

14–18 October 2019

## Plan

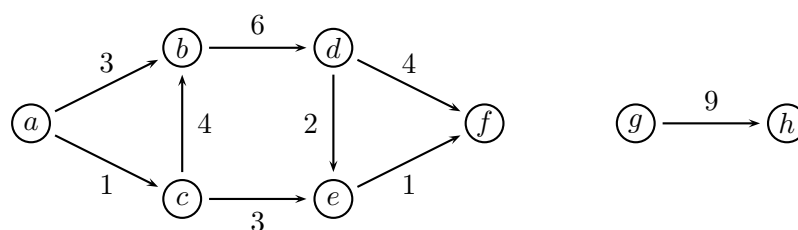
The exam is not far away, so keep up with tutorials; as always, try tackling the problems *before* the tute.

## The exercises

74. Use the dynamic-programming algorithm developed in Lecture 18 to solve this instance of the coin-row problem: 20, 50, 20, 5, 10, 20, 5.
75. In Week 12 we will meet the concept of *problem reduction*. This question prepares you for that. First, when we talk about the length of a path in an un-weighted directed acyclic graph (DAG), we mean the number of edges in the path. (You could also consider the un-weighted graph weighted, with all edges having weight 1)

Show how to reduce the coin-row problem to the problem of finding a longest path in a DAG. That is, give an algorithm that transforms any coin-row instance into a longest-path-in-DAG instance in such a way that a solution to the latter provides a solution to the former. Hint: If there are  $n$  coins, use  $n + 1$  nodes; let an edge with weight  $i$  correspond to picking a coin with value  $i$ .

76. Consider the problem of finding the length of a “longest” path in a *weighted*, not necessarily connected, DAG. We assume that all weights are positive, and that a “longest” path is a path whose edge weights add up to the maximal possible value. For example, for the following graph, the longest path is of length 15:



Use a dynamic programming approach to the problem of finding longest path in a weighted dag.

77. Design a dynamic programming algorithm for the version of the knapsack problem in which there are unlimited numbers of copies of each item. That is, we are given items  $I_1, \dots, I_n$  that have values  $v_1, \dots, v_n$  and weights  $w_1, \dots, w_n$  as usual, but each item  $I_i$  can be selected several times. Hint: This actually makes the knapsack problem a bit easier, as there is only one parameter (namely the remaining capacity  $w$ ) in the recurrence relation.
78. Work through Warshall’s algorithm to find the transitive closure of the binary relation given by this table (or directed graph):

	$a$	$b$	$c$	$d$
$a$	0	0	1	1
$b$	0	0	1	0
$c$	1	0	0	0
$d$	0	0	0	0

