THE UNIVERSITY OF
MELBOURNE

COMP90038
Algorithms and Complexity

Lecture 2: Review of Basic Concepts
(with thanks to Harald Søndergaard)

Toby Murray

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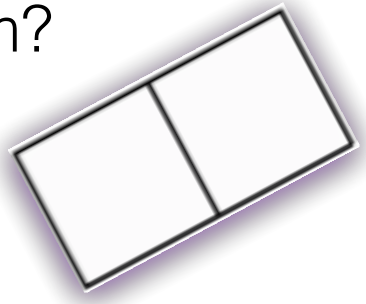
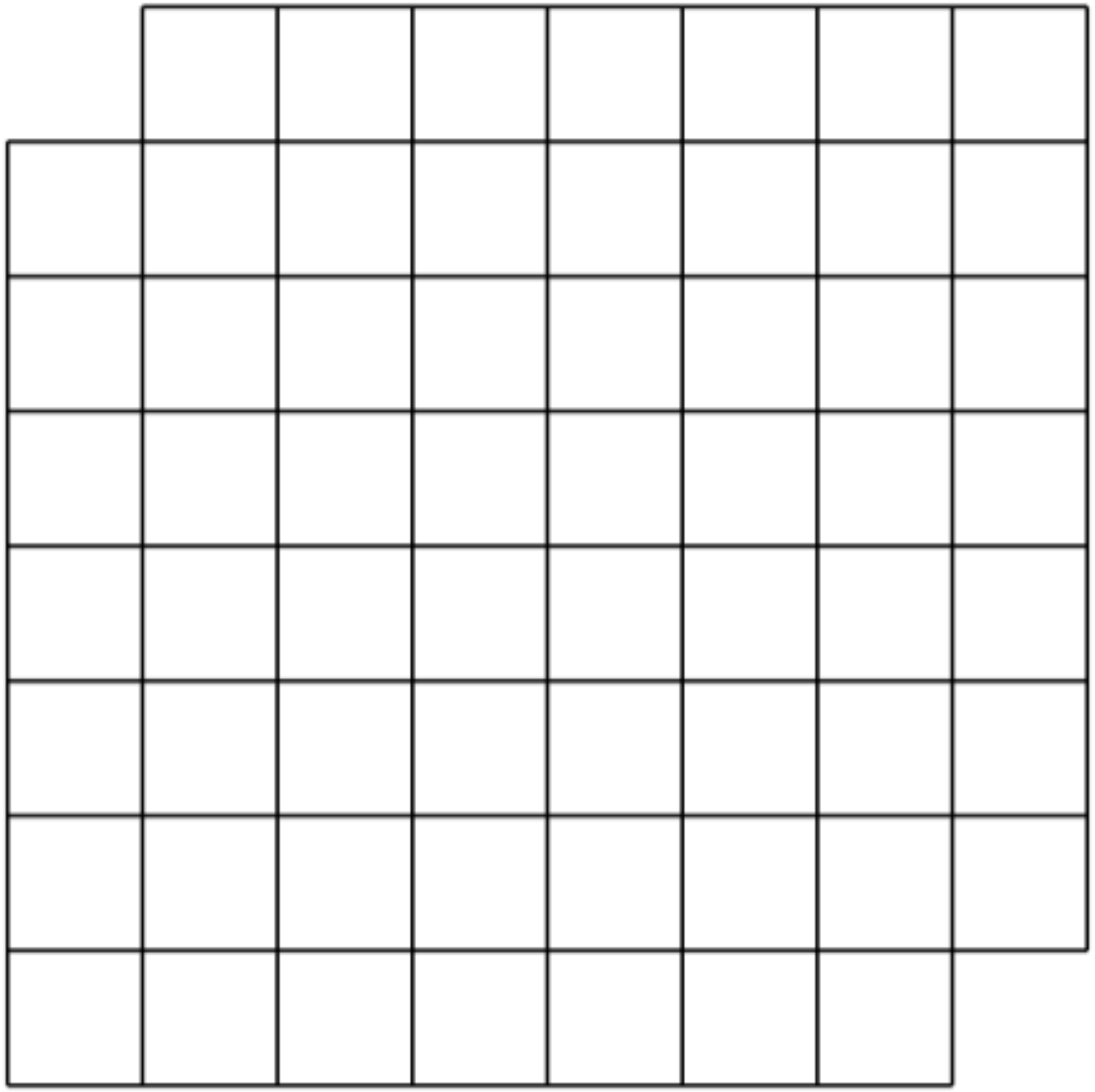
🐦 @tobycmurray

Approaching a problem

- Can we cover this board with 31 tiles of the following form?
- This is the **mutilated checkerboard problem**.
- There are only finitely many ways we can arrange the 31 tiles, so there is a brute-force (and very inefficient) way of solving the problem.





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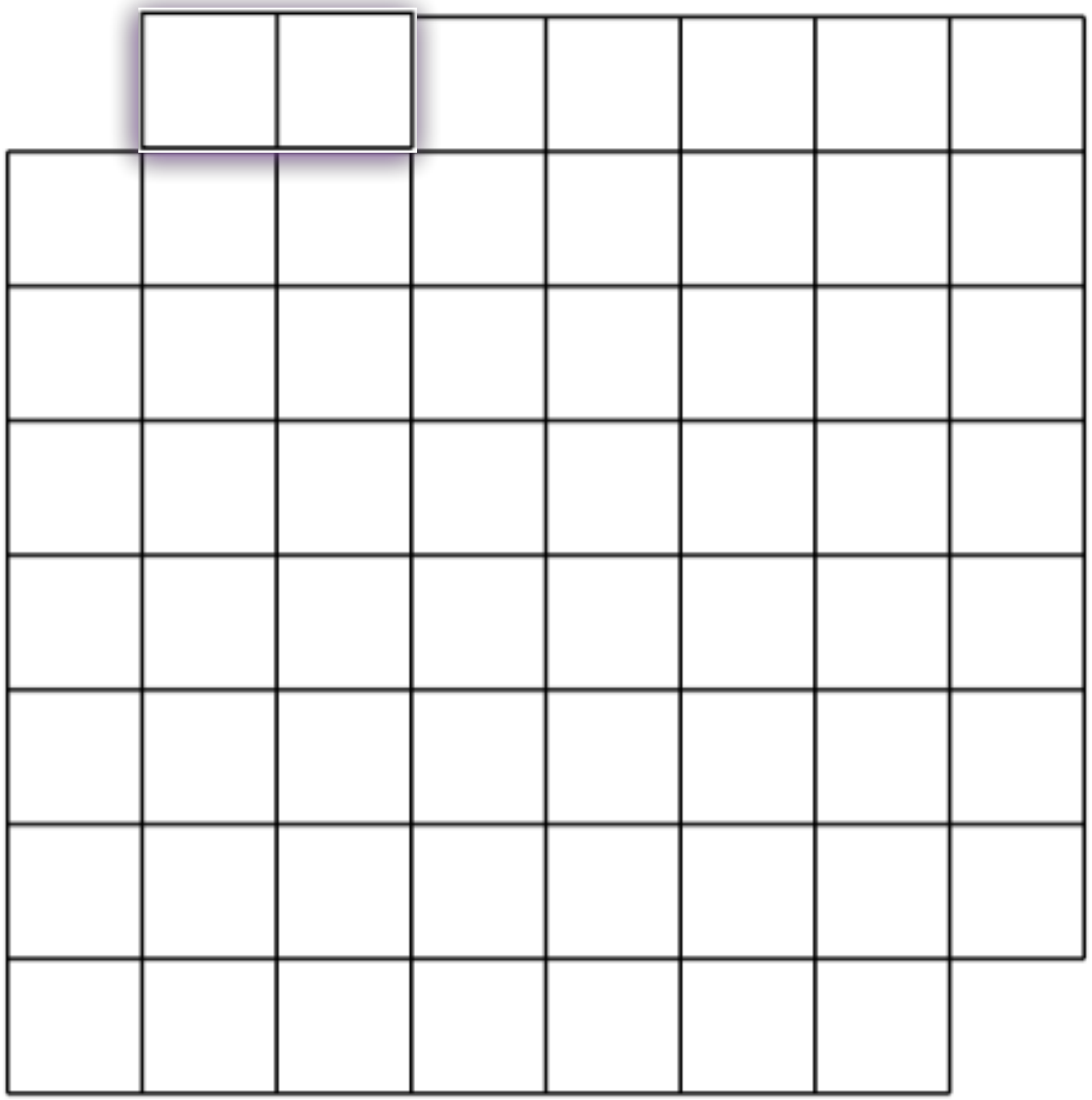

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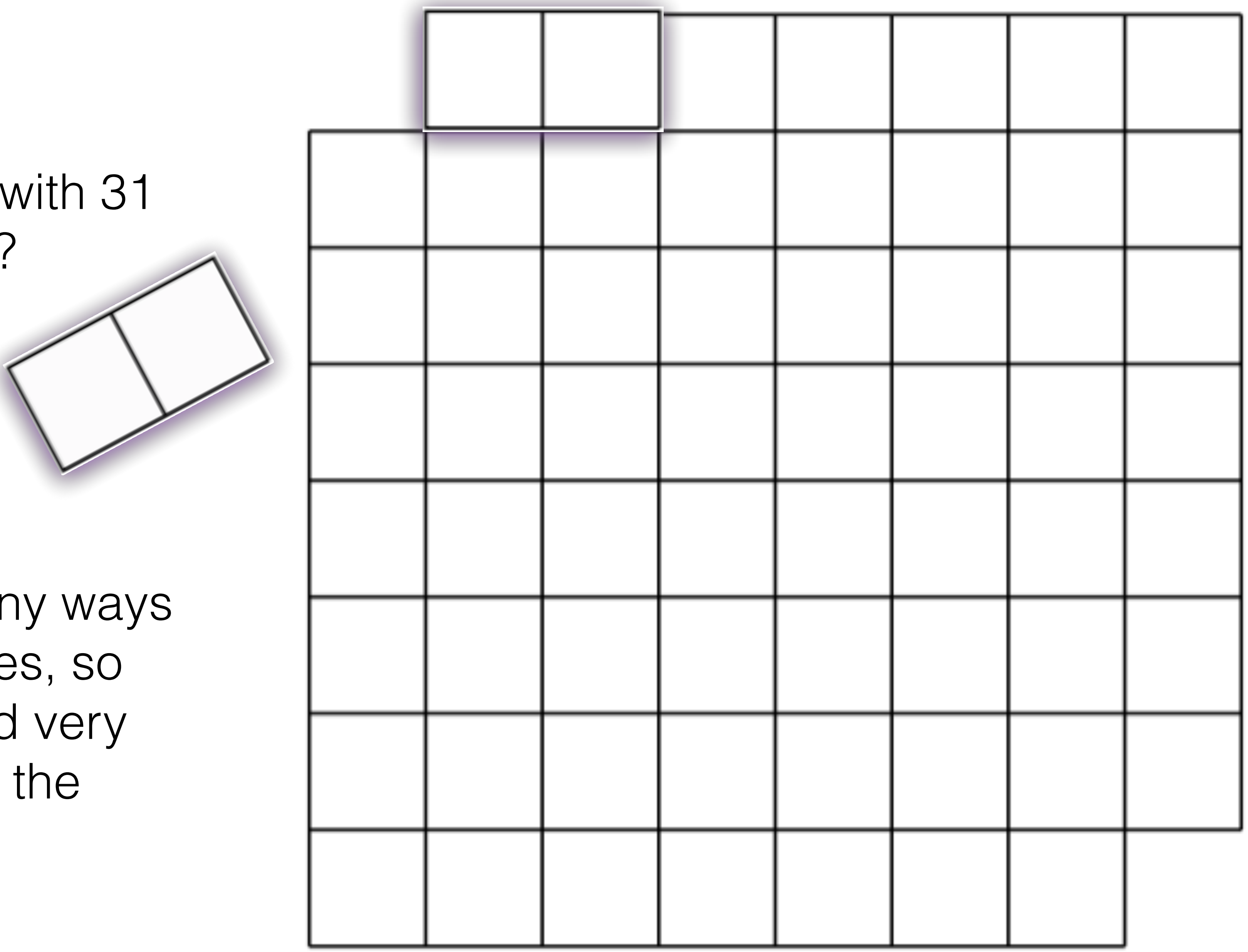

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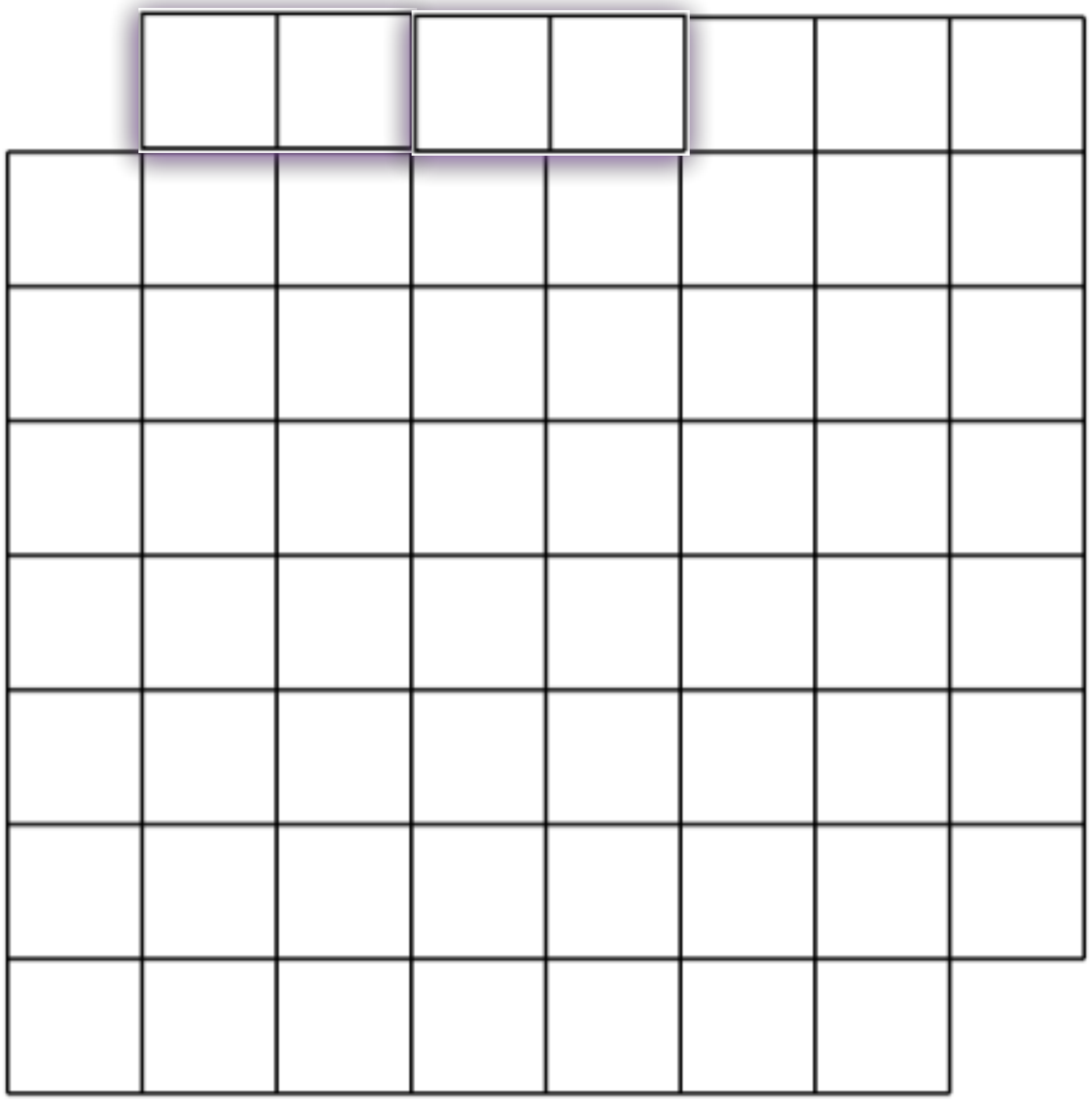

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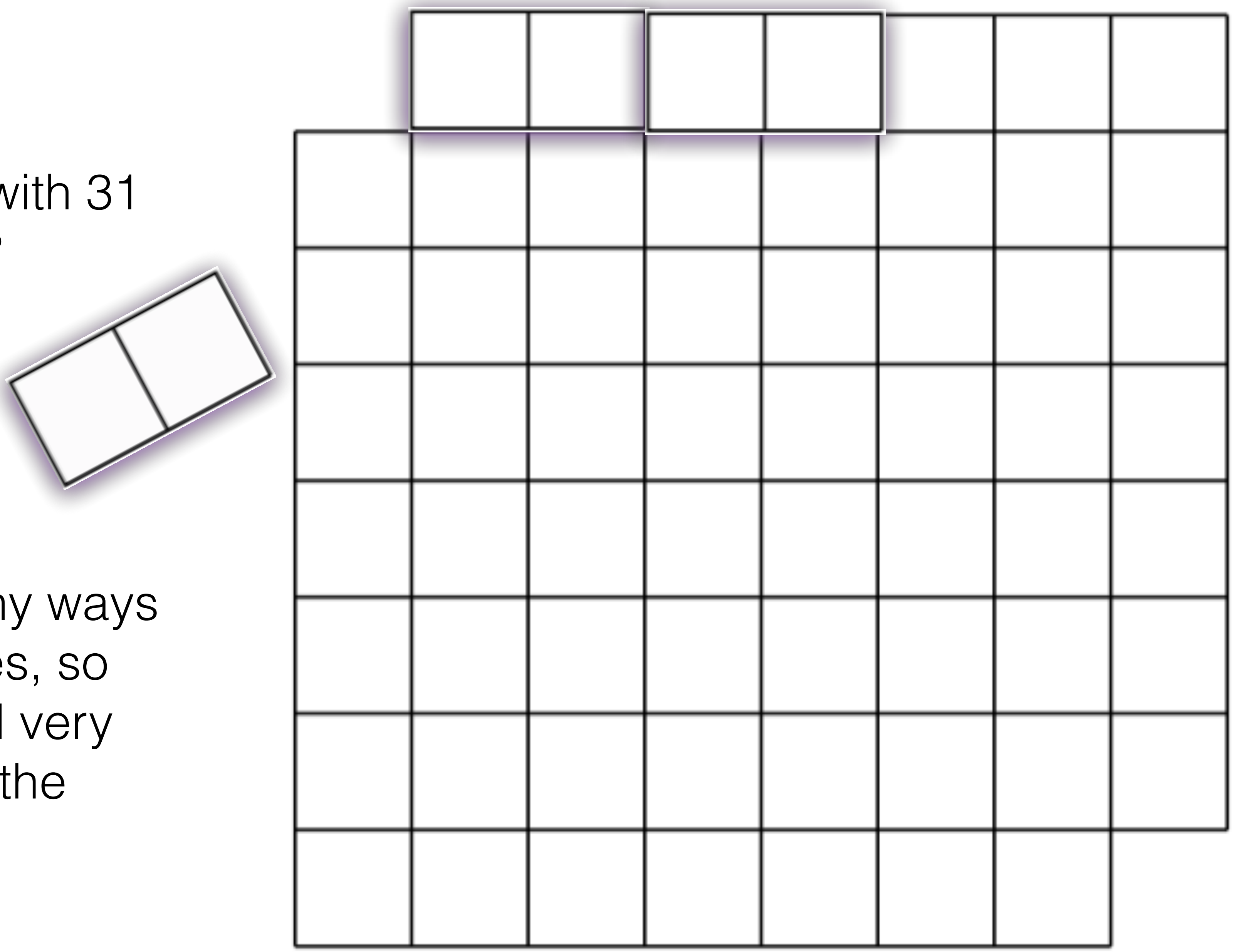

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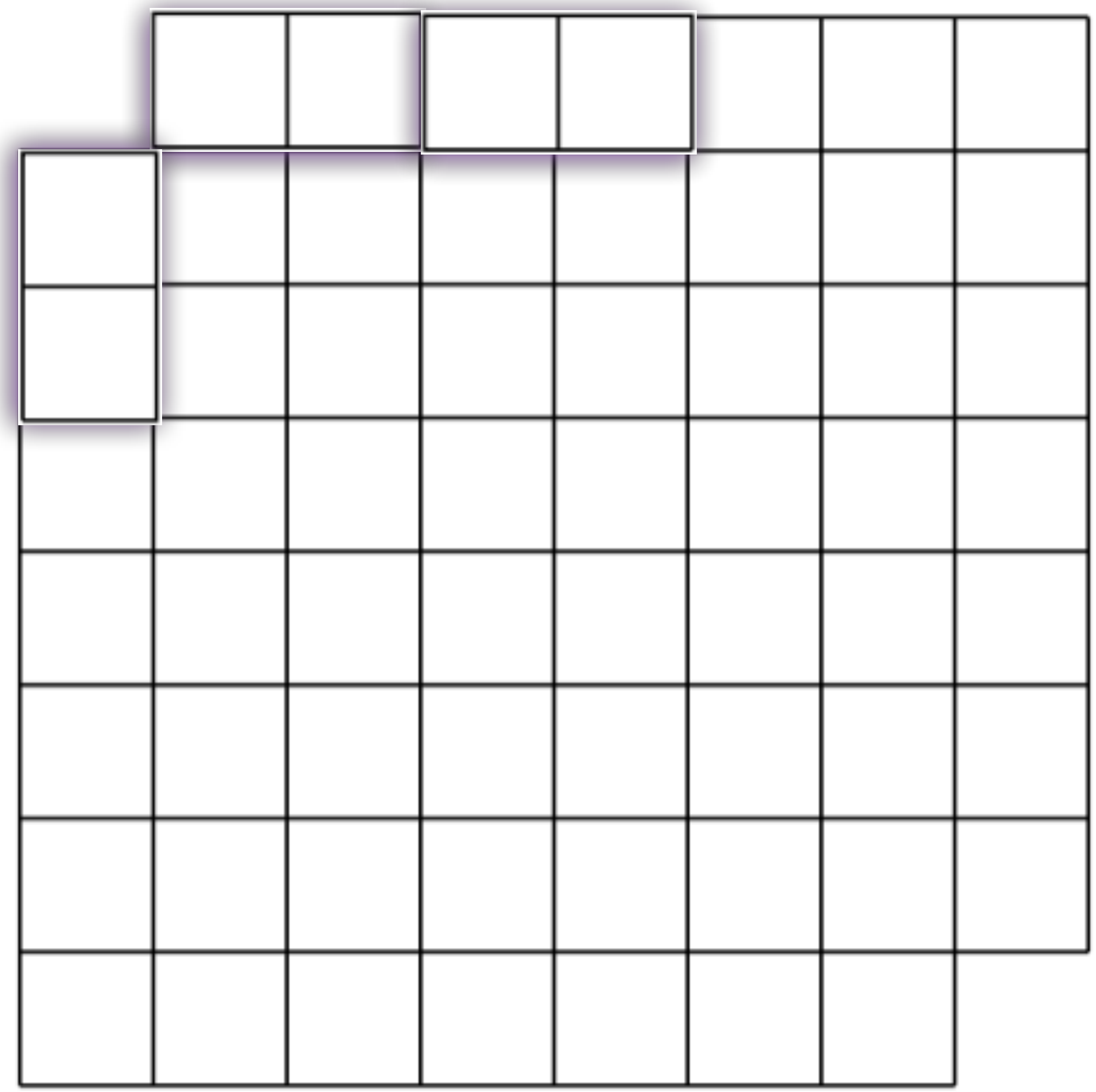

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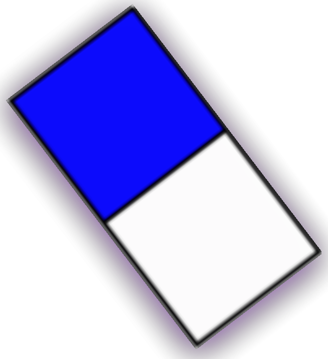
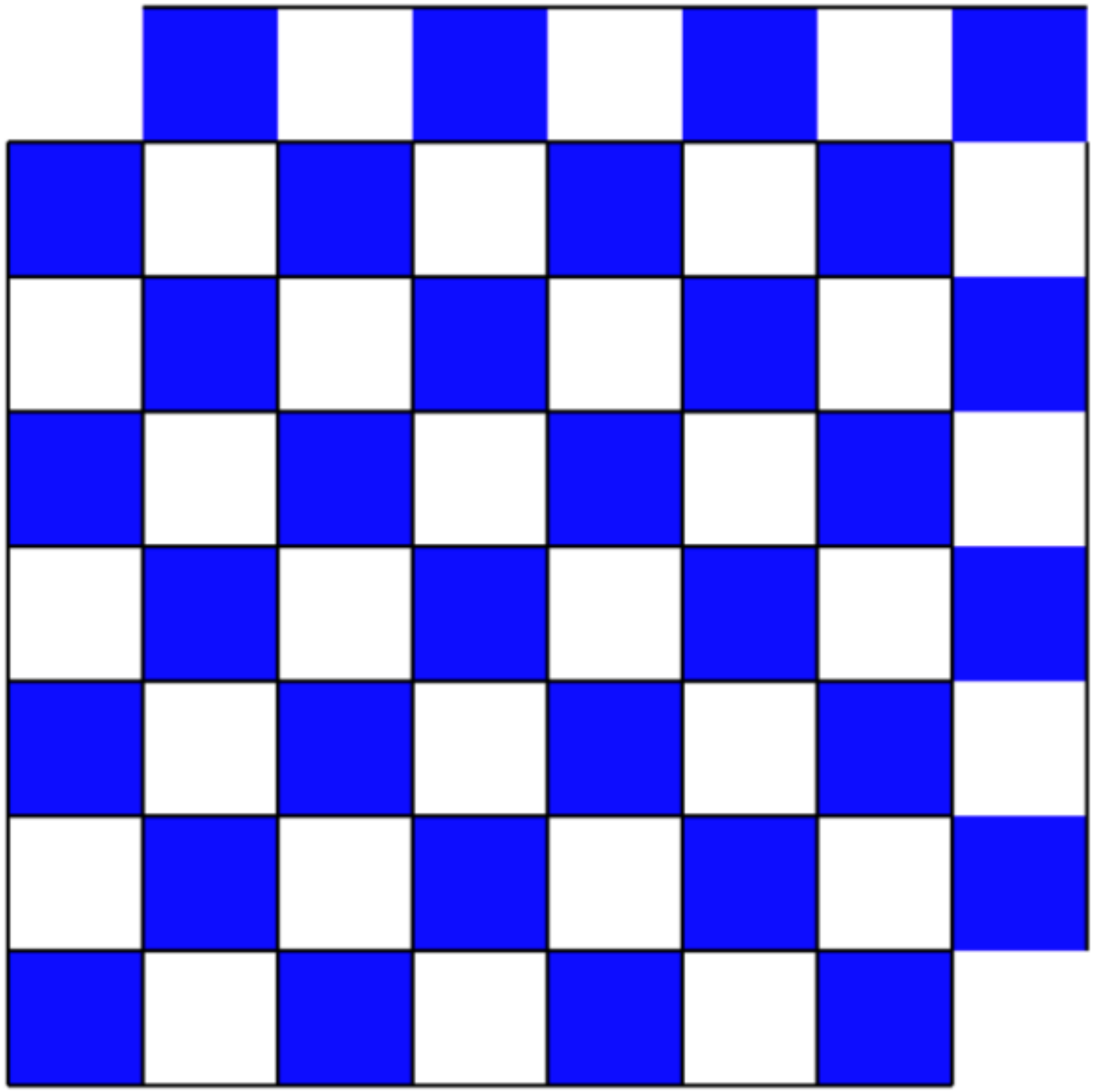


- Can we cover this board with 31 tiles of the form shown?
- Why can we quickly determine that the answer is no?
- **Hint:** Using the way the squares are coloured helps.



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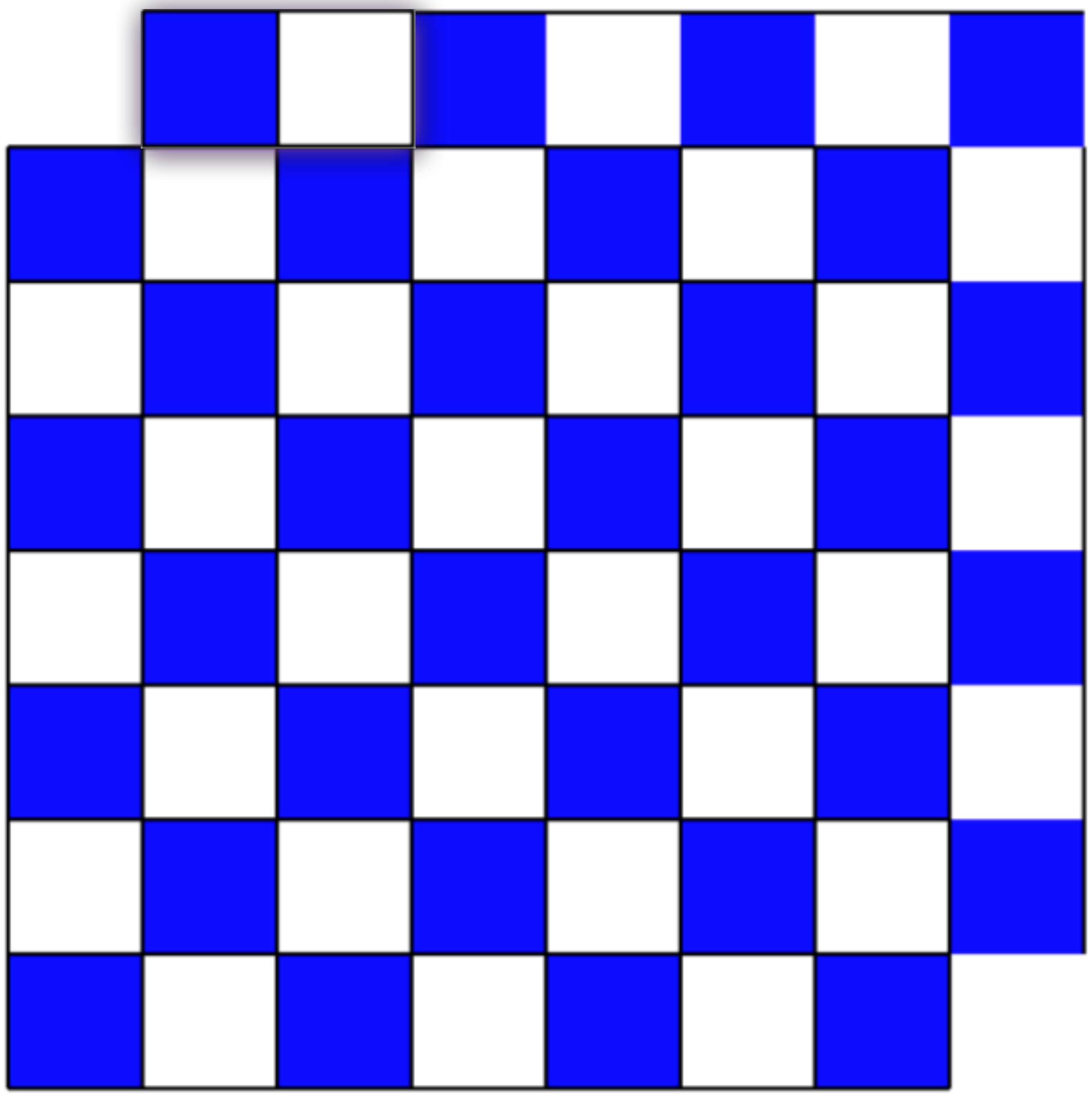

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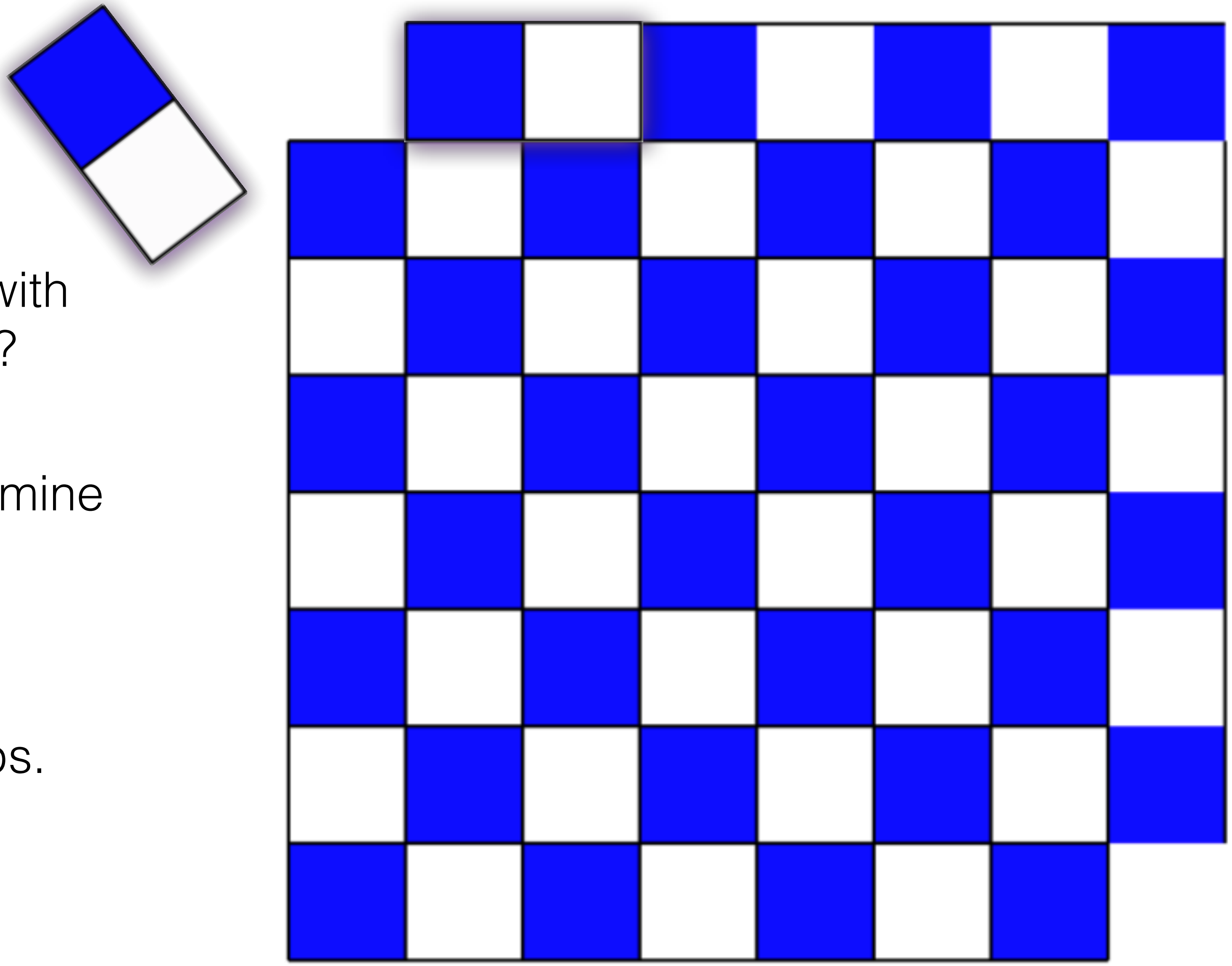

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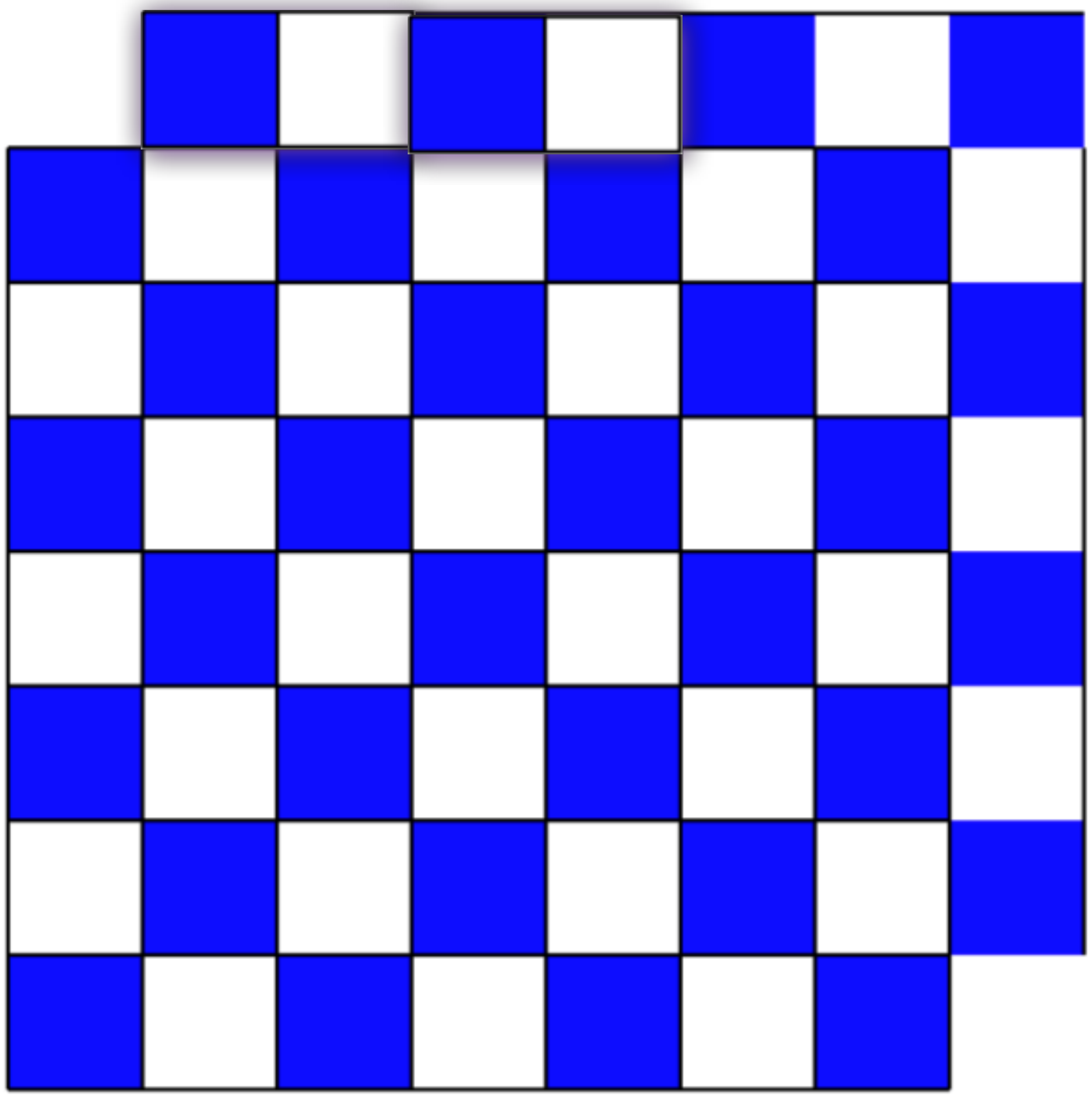

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Algorithms and Data Structures

- **Algorithms**: for solving problems, transforming data.
- **Data structures**: for storing data; arranging data in a way that suits an algorithm.
    - **Linear** data structures: stacks and queues
    - Trees and graphs
    - Dictionaries
- Which data structures are you familiar with?

Exercise

- Pick you favourite data structure and describe:
    - How to insert and item into the data structure
    - How to find an item
    - How to handle duplicate items

Primitive Data Structures: The Array

- An array corresponds to a sequence of consecutive cells in memory.
- Depending on programming language: `A[0]` up to `A[n-1]`, or `A[1]` up to `A[n]`.
- Locating a cell, and storing or retrieving data at that cell is very fast.
- The downside of an array is that maintaining a contiguous bank of cells with information can be difficult and time-consuming.

6	9	2	3	7	5	8
0	1	2	3	4	5	6

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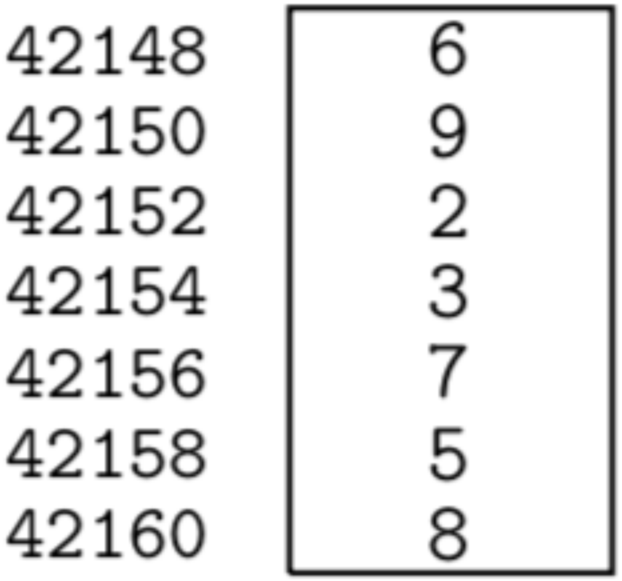

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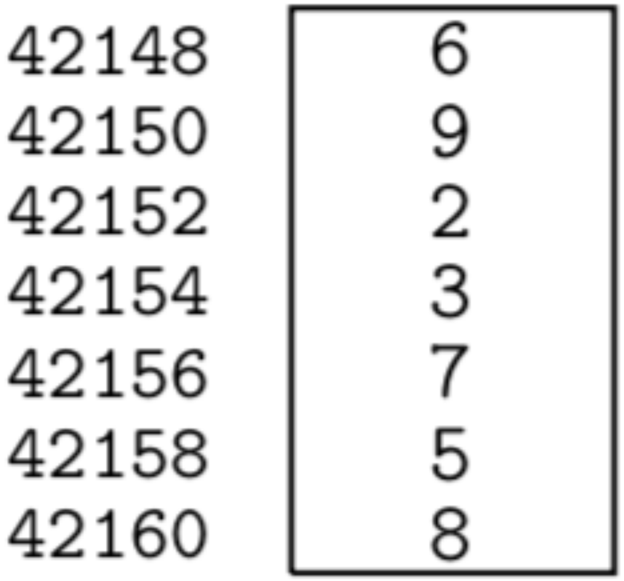
How many bytes does each integer occupy here?



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How many bytes does each integer occupy here?

Answer: 2 (16-bit integers)

Primitive Data Structures: The Linked List

An array **`x`**:

2	3	5	7

Primitive Data Structures: The Linked List

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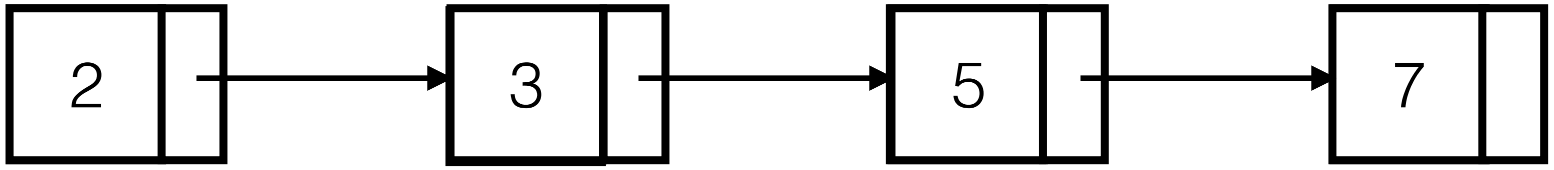

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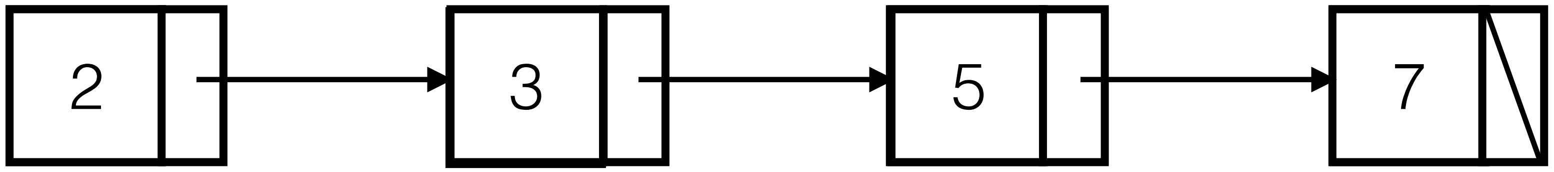

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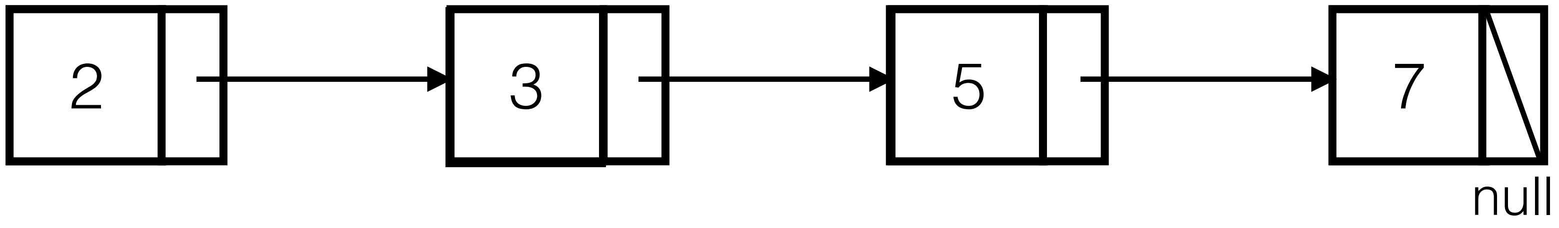

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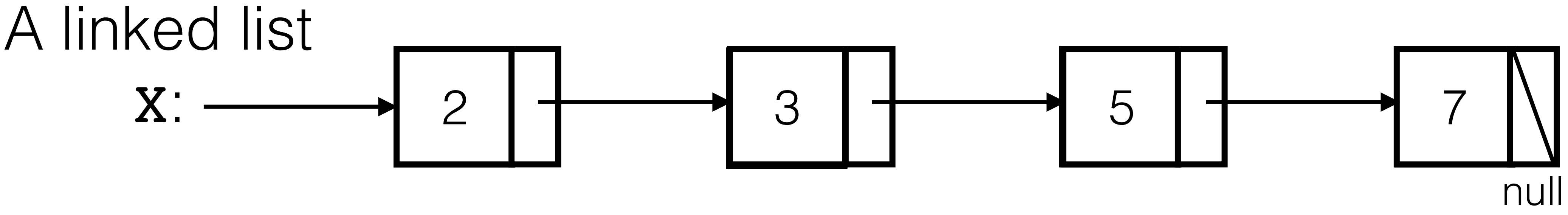

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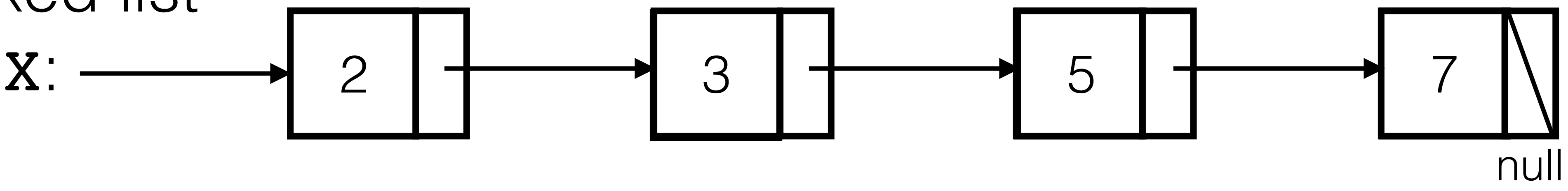

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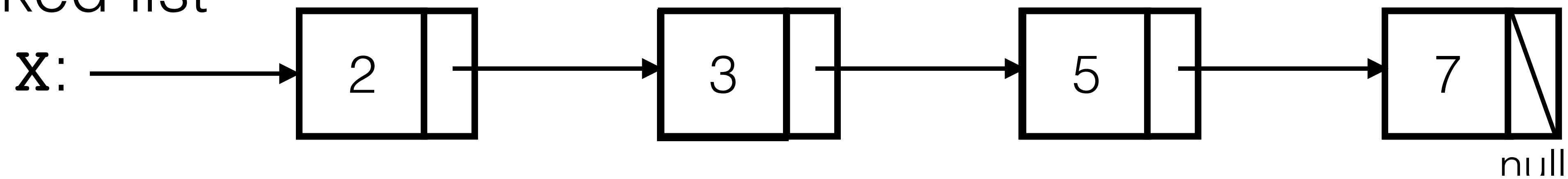

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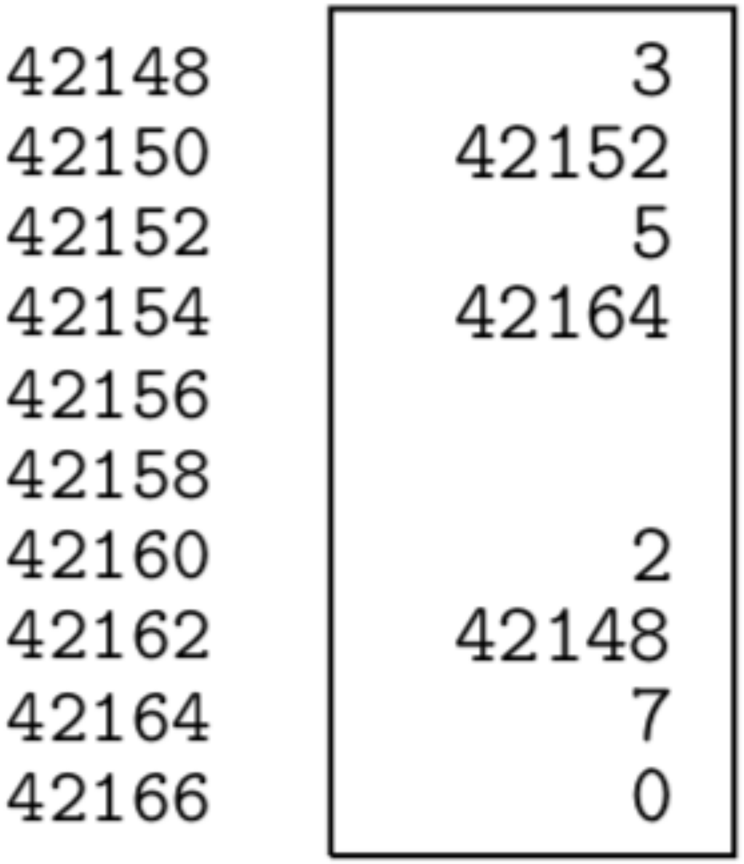

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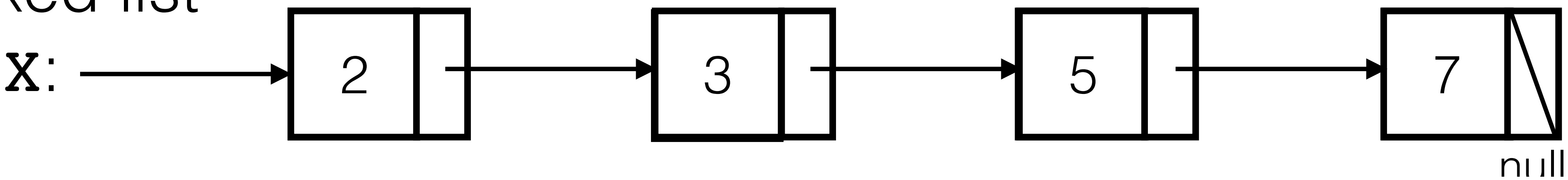

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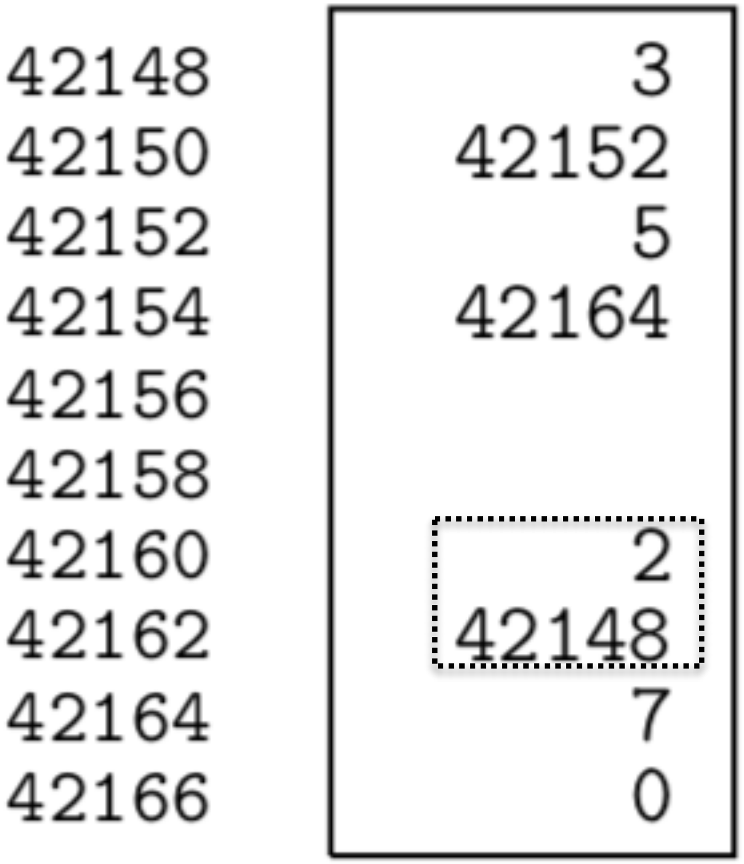

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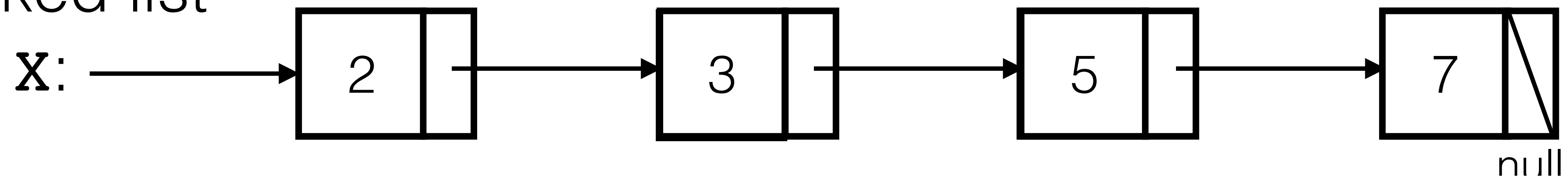

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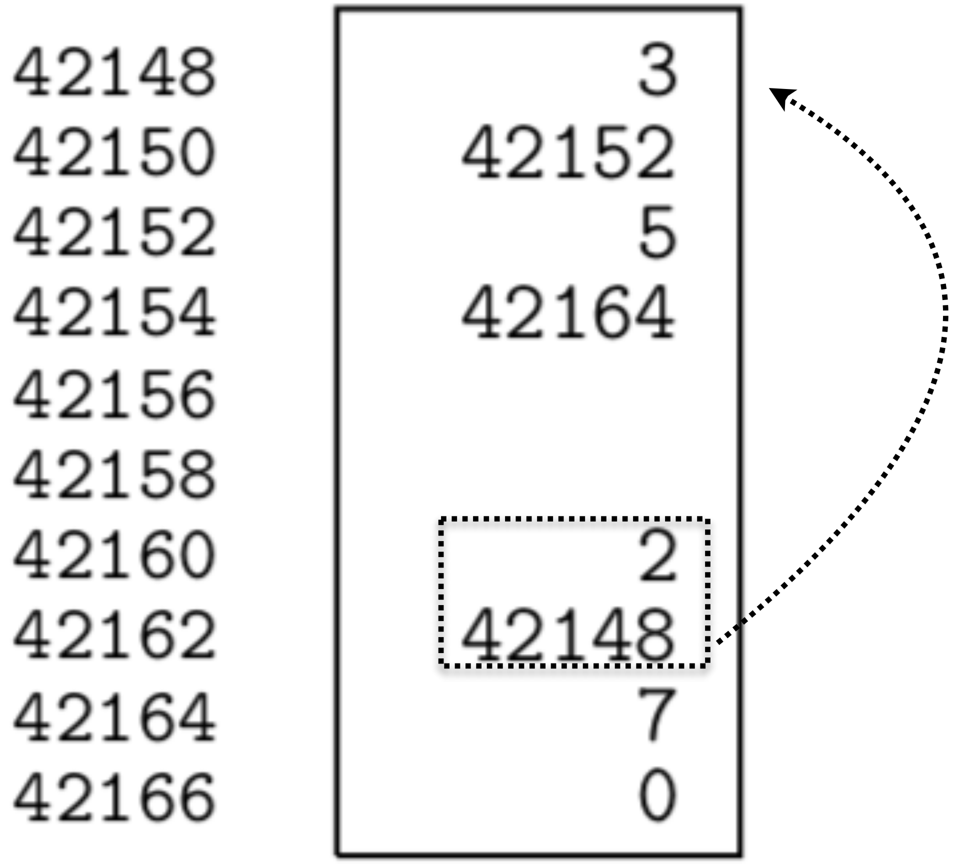

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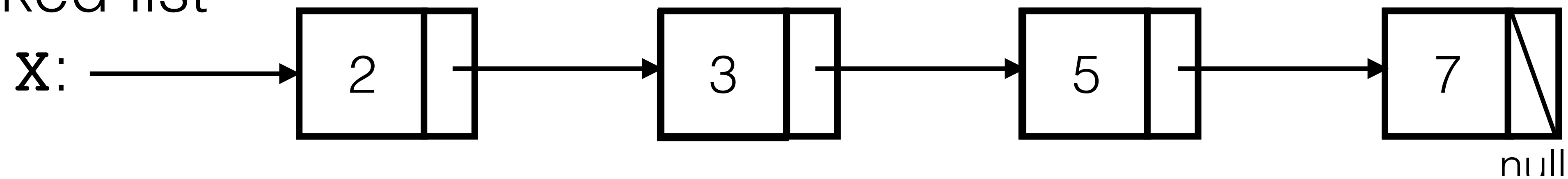

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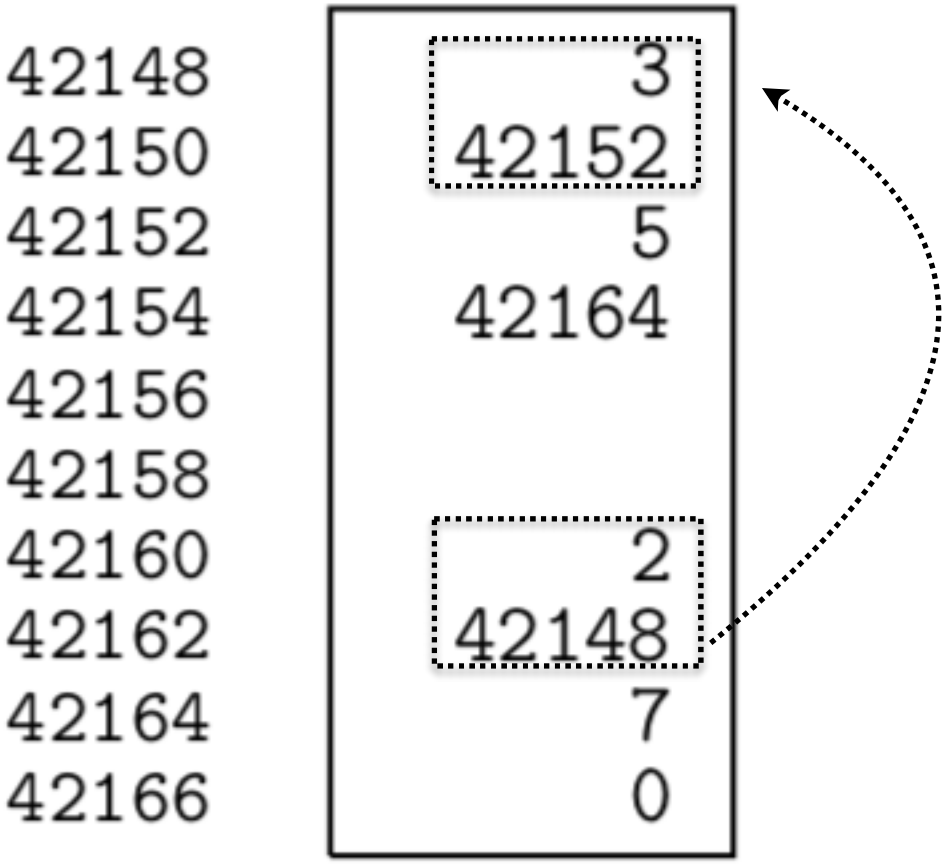

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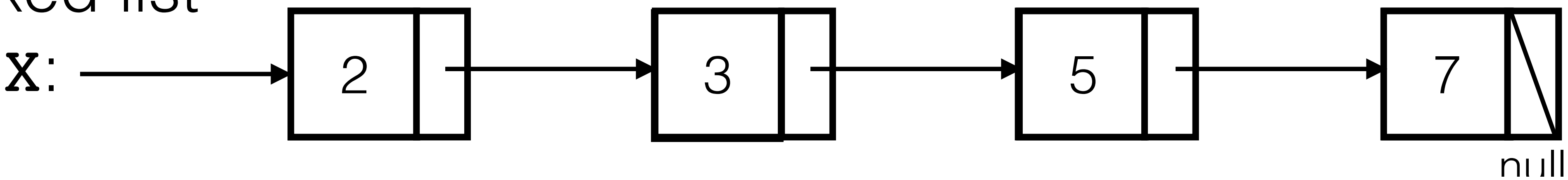

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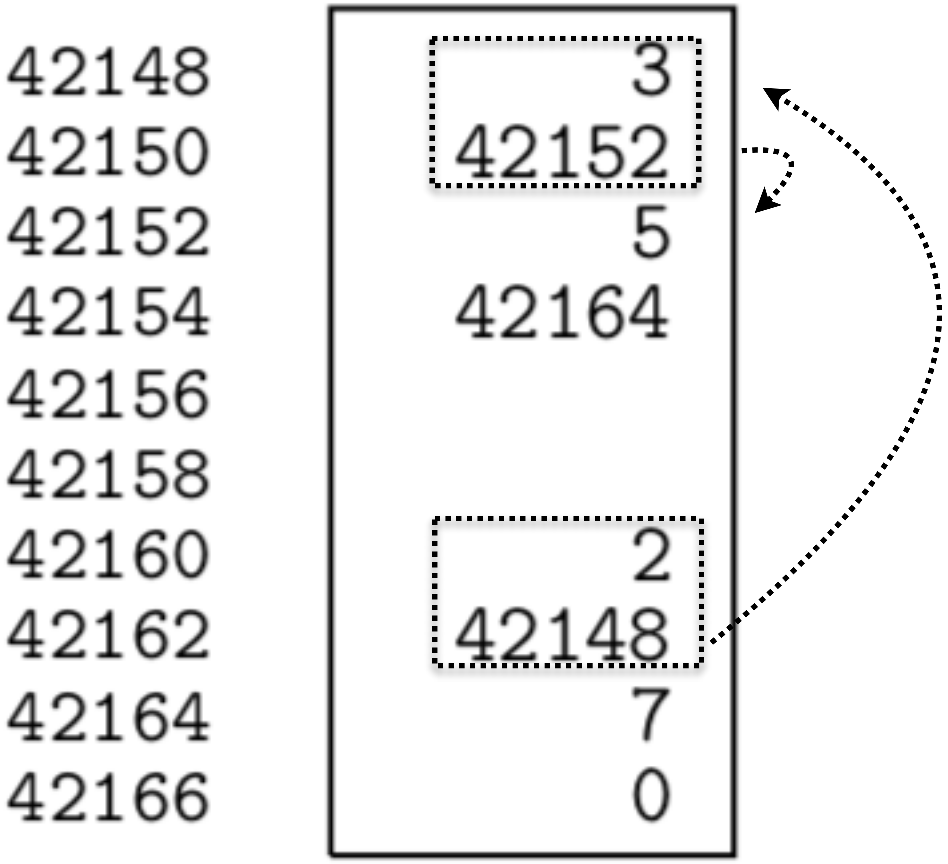

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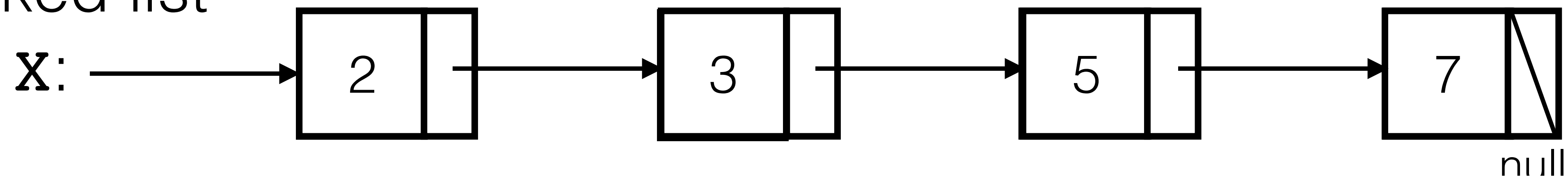

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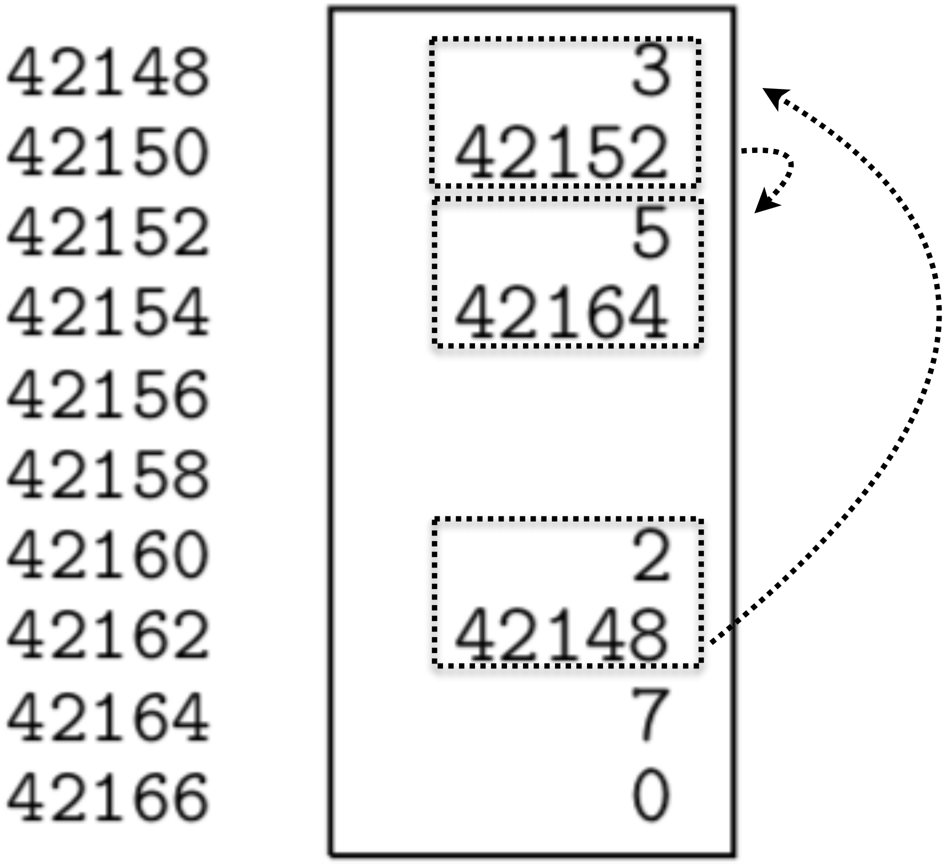

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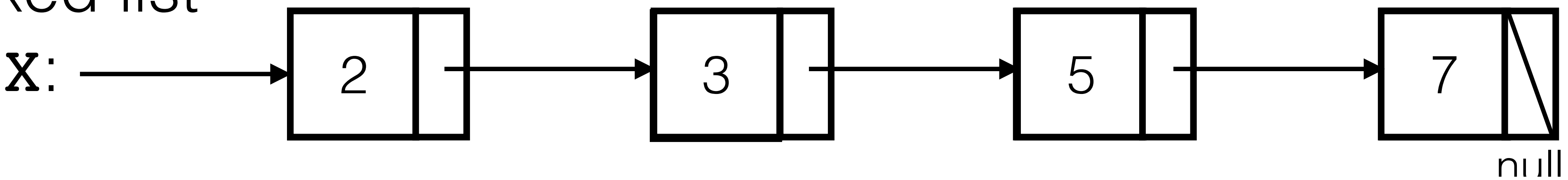

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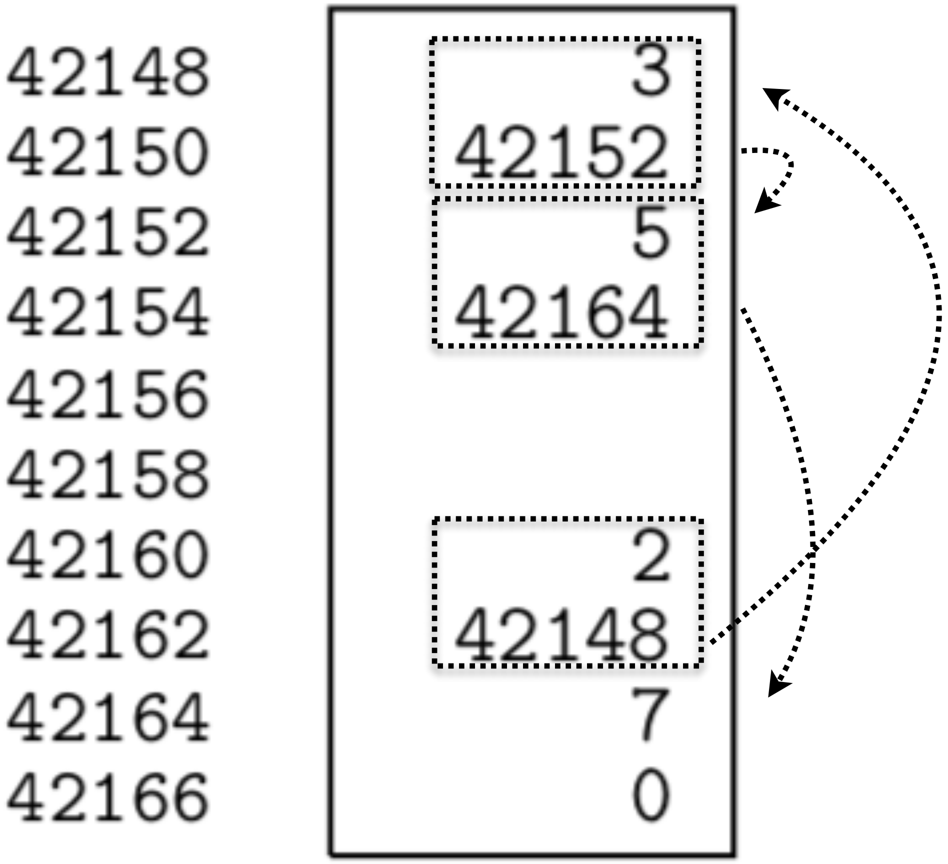

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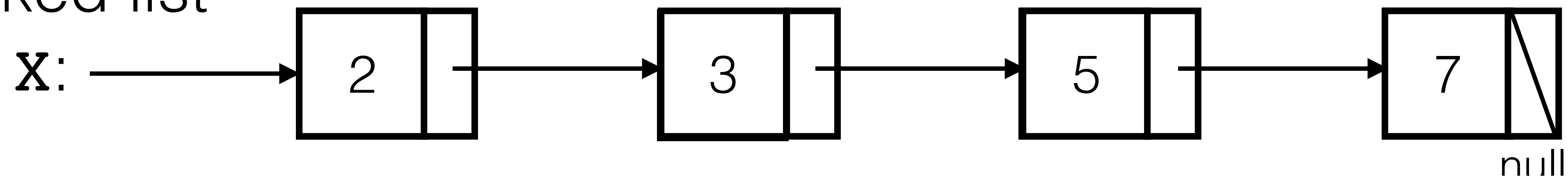

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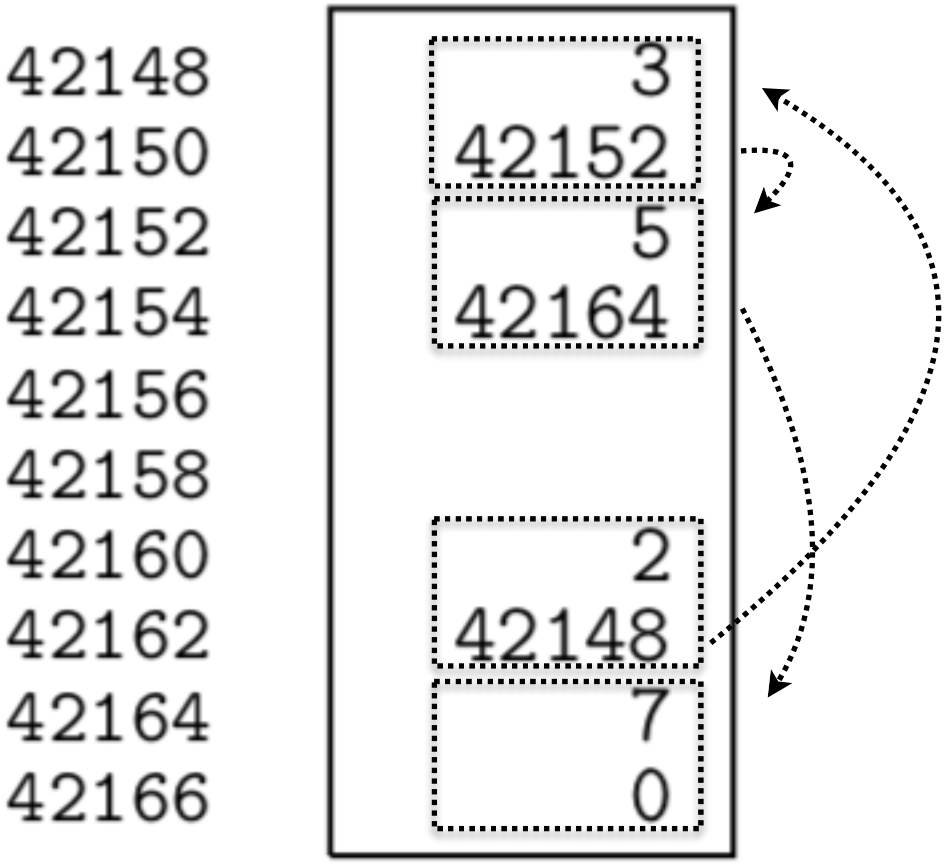

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Terminology

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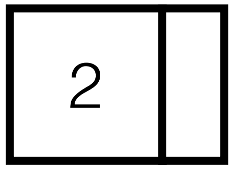
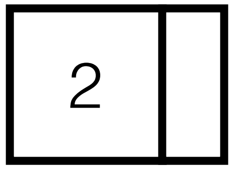


Terminology

node



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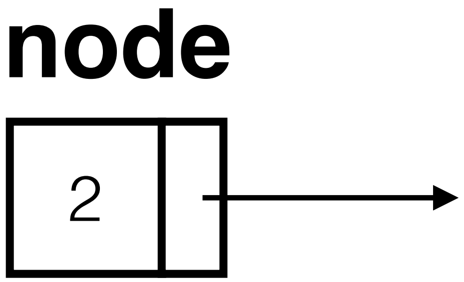

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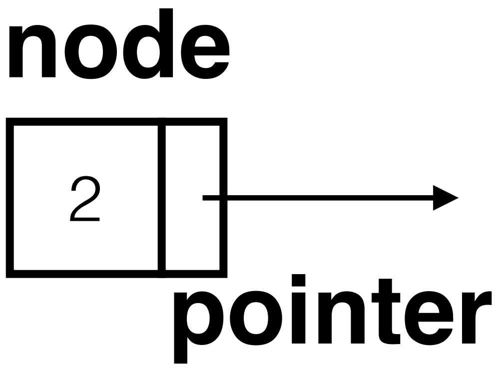

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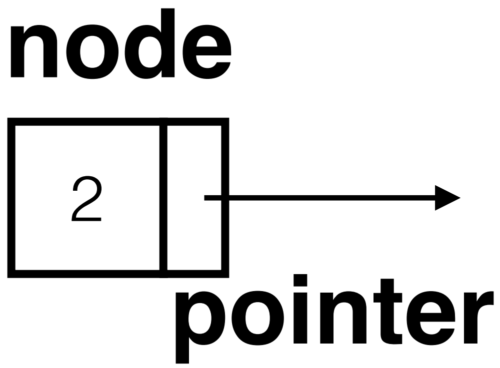

(in Java: “reference”)

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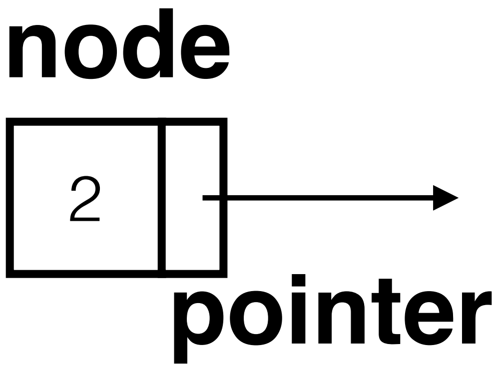
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pointer

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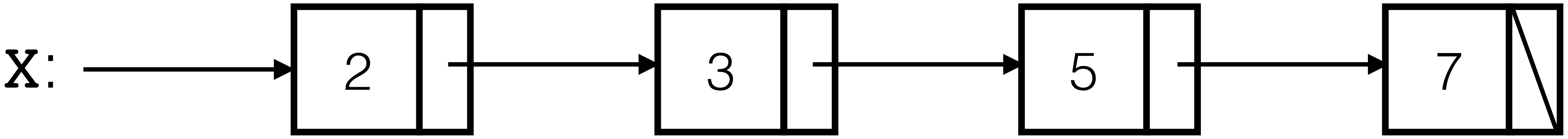

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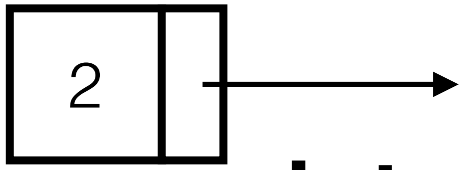

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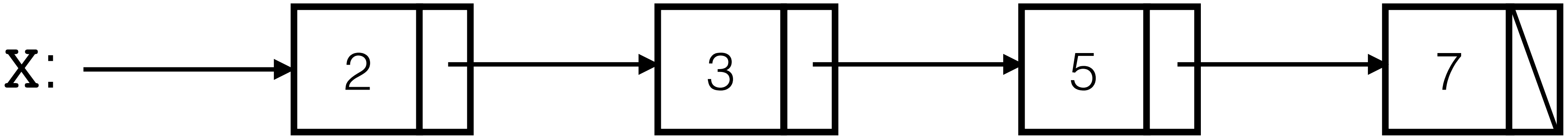
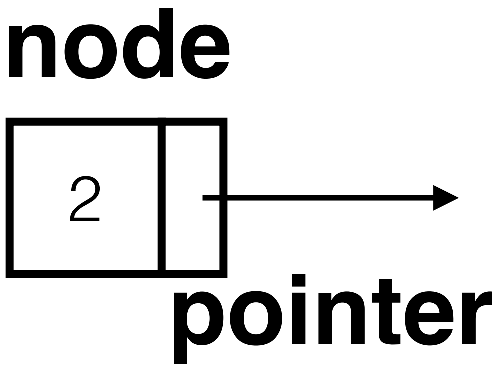


`x` is (a pointer to) the **head node** of the list

Terminology



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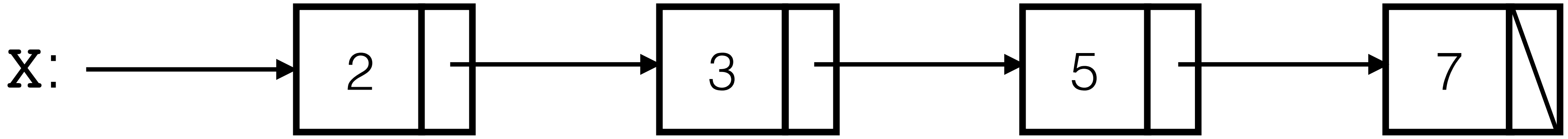

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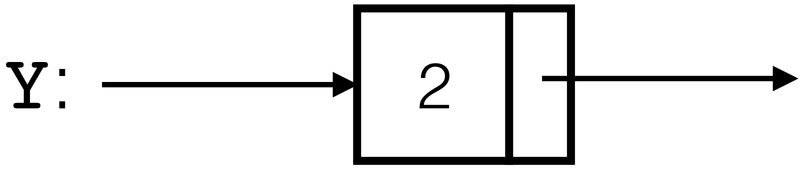

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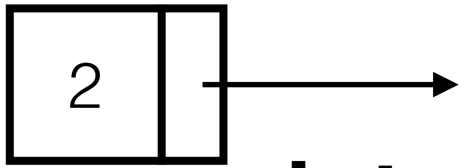

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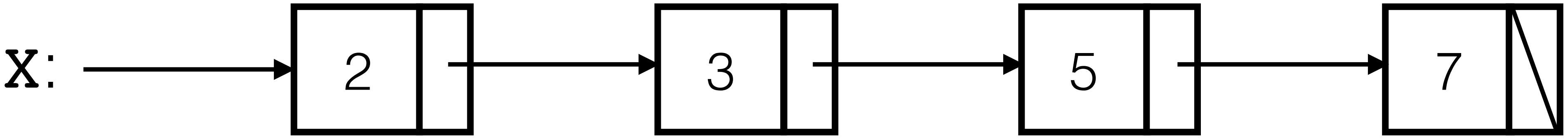

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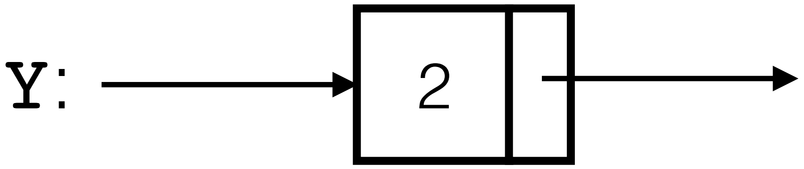

“`Y.val`” *refers to*

Terminology

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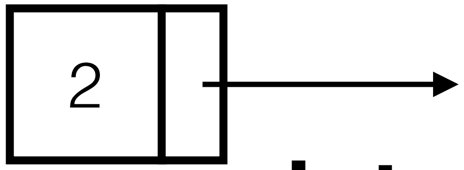
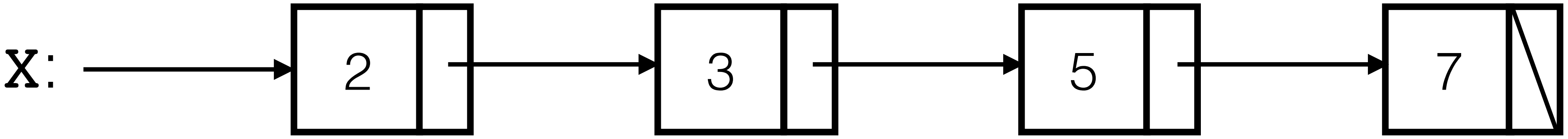
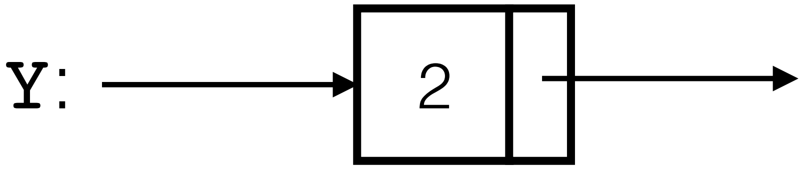


pointer

(in Java: “reference”)

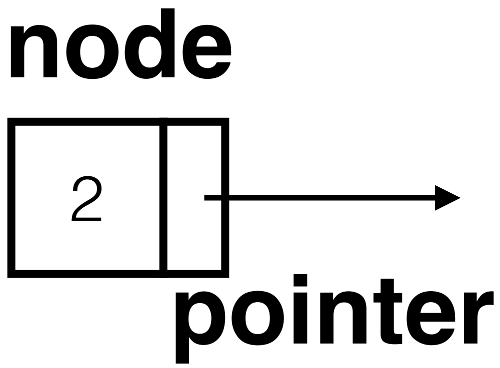


`X` is (a pointer to) the **head node** of the list



“`Y.val`” *refers to*

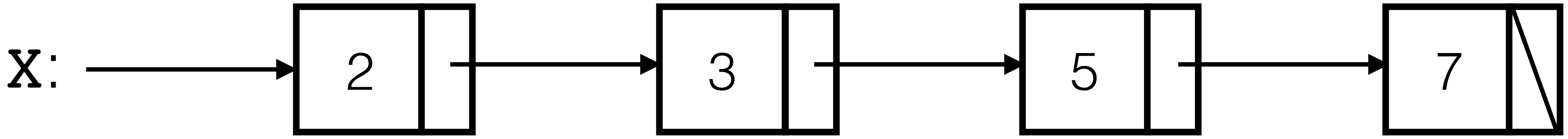
Terminology



node

pointer

(in Java: “reference”)



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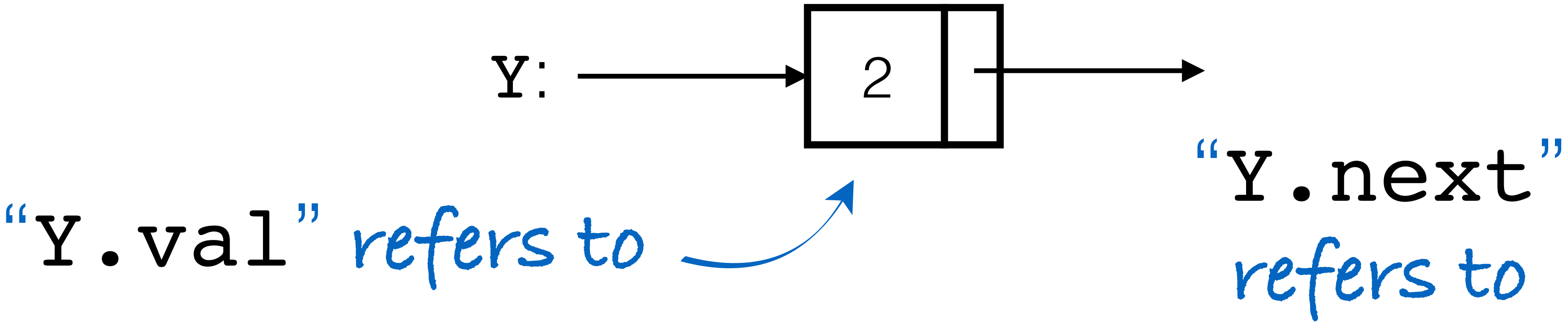
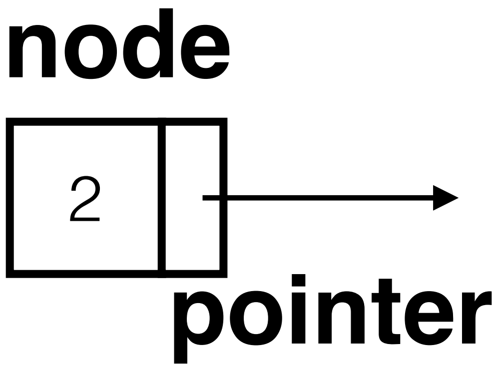


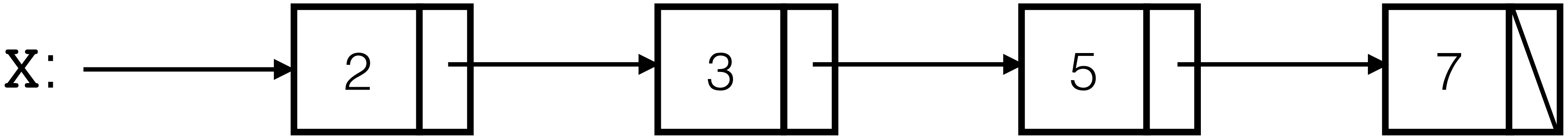
“`Y.val`” refers to

“`Y.next`” refers to

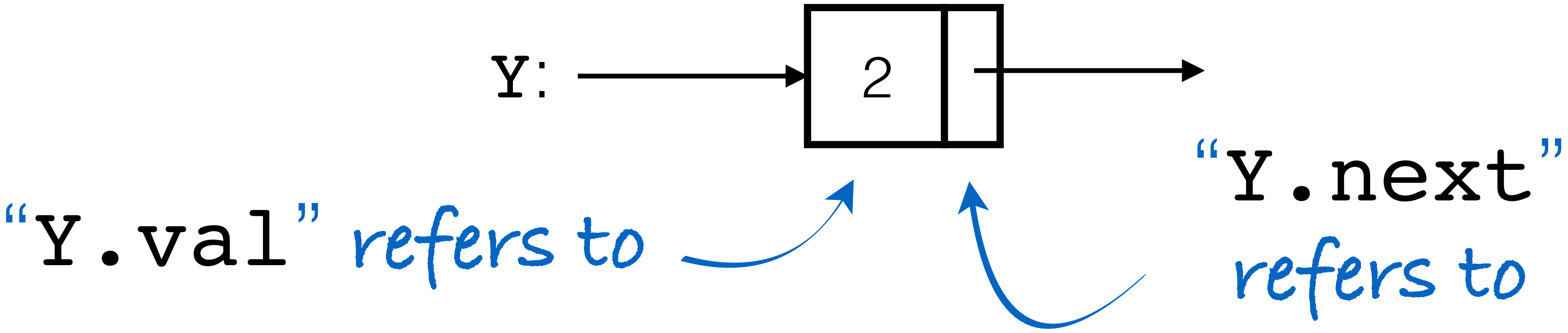
Terminology



(in Java: “reference”)



`X` is (a pointer to) the **head node** of the list



“`Y.val`” *refers to*

“`Y.next`” *refers to*

Linked List

- Often we use a dummy head node that points to the first object, or to a special `null` object that represents an empty list. This makes it easier to write functions that insert or delete elements.
- Inserting and deleting elements is very fast: just move a few links around.
- Finding the *i*th element can be time-consuming.

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item `x`.

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Iterative Processing: Array

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```

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Iterative Processing: Array

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```

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item `x`.

```
function find(A,x,n)
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        $j \leftarrow j+1$
    return -1
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item `x`.

A: Y

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item `x`.

A: Y   x: 7

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item `x`.

A: Y x: 7 n: 7

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
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```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item `x`.

A: Y x: 7 n: 7 j: 0

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Y:

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

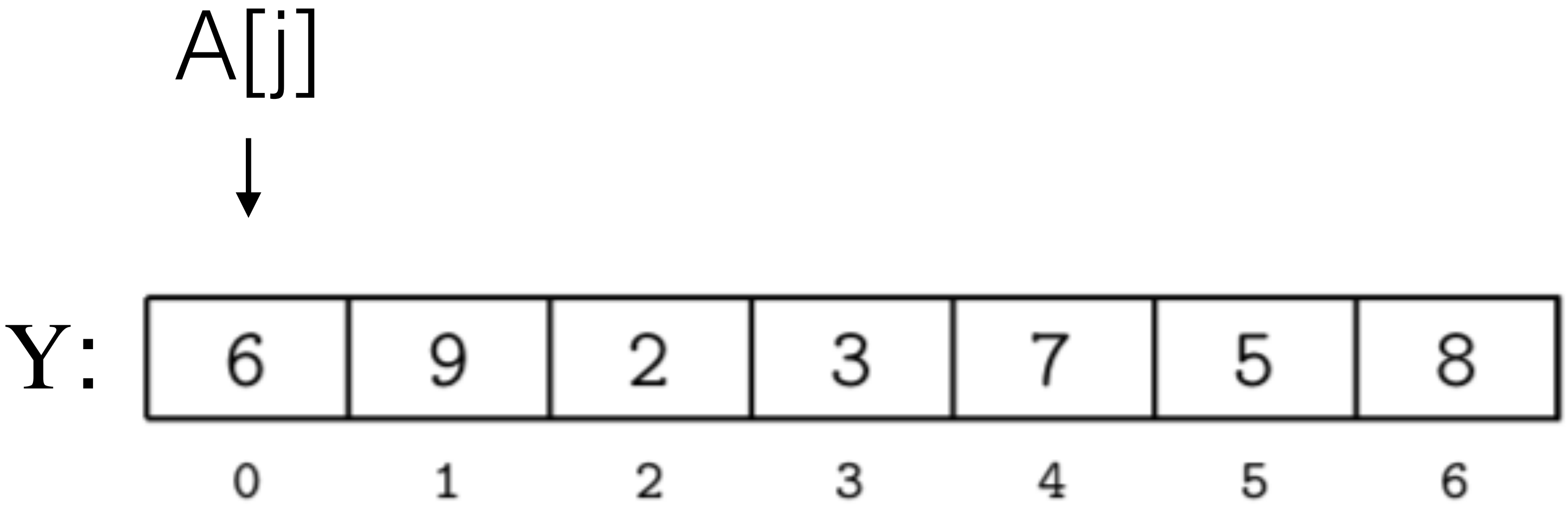
Let's trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item `x`.

A: Y x: 7 n: 7 j: 0

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



6	9	2	3	7	5	8
0	1	2	3	4	5	6

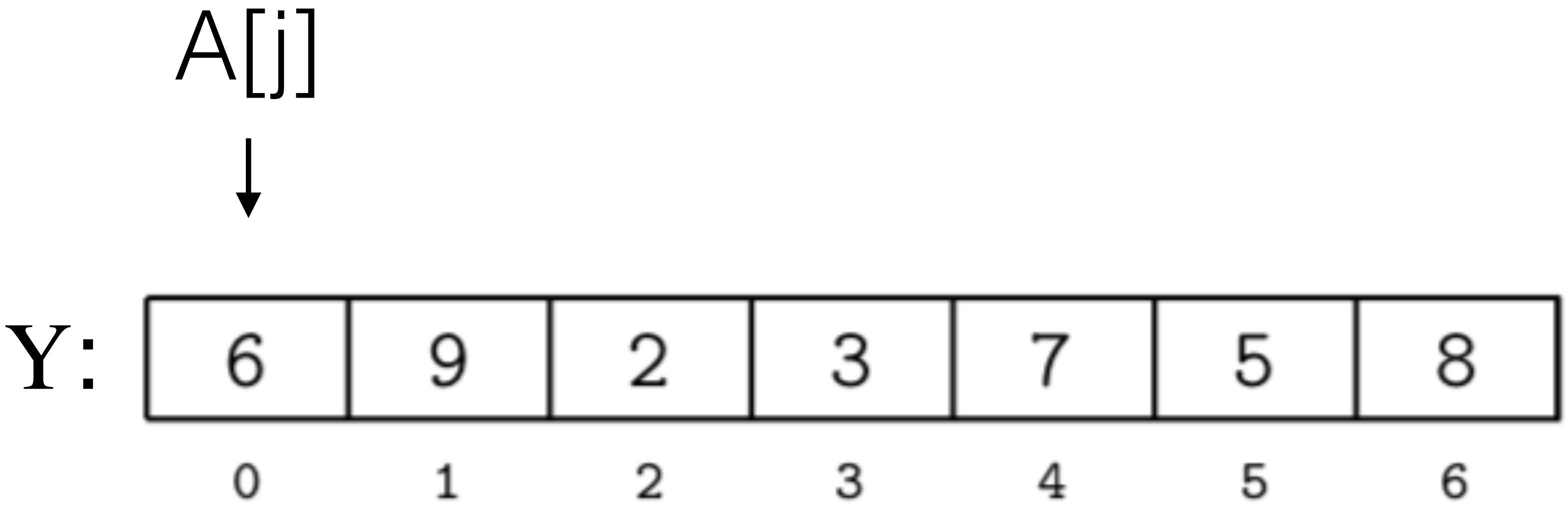
Let's trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 7 j: 1

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



Y:							
	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item `x`.

A: Y x: 7 n: 7 j: 1

```
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        if $A[j] = x$
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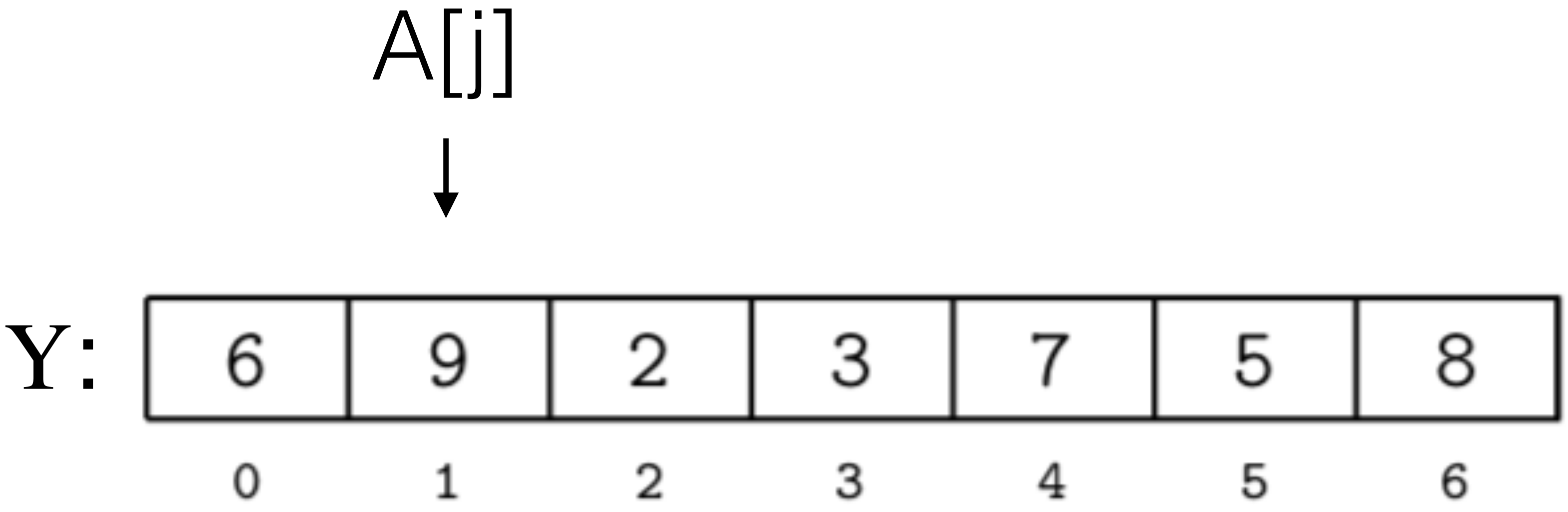
```



A[j]

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,7)

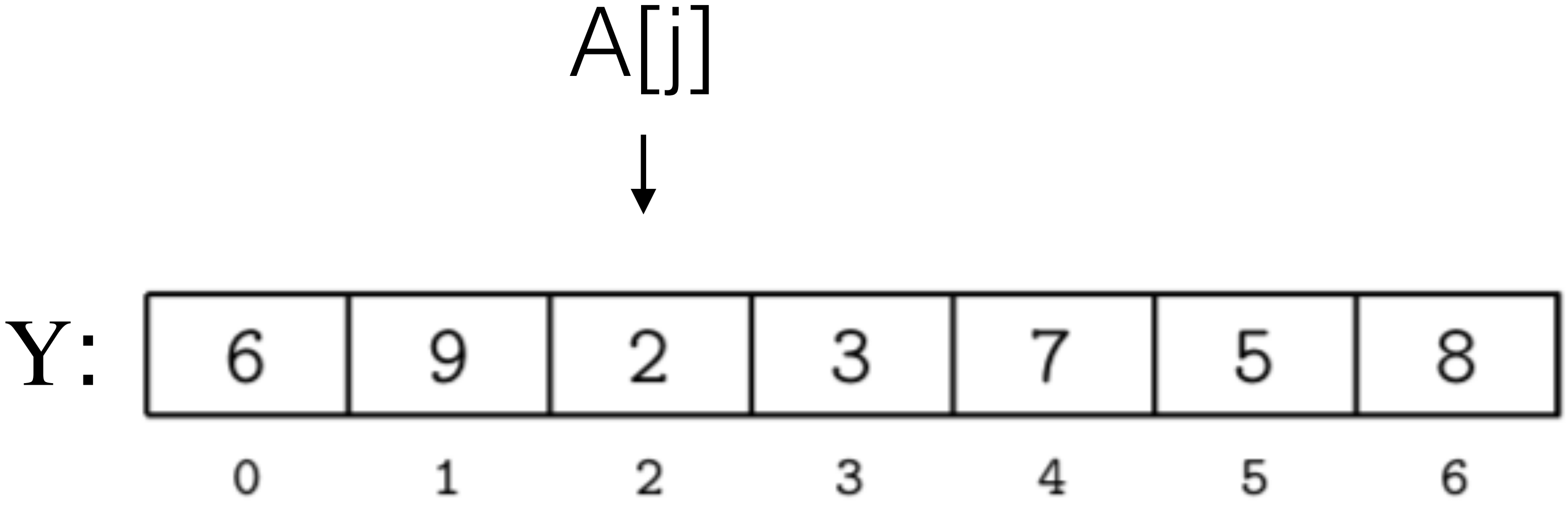
Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y   x: 7   n: 7   j: 2

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



A[j]

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

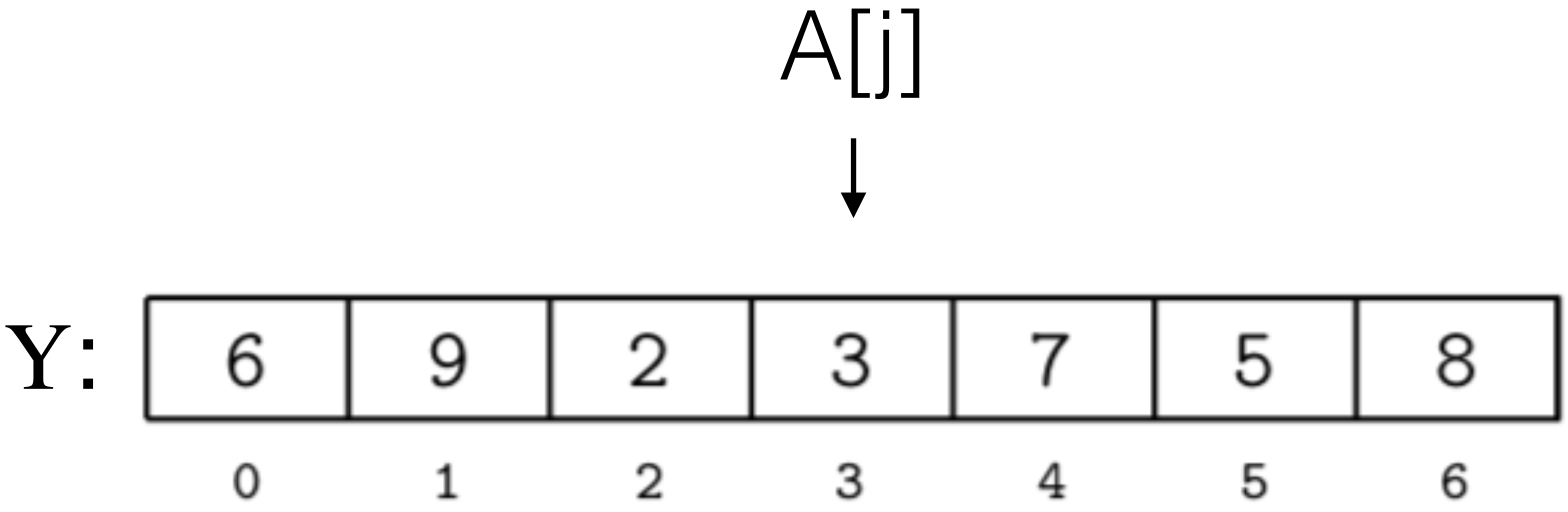
Let's trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **`x`**.

A: Y x: 7 n: 7 j: 3

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



A[j]

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

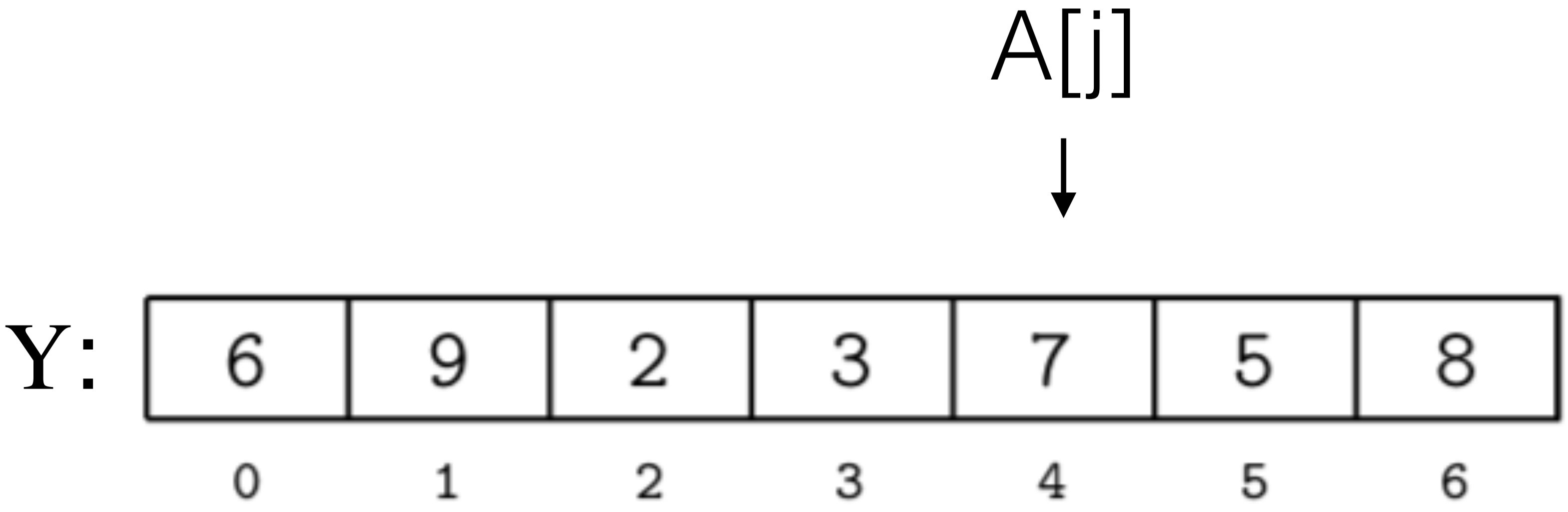
Let's trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 7 j: 4

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



A[j]

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,7)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y   x: 7   n: 7   j: 4

```
function find(A,x,n)
    j ← 0
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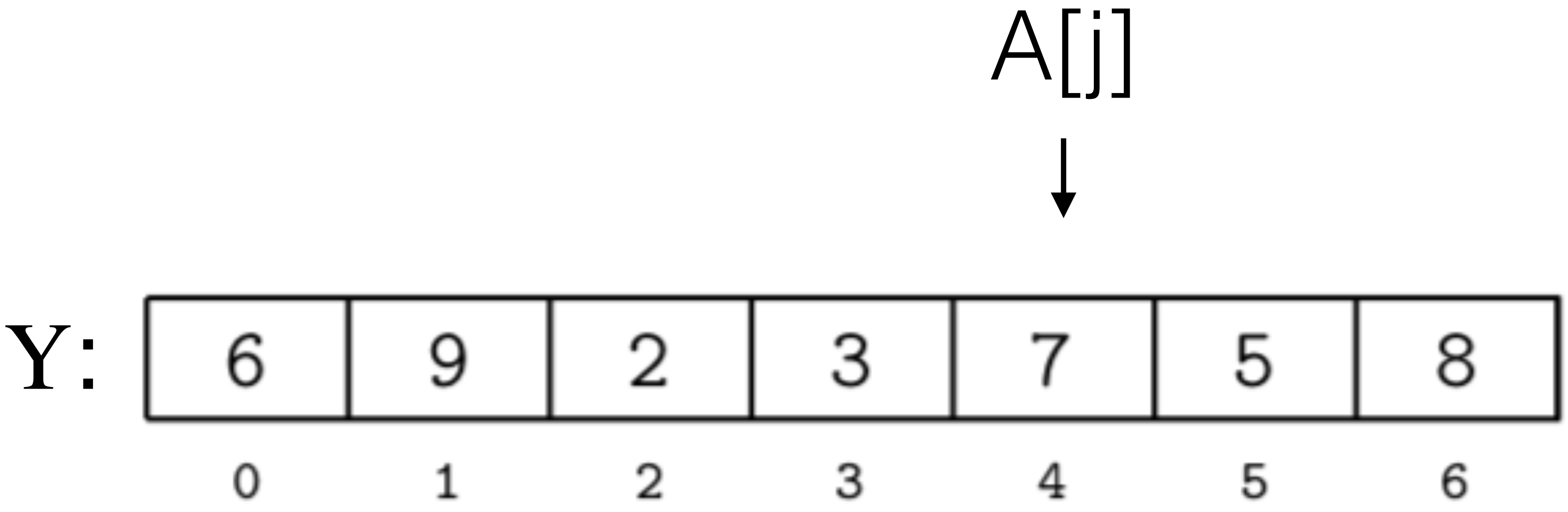

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,7)

(returns 4)

Iterative Processing: List

- Walk through a linked list.
- For example, to locate item `x`.

```
function find(head,x)
    p ← head
    while p ≠ null
        if p.val = x
            return p
        p ← p.next
    return null
```

Iterative Processing: List

- Walk through a linked list.
- For example, to locate item `x`.

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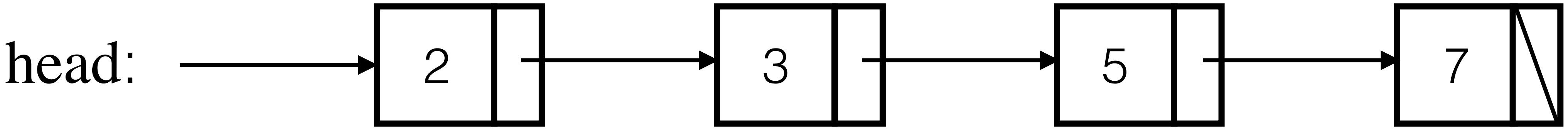

Iterative Processing: List

- Walk through a linked list.
- For example, to locate item `x`.

(note similarity to array version)

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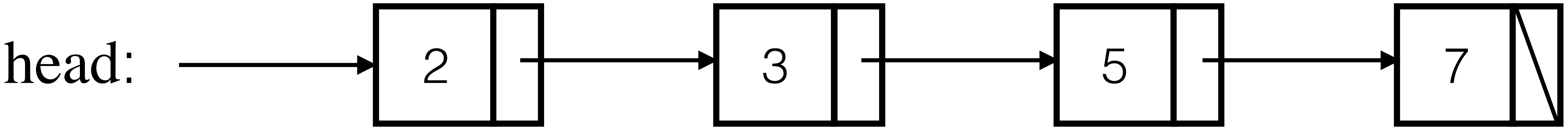

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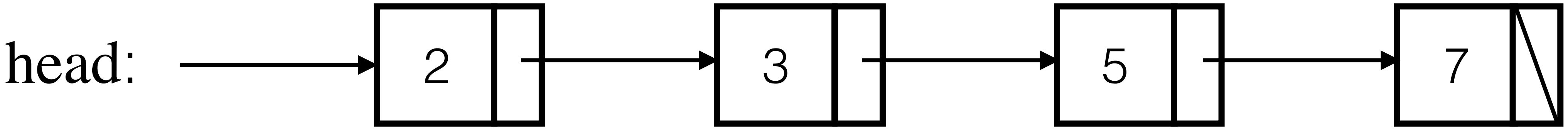

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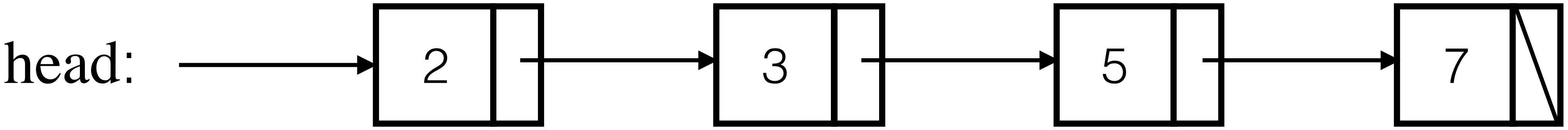
```

p:



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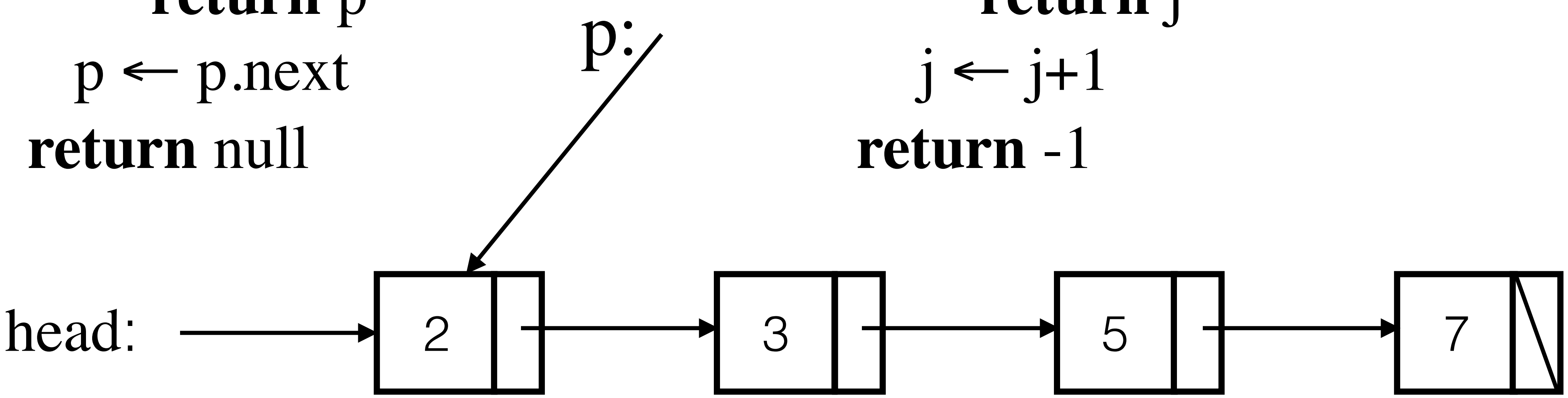

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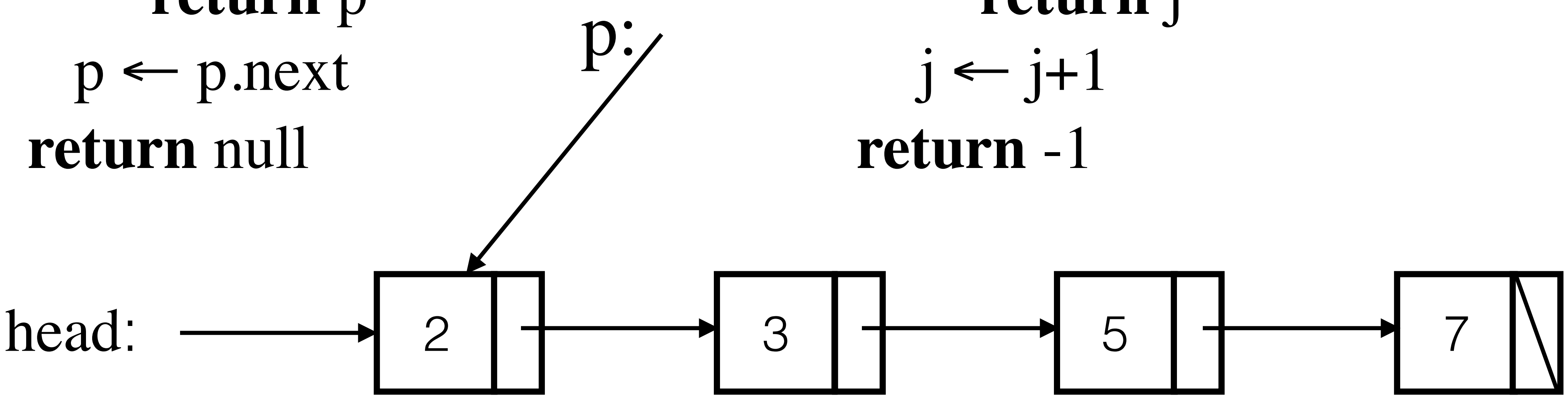

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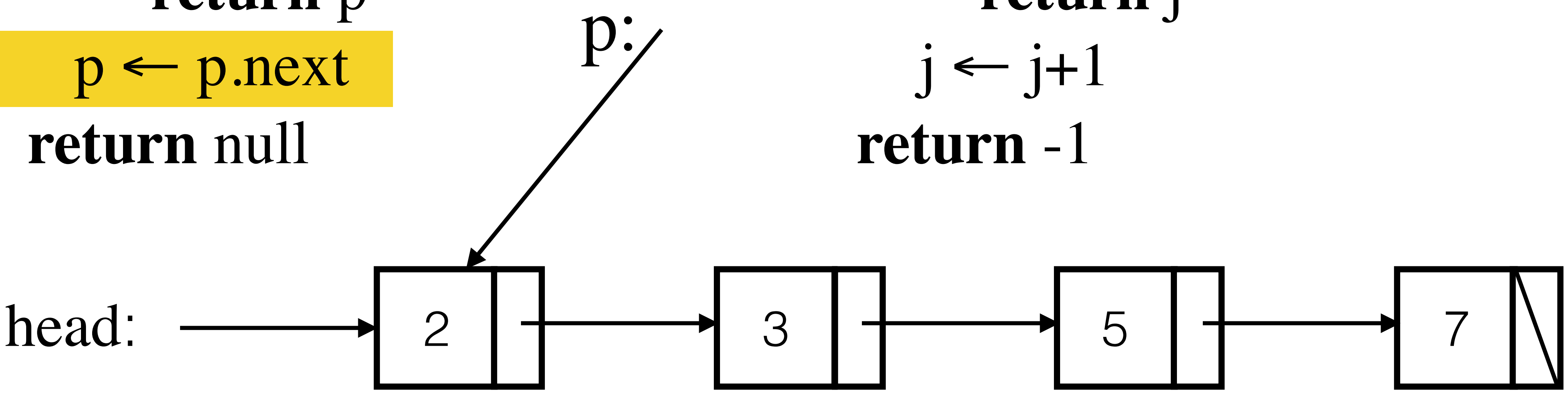

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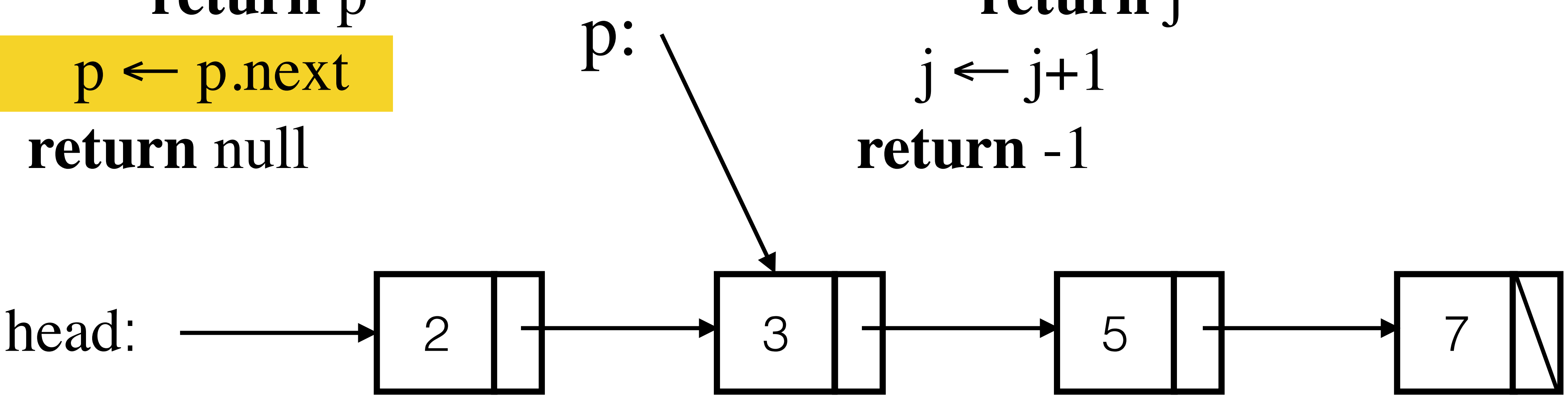

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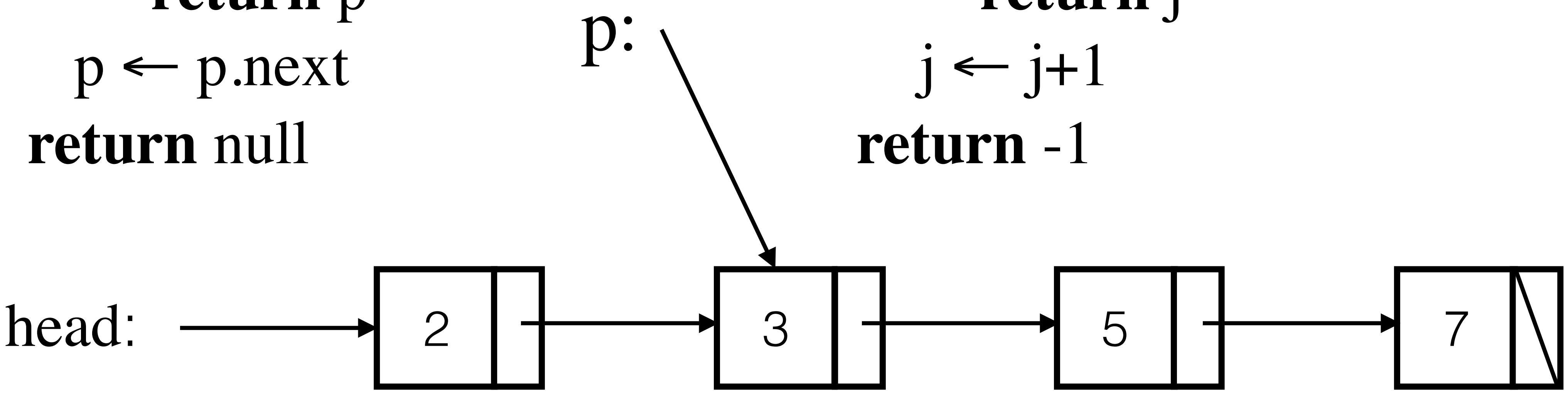

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Initial call: find(A,x,0,n-1)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Recursive Processing: Array

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function find(A,x,lo,hi)
    if lo > hi
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        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

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function find(A,x,lo,hi)
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```

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let’s trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y   x: 7

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y   x: 7   lo: 0

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y x: 7 lo: 0 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

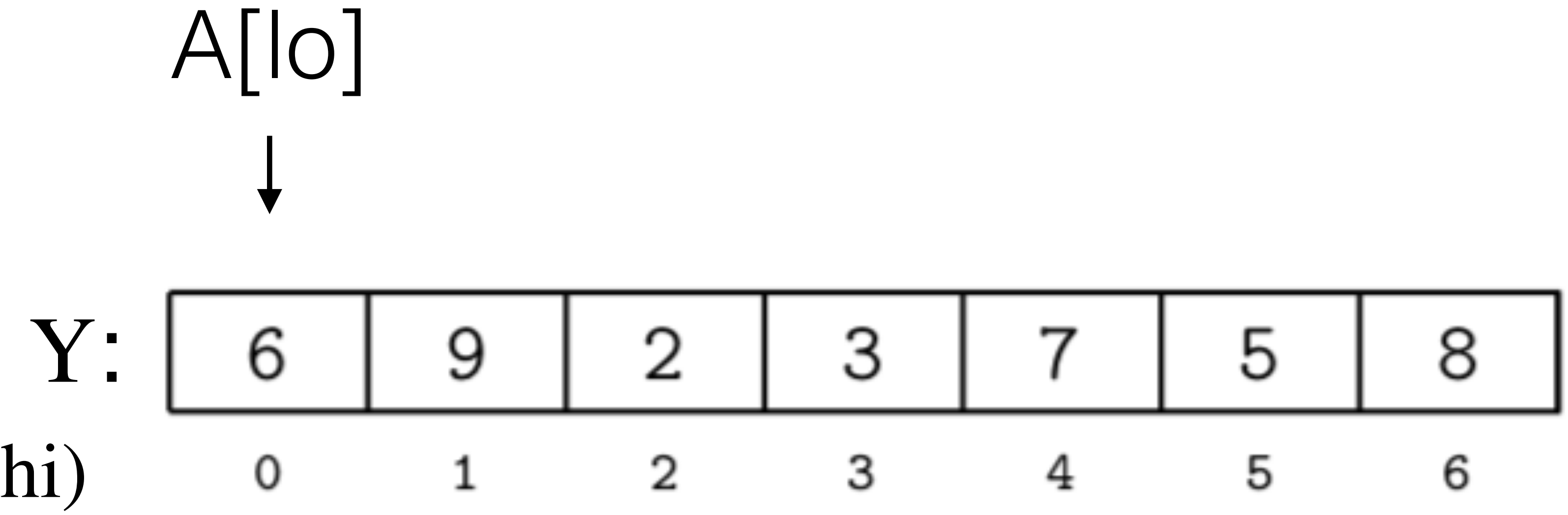
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y   x: 7   lo: 0   hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



A[lo]

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let’s trace the execution of find(Y,7,0,6)

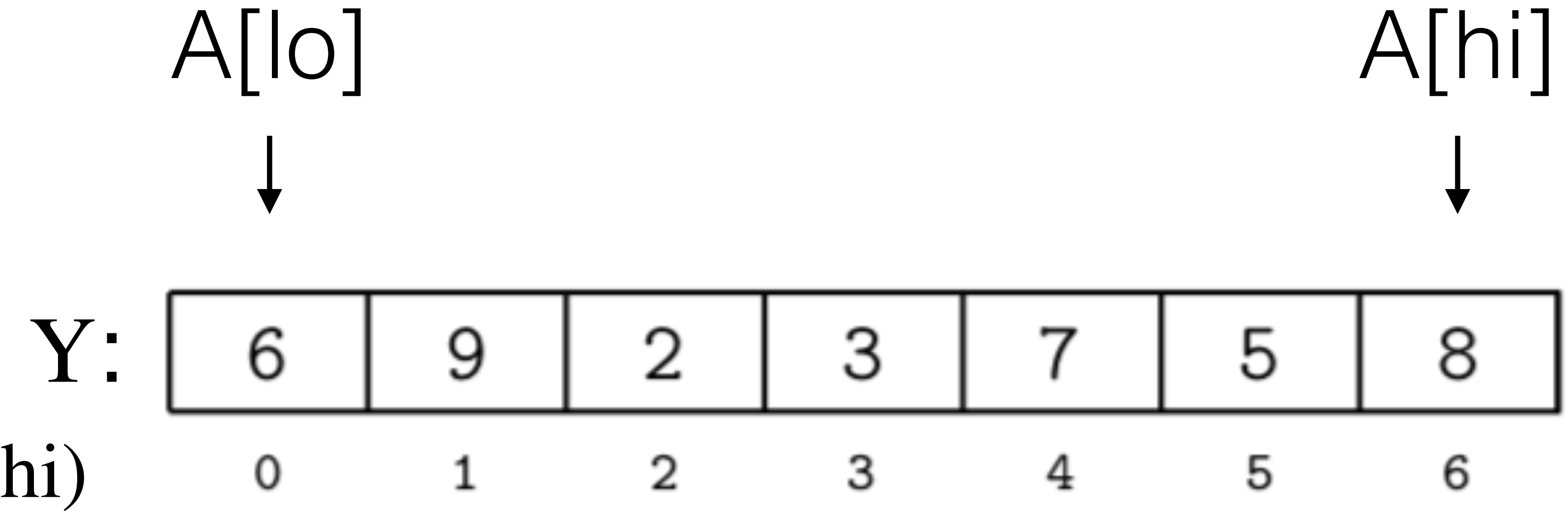
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y x: 7 lo: 0 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



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Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y x: 7 lo: 1 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
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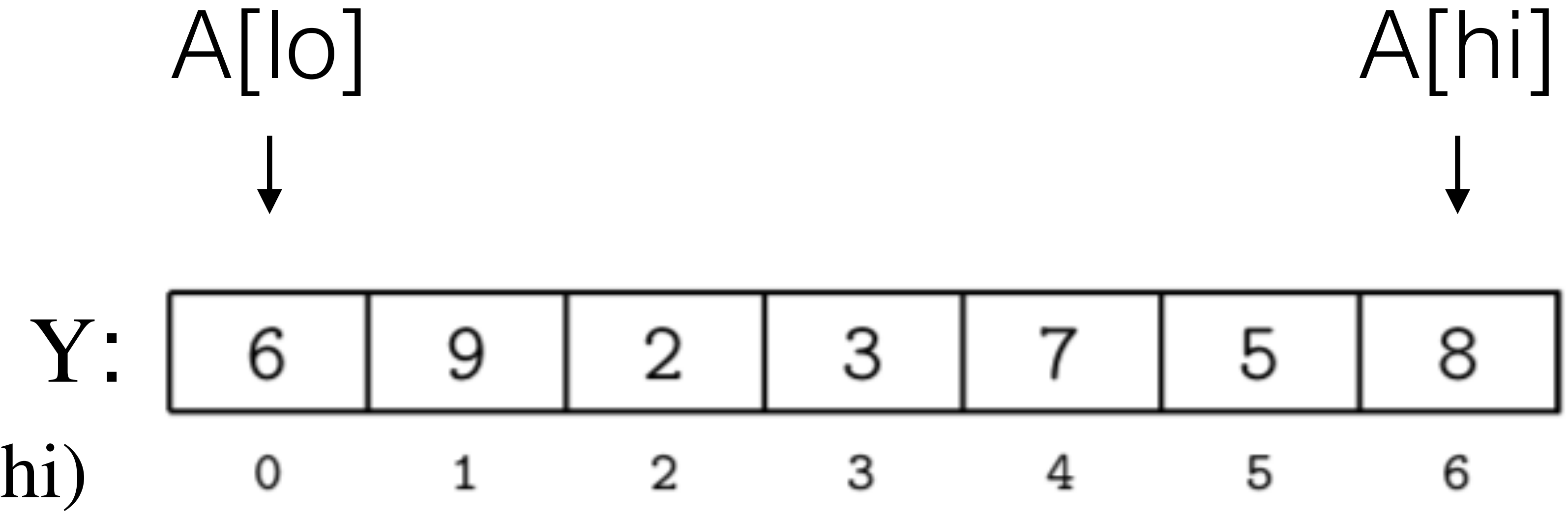

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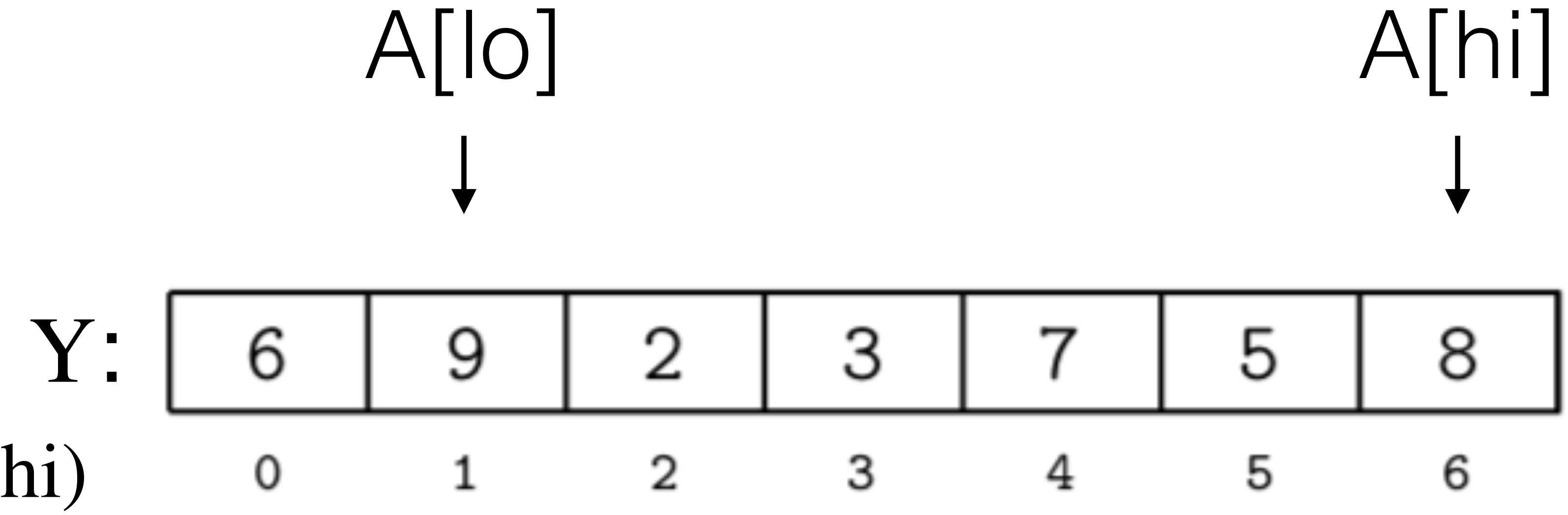

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Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance
- For example, to locate item `x`.

A: Y x: 7 lo: 1 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
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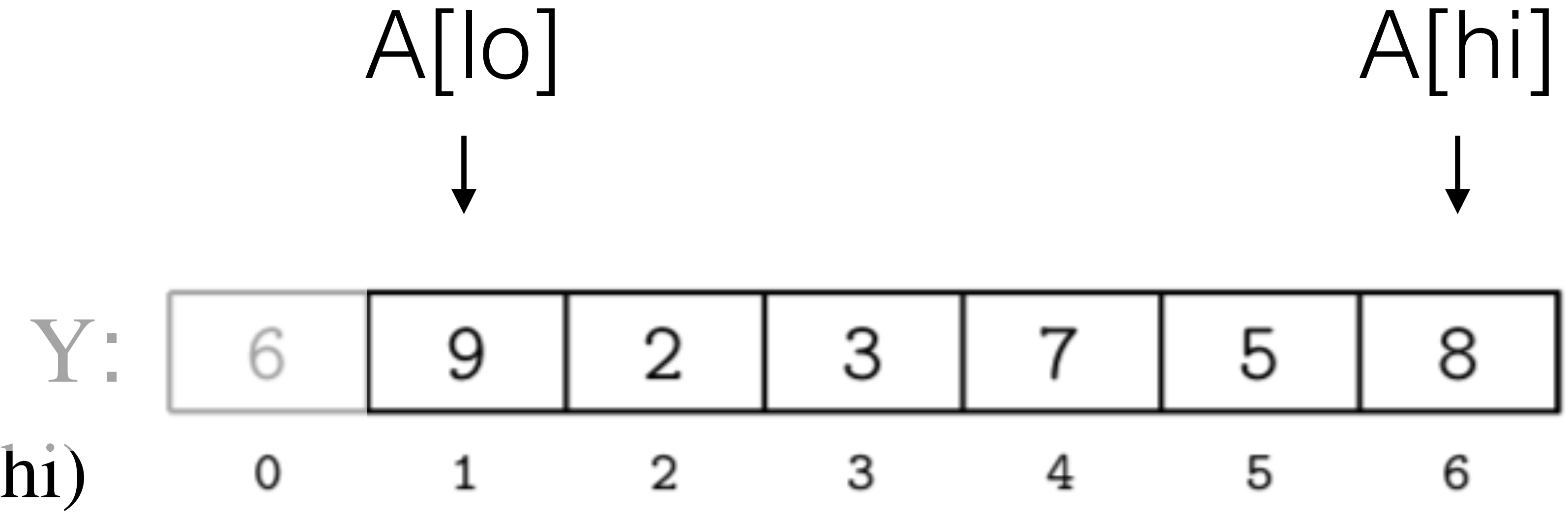

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	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y   x: 7   lo: 2   hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
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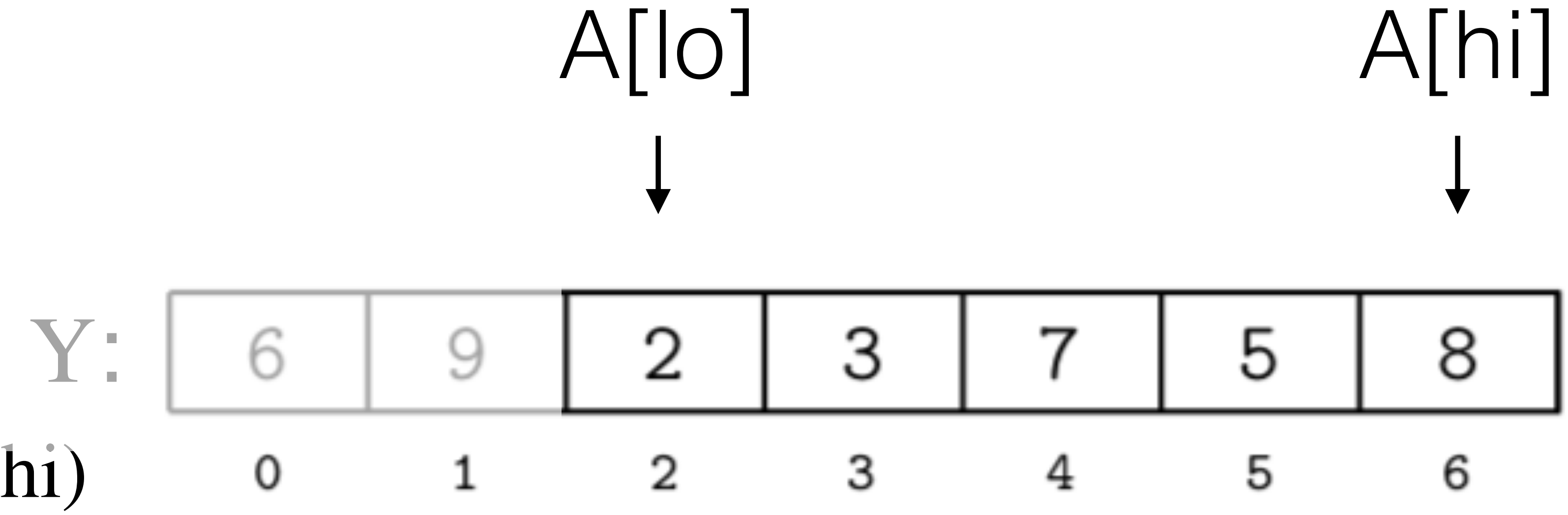

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	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

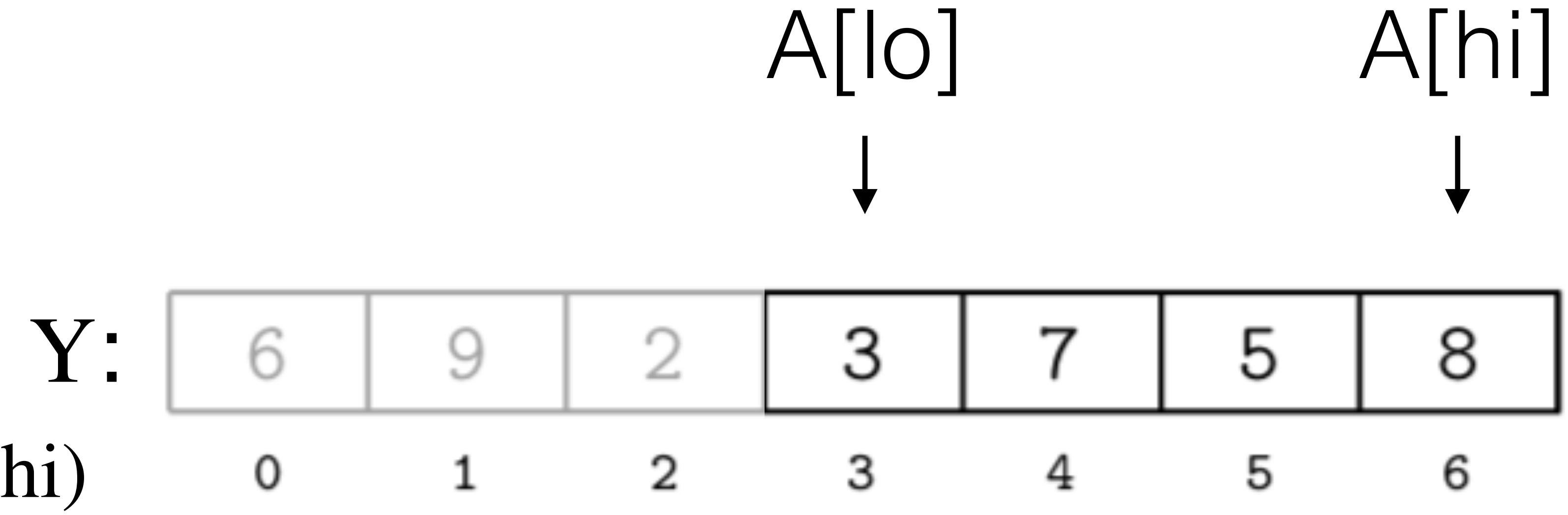
Let’s trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance
- For example, to locate item `x`.

A: Y   x: 7   lo: 3   hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



A[lo] ↓   A[hi] ↓

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 4 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
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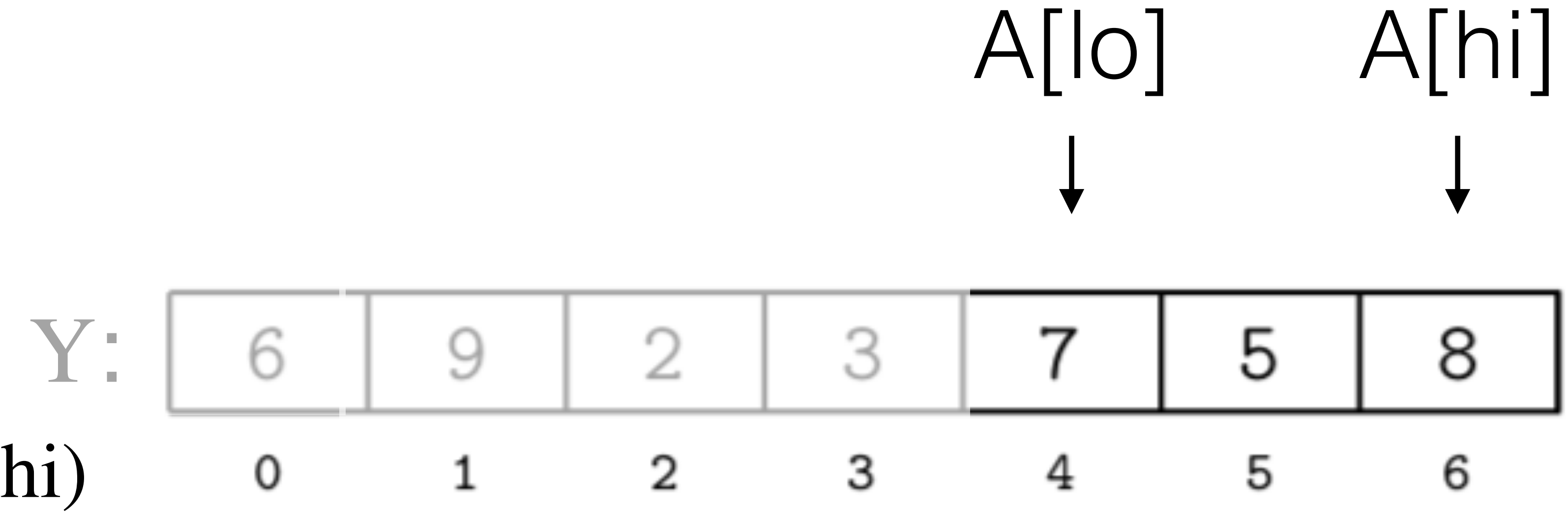

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Initial call: find(A,x,0,n-1)

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Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y x: 7 lo: 4 hi: 6

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function find(A,x,lo,hi)
    if lo > hi
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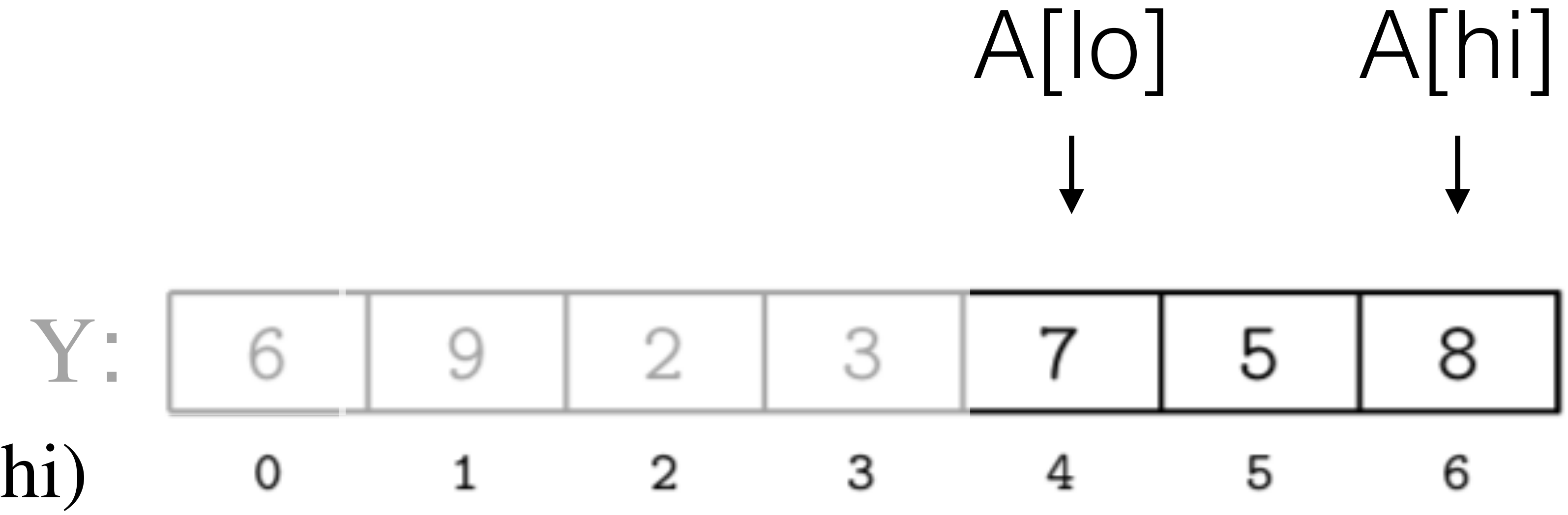

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

(returns 4)

Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

```
function find(p,x)
    if p = null
        return p
    else if p.val = x
        return p
    else
        return find(p.next,x)
```

Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

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function find(p,x)
    if p = null
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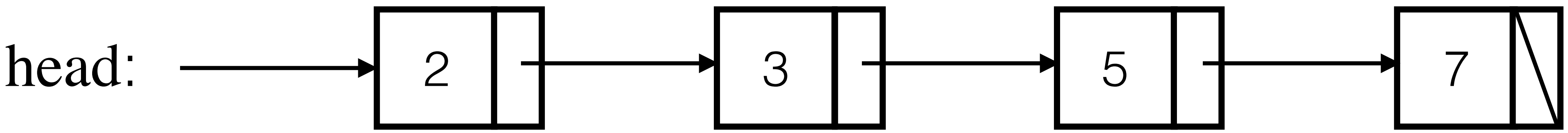

Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

```
function find(p,x)
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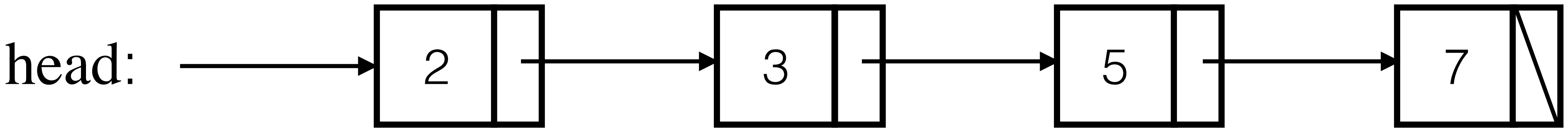
```

Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

(note similarity to array version)

```
function find(p,x)
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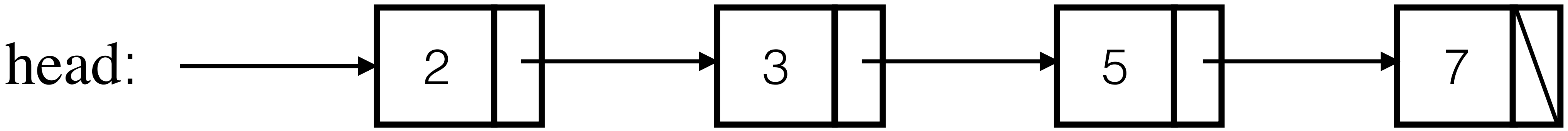

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p:

```
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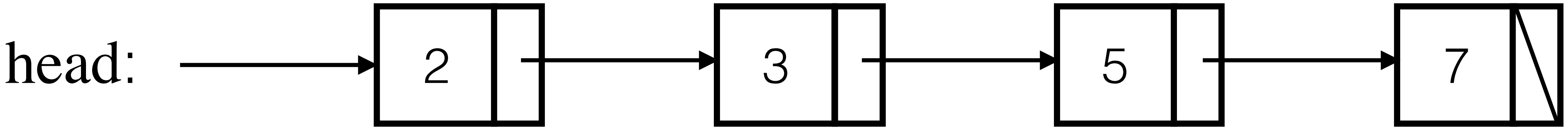

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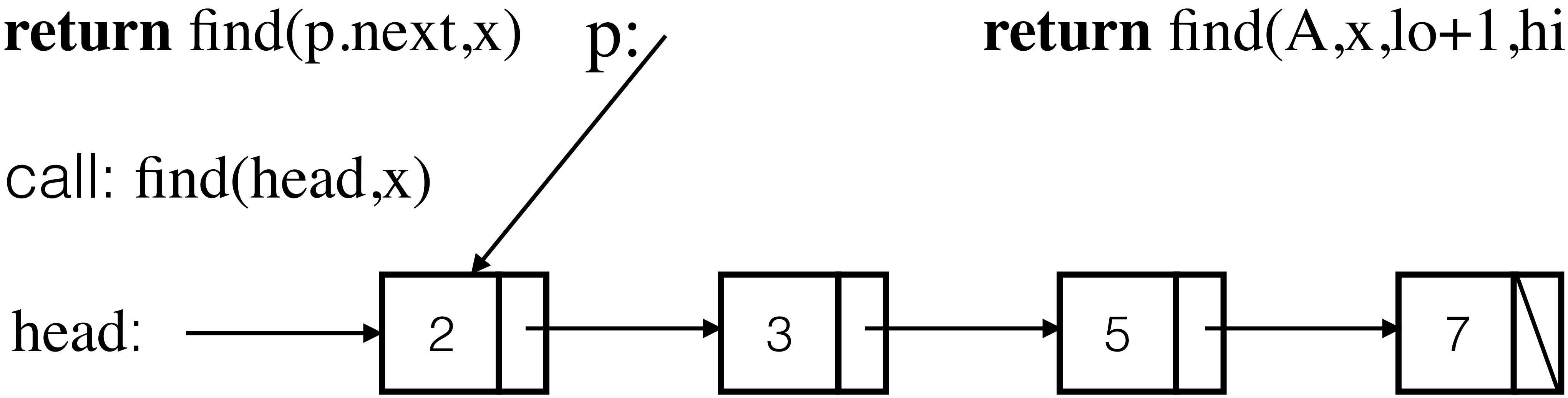

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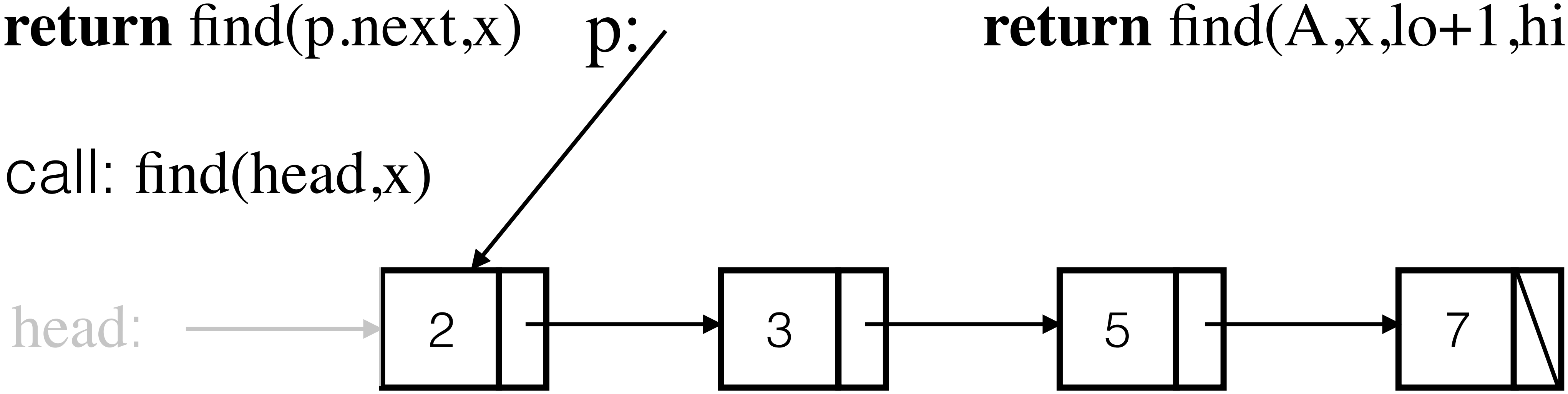

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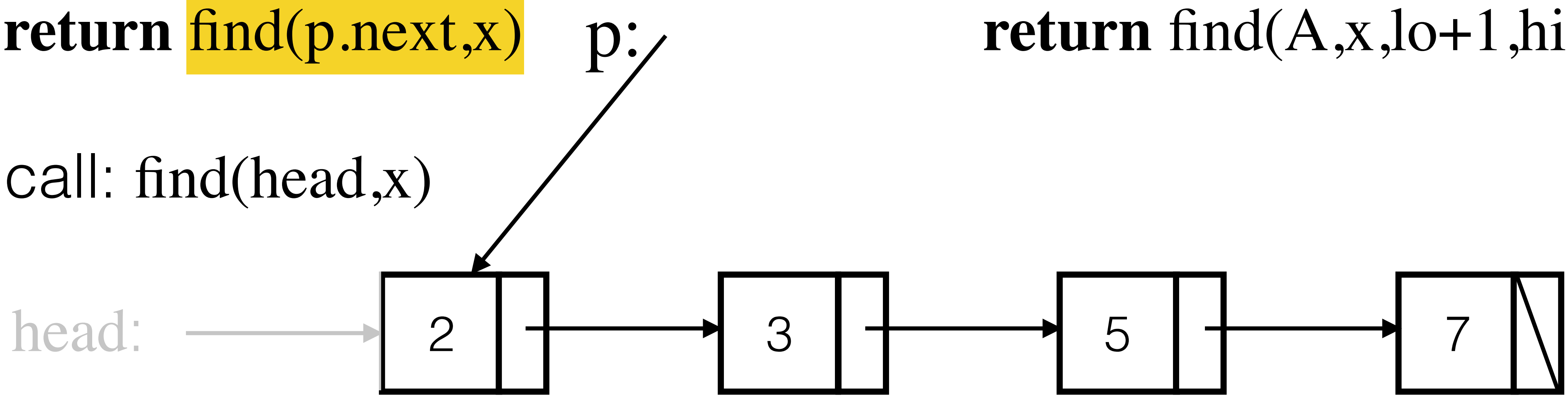

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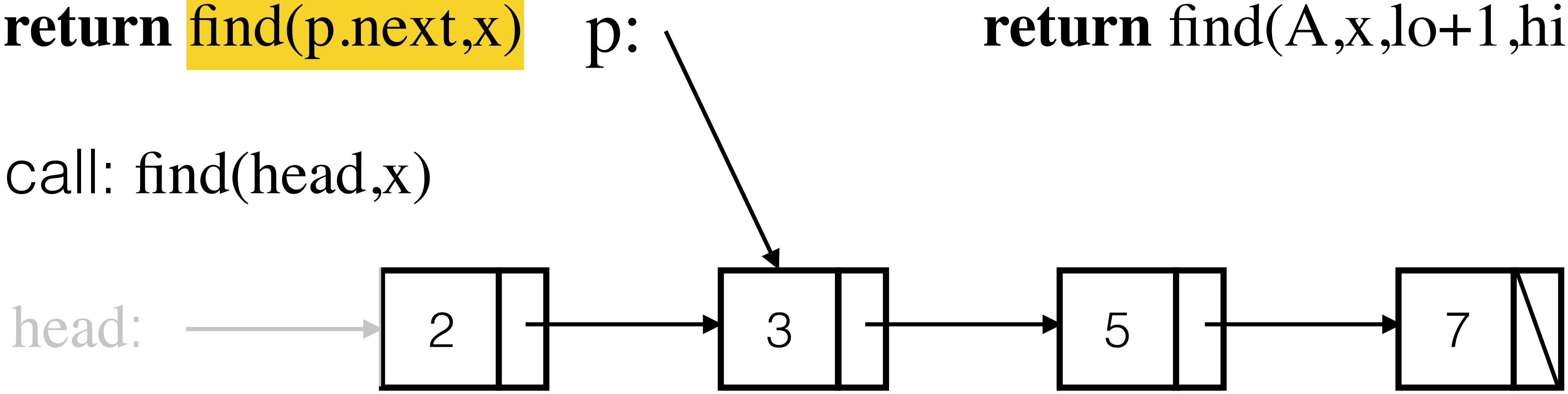

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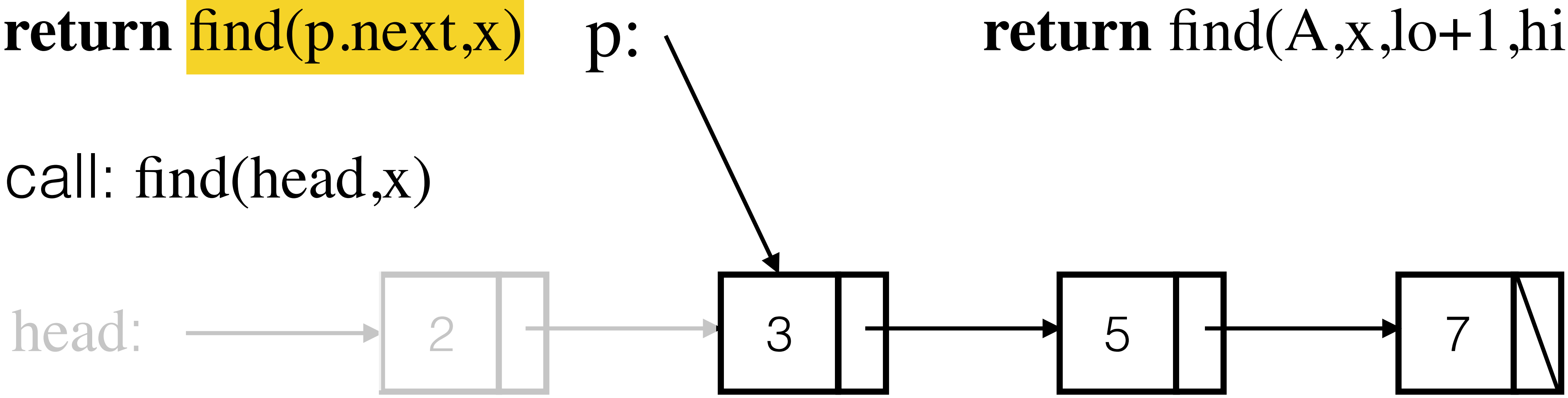

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```

We will discuss recursion properly in week 3

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function find(A,x,lo,hi)
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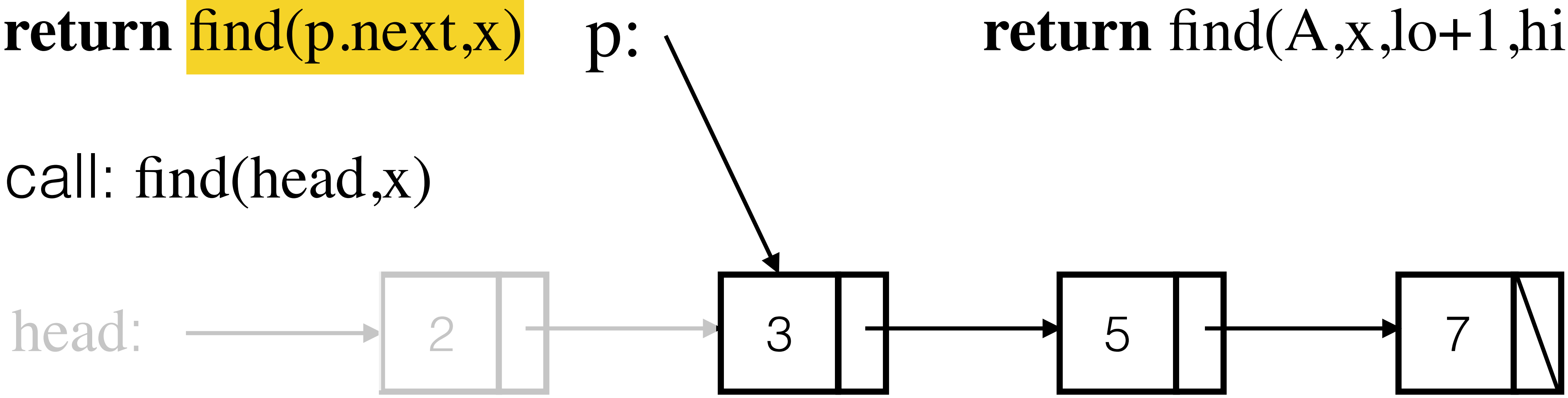

Abstract DataTypes

- A collection of data items, and a family of operations that operate on that data
- Think of an ADT as a set of contracts, an **interface**
- We must still **implement** these promises, but it is an advantage to separate the implementation of the ADT from the "concept" (i.e. the interface it provides)
- Good programming practice is to support this separation
    - Nothing outside of the definition of the ADT should refer to anything inside, except through function calls and basic operations

Fundamental Data Structure: The Stack

- Last-In-First-Out (LIFO)
- Operations:
    - CreateStack
    - Push
    - Pop
    - Top
    - EmptyStack?
    - …
- Usually implemented as an ADT

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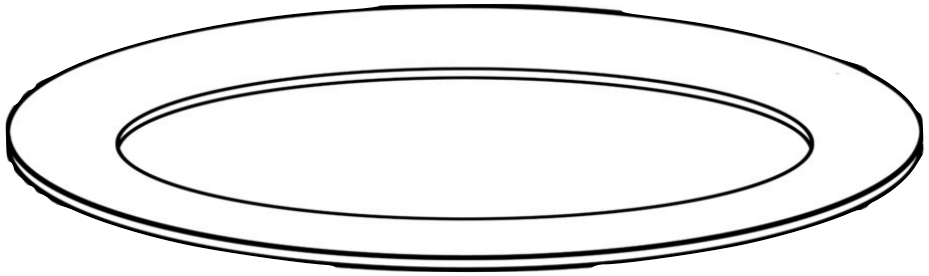

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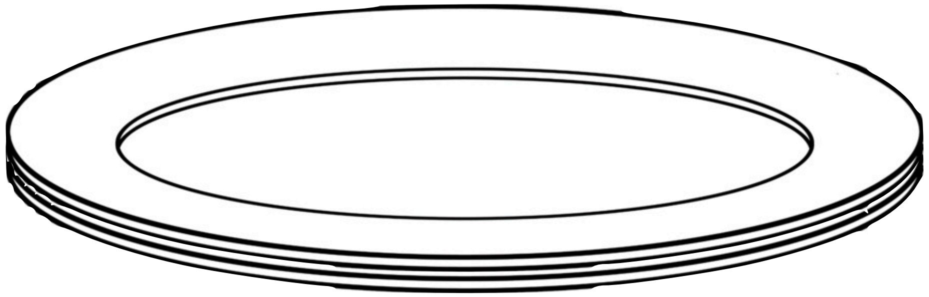

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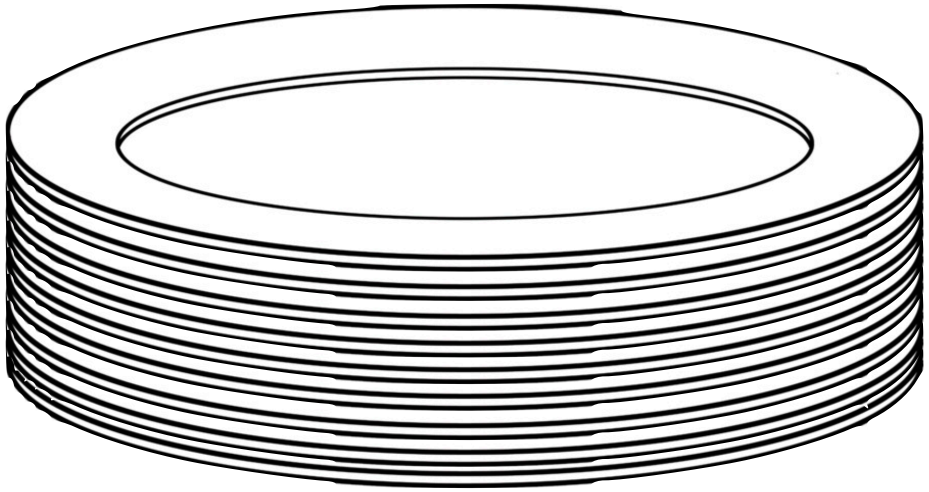

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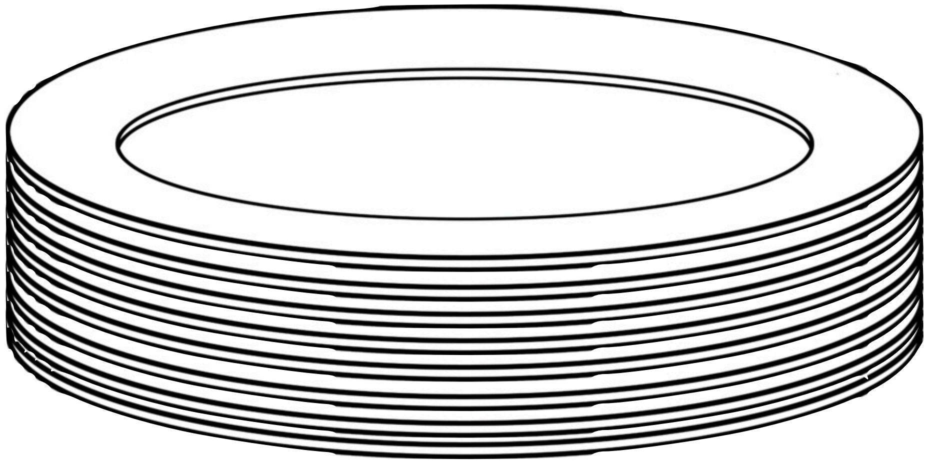

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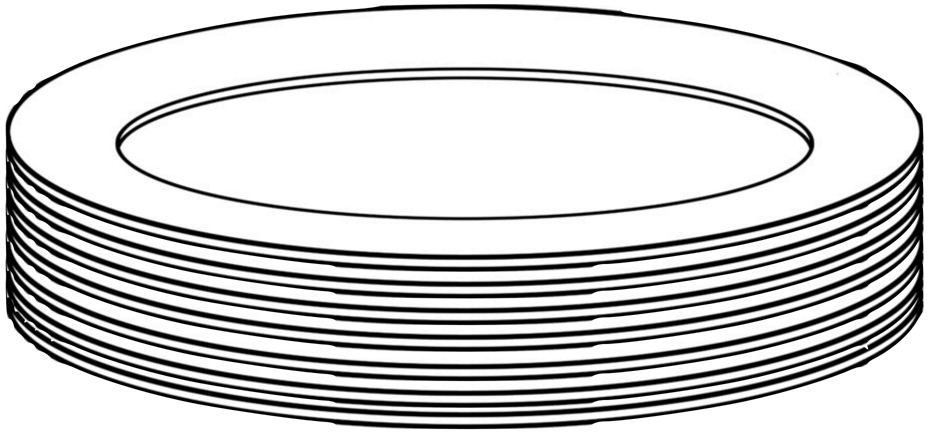

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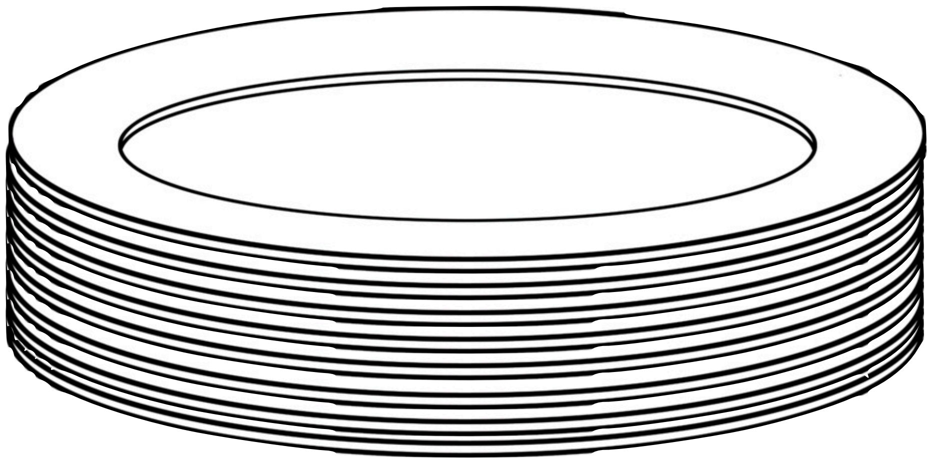

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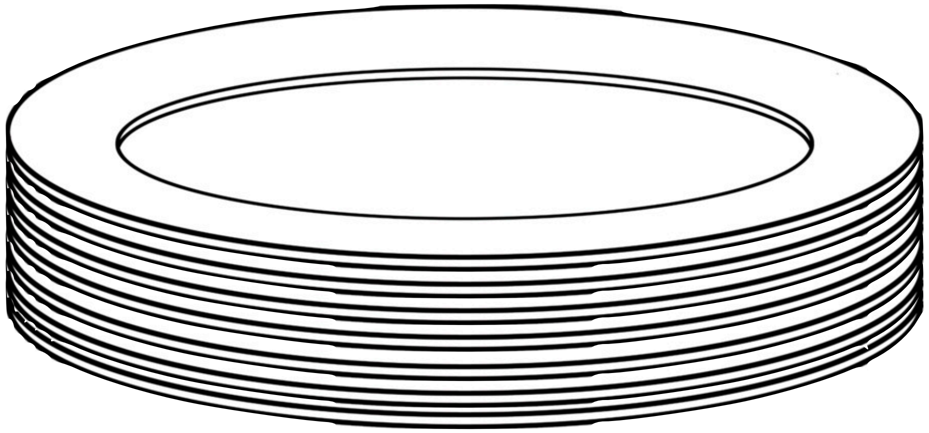

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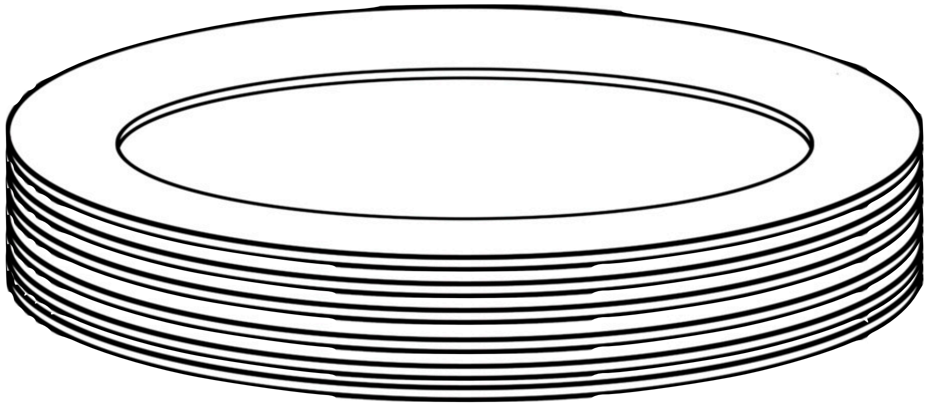


Stack Implementation: Array

Stack Implementation: Array

6	9	2	3	7		
0	1	2	3	4	5	6

Stack Implementation: Array

6	9	2	3	7		
0	1	2	3	4	5	6

top: 5

Stack Implementation: Array

6	9	2	3	7		
0	1	2	3	4	5	6

top: 5

Push(5)

Stack Implementation: Array

6	9	2	3	7	5	
0	1	2	3	4	5	6

top: 5

Push(5)

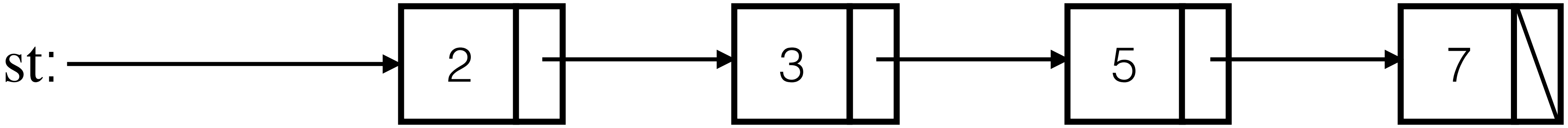
Stack Implementation: Array

6	9	2	3	7	5	
0	1	2	3	4	5	6

top: 6

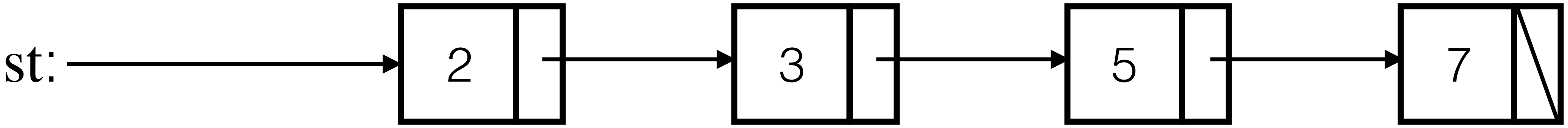
Push(5)

Stack Implementation: Linked List



```
function push(st,x)
    elt ← new node
    elt.val ← x
    elt.next ← st
    st ← elt
    return st
```

Stack Implementation: Linked List



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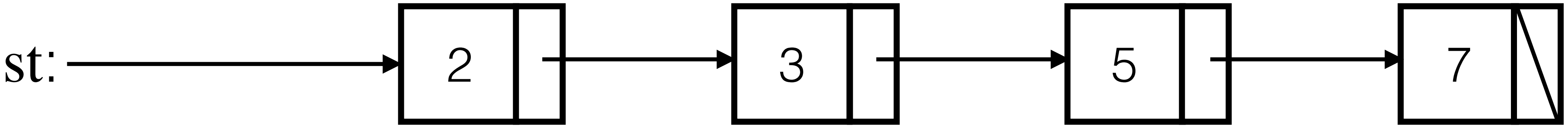
Stack Implementation: Linked List



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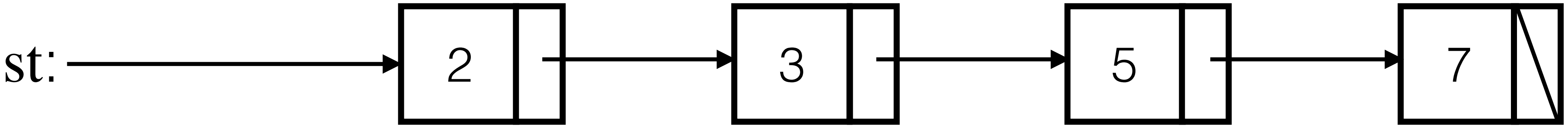

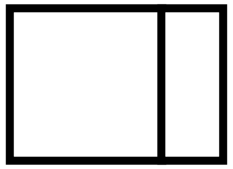

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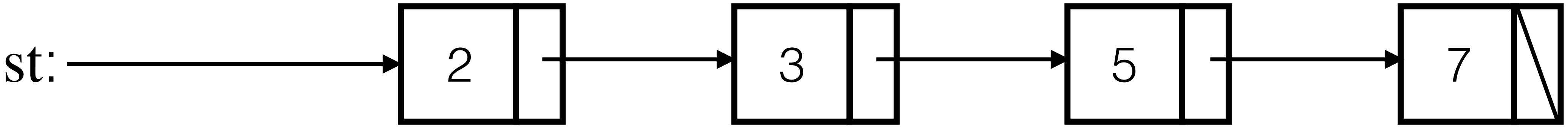

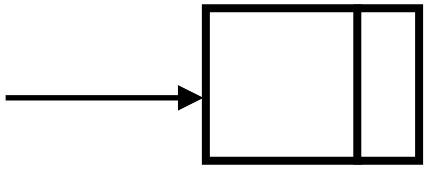

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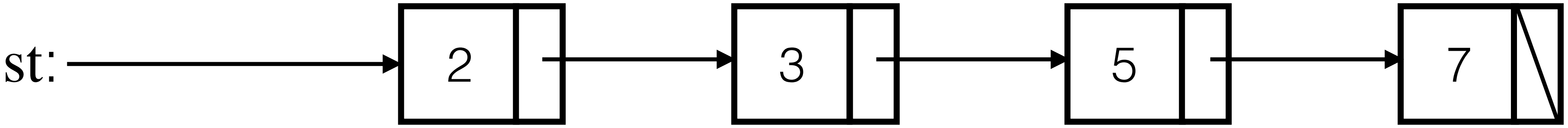
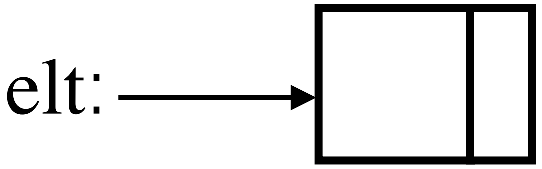
Stack Implementation: Linked List





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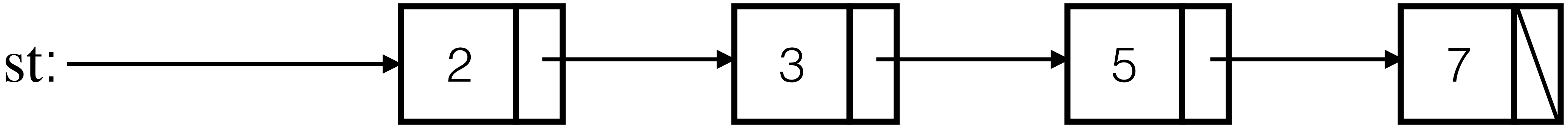

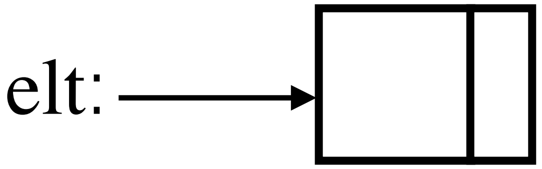

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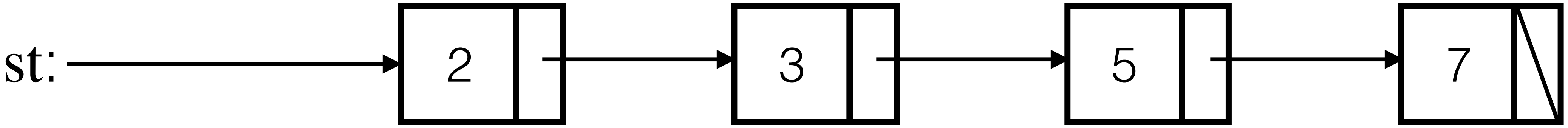

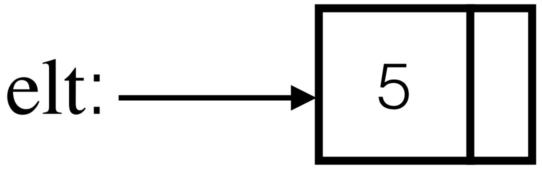

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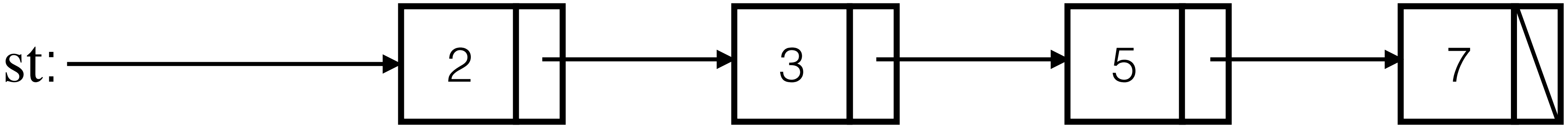

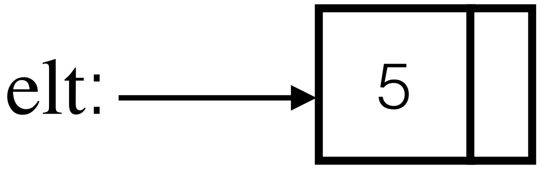

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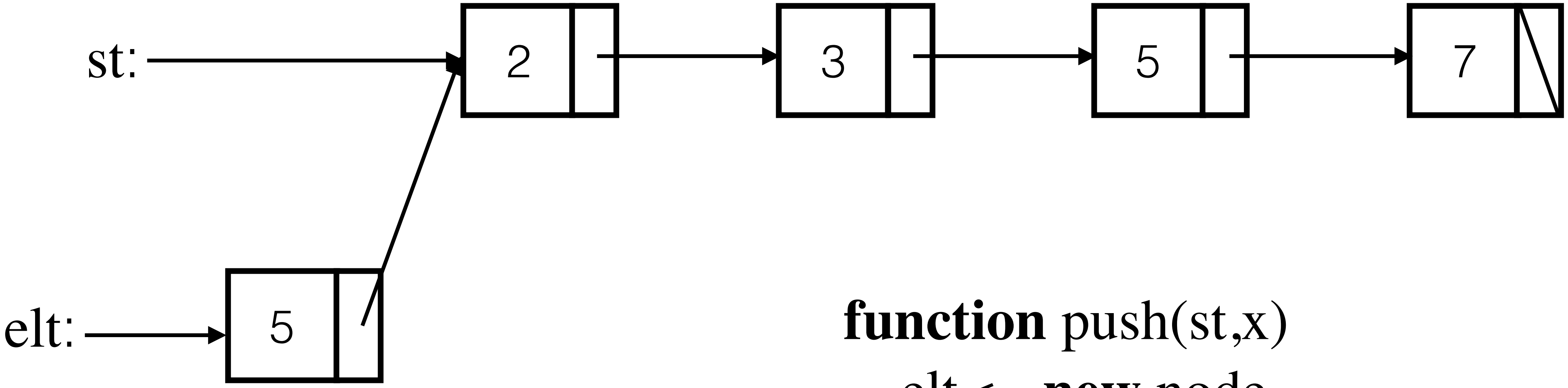

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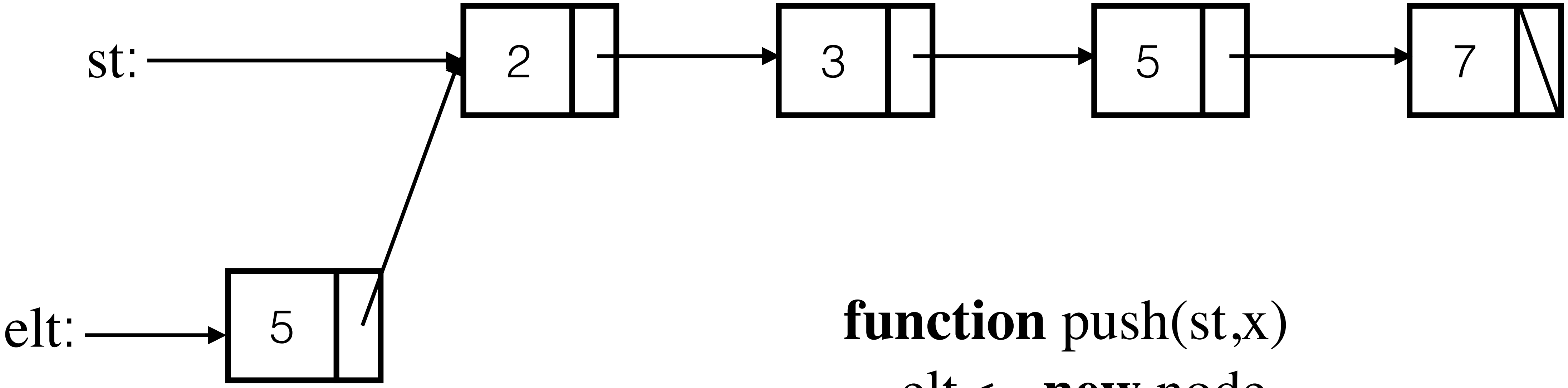

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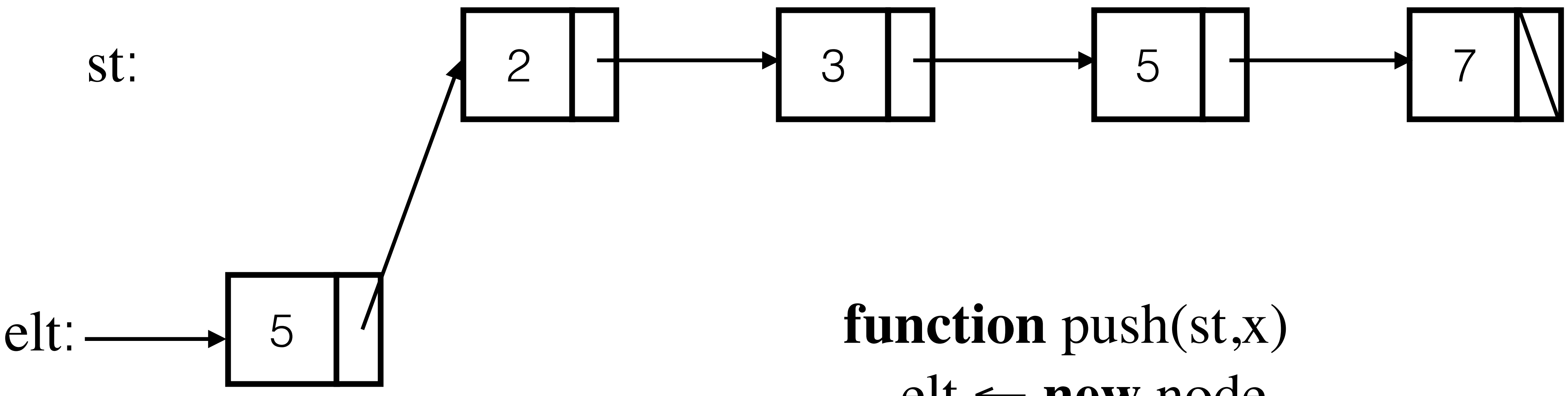

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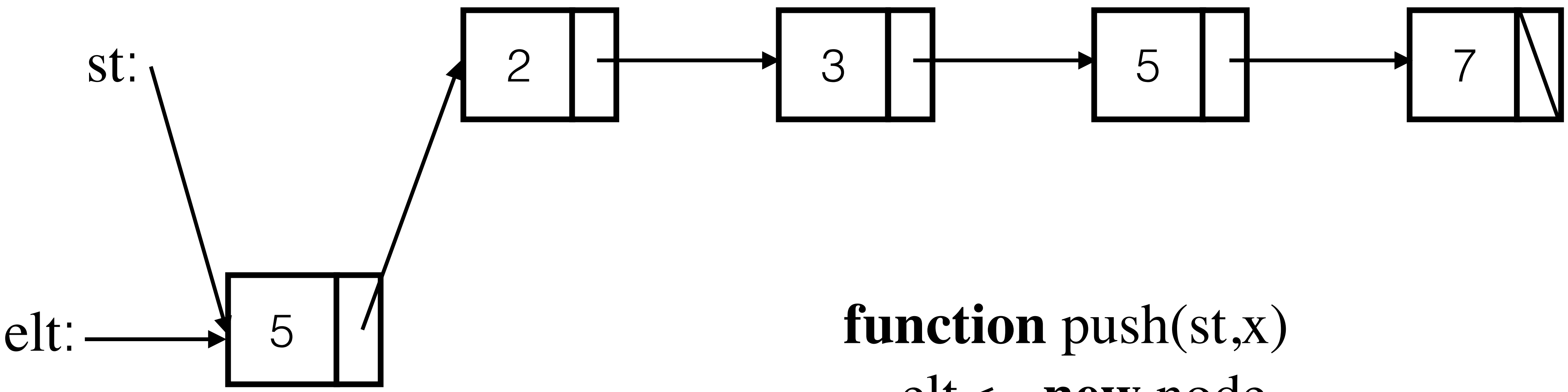

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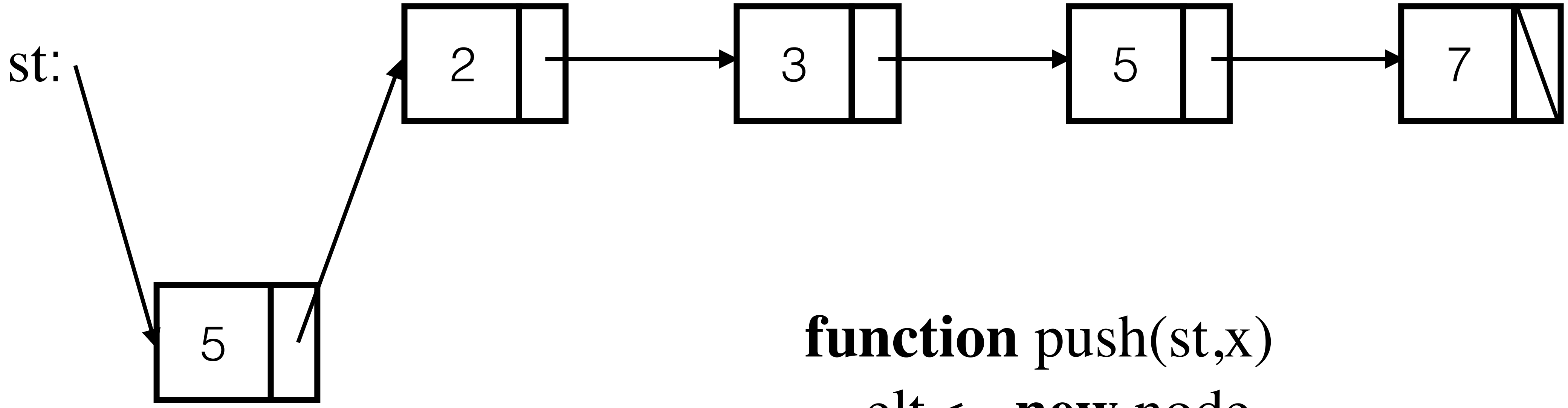

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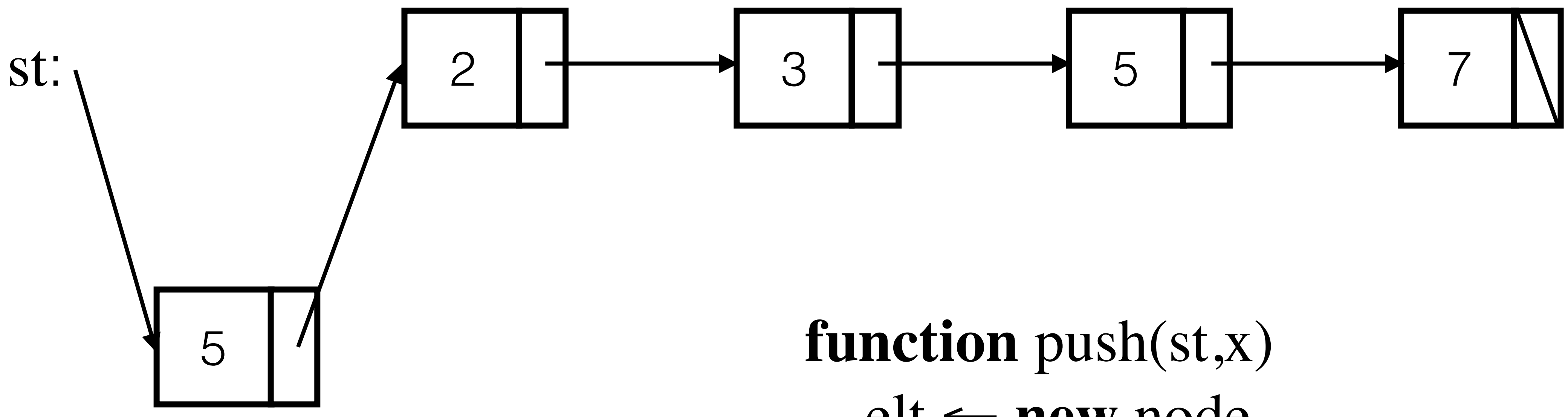

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See
https://www.cs.usfca.edu/~galles/visualization/Algorithms.html
for more visualisations

Pseudo Code

- On the previous slide, we assumed that a “node” has two attributes: a “val” which is its value, and a “next” which points to the rest of the list.
- There is no standard for pseudo-code. Use the examples in Levitin as a guide. Cormen et al. pages 20–22 (in Reading Resources) has a list of standard conventions used with pseudo-code which are good to follow, except we use ← as the assignment operator.

Fundamental Data Structure: Queues

- First-In-First-Out (FIFO)
- Operations:
  - CreateQueue
  - Enqueue
  - Dequeue
  - Head
  - EmptyQueue?
  - …

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Other Data Structures

- We will meet many other (abstract) data structures, e.g.
    - The priority queue
    - Various types of “tree”
    - Various types of “graph”
- If you check out algorithm animation tools or advanced algorithm books, you will meet exotic data structures such as splay trees and skip lists.

Next Week

- Algorithm analysis—how to reason about an algorithm's resource consumption.