A) + P(B) – P(A^nB)
P(Both Kings or Both Blacks) = (4/52)*(3/51)+(26/52)*(25/51)-(2/52)*(1/51)

Type: *Problems on Odds*

Q20. From a bag containing 4 white and 5 black balls a man draws 3 at random. What are the odds against these being all black? [Find probability and then use formula: 6]
A. 5/37 B. 37/5 C. 11/13 D. 5/42

Solution:

P(All three balls drawn are black)
= P(First ball is black AND Second ball is black AND Third ball is black)
= (5/9)*(4/8)*(3/7) = 5/42

Odds **against** all three balls drawn are black = (42-5)/5 = **37/5**

Type: *'Puzzle like' questions*

Q21. Three ants are sitting at the three corners of an equilateral triangle. Each ant starts randomly picks a direction and starts to move along the edge of the triangle. What is the probability that none of the ants collide?

Solution:

The ants can only avoid a collision if they all decide to move in the same direction (either clockwise or anti-clockwise). Each ant has the option to either move clockwise or anti-clockwise. There is a one in two chance that an ant decides to pick a particular direction.

P(No collision) = P(All ants go in a clockwise direction) OR P(All ants go in an anti-clockwise direction) = 0.5 * 0.5 * 0.5 + 0.5 * 0.5 * 0.5 = **0.25**

Q22. There are two boxes,one containing 39 red balls & the other containing 26 green balls. You are allowed to move the balls between the boxes so that when you choose a box random & a ball at random from the chosen box,the probability of getting a red ball is maximized. This maximum probability is

A. 60 B. 50 C. 80 D. 30

Solution:

Move 38 balls from first box to second box so that probability is maximized in 1st box (P = 1 in first box)

Probability of selecting red ball = 1/2 * 1 + 1/2 * 38/64 = **0.8**

Q23. If the probability of observing a car in 30 minutes on a highway is 0.95, what is the probability of observing a car in 10 minutes (assuming constant default probability)

Solution:

Here 0.95 is the probability for 1 or more cars or in other words probability of finding at least one car, not the probability of seeing just one car.

P(Finding no car in 30 minutes) = 0.05

P(Finding no car in first 10 min) AND P(Finding no car in another 10 min) AND P(Finding no car in another 10 min) = 0.05

P(Finding no car in 10 min) = Cube Root(0.05) = 0.369

P(FInding a car) = **0.63**

Q24. You meet a man on the street and he says, “I have two children and one is a son born on a Tuesday.” What is the probability that the other child is also a son?

Solution:

It seems like the probability should be 1/2 and the "Tuesday" piece of information shouldn't be relevant. But the problem says, "any piece of information that affects the selection will also affect the probability". So "one is a son born on a Tuesday" changes the probability. You can see this if you write out all different possible combinations:

1. Possibilities: 1-7 - First Child: Boy born on Tuesday; Second Child: Boy born on each of the seven days.

2. Possibilities 8-14 - First Child: Boy born on Tuesday; Second Child: Girl born on each of the seven days.

3. Possibilities 15-21 - First Child: Boy born on each of the seven days; Second Child: Boy born on Tuesday.

4. Possibilities 22-28 - Girl born on each of the seven days; Second Child: Boy born on Tuesday.

So we have 28 different possibilities, of which one is a repeat if you check carefully (both children being boys born on Tuesday got counted twice above). So eliminate that one and you have 27 different possibilities.

Counting up each of the possibilities that has the other child being a son and there's 13. So the answer is 13/27, or about a **48%** chance.

Question Bank
Q1. Saina has ‘n‘ paise with her. Neha has ‘a‘ paise with her. Both n and a are less than 100 and a!=n. What is the probability that (a^2 + n^2) is also less than 100?

A. 57/9702 B. 60/9702 C. 64/9702 D. 67/9702

Solution:

Total cases = 99*98 [Since a cannot be n]

Favorite Cases:

(2, 1), (1, 2)	[2]
(3, 1), (3, 2), (1, 3), (2, 3)	[4]
(4, 1), (4, 2), (4, 3), (1, 4), (2, 4), (3, 4)	[6]
(5, 1), (5, 2), (5, 3), (5, 4), (1, 5), (2, 5), (3, 5), (4, 5)	[8]
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6)	[10]
(7, 1), (7, 2), (7, 3), (7, 4), (7, 5), (7, 6), (1, 7), (2, 7), (3, 7), (4, 7), (5, 7), (6, 7)	[12]
(8, 1), (8, 2), (8, 3), (8, 4), (8, 5), (1, 8), (2, 8), (3, 8), (4, 8), (5, 8)	[10]
(9, 1), (9, 2), (9, 3), (9, 4), (1, 9), (2, 9), (3, 9), (4, 9)	[8]

Count = 60

P = **60/9702**

Q2. A habitual clock watcher looks at a Digital Clock which shows time in 12 hour format : HH:MM. For example 4 minutes past 4pm is show as 4:04. Both 12AM and 12PM is shown as 12:00. What is the probability that each of the digit in the clock shows same number when the person randomly

looks for the time.

A. 1/60 B. 1/72 C. 1/120 D. 1/144

Solution:

Total possible readings = 12*60

Favorite: (1:11), (2:22), (3:33), (4:44), (5:55), (11:11)

P = **1/120**

Q3. A drawer holds 4 red hats and 4 blue hats. What is the probability of getting exactly three red hats or exactly three blue hats while drawing four hats randomly and immediately returning every hat to the drawer before taking out the next?

A. 1/2 B. 1/4 C. 1/8 D. None

Solution:

P(Exactly 3 red or exactly 3 blue) = P(Exactly 3 red) + P(Exactly 3 blue)

P(Exactly 3 red) =

P(First Red AND Second Red AND Third Red AND Fourth Blue)
OR
P(First Red AND Second Red AND Third Blue AND Fourth Red)
OR
P(First Red AND Second Blue AND Third Red AND Fourth Red)
OR
P(First Blue AND Second Red AND Third Red AND Fourth Red)

= (4/8)*(4/8)*(4/8)*(4/8) + ... 4 times = 1/4

Similarly P(Exactly 3 Blue) = 1/4

P(Exactly 3 red or exactly 3 blue) = P(Exactly 3 red) + P(Exactly 3 blue) = **1/2**

Q4. The probability that A will solve a problem is 1/5. What is the probability that he solves at least one problem out of ten problems?

A. (1/5)^10 B. (1/5)^9 C. (4/5) D. 1-(4/5)^10

Solution:

P(A solves at least one problem) = 1 – P(He does not solve any problem)

= **1-(4/5)^10**

Q5. The probability that A speaks truth is 3/4, while this probability for B is 5/6. The probability that they contradict each other when asked to speak on a fact is:

A. 3/20	B. 1/3	C. 7/20	D. 4/5

Solution:

P[They contradict] = P[A speaks truth AND B lies OR A lies and B speaks truth]
= (3/4)(1/6) OR (1/4)(5/6) = 8/24 = **1/3**

Q6. What is the probability that a 9 digit number formed using 1,2,3,4,5,6,7,8,9 is divisible by 36. (Without repetition)

Solution:

Total: 9!

Fav: If a number has to be divisible by 36, it should be divisible by both 9 and 4.

1. 1 + 2 + ... + 9 = 45 is divisible by 9.

2. To make the number divisible by 4 options for last 2 digits are 12, 16, 24, 28, 32, 36, 48, 52, 56, 64, 68, 72, 76, 84, 92, 96

and i each of the above 16 cases, the remaining 7 digits can be done in 7! ways.

So Fav = 16*7!

Probability = **16*7!/9!**

Q7. An elevator starts with 5 person on board and goes up 5 floors. What is the probability that all 5 passengers are on a different floor?

Solution:

Total: (5)*(5)*(5)*(5)*(5)

Fav: 5!

Probability = **5!/5^5**

Q8. Find Probability that a leap year is chosen at random will have 53 Sundays?

Solution:

A Leap Year has 366 Days which includes 52 complete weeks and 2 odd days..

Total possibilities for 2 odd days: (M, T), (T, W), (W, Th), (Th, F), (F, S), (S, Su), (Su, M)   [7]

Fav: (S, Su), (Su, M)   [2]

Probability = 2/7

Q9. 15 people sit round a circular table what are the odds against two particular people sitting together.
A. 7:1   B. 6:1   C. 5:1   D. 1:5

Solution:

Total: 14!

Fav: Considering those 2 persons as single entity AND Arranging 14 entities AND Swapping Bigger entity

= 1*13!*2

P(2 particular persons are always together) = 1/7

Odds against(2 particular persons are always together) = **6/1**

Q10. Set A {1,3,10,17,19}, B{9,12,15,18,21} and set C{4,7,10,13,16,19} probability of "a num from A + a num from B > a num from C".

Solution:

Total: 5*5*6   [150]

False Cases:

1+9!> 10,13,16,19
1+12!> 13,16,19
1+15!> 16,19
1+18!> 19
3+9!> 13,16,19
3+12!> 16,19
3+15!>19
10+9!>19   [17]

Probability=1-(17/150)=**133/150**

Q11. The probability of a bomb hitting a bridge is 1/2 and two direct hits are required to destroy it. The least no of bombs required so that the probability of the bridge being destroyed is greater than 0.9 is?

Solution:

Probability of the bridge being destroyed is greater than 0.9
= Probability of the bridge being not destroyed is lesser than 0.1
= P(out of n shots 0 or 1 shot hits the target) < 0.1

Since a shot can either hit or miss, it is a Binomial Distribution.

P(Exact 'r' success out of 'n' attempts for a Binomial Distribution) = nCr/2^n

P(out of n shots 0 or 1 shot hits the target) < 0.1

[nC0+nC1]/2^n < 0.1

The above equation is satisfied for **n=7** for the first time.

Q12. 6 Red chips, 7 Green and 8 Blue chips are in a bag if 5 are drawn with replacement. What is the probability that at least 3 are red?

Solution:

Exactly 3 Red: RRRGG*(5!/3!2!) OR RRRBB*(5!3!2!) OR RRRGB*(5!/3!)
OR
Exactly 4 Red: RRRRG*(5!4!) OR RRRRB*(5!/4!)
OR
Exactly 5 Red: RRRRR*1

=
(6/21)*(6/21)*(6/21)*(7/21)*(7/21)* (5!/3!2!) + (6/21)*(6/21)*(6/21)*(8/21)*(8/21)* (5!/3!2!) +
(6/21)*(6/21)*(6/21)*(8/21)*(8/21)* (5!/3!)

+ (6/21)*(6/21)*(6/21)*(6/21)*(7/21)* (5!/4!) + (6/21)*(6/21)*(6/21)*(6/21)*(8/21)* (5!/4!)

+ (6/21)*(6/21)*(6/21)*(6/21)*(6/21)