4. (a). 矩阵形式的 var(2) 模型

定义 $y_t = \begin{pmatrix} a_t \\ b_t \end{pmatrix} = \begin{pmatrix} 0.5 & 0.7 \\ -0.2 & 0.4 \end{pmatrix} \begin{pmatrix} a_{t-1} \\ b_{t-1} \end{pmatrix} + \begin{pmatrix} 0.3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a_{t-2} \\ b_{t-2} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$

故 var(2) 模型的矩阵形式为 $y_t = \Gamma_1 y_{t-1} + \Gamma_2 y_{t-2} + \varepsilon_t$.

其中 $\Gamma_1 = \begin{pmatrix} 0.5 & 0.7 \\ -0.2 & 0.4 \end{pmatrix}$ $\Gamma_2 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0 \end{pmatrix}$ $\varepsilon_t = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$

(b). b_t 的增加 1 个单位的冲击对 a_{t+2} 的影响

$y_{t+2} = \Gamma_1 y_{t+1} + \Gamma_2 y_t + \varepsilon_{t+2}$ 又 $y_{t+1} = \Gamma_1 y_t + \Gamma_2 y_{t-1} + \varepsilon_{t+1}$

$= \Gamma_1 (\Gamma_1 y_t + \Gamma_2 y_{t-1} + \varepsilon_{t+1}) + \Gamma_2 y_t + \varepsilon_{t+2}$

$= \Gamma_1^2 y_t + \Gamma_1 \Gamma_2 y_{t-1} + \Gamma_1 \varepsilon_{t+1} + \Gamma_2 y_t + \varepsilon_{t+2}$

$= (\Gamma_1^2 + \Gamma_2) y_t + \Gamma_1 \Gamma_2 y_{t-1} + \Gamma_1 \varepsilon_{t+1} + \varepsilon_{t+2}$

我们只关心 b_t 增加一个单位对 a_{t+2} 的影响，因此设定 $y_t = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$y_{t+2} = (\Gamma_1^2 + \Gamma_2) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\Gamma_1^2 = \begin{pmatrix} 0.5 & 0.7 \\ -0.2 & 0.4 \end{pmatrix} \begin{pmatrix} 0.5 & 0.7 \\ -0.2 & 0.4 \end{pmatrix} = \begin{pmatrix} 0.11 & 0.63 \\ -0.18 & 0.02 \end{pmatrix}$

$\Gamma_1^2 + \Gamma_2 = \begin{pmatrix} 0.41 & 0.63 \\ -0.18 & 0.02 \end{pmatrix}$

$(\Gamma_1^2 + \Gamma_2) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.63 \\ 0.02 \end{pmatrix}$ 故增加 0.63 个单位.