1. Consider a Gaussian mixture model in 2-dimensional feature space.
   $g(x) = \sum_{k=1}^{2} \pi_k \, g_k(x)$,

   where $g_k(x) = N(\mu_k \, \mathrm{I}.\sigma^2)$ and $\pi_k \geq 0 \forall k$ with $\pi_1 + \pi_2 = 1$. Here $\{\mu_k, \pi_k\}, k = 1,2$ and σ^2 are unknown parameters.

   Suppose we have a sample $x_1, x_2, \ldots, x_N \sim g(x)$ and we want to fit the mixture model to this dataset. Write down the likelihood of the data. Further, develop a computational algorithm for computing the maximum likelihood estimates. Clearly show each step involved in the computing the parameters. 6

2. Following approach was taken to generate samples from the mixture distribution $g(x)$. Set the initial seed value to 1 for this exercise. Generate 50 samples (cluster A) from the bivariate normal distribution with $\mu_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\sigma^2 = 0.5$ and 50 samples (cluster B) from the bivariate normal distribution with $\mu_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\sigma^2 = 0.5$. These 100 samples collectively mimic the mixture distribution, $g(x)$, for $\pi_1 = \pi_2 = \frac{1}{2}$. Generate scatter plot for these samples, use different colors for observations from each of the clusters (A and B). Also, apply K-means clustering (K=2) to the above 100 samples and use different colors to highlight the observations assigned into different clusters using K-means. How many observations have been given wrong cluster labels? 4