When Manually Counting is not Sensible!

Q1. How many numbers between 1-1000 have digit '7' in them?

Q1. Permutation Question(Numbers are permutation questions)
Q2. Restriction is that count only numbers with at least one digit as '7'.
Q3. Digits can be repeated. (Ex we have to count 774). So n**P**r is not applicable.

Step 1:

777 OR
77_ OR
-77 OR
7_7 OR
7_ _ OR
_7_ OR
_ _ 7

Step 2: Each _ can be filled in 9 ways (0 to 9 except 7.)

Step 3: 1 + 9 + 9 + 9 + 9*9 + 9*9 + 9*9 = **271**

Q2. In how many ways 8 different letters can be posted in 5 letter boxes?

Q1. Permutation Question(If you change the order of posting it will become a different posting.)
Q2. No restriction. Any letter can go to any box.
Q3. Letters can be repeated. (More than one letter can be put in a box. So n**P**r is not applicable.

Step 1:

[Post 1st letter] AND [Post 2nd letter] AND [Post 3rd letter] ....

Step 2: Each letter can be posted in 5 ways.

Step 3: (5)*(5)*(5) ..... = **5^8**

Q3. How many 5 digit no. can be formed with digits 1, 2, 3,4,5,6 which are divisible by 4 and digits not repeated?
A. 144 B. 168 C. 192 D. None

Q1. Permutation Question(Numbers are permutation questions)
Q2. Restriction is that count only multiples of 4.
Q3. Digits can not be repeated. So n**P**r is applicable.

Step 1:

__ AND __ AND __ AND 12 OR
__ AND __ AND __ AND 16 OR
__ AND __ AND __ AND 24 OR
__ AND __ AND __ AND 32 OR
__ AND __ AND __ AND 36 OR

__ AND __ AND __ AND 52 OR
__ AND __ AND __ AND 56 OR
__ AND __ AND __ AND 64 OR

Step 2: Three __ in each can be filled in 4**P**3 ways. (After using 2 digits only 4 digits will be left)

Step 3: 4**P**3 + 4**P**3 + 4**P**3 + ... 8 times = **192**

Q4. How many 5 digit numbers can be formed with digits 1, 2, 3,4,5,6 which are divisible by 4 and digits repeated?

Q1. Permutation Question(Numbers are permutation questions)
Q2. Restriction is that count only multiples of 4.
Q3. Digits can be repeated. So n**P**r is not applicable.

Step 1:

__ AND __ AND __ AND 12 OR
__ AND __ AND __ AND 16 OR
__ AND __ AND __ AND 24 OR
__ AND __ AND __ AND 32 OR
__ AND __ AND __ AND 36 OR
__ AND __ AND __ AND 44 OR
__ AND __ AND __ AND 52 OR
__ AND __ AND __ AND 56 OR
__ AND __ AND __ AND 64 OR

Step 2: Each __ in each can be filled in 6 ways.

Step 3: 6*6*6 + 6*6*6 + ... 9 times = 9*216 = **1944**

Q5. A team of 8 students go to an excursion, in two cars, of which one can seat 5 and the other only 4. In how many ways can they travel?
A. 9 B. 26 C. 126 D. 3920

Q1. Combination Question.
Q2. No restriction. Any student can go to any car.
Q3. People can't be repeated. So n**C**r is applicable.

Step 1: [5 in big car AND 3 in small car] OR [4 in big car AND 4 in small car]

Step 2: 8C5 AND 3C3 OR 8C4 AND 4C4

Step 3: 8C5 * 3C3 + 8C4 * 4C4 = **126**

Q6. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
A. 63 B. 90 C. 126 D. 45

Q1. Combination Question.

Q2. No restriction.
Q3. People can't be repeated. So n**C**r is applicable.

Step 1: [Select 5 men from 7 men] AND [Select 2 women from 5 women]

Step 2: 7C5 AND 3C2

Step 3: 7C5 + 3C2 = **63**

Q7. A cultural committee of eight has to be formed from nine Asians and five Africans. One of the Africans' names is Randy. In how many ways can it be done when the committee consists:

1. Randy as a must member.
2. Exactly 3 Africans
3. At least 3 Africans
4. At most 3 Africans

Q1. Combination Question.
Q2. Restriction in each sub question.
Q3. People can't be repeated. So n**C**r is applicable.

1. Select Randy AND Select 7 more = **1 * 13C7**

2. Select 3 Africans AND Select 5 more Asians = **5C3 * 9C5**

3. Select exactly 3 Africans OR Select exactly 4 Africans OR Select exactly 5 Africans
= **5C3 * 9C5 + 5C4 * 9C4 + 5C5 * 9C3**

4. Select exactly 3 Africans OR Select exactly 2 Africans OR Select exactly 1 African OR Select exactly 0 african.
= **5C3 * 9C5 + 5C2 * 9C6 + 5C1 * 9C7 + 5C0 * 9C8**

Q8. How many words can be formed by using all the letters of the word 'QUANTECH'

Q1. Permutation Question(Words are permutation questions)
Q2. No restriction.
Q3. Letters cannot be repeated. (Else you can be infinitely long words)

Answer = 8P8 = **8!**

Q9. How many different permutations can be made out of the letters of the word, 'ASSISTANTS' ?

Answer = **10!/4!4!2!**

[nPn = n!/p!q!r! when of 'n' items 'p' are of first kind, 'r' are of second kind and so on.]

Step1: [Select 4 consonants from 12 consonants] AND [Select 3 vowels from 4 vowels} AND [Arrange chosen 7 letters]

Step2: 12C4 AND 4C3 AND 7P7

Step3: **12C4 * 4C3 * 7!**

Q10. How many words can be formed by using all the letters of the word 'LAUGHTER' so that the vowels always come together?

A. 3660 B. 3980 C. 4320 D. None of these

Q1. Permutation Question(Words are permutation questions)
Q2. Restriction is that vowels must be together.
Q3. Letters cannot be repeated. (Else you can be infinitely long words)

Step1: Consider 3 vowels as on entity AND Arrange 6 entities AND For each arrangement arrange vowels entity.

Step 2: 1 AND 6P6 AND 3P3

Step 3: 6! * 3! = **4320**

Q11. How many words can be formed from letters of the word 'LAUGHTER', so that:

(i). Three vowels are not together? (ii) None of the vowels are together

1. No of words where 3 vowels are not together = Total words possible - No. of words where 3 vowels are together

   = **8! - 6! * 3!** [Refer to previous solution]

2.

Q1. Permutation Question(Words are permutation questions)
Q2. Restriction is that vowels must never together.
Q3. Letters cannot be repeated. (Else you can be infinitely long words)

Step 1: Arrange 5 consonants AND In 6 places available around 5 consonants place 3 vowels.

Step 2: 5P5 AND 6P3

Step3: **5! * 6P3**

Q12. In a meeting there are 10 couples (10 men and 10 women). Each person shakes hand with every other except his or her own spouse. How many handshakes were exchanged?

Q1.Combination question. (Handshakes are combinations)
Q2. Restriction is that husbands must not shake hands with their wives.
Q3. People cannot be repeated. So n**C**r is applicable.

Total valid handshakes = Total possible handshakes - Invalid handshakes = 20C2 - 10 = **180**

Type: Distribution

Q13. In how many ways 10 identical chocolates can be distributed to 4 children?

a+b+c+d = 10 .. (1)

This can be done in 1**3C3** ways.

Q14. In how many ways 10 identical chocolates can be distributed to 4 children in such a way that a child gets at least one chocolate.

a+b+c+d = 10 .. (1)

Distributing one chocolate each for 4 children

a+b+c+d = 6 .. (2)

This can be done in **9C3** ways.

Type: Circular Arrangement

Q15. In how many ways can 21 differently colored beads be strung on a necklace?

A. 21! B. 20! C. 21!/2 D. 20!/2

Since clockwise and anti-clockwise arrangements of colored beads does not make difference (other can be got by inverting the necklace), number of arrangements will be **20!/2**

Q16. Twelve friends go out for a dinner to a restaurant where they find two circular tables, one with 7 chairs and the other with 5 chairs. In how many ways can the group settle down themselves for the dinner?

A. 12!/7!5! B. 12!/35 C. 12! D. 12!5!7!

Count(Arrange twelve friends) =

Count(Select 7 people to larger table AND select 5 people to smaller table AND arrange 7 people in larger table AND arrange 5 people in the smaller table)

= 12C7 AND 5C5 * (7-1)! * (5-1)!

= **12!/35**

Type: Applications

Q17. If all the letters of the word MASTER are taken and permuted and arranged in alphabetical order as in a dictionary, then what is the rank of the word MASTER?

Alphabetical Order:

A, E, M, R, S, T

Words of the form A __ __ __ __ __ (5!)

Words of the form E __ __ __ __ __ (5!)

Words of the form M A E __ __ __ (3!)

Words of the form M A R __ __ __ (3!)

Words of the form M A S E __ __ (2!)

Words of the form M A S R __ __ (2!)

come before word “Master”. So the rank of ‘master’ will be 120 + 120 + 6 + 6 + 2 + 2 + 1 = **257**

Q18. How many squares and rectangles are there in a 8 x 8 chess board?

Use the below formula.

Number of squares in a square of n x n side = $1^2+2^2+3^2+4^2+$ ..... n^2

Number of rectangles in a square of n x n side = $1^3+2^3+3^3+4^3+$ ..... n^3
(This includes the number of squares.)

Q19. How many squares and rectangles are there in a 5 x 7 grid?

Use the below formula.

Number of squares in a rectangle of m x n side = mn + (m−1)(n−1) + (m−2)(n−2) + ... until any of (m−x) or (n−x) comes to 1.

Number of rectangles in a rectangle of m x n side = (1+ 2+3+..... +m)(1+ 2+3+..... +n)

Q20. Find the sum of all numbers that can be formed by taking all the digits at a time from 4, 5, 6, 7, 8 without repetition. [Use Formula: 6]
A. 133320 B. 244420 C. 332244 D.7999920

If all the possible n–digit numbers using n distinct digits are formed, the sum of all the numbers so formed is equal to **Sum = (n−1)! * (sum of the n digits)*(11111 ..... n times)**

Sum = (5−1)![11111](4+5+6+7+8) = **7999920**

Q21. In how many ways an Ant can move from one corner of a chessboard to the opposite corner, if it is only allowed to move along edges of squares, without retracing steps? [Only allowed to move upward or forward]

However ant travels it has to cover 8 horizontal edges and 8 vertical edges.

No. of ways in which an ant can travel from one corner to another = No. of arrangements of (HHHHHHHHVVVVVVVV) = **16!/8!8!**

Question Bank

Q1. How many words of 4 consonants and 3 vowels can be made from 12 consonants and 4 vowels, if all the letters are different?
A. $^{16}C_7 * 7!$ B. $^{12}C_4 * {}^4C_3 * 7!$ C. $^{12}C_3 * {}^4C_4$ D. $^{12}C_4 * {}^4C_3$

Q2. Mr Smith and Mrs Smith have invited 9 of their friends and their spouses for a party. If Mr Smith never stands next to Mrs Smith, how many ways the group can be arranged in a row for a photograph?

Q1. Permutation Question(Arrangements are permutation questions)
Q2. Restriction is that count Mr. Smith can not sit with Mrs. Smith
Q3. People can't be repeated. So n**P**r is applicable.

Step 1: Keep Aside Mr. Smith AND Arrange 19 other people AND Place Mr. Smith

Step 2: 1 way AND 19! ways AND 18 [Around 19 people there will be 20 places. But on either side of Mrs. Smith, mr. Smith cannot go. So 18 places available now]

Step 3: **19! * 18**

Q3. How many ways last 3 digits of a mobile number can be formed in such a way that there is exactly one '7' in them and no digit is repeated?

Q1. Permutation Question(Numbers are permutation questions)
Q2. Restriction is that count only numbers with exactly one digit as '7'.
Q3. Digits can't be repeated. So n**P**r is applicable.

Step 1:

7_ _ OR
_7_ OR
_ _ 7

Step 2: Two __ can together be done in 9P2 ways.

Step 3: 9P2 + 9P2 + 9P2 = **216**

Q4. How many 8's are there n the numbers between 145-479?

Forms which are of interest to us:- (__, 8, 8), (__, __, 8) and (__, 8, __)

Consider:- (__, 8, 8) where __ can be filled with 1, 2, 3. So totally 3*2 = **6** 8s.

Consider:- (__, __, 8) which I will split as (1, __, 8) where 2nd digit can be filled in **5**(4, 5, 6, 7, 9) ways, (2, __, 8) where 2nd digit can be filled in **9**(0 to 9 except 8) ways, (3, __, 8) where 2nd digit can be filled in **9** (0 to 9 except 8) ways and (4, __, 8) where 2nd digit can be filled in **8**(0 to 7) ways.

Consider:- (__, 8, __) which I will split as (1, 8, __) where 3rd digit can be filled in **9**(0 to 9 except 8) ways, (2, 8, __) where 3rd digit can be filled in **9**(0 to 9 except 8) ways and (3, 8, __) where 3rd digit can be filled in **9**(0 to 9 except 8) ways.

Total = 6+31+27=**64**

Q5. How many 3 digit numbers are there in such a way that product of their digits is 48? [E Litmus]

Q1. Permutation Question(Numbers are permutation questions)
Q2. Restriction is that product of the digits must be 48.
Q3. Digits can be repeated.

Step 1:

8 6 1 [6 arrangements] OR

8 3 2 [6 arrangements] OR

4 6 2 [6 arrangements] OR

4 3 4 [3 arrangements]

Answer: 6 + 6 + 6 + 3 = **21**

Q6. In how many ways 9 people, two of whom are Chakkar and Ghanchakkar, can be arranged in such a way that 2 or fewer people in between Chakkar and Ghanchakkar? [E Litmus]

Case 1 : 0 people in between Chankkar and Ghanchakkar = Chakkar and Ghanchakkar always together =

= 8! * 2! [Refer to Q12]

Case 2: 1 person between Chakkar and Ghanchakkar

= Select one person to come between them(7C1 ways) AND Consider the 3 as one entity AND Arrange 7 entities AND for each arrangement you can exchange position of Chakkar and Ghanchakkar

= 7C1 * 1 * 7! * 2

Case 3: Two persons between Chakkar and Ghanchakkar

= Select two people to come in between Chakkar and Ghanchakkar AND consider the 4 people as one entity AND arrange 6 entities AND in each arrangement interchange Chakkar and Ghanchakkar and interchange middle two people.

= 7C2 * 6! * (2 * 2)

Answer = **8! * 2! + 7C1 * 1 * 7! * 2 + 7C2 * 6! * (2 * 2)**

Q7. If all the words that can be made from letters of "master" is sorted in alphabetical order, what will be the 49th word?

Alphabetical order is A, E, M, R, S, T

Words starting with A will be of the type: A,_,_,_,_,_ and will be 5! = 120 in numbers.

so 49th word will be one among the words starting in A.

Number of words starting with AE_,_,_,_ = 4! = 24
Number of words starting with AM_,_,_,_ = 4! = 24

So 49th word will be **AREMST**

Q8. All the words that can be made from letters of the word PARROT is permuted and arranged in dictionary order. What will be the 20th word?

Alphabetical order: A, O, P, R, R, T

Words starting with A = 5!/2! = 60 [Note that R is repeated twice in O, P, R, R, T]. So 20th word must also be starting from A.

1: Words starting with AO = 4!/2! = 12 [R is repeated twice in P, R, R, T]

Words starting with AP will have another 12 words which will take the count to 24. So 20th word must be starting with AP.

2: Words starting with APO: 3!/2! = 3 words.

Words starting with APR: 3! = 6 words will take the count to 21. So 20th word will be starting from APR.

3: Words starting with APRO: 2! = 2 words.

4: Words starting with APRR: 2! = 2 words.

Total of 19 words so far.

So 20th word will be **APRTOR**

Q9. In how many ways can 5 students and 5 teachers sit around a circular table so that no teachers sit together?
A. $(4!)^2$ B. $(5!)^2$ C. 4!5! D. 5! x^6P_5

Firstly 5 students can be arranged around the table in 4! ways. Between 5 students now there will be 5 places available where 5 teachers can be arranged in 5! ways.

[Note that teachers' position is defined w.r.t between who two students the teacher is seated. So shifting teachers by 1 or 2 positions will alter the arrangement. So arranging teachers between students is not circular but linear arrangement]

So arranging students AND teachers can be done in **4!.5!** ways.

Q10. A manufacturer manufactures 6 different flavors of chocolate. The chocolates are sold in boxes of 10. How many different boxes of chocolates can be made? (Note: A box is considered different from other only if it contains a different number of chocolates of at least one type, regardless of the order)
A. 10^6 B. 3003 C. 3000 D. 6^10

Taking one variable each for 6 different chocolates, equation is

a+b+c+d+e+f = 10 ... (1)

The above distribution can be done in 15C5 = **3003** ways.

Q11. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?
A. 32 B. 48 C. 96 D.64

Q1. Selection (Combination) question.
Q2. Restriction is that at least one black ball has to be drawn.
Q3. Repetition is not allowed.

Step 1: At least one black = (One black & 2 other colors) OR (two black & one other color) OR (3 black)

Step 2: **(3C1 * 6C2) + (3C2 *6C1) + (3C3)**

Q12.In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?
A. 2880 B.810 C. 1440 D. 5760 E. 50400

Solution:

Vowels are: O, O, A, I, O

Treating those 5 vowels as one entity and other consonants (C, R, P, R, T, N) as different entities we have totally 7 entities.

Step 1: Arrange 7 entities & Arrange vowels entity

Step 2: **(7!/2!) * (5!/3!)** [2 R's and 3 O's repeated]

Q13. a,b,c,d are positive integers. Find the no of possible solutions to the equation a+b+c+d=10.

Solution: Distribution problem with equation a + b + c + d = 6 [Each should be at least one for positive integer solution]

So **9C3**

Q14. How many ways can 4 prizes be given away to 3 boys, if each boy is eligible for all prizes?
A. 256 B. 12 C. 81 D. None of these

Solution:

Q1: Permutation
Q2: No restriction.
Q3: Multiple prizes can be given to same boy. So no nPr.

Step 1: Give 1st prize AND Give 2nd prize AND Give 3rd prize AND Give 4th prize

Step 2: (3) * (3) * (3) * (3) = **81**

Q15. Determine the number of ways to distribute 10 (indistinguishable) orange drinks, 1 lemon drink, and 1 lime drink to four thirsty students such so that each student gets at least one drink, and the lemon and lime drinks go to different students.

Solution:

Task 1: Give out lemon drink in 4 ways (any 4 children)

Task 2: Give out lime drink in 3 ways (any of the remaining 3)

Task 3: Give one orange drink each for the remaining 2 children in 1 way (orange drinks are identical)

Task 4: Distribute the remaining 8 orange drinks in all possible ways (a + b + c + d = 8) which can be done in 11C3 ways

Answer = **4*3*1*11C3 = 1980**

Q16. Find number of triangles that can be formed from 24 points in which 7 are collinear.

Solution:

Q1. Combination (any order of points makes same triangle)
Q2. Certain restrictions given.
Q3. Points cannot be repeated. So nCr is applicable.

Solution: **24C3 - 7C3** [If all of 3 chosen points came from collinear points they don't make triangle]

Q17. How many quadrilaterals can be formed from 25 points of which 7 are collinear?

Solution:

Q1. Combination (any order of points makes same triangle)
Q2. Certain restrictions given.
Q3. Points cannot be repeated. So nCr is applicable.

In case all 25 points were non collinear you could have made 25C4 quadrilaterals. But because 7 points are collinear 7C4 of them are not valid. But there are also some cases in 25C4 where 3 of the chosen points were from those 7 collinear and one from non collinear. They would have formed triangles. So we have to remove 7C3*18C1

Answer = **25C4-7C4- 7C3*18C1**

Q18. In how many rearrangements of the word ERASED in the letter 'A' positioned in between the 2 'E's?

Q1. Permutation Question(Words are permutation questions)
Q2. Restriction.
Q3. Letters cannot be repeated. (Else you can be infinitely long words)

Case 1. (EAE)RSD: 4!

OR

Case 2. 3C1 * 3! * 2! [E A _ E] [2 other letters]

OR

Case 3. 3C2 * 2! * 3! [E A _ _ E] [One other letter]

OR

Case 4. E in the extremes: 3!

Answer = **4! + 3C1*3!2! + 3C2*2!3! + 3!**