Group Representation

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1 Atiyah-Segal Completion Theorem

Theorem 1.1 (Atiyah-Segal Completion Theorem)**.** Let G be a compact Lie group and X be a compact G-space. Consider the projection map $EG \times X \to X$, which induces a natural homomorphism $\alpha : K_G(X) \to K(EG \times_G X)$ by regarding a G-bundle over X as a bundle over the homotopy quotient X_G.

The map α induces an isomorphism between the completion of the equivariant K-theory with respect to the augmentation ideal $I(G)$ and the ordinary K-theory of the homotopy quotient:

$$\alpha : \widehat{K_G(X)}_{I(G)} \xrightarrow{\cong} K(EG \times_G X). \tag{1}$$

In particular, for $X = \mathrm{pt}$, we have an isomorphism $\widehat{R(G)}_{I(G)} \cong K(BG)$.