

When Manually Counting is not Sensible!

Q1. How many numbers between 1–1000 have digit '7' in them?

Q1. Permutation Question(Numbers are permutation questions)

Q2. Restriction is that count only numbers with at least one digit as '7'.

Q3. Digits can be repeated. (Ex we have to count 774). So nPr is not applicable.

Step 1:

777 OR

77_ OR

-77 OR

7_7 OR

7__ OR

7 OR

__7

Step 2: Each _ can be filled in 9 ways (0 to 9 except 7.)

Step 3: $1 + 9 + 9 + 9 + 9*9 + 9*9 + 9*9 = 271$

Q2. In how many ways 8 different letters can be posted in 5 letter boxes?

Q1. Permutation Question(If you change the order of posting it will become a different posting.)

Q2. No restriction. Any letter can go to any box.

Q3. Letters can be repeated. (More than one letter can be put in a box. So nPr is not applicable.

Step 1:

[Post 1st letter] AND [Post 2nd letter] AND [Post 3rd letter]

Step 2: Each letter can be posted in 5 ways.

Step 3: $(5)*(5)*(5) \dots = 5^8$

Q3. How many 5 digit no. can be formed with digits 1, 2, 3,4,5,6 which are divisible by 4 and digits not repeated?

A. 144

B. 168

C. 192

D. None

Q1. Permutation Question(Numbers are permutation questions)

Q2. Restriction is that count only multiples of 4.

Q3. Digits can not be repeated. So nPr is applicable.

Step 1:

__ AND __ AND __ AND 12 OR

__ AND __ AND __ AND 16 OR

__ AND __ AND __ AND 24 OR

__ AND __ AND __ AND 32 OR

__ AND __ AND __ AND 36 OR

__ AND __ AND __ AND 52 OR

__ AND __ AND __ AND 56 OR

__ AND __ AND __ AND 64 OR

Step 2: Three ___ in each can be filled in $4P_3$ ways. (After using 2 digits only 4 digits will be left)

Step 3: $4P_3 + 4P_3 + 4P_3 + \dots 8 \text{ times} = 192$

Q4. How many 5 digit numbers can be formed with digits 1, 2, 3,4,5,6 which are divisible by 4 and digits repeated? [TCS 2012]

Q1. Permutation Question(Numbers are permutation questions)

Q2. Restriction is that count only multiples of 4.

Q3. Digits can be repeated. So nPr is not applicable.

Step 1:

___ AND ___ AND ___ AND 12 OR
___ AND ___ AND ___ AND 16 OR
___ AND ___ AND ___ AND 24 OR
___ AND ___ AND ___ AND 32 OR
___ AND ___ AND ___ AND 36 OR
___ AND ___ AND ___ AND 44 OR
___ AND ___ AND ___ AND 52 OR
___ AND ___ AND ___ AND 56 OR
___ AND ___ AND ___ AND 64 OR

Step 2: Each ___ in each can be filled in 6 ways.

Step 3: $6*6*6 + 6*6*6 + \dots 9 \text{ times} = 9*216 = 1944$

Q5. Mr Smith and Mrs Smith have invited 9 of their friends and their spouses for a party. If Mr Smith never stands next to Mrs Smith, how many ways the group can be arranged in a row for a photograph?

Q1. Permutation Question(Arrangements are permutation questions)

Q2. Restriction is that count Mr. Smith can not sit with Mrs. Smith

Q3. People can't be repeated. So nPr is applicable.

Step 1: Keep Aside Mr. Smith AND Arrange 19 other people AND Place Mr. Smith

Step 2: 1 way AND $19!$ ways AND 18 [Around 19 people there will be 20 places. But on either side of Mrs. Smith, Mr. Smith cannot go. So 18 places available now]

Step 3: $19! * 18$

Q6. A team of 8 students go to an excursion, in two cars, of which one can seat 5 and the other only 4. In how many ways can they travel?

A. 9 B. 26 C. 126 D. 3920

Q1. Combination Question.

Q2. No restriction. Any student can go to any car.

Q3. People can't be repeated. So nCr is applicable.

Step 1: [5 in big car AND 3 in small car] OR [4 in big car AND 4 in small car]

Step 2: $8C_5$ AND $3C_3$ OR $8C_4$ AND $4C_4$

Step 3: $8C_5 * 3C_3 + 8C_4 * 4C_4 = 126$

Q7. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?

- A. 63 B. 90 C. 126 D. 45

Q1. Combination Question.

Q2. No restriction.

Q3. People can't be repeated. So nCr is applicable.

Step 1: [Select 5 men from 7 men] AND [Select 2 women from 3 women]

Step 2: 7C_5 AND 3C_2

Step 3: ${}^7C_5 + {}^3C_2 = 63$

Q8. A cultural committee of eight has to be formed from nine Asians and five Africans. One of the Africans' names is Randy. In how many ways can it be done when the committee consists:

- | | |
|----------------------------|-----------------------|
| 1. Randy as a must member. | 2. Exactly 3 Africans |
| 3. At least 3 Africans | 4. At most 3 Africans |

Q1. Combination Question.

Q2. Restriction in each sub question.

Q3. People can't be repeated. So nCr is applicable.

1. Select Randy AND Select 7 more = $1 * {}^{13}C_7$

2. Select 3 Africans AND Select 5 more Asians = ${}^5C_3 * {}^9C_5$

3. Select exactly 3 Africans OR Select exactly 4 Africans OR Select exactly 5 Africans
= ${}^5C_3 * {}^9C_5 + {}^5C_4 * {}^9C_4 + {}^5C_5 * {}^9C_3$

4. Select exactly 3 Africans OR Select exactly 2 Africans OR Select exactly 1 African OR Select exactly 0 african.
= ${}^5C_3 * {}^9C_5 + {}^5C_2 * {}^9C_6 + {}^5C_1 * {}^9C_7 + {}^5C_0 * {}^9C_8$

Q9. How many words can be formed by using all the letters of the word 'QUANTECH'?

Q1. Permutation Question(Words are permutation questions)

Q2. No restriction.

Q3. Letters cannot be repeated. (Else you can be infinitely long words)

Answer = $8P8 = 8!$

Q10. How many different permutations can be made out of the letters of the word, 'ASSISTANTS' ?

Answer = $10!/4!4!2!$

[$nPr = n!/p!q!r!$ when of 'n' items 'p' are of first kind, 'r' are of second kind and so on.]

Q11. How many words of 4 consonants and 3 vowels can be made from 12 consonants and 4 vowels, if all the letters are different?

- A. ${}^{16}C_7 * 7!$ B. ${}^{12}C_4 * {}^4C_3 * 7!$ C. ${}^{12}C_3 * {}^4C_4$ D. ${}^{12}C_4 * {}^4C_3$

Q1. Permutation Question(Words are permutation questions)

Q2. No restriction.

Q3. Letters cannot be repeated. (Else you can be infinitely long words)

Step1: [Select 4 consonants from 12 consonants] AND [Select 3 vowels from 4 vowels} AND [Arrange chosen 7 letters]

Step2: $12C4$ AND $4C3$ AND $7P7$

Step3: $12C4 * 4C3 * 7!$

Q12. How many words can be formed by using all the letters of the word 'LAUGHTER' so that the vowels always come together?

- A. 3660 B. 3980 C. 4320 D. None of these

Q1. Permutation Question(Words are permutation questions)

Q2. Restriction is that vowels must be together.

Q3. Letters cannot be repeated. (Else you can be infinitely long words)

Step1: Consider 3 vowels as on entity AND Arrange 6 entities AND For each arrangement arrange vowels entity.

Step 2: 1 AND $6P6$ AND $3P3$

Step 3: $6! * 3! = 4320$

Q13. How many words can be formed from letters of the word 'LAUGHTER', so that:

- (i). Three vowels are not together? (ii) None of the vowels are together

1. No of words where 3 vowels are not together = Total words possible – No. of words where 3 vowels are together
= $8! - 6! * 3!$ [Refer to previous solution]

2.

Q1. Permutation Question(Words are permutation questions)

Q2. Restriction is that vowels must never together.

Q3. Letters cannot be repeated. (Else you can be infinitely long words)

Step 1: Arrange 5 consonants AND In 6 places available around 5 consonants place 3 vowels.

Step 2: $5P5$ AND $6P3$

Step3: $5! * 6P3$

Q14. In a meeting there are 10 couples (10 men and 10 women). Each person shakes hand with every other except his or her own spouse. How many handshakes were exchanged?

Q1.Combination question. (Handshakes are combinations)

Q2. Restriction is that husbands must not shake hands with their wives.

Q3. People cannot be repeated. So nCr is applicable.

Total valid handshakes = Total possible handshakes – Invalid handshakes = $20C2 - 10 = 180$

Type: Distribution

Q15. In how many ways 10 identical chocolates can be distributed to 4 children?

$$a+b+c+d = 10 \dots (1)$$

This can be done in $^{13}C_3$ ways.

Q16. In how many ways 10 identical chocolates can be distributed to 4 children in such a way that a child gets at least one chocolate. [TCS 2012]

$$a+b+c+d = 10 \dots (1)$$

Distributing one chocolate each for 4 children

$$a+b+c+d = 6 \dots (2)$$

This can be done in 9C_3 ways.

Type: Circular Arrangement

Q17. In how many ways can 21 differently colored beads be strung on a necklace?

- A. $21!$ B. $20!$ C. $21!/2$ D. $20!/2$

Since clockwise and anti-clockwise arrangements of colored beads does not make difference (other can be got by inverting the necklace), number of arrangements will be $20!/2$

Q18. Twelve friends go out for a dinner to a restaurant where they find two circular tables, one with 7 chairs and the other with 5 chairs. In how many ways can the group settle down themselves for the dinner?

- A. $12!/7!5!$ B. $12!/35$ C. $12!$ D. $12!5!7!$

Count(Arrange twelve friends) =

Count(Select 7 people to larger table AND select 5 people to smaller table AND arrange 7 people in larger table AND arrange 5 people in the smaller table)

$$= {}^{12}C_7 \text{ AND } {}^5C_5 * (7-1)! * (5-1)!$$

$$= 12!/35$$

Q19. In how many ways can 5 students and 5 teachers sit around a circular table so that no teachers sit together?

- A. $(4!)^2$ B. $(5!)^2$ C. $4!5!$ D. $5! \times {}^6P_5$

Firstly 5 students can be arranged around the table in $4!$ ways. Between 5 students now there will be 5 places available where 5 teachers can be arranged in $5!$ ways.

[Note that teachers' position is defined w.r.t between who two students the teacher is seated. So shifting teachers by 1 or 2 positions will alter the arrangement. So arranging teachers between students is not circular but linear arrangement]

So arranging students AND teachers can be done in $4!5!$ ways.

Type: Applications

Q20. If all the letters of the word MASTER are taken and permuted and arranged in alphabetical order as in a dictionary, then what is the rank of the word MASTER?

Alphabetical Order:

A, E, M, R, S, T

Words of the form A _ _ _ _ (5!)

Words of the form E _ _ _ _ (5!)

Words of the form M A E _ _ (3!)

Words of the form M A R _ _ (3!)

Words of the form M A S E _ (2!)

Words of the form M A S R _ (2!)

come before word "Master". So the rank of 'master' will be $120 + 120 + 6 + 6 + 2 + 2 + 1 = 257$

Q21. All the words that can be made from letters of the word PARROT is permuted and arranged in dictionary order. What will be the 20th word? [TCS 2012]

Alphabetical order: A, O, P, R, R, T

Words starting with A = $5!/2! = 60$ [Note that R is repeated twice in O, P, R, R, T]. So 20th word must also be starting from A.

1: Words starting with AO = $4!/2! = 12$ [R is repeated twice in P, R, R, T]

Words starting with AP will have another 12 words which will take the count to 24. So 20th word must be starting with AP.

2: Words starting with APO: $3!/2! = 3$ words.

Words starting with APR: $3! = 6$ words will take the count to 21. So 20th word will be starting from APR.

3: Words starting with APRO: $2! = 2$ words.

4: Words starting with APRR: $2! = 2$ words.

Total of 19 words so far.

So 20th word will be **APRTOR**

Q22. How many squares and rectangles are there in a 8 x 8 chess board?

Use the below formula.

Number of squares in a square of n x n side = $1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2$

Number of rectangles in a square of n x n side = $1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3$
(This includes the number of squares.)

Q23. How many squares and rectangles are there in a 5 x 7 grid?

Use the below formula.

Number of squares in a rectangle of m x n side = $mn + (m-1)(n-1) + (m-2)(n-2) + \dots$ until any of (m-x) or (n-x) comes to 1.

Number of rectangles in a rectangle of m x n side = $(1+2+3+\dots+m)(1+2+3+\dots+n)$

Q24. Find the sum of all numbers that can be formed by taking all the digits at a time from 4, 5, 6, 7, 8 without repetition. [Use Formula: 6]

A. 133320

B. 244420

C. 332244

D. 7999920

If all the possible n-digit numbers using n distinct digits are formed, the sum of all the numbers so formed is equal to **Sum = (n-1)! * (sum of the n digits) * (1111 n times)**

Sum = $(5-1)![11111](4+5+6+7+8) = 7999920$

Q25. In how many ways an Ant can move from one corner of a chessboard to the opposite corner, if it is only allowed to move along edges of squares, without retracing steps? [Only allowed to move upward or forward]

However ant travels it has to cover 8 horizontal edges and 8 vertical edges.

No. of ways in which an ant can travel from one corner to another = No. of arrangements of (HHHHHHHHVVVVVVVV) = **16!/8!8!**

Question Bank

Q1. How many ways last 3 digits of a mobile number can be formed in such a way that there is exactly one '7' in them and no digit is repeated?

Q1. Permutation Question (Numbers are permutation questions)

Q2. Restriction is that count only numbers with exactly one digit as '7'.

Q3. Digits can't be repeated. So nPr is applicable.

Step 1:

7__ OR

7 OR

__7

Step 2: Two __ can together be done in $9P2$ ways.

Step 3: $9P2 + 9P2 + 9P2 = 216$

Q2. How many 8's are there in the numbers between 145-479?

[TCS 2012]

Forms which are of interest to us: - (__ , 8, 8), (__ , __ , 8) and (__ , 8, __)

Consider: - (__ , 8, 8) where __ can be filled with 1, 2, 3. So totally $3*2 = 6$ 8s.

Consider: - (__ , __ , 8) which I will split as (1, __ , 8) where 2nd digit can be filled in 5 (4, 5, 6, 7, 9) ways, (2, __ , 8) where 2nd digit can be filled in 9 (0 to 9 except 8) ways, (3, __ , 8) where 2nd digit can be filled in 9 (0 to 9 except 8) ways and (4, __ , 8) where 2nd digit can be filled in 8 (0 to 7) ways.

Consider:- (__, 8, __) which I will split as (1, 8, __) where 3rd digit can be filled in 9(0 to 9 except 8) ways, (2, 8, __) where 3rd digit can be filled in 9(0 to 9 except 8) ways and (3, 8, __) where 3rd digit can be filled in 9(0 to 9 except 8) ways.

$$\text{Total} = 6 + 31 + 27 = 64$$

Q3. How many 3 digit numbers are there in such a way that product of their digits is 48? [E Litmus]

Q1. Permutation Question(Numbers are permutation questions)

Q2. Restriction is that product of the digits must be 48.

Q3. Digits can be repeated.

Step 1:

8 6 1 [6 arrangements] OR

8 3 2 [6 arrangements] OR

4 6 2 [6 arrangements] OR

4 3 4 [3 arrangements]

$$\text{Answer: } 6 + 6 + 6 + 3 = 21$$

Q4. In how many ways 9 people, two of whom are Chakkar and Ghanchakkar, can be arranged in such a way that 2 or fewer people in between Chakkar and Ghanchakkar? [E Litmus]

Case 1 : 0 people in between Chankkar and Ghanchakkar = Chakkar and Ghanchakkar always together =

$$= 8! * 2! \quad \text{[Refer to Q12]}$$

Case 2: 1 person between Chakkar and Ghanchakkar

= Select one person to come between them(7C_1 ways) AND Consider the 3 as one entity AND Arrange 7 entities AND for each arrangement you can exchange position of Chakkar and Ghanchakkar

$$= {}^7C_1 * 1 * 7! * 2$$

Case 3: Two persons between Chakkar and Ghanchakkar

= Select two people to come in between Chakkar and Ghanchakkar AND consider the 4 people as one entity AND arrange 6 entities AND in each arrangement interchange Chakkar and Ghanchakkar and interchange middle two people.

$$= {}^7C_2 * 6! * (2 * 2)$$

$$\text{Answer} = 8! * 2! + {}^7C_1 * 1 * 7! * 2 + {}^7C_2 * 6! * (2 * 2)$$

Q5. If all the words that can be made from letters of “master” is sorted in alphabetical order, what will be the 49th word?

Alphabetical order is A, E, M, R, S, T

Words starting with A will be of the type: A, __, __, __, __ and will be $5! = 120$ in numbers.

so 49th word will be one among the words starting in A.

$$\text{Number of words starting with AE}_{_,_,_} = 4! = 24$$

$$\text{Number of words starting with AM}_{_,_,_} = 4! = 24$$

So 49th word will be **AREMST**

Q6. A manufacturer manufactures 6 different flavors of chocolate. The chocolates are sold in boxes of 10. How many different boxes of chocolates can be made? (Note: A box is considered different from other only if it contains a different number of chocolates of at least one type, regardless of the order)

A. 10^6

B. 3003

C. 3000

D. 6^{10}

Taking one variable each for 6 different chocolates, equation is

$$a+b+c+d+e+f = 10 \dots (1)$$

The above distribution can be done in ${}^{15}C_5 = \mathbf{3003}$ ways.