Mat 320 - Stout Name:______________________

QUIZ 9: ABSTRACT ALGEBRA

A *group homomorphism* is a set map f from a group $(G, \cdot_G)$ to group a group $(H, \cdot_H)$ such that it satisfies Property 1 below

$$\underline{\text{Property 1}} := [f(x \cdot_G y) = f(x) \cdot_H f(y) \text{ for all } x, y \text{ in } G]$$

Problem 1: Let $f : G \to H$ be a group homomorphism. Define the kernel of f to be

$$\ker(f) := \{x \in G \mid f(x) = 1_H\}$$

where 1_H is the identity element in H. Show that $\ker(f)$ is a normal subgroup of G. For this you must show that it is a subgroup and then prove that it is normal.

Problem 2: Let $f : G \to H$ be a *group isomorphism* (i.e., a bijective group homomorphism). Prove that the inverse set map $f^{-1} : H \to G$ which is defined by $f^{-1}(h) = g \iff f(g) = h$ also satisfies Property 1 above.