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COMP90038 Algorithms and Complexity

Lecture 4: Analysis of Algorithms

(with thanks to Harald Søndergaard)

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@tobycmurray

Last Time: Time Complexity

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- Measure **input size** by natural number *n*
- Measure **execution time** as number of **basic operations** performed
- **Time complexity** *t(n)* for an algorithm: number of **basic operations** as a function of *n*
- How to **compare** different *t(n) ?*
  - Asymptotic growth rate
  - *O*(g(n)), Ω(*g*(*n*)), Θ(*g*(*n*))

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Problem	Size Measure	Basic Operation
Search in a list of *n* items	*n*	Key comparison
Multiply two matrices of floats	Matrix size (rows x columns)	Float multiplication
Compute a^n	log *n*	Float multiplication
Graph problem	Number of nodes and edges	Visiting a node

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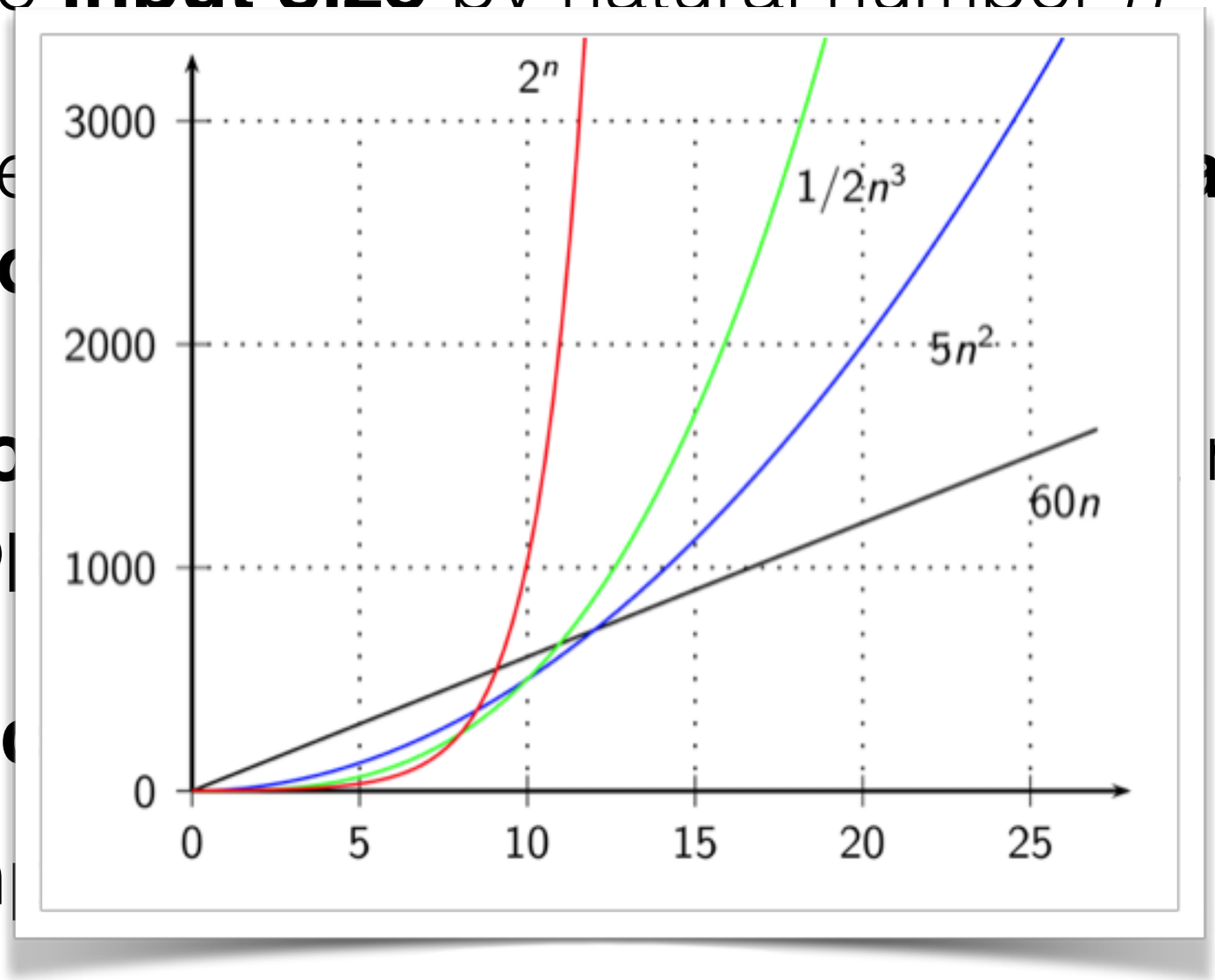

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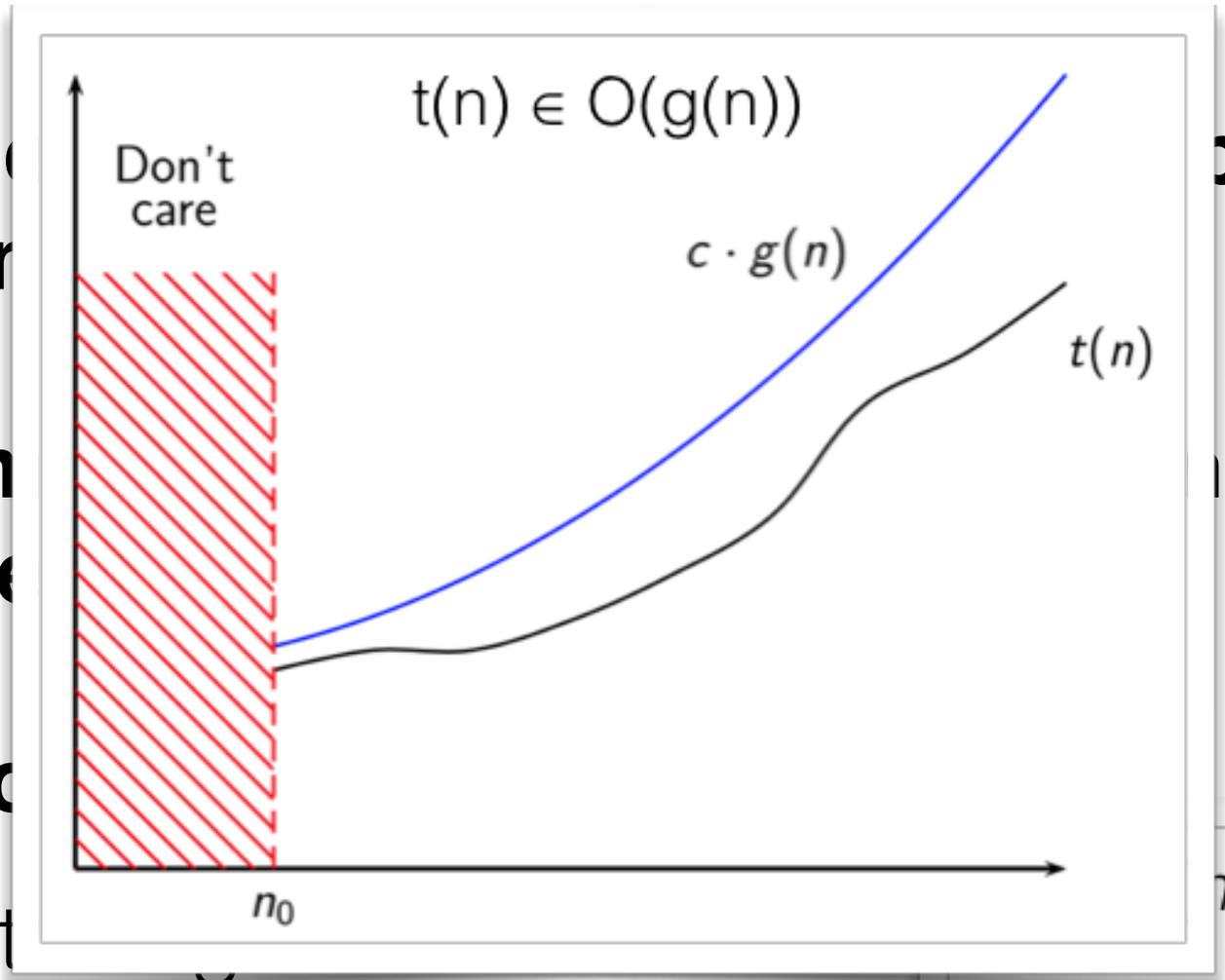

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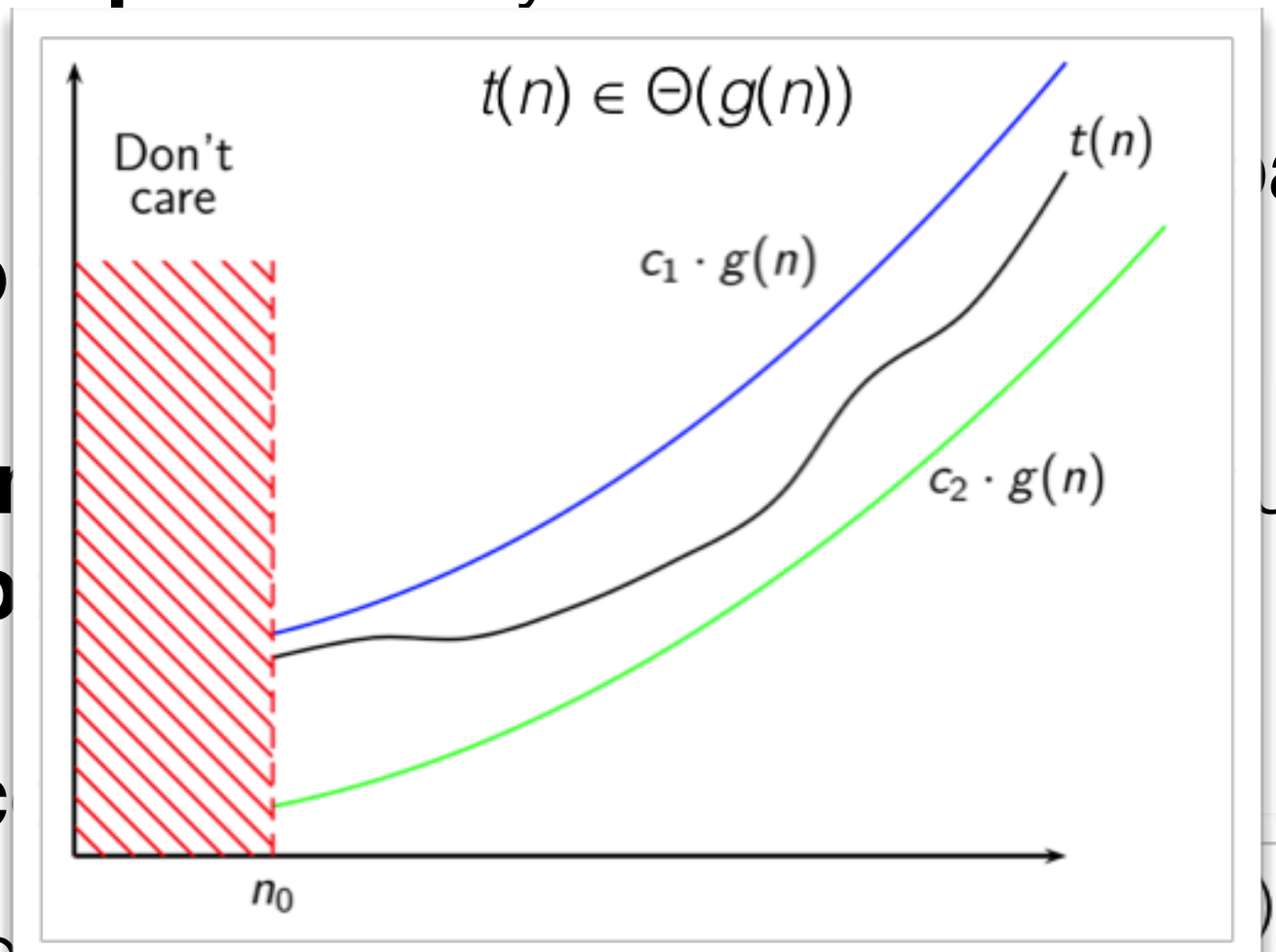

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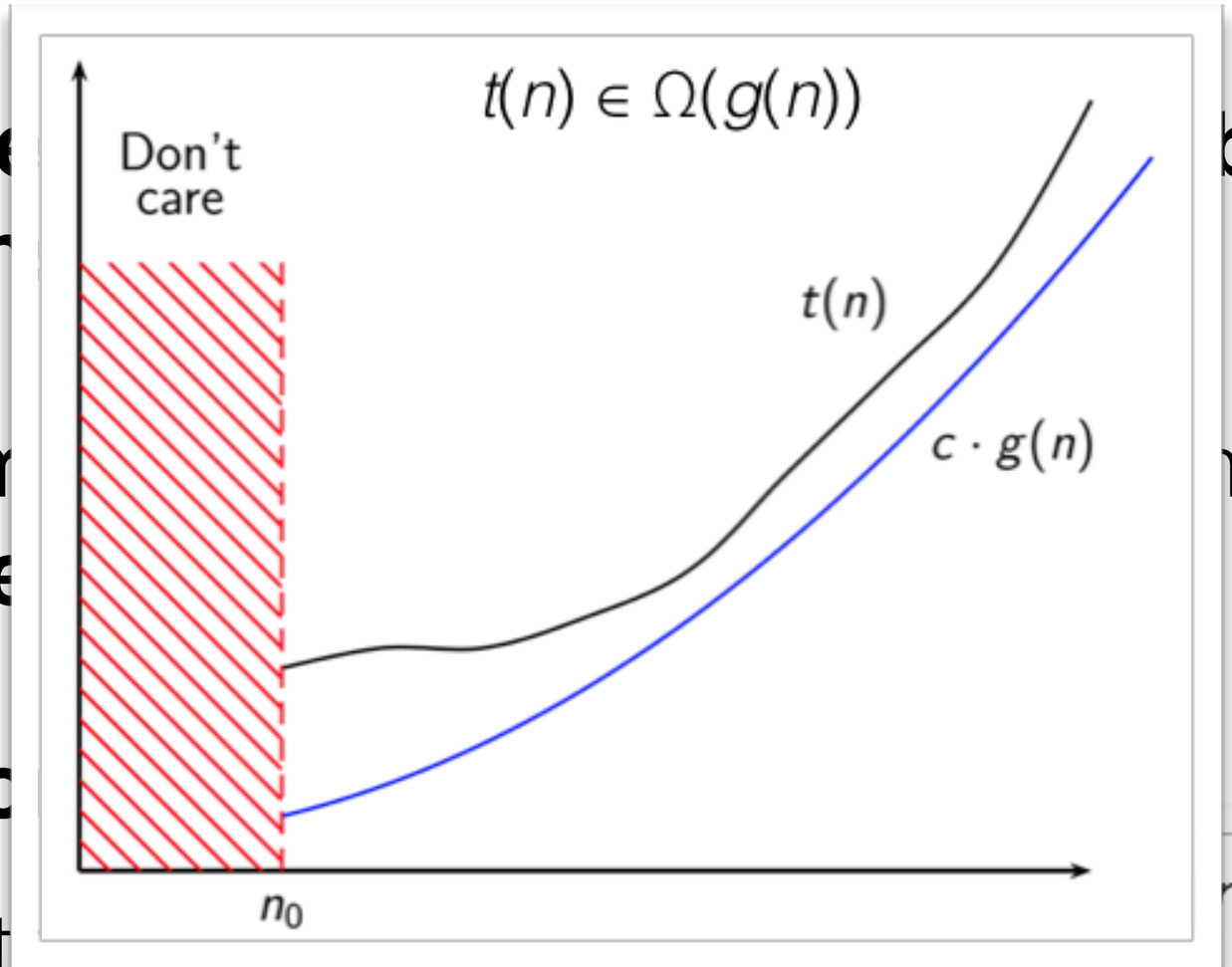

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Establishing Growth Rate

- In the last lecture we proved $t(n) \in O(g(n))$ for some cases of t and g, using the definition of O directly:

  $n > n_0 \Rightarrow t(n) < c \cdot g(n)$ for some c and n_0.

- $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & f \text{ grows asymptotically slower than } g \\ c & f \text{ and } g \text{ have same order of growth} \\ \infty & f \text{ grows asymptotically faster than } g \end{cases}$

- Use this to show that $1000n \in O(n^2)$

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$$\lim_{n \to \infty} \frac{1000n}{n^2}$$

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O, Ω, Θ

What this tells us about how $f(n)$ relates to $O(g(n))$, $\Omega(g(n))$ and $\Theta(g(n))$

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L'Hôpital's Rule

$$\lim_{h \to \infty} \frac{t(n)}{g(n)} = \lim_{h \to \infty} \frac{t'(n)}{g'(n)}$$

where t' and g' are the derivatives of t and g

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Finding Largest Element in an Array

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function MaxElement(A[·], n)
    max ← A[0]
    for i ← 1 to n − 1 do
        if A[i] > max then
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(where n is length of the array)

A:

23	12	42	6	69	18	3
0	1	2	3	4	5	6

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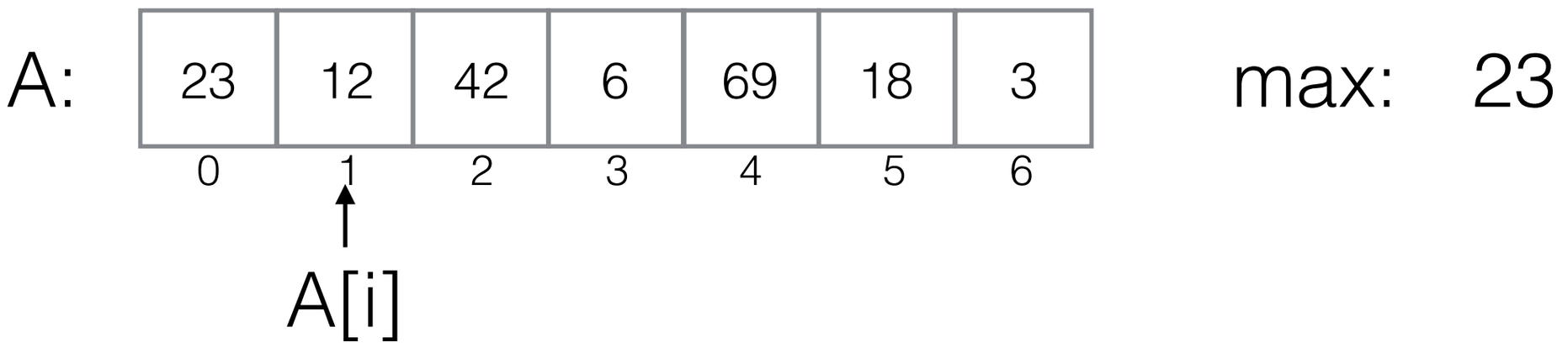

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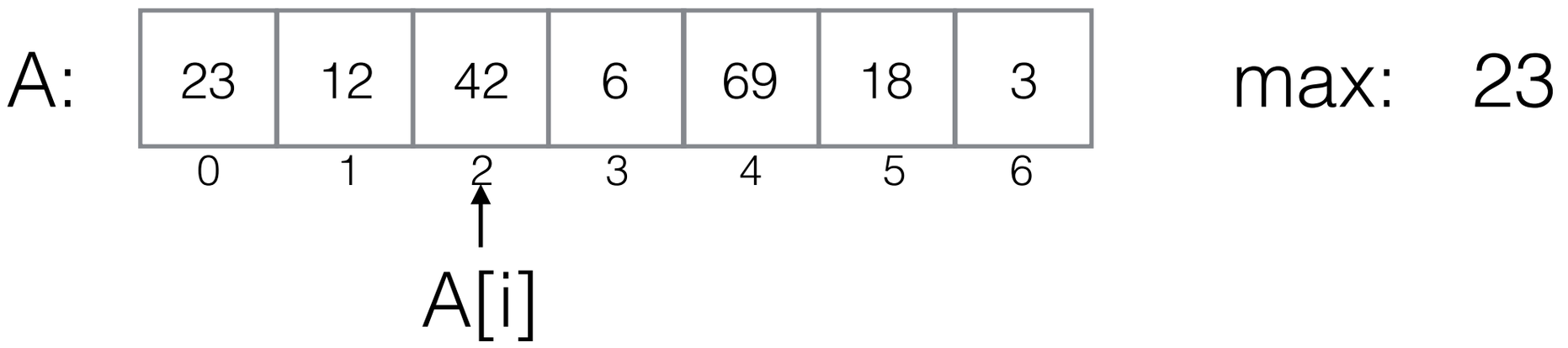

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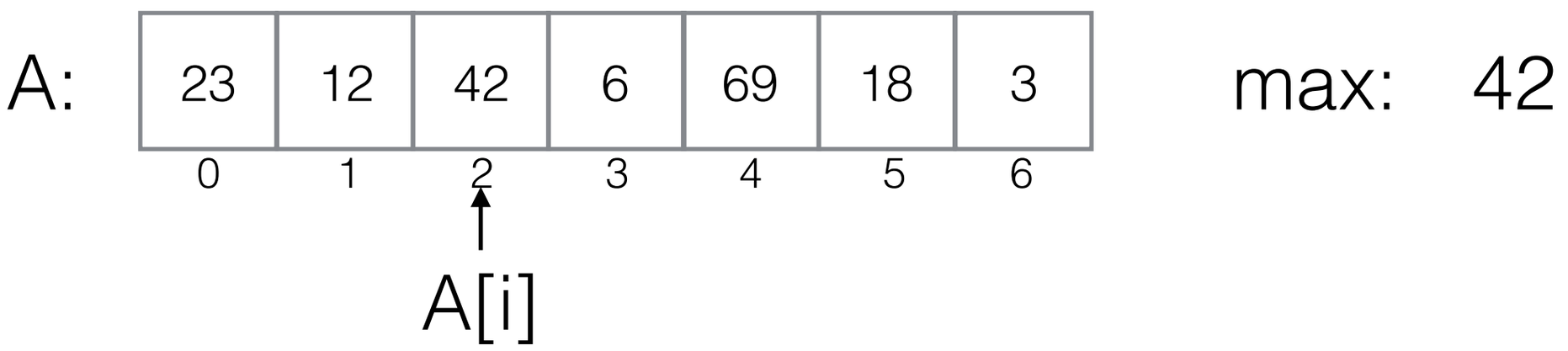

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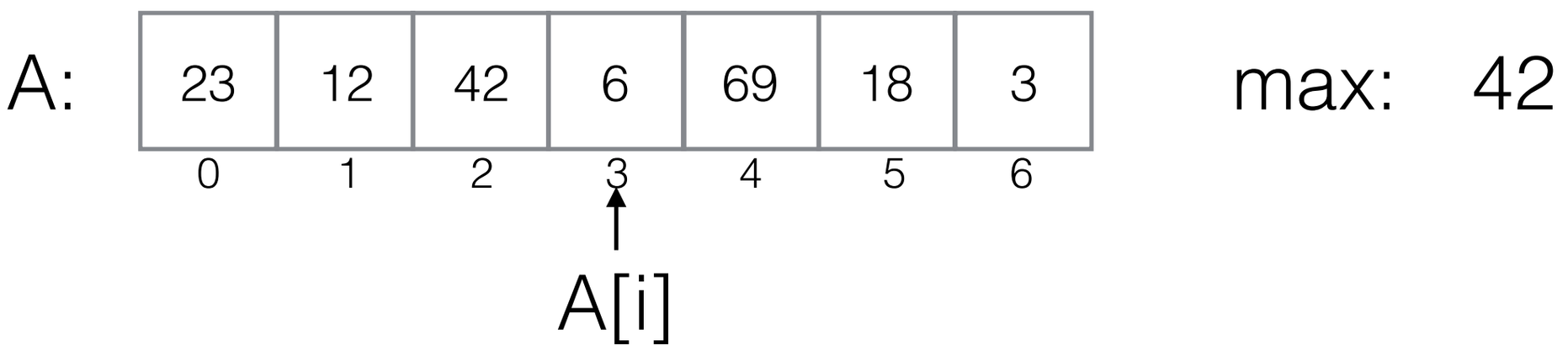

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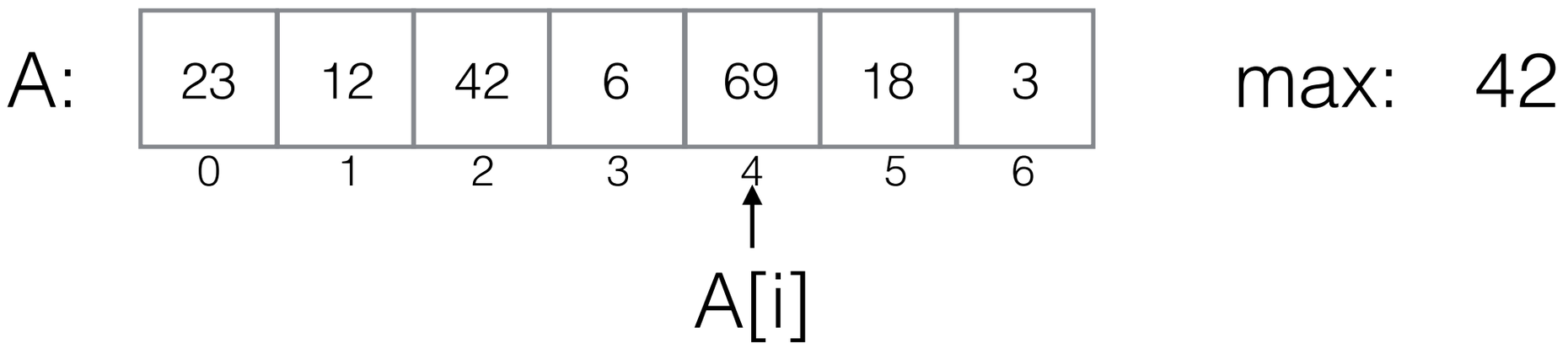

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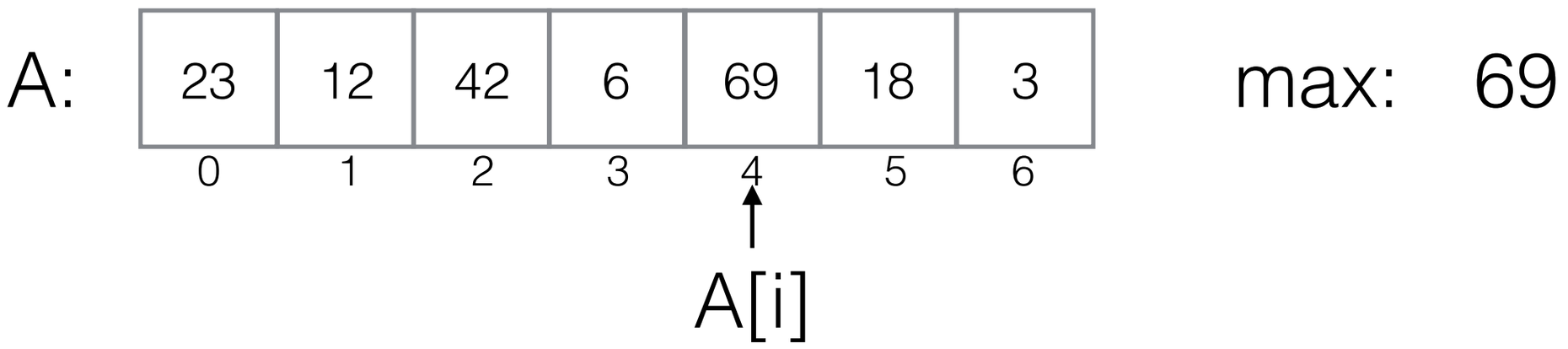

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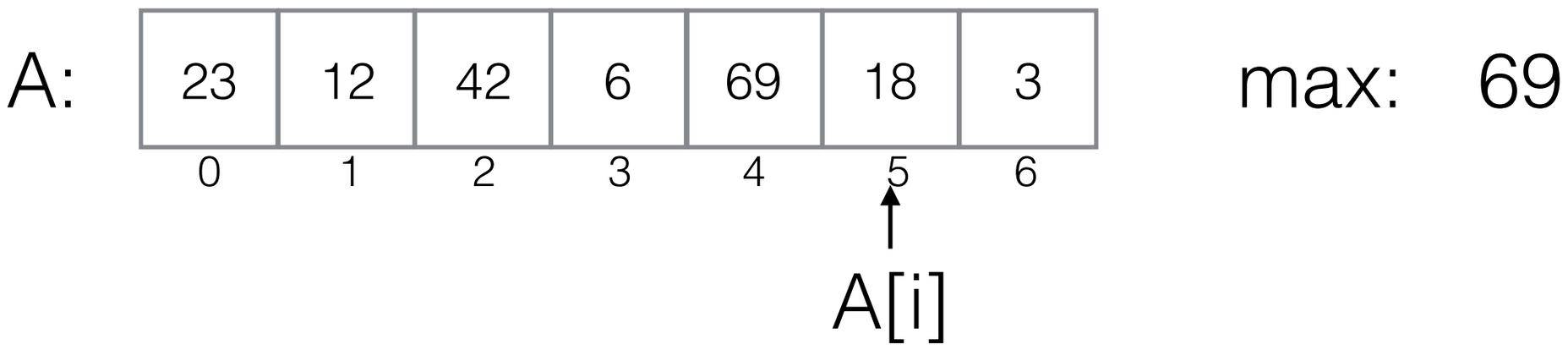

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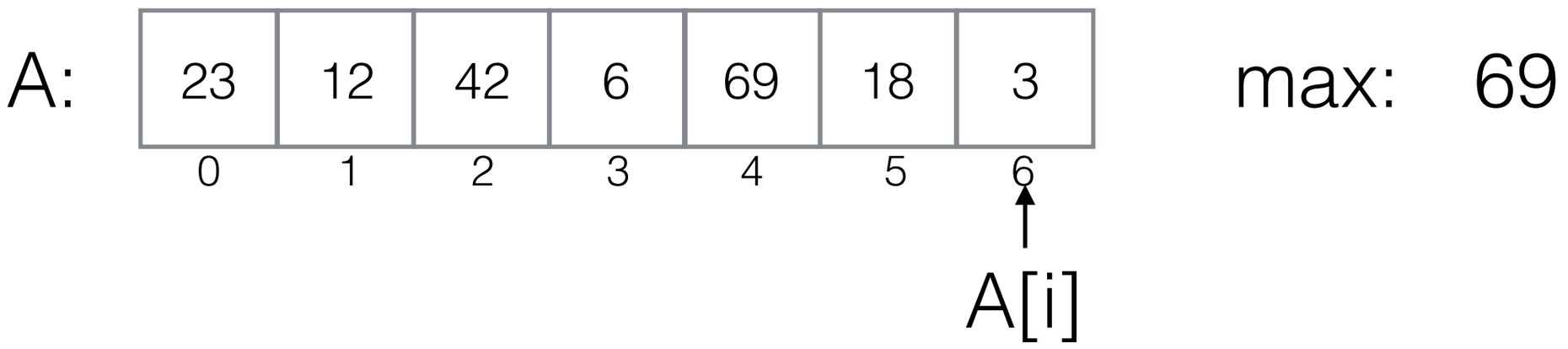

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Count the number of basic operations executed for an array of size n:

Example:
Finding Largest Element in an Array

THE UNIVERSITY OF MELBOURNE

```
function MaxElement(A[·], n)
    max ← A[0]
    for i ← 1 to n − 1 do
        if A[i] > max then
            max ← A[i]
    return max
```

(where *n* is length of the array)

Size of input, *n*:
length of the array

Basic operation: comparison “A[i] > max”

Count the number of basic operations executed for an array of size *n:*

$$1$$

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$$C(n) = \sum_{i=1}^{n-1} 1$$

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(where *n* is length of the array)

Size of input, *n*:
length of the array

Basic operation: comparison “A[i] > max”

Count the number of basic operations executed for an array of size *n:*

$$C(n) = \sum_{i=1}^{n-1} 1 = ((n-1) - 1 + 1)$$

Example:
Finding Largest Element in an Array

THE UNIVERSITY OF MELBOURNE

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(where *n* is length of the array)

Size of input, *n*: length of the array

Basic operation: c

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

Count the number of basic
for an array of size *n:*

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Finding Largest Element in an Array

THE UNIVERSITY OF MELBOURNE

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Count the number of basic
for an array of size *n:*

$$C(n) = \sum_{i=1}^{n-1} 1 = ((n-1) - 1 + 1) = n - 1$$

Example:
Finding Largest Element in an Array

THE UNIVERSITY OF MELBOURNE

function MaxElement($A[\cdot], n$)
    $max \leftarrow A[0]$
    for $i \leftarrow 1$ to $n-1$ **do**
        if $A[i] > max$ **then**
            $max \leftarrow A[i]$
    return *max*

(where n is length of the array)

Size of input, n:
length of the array

Basic operation: c

$$\sum_{i=l}^{u} 1 = \underbrace{1+1+\cdots+1}_{u-l+1 \text{ times}} = u-l+1$$

Count the number of basic
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Example: Selection Sort

THE UNIVERSITY OF MELBOURNE

```
function SelSort(A[·], n)
    for i ← 0 to n − 2 do
        min ← i
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```

23	12	42	6	69	18	3
0	1	2	3	4	5	6

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THE UNIVERSITY OF MELBOURNE

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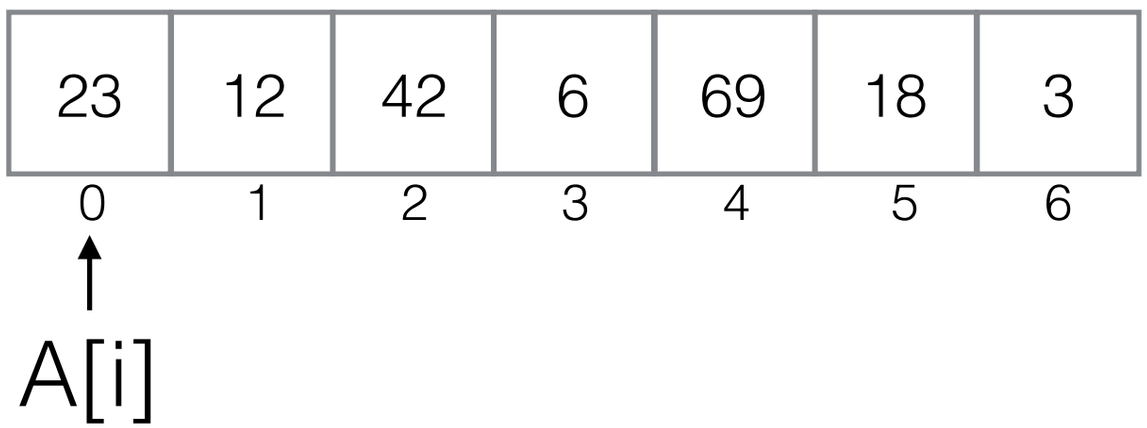

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THE UNIVERSITY OF MELBOURNE

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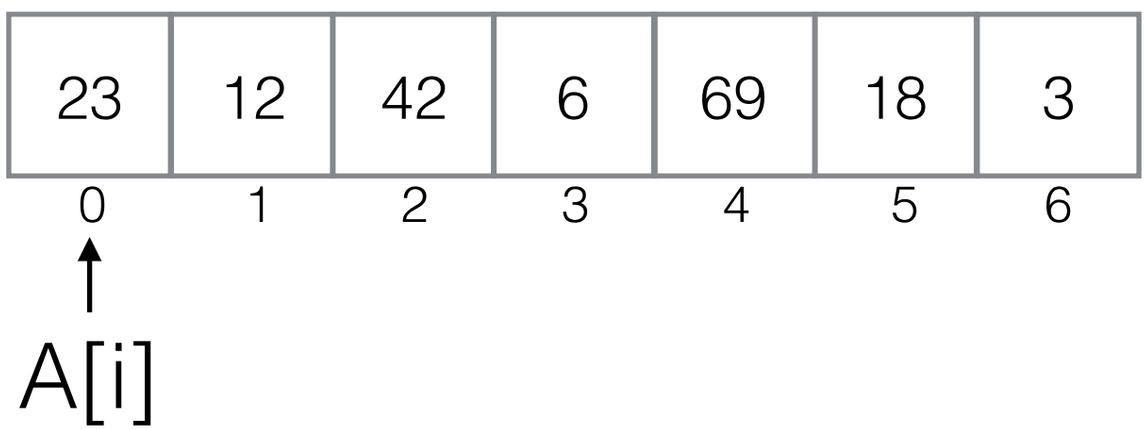

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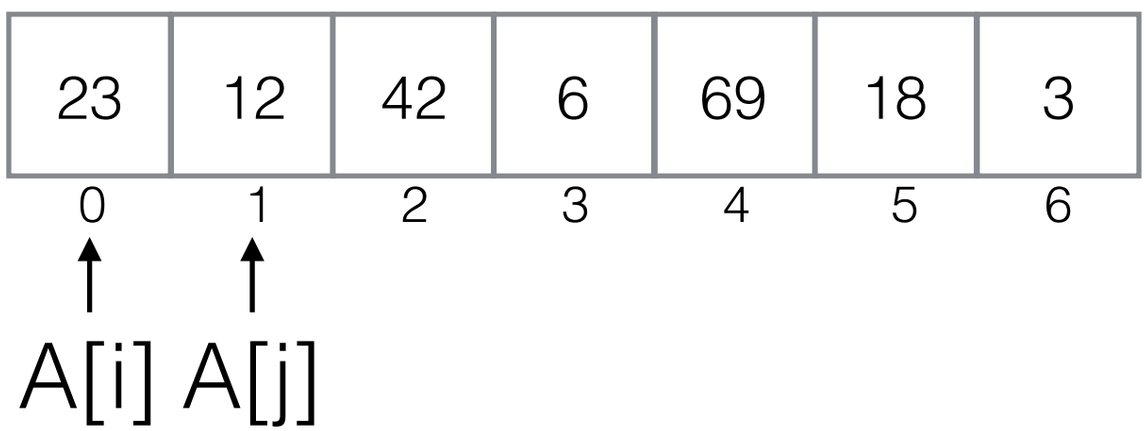

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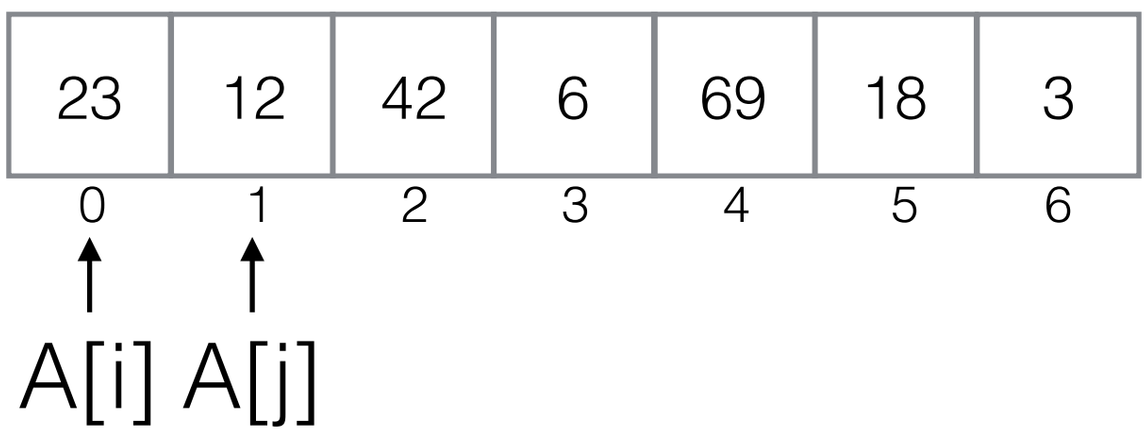

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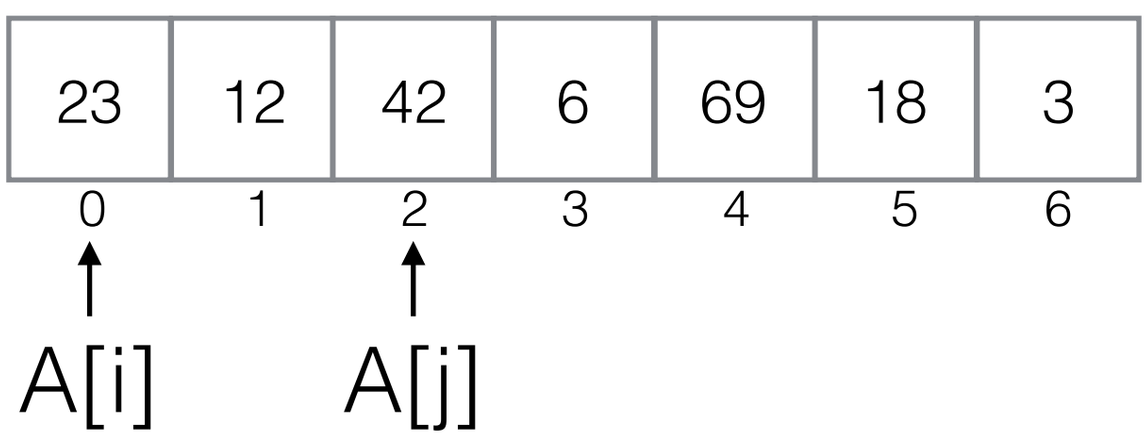

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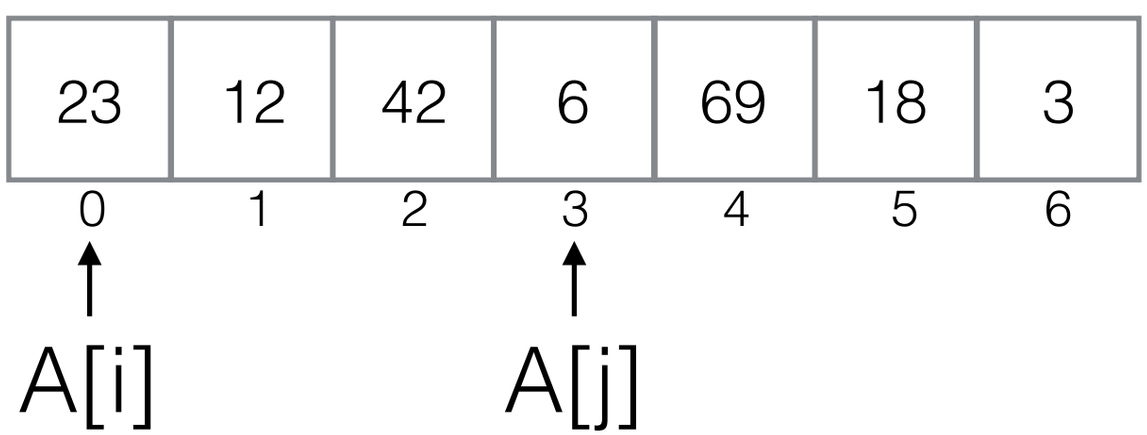

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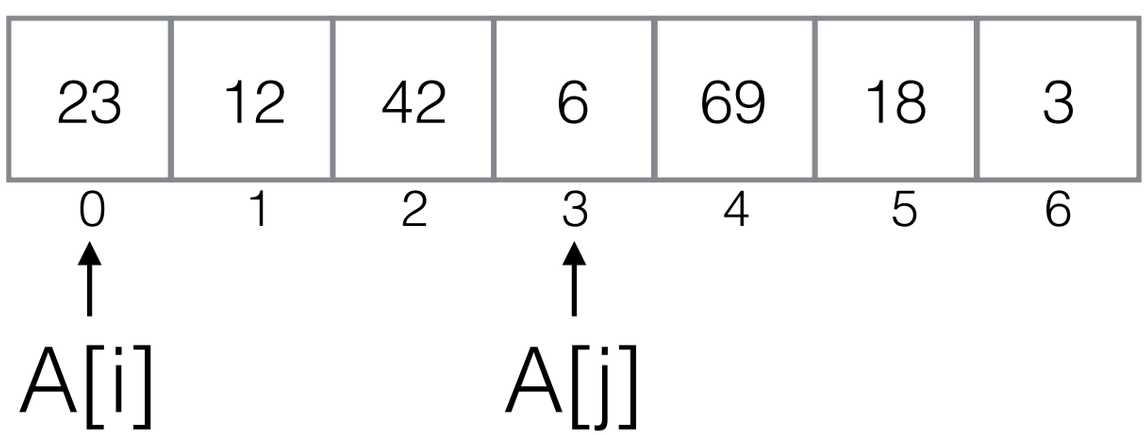

min: 3

Example: Selection Sort

THE UNIVERSITY OF MELBOURNE

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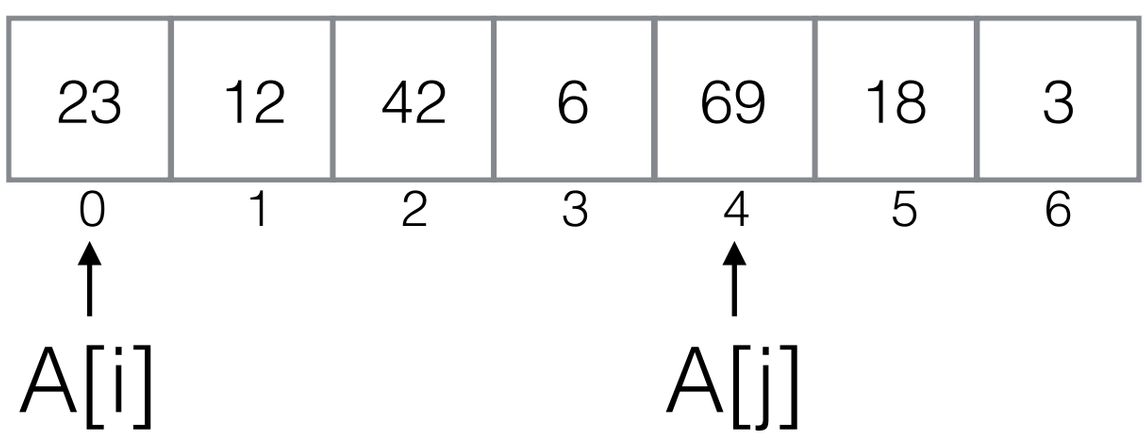

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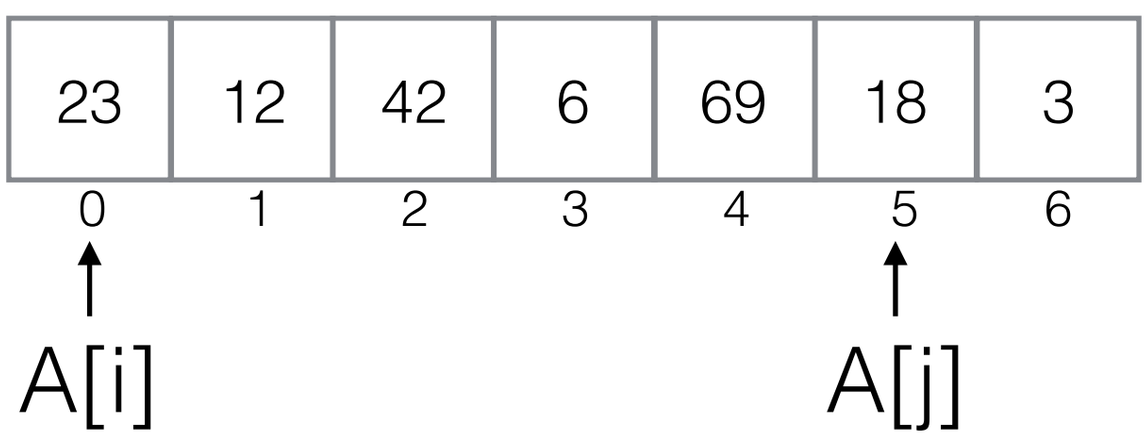

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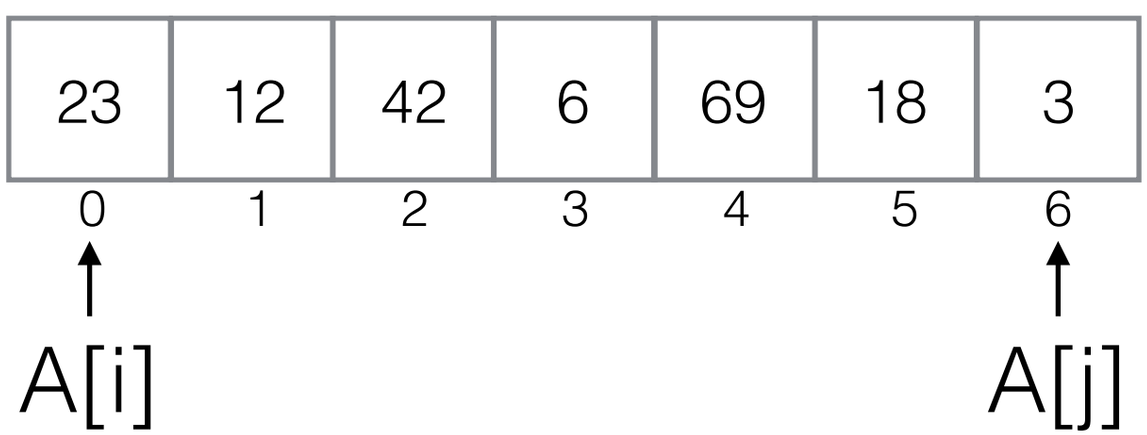

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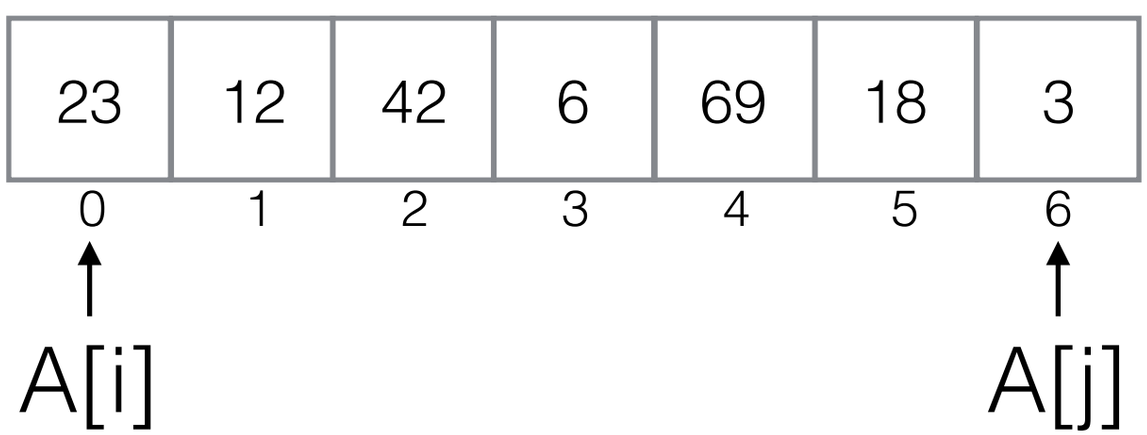

min: 6

Example: Selection Sort

THE UNIVERSITY OF MELBOURNE

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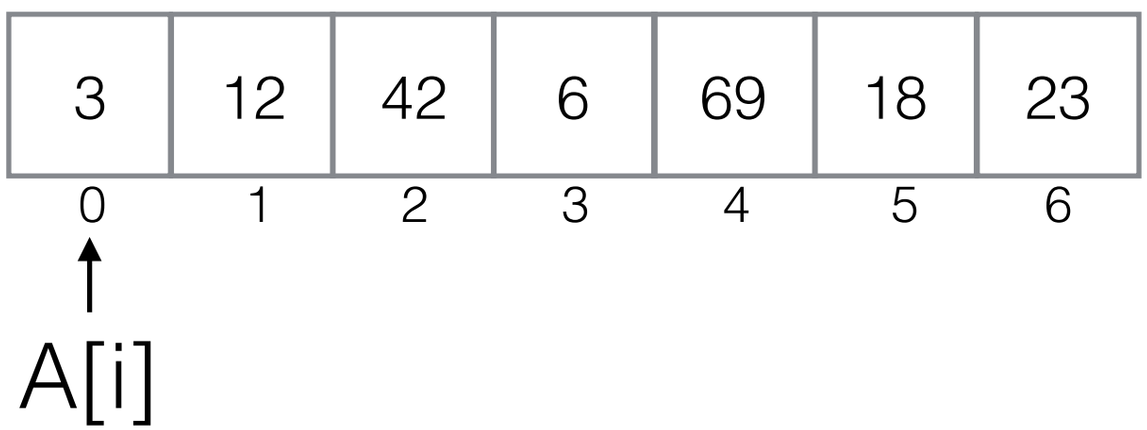

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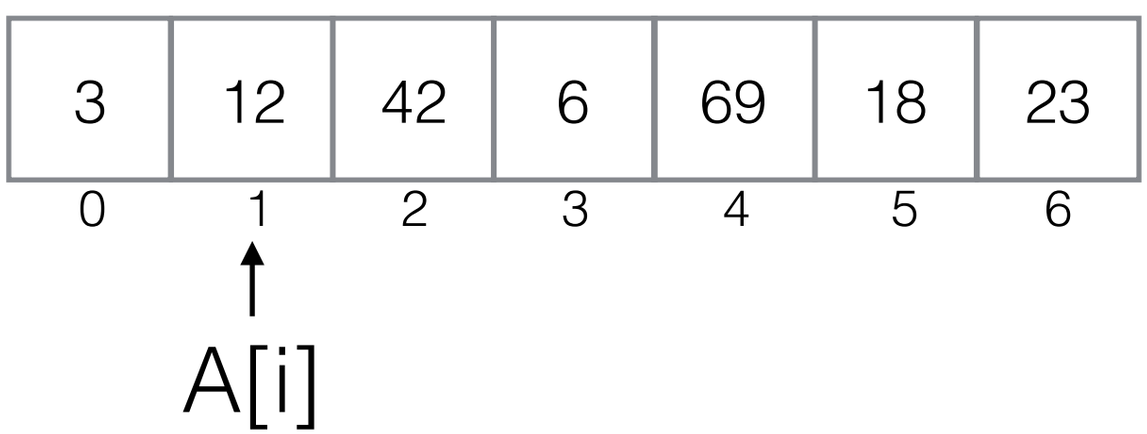

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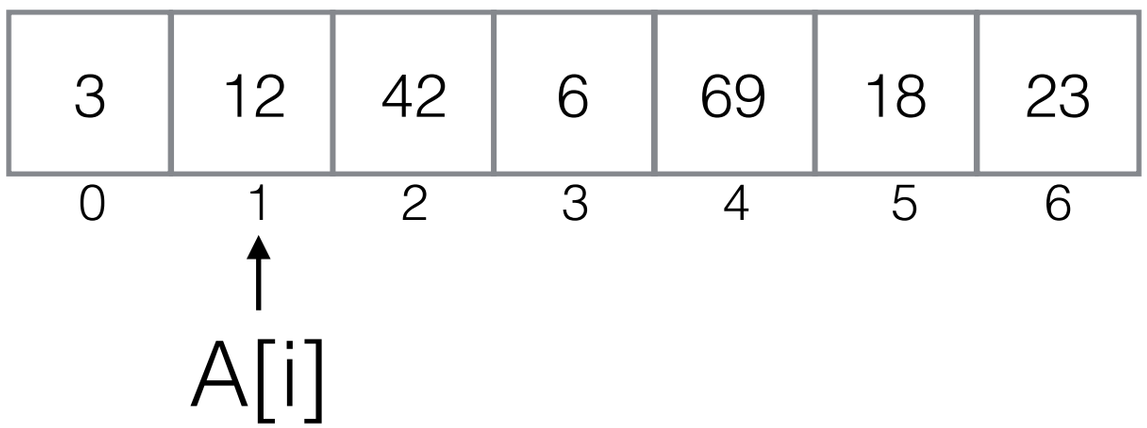

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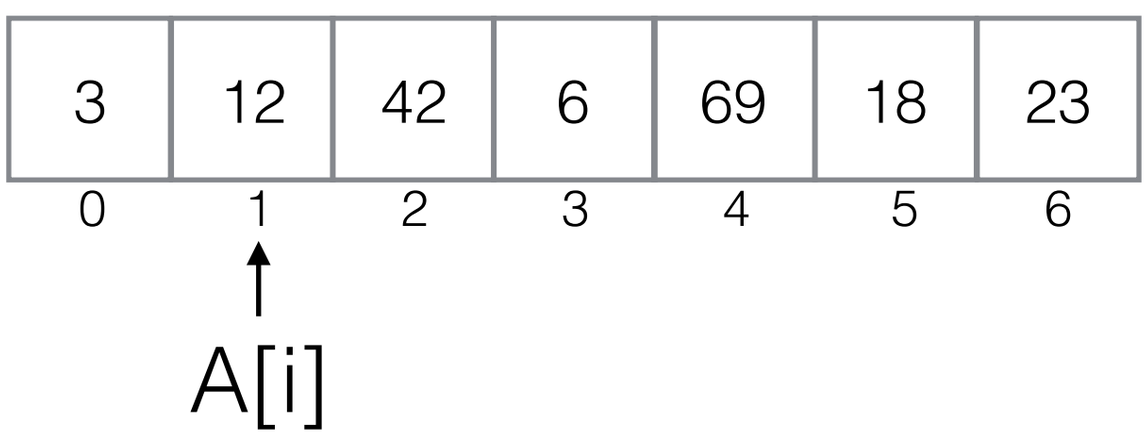

min: 1

Example: Selection Sort

THE UNIVERSITY OF MELBOURNE

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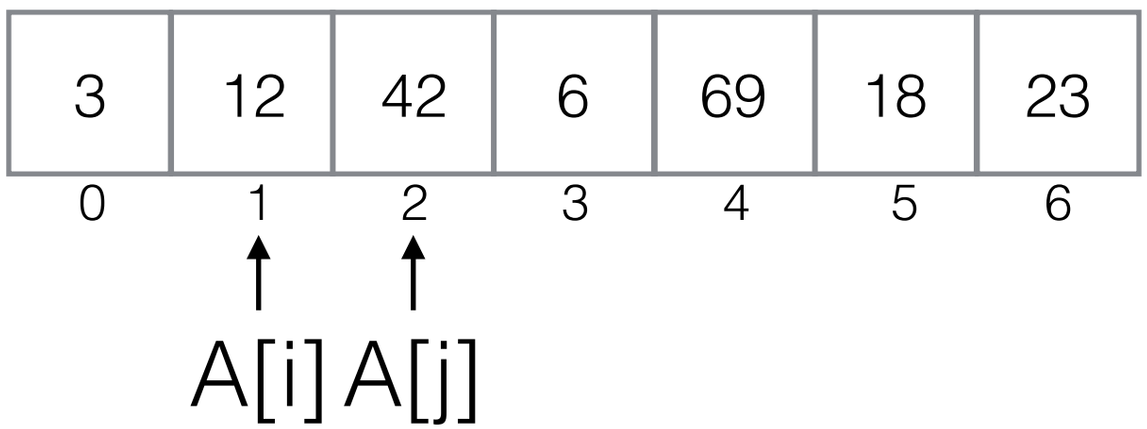

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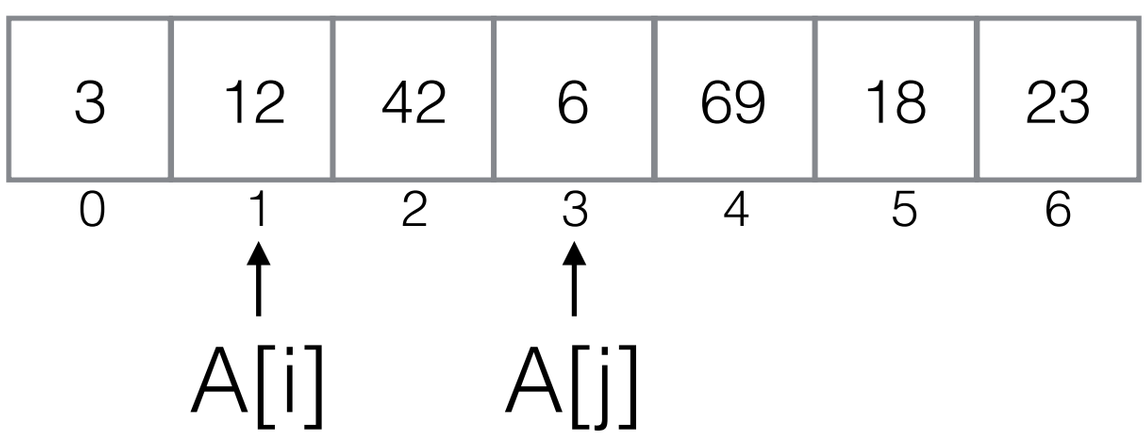

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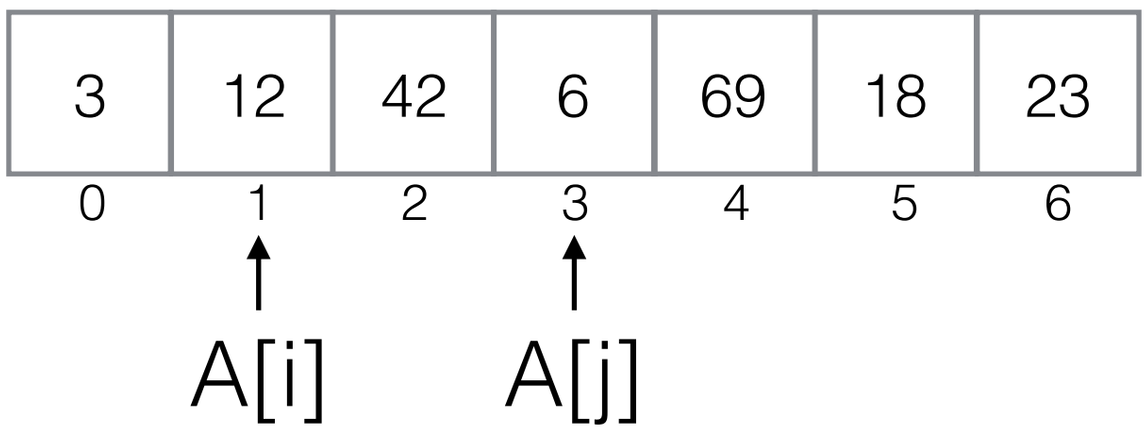

min: 3

Example: Selection Sort

THE UNIVERSITY OF MELBOURNE

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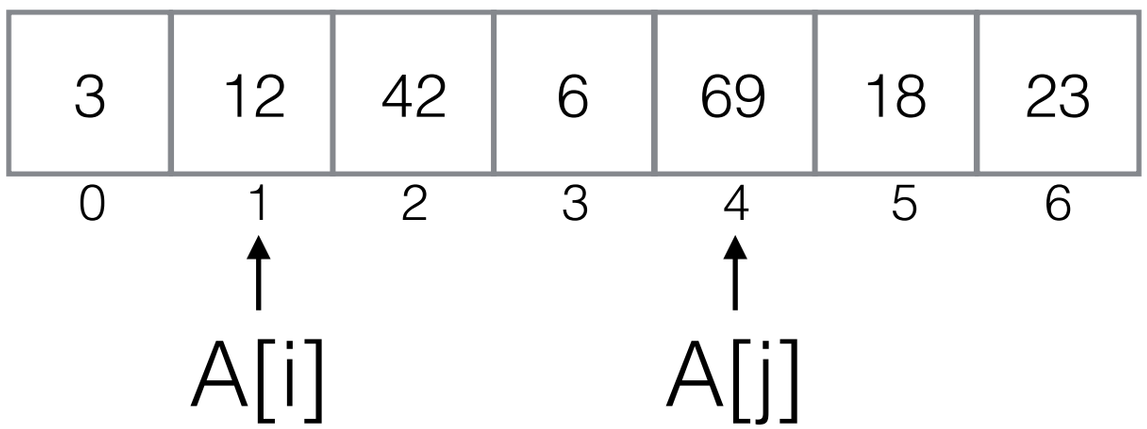

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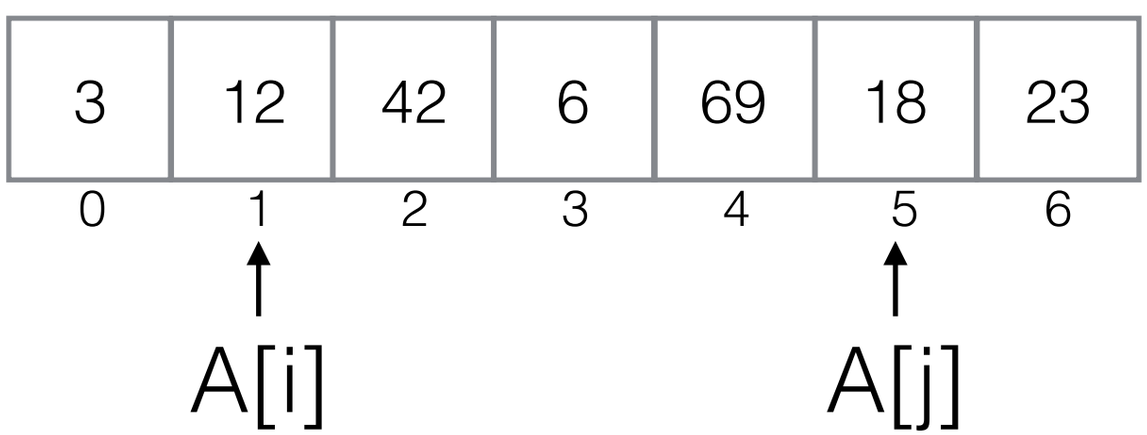

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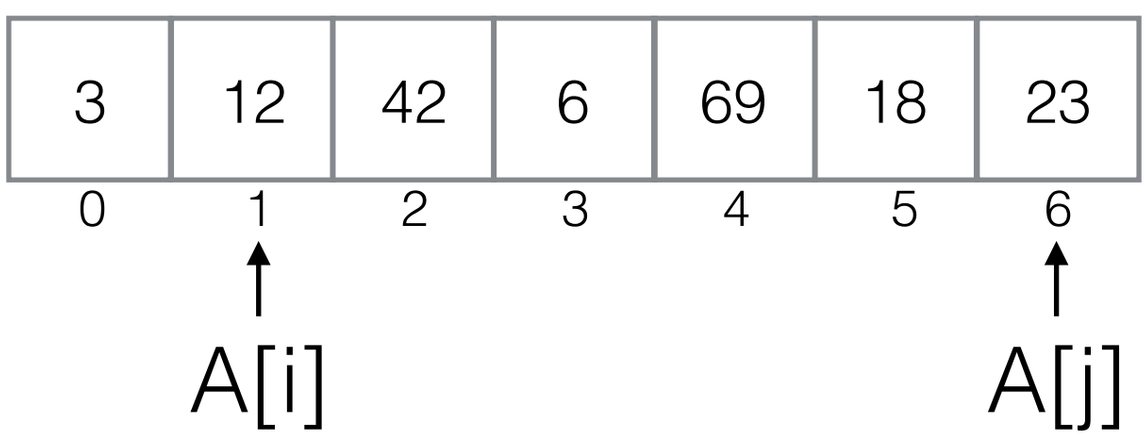

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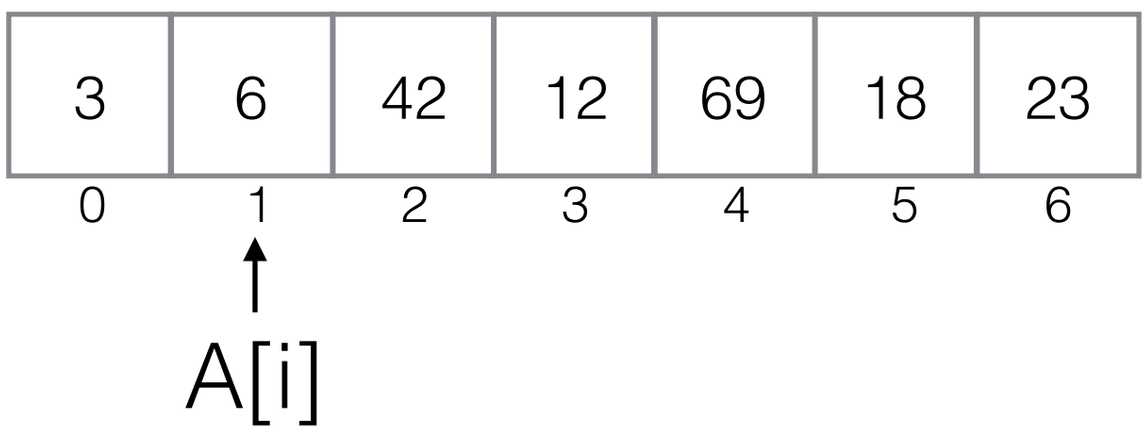

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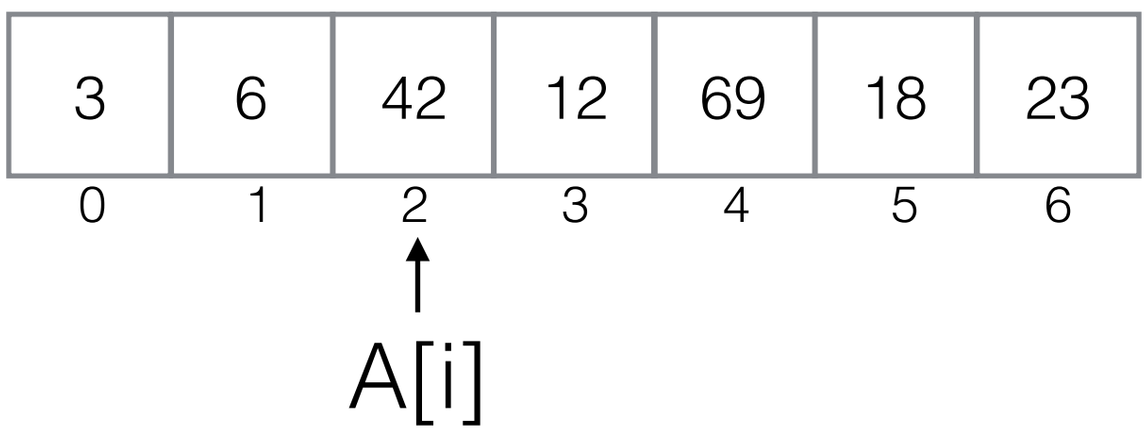

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Example: Selection Sort

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Input size n: length of the array

Basic operation: comparison A[j] < A[min]

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i)$$

Example: Selection Sort

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function SELSORT($A[\cdot], n$)
  for $i \leftarrow 0$ to $n-2$ **do**
    $min \leftarrow i$
    for $j \leftarrow i+1$ to $n-1$ **do**
      if $A[j] < A[min]$ **then**
        $min \leftarrow j$
    swap $A[i]$ and $A[min]$

Input size n: length of the array

Basic operation: comparison A[j] < A[min]

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i) = \sum_{i=0}^{n-2} (n-1) - \sum_{i=0}^{n-2} i$$

Example: Selection Sort

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```
function SelSort(A[·], n)
    for i ← 0 to n − 2 do
        min ← i
        for j ← i + 1 to n − 1 do
            if A[j] < A[min] then
                min ← j
        swap A[i] and A[min]
```

Input size n: length of the array

Basic operation: comparison A[j] < A[min]

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i) = \sum_{i=0}^{n-2} (n-1) - \sum_{i=0}^{n-2} i$$

$$= (n-1)^2 - \frac{(n-2)(n-1)}{2}$$

Example: Selection Sort

```
function SelSort(A[·], n)
    for i ← 0 to n − 2 do
        min ← i
        for j ← i + 1 to n − 1 do
            if A[j] < A[min] then
                min ← j
        swap A[i] and A[min]
```

Input size n: length of the array

Basic operation: comparison A[j] < A[min]

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i) = \sum_{i=0}^{n-2} (n-1) - \sum_{i=0}^{n-2} i$$

$$= (n-1)^2 - \frac{(n-2)(n-1)}{2} = \frac{n(n-1)}{2} \in \Theta(n^2)$$

Example: Matrix Multiplication

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function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} & \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

$$A = \begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad C = \begin{bmatrix} & \\ & \end{bmatrix}$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
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      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 0

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} & \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
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    for $j \leftarrow 0$ to $n-1$ **do**
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      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 0

$$A = \begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 0 & \\ & \end{bmatrix}$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 0

k: 0

$$A = \begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 0 & \\ & \end{bmatrix}$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 0

k: 0

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} 40 & \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 0

k: 1

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} 40 & \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 0

k: 1

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} 103 & \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 1

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} 103 & \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 1

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} 103 & 0 \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 1

k: 0

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} 103 & 0 \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 1

k: 0

$$A = \begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 103 & 10 \\ & \end{bmatrix}$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 1

k: 1

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} 103 & 10 \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

i: 0

j: 1

k: 1

$$\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 9 & 6 \end{bmatrix} \quad \begin{bmatrix} 103 & 52 \\ & \end{bmatrix}$$

A B C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

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  for $i \leftarrow 0$ to $n-1$ **do**
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      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

Basic operation: multiplication A[i,k] * B[k,j]

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
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        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

Basic operation: multiplication A[i,k] * B[k,j]

$$M(n) =$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

Basic operation: multiplication A[i,k] * B[k,j]

$$M(n) = \qquad\qquad 1$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

Basic operation: multiplication A[i,k] * B[k,j]

$$M(n) = \sum_{k=0}^{n-1} 1$$

Example: Matrix Multiplication

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function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
      $C[i,j] \leftarrow 0.0$
      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

Basic operation: multiplication A[i,k] * B[k,j]

$$M(n) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

function MATRIXMULT($A[\cdot,\cdot], B[\cdot,\cdot], n$) ▷ For $n \times n$ matrices
  for $i \leftarrow 0$ to $n-1$ **do**
    for $j \leftarrow 0$ to $n-1$ **do**
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      for $k \leftarrow 0$ to $n-1$ **do**
        $C[i,j] \leftarrow C[i,j] + A[i,k] \cdot B[k,j]$
  return C

Basic operation: multiplication A[i,k] * B[k,j]

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

Example: Matrix Multiplication

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$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

Example: Matrix Multiplication

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$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

Example: Matrix Multiplication

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$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n-1) - 0 + 1)$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

Example: Matrix Multiplication

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$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n-1) - 0 + 1) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n-1) - 0 + 1) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n \cdot 1)$$

Example: Matrix Multiplication

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$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n-1) - 0 + 1) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n \cdot 1)$$

$$\sum_{i=l}^{u} c a_i = c \sum_{i=l}^{u} a_i$$

Example: Matrix Multiplication

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n-1) - 0 + 1) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n \cdot 1)$$

$$= \sum_{i=0}^{n-1} \left(n \cdot \sum_{j=0}^{n-1} 1 \right)$$

$$\sum_{i=l}^{u} c a_i = c \sum_{i=l}^{u} a_i$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n-1) - 0 + 1) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n \cdot 1)$$

$$= \sum_{i=0}^{n-1} \left(n \cdot \sum_{j=0}^{n-1} 1 \right) = \sum_{i=0}^{n-1} n^2$$

$$\sum_{i=l}^{u} c a_i = c \sum_{i=l}^{u} a_i$$

Example: Matrix Multiplication

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$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n-1) - 0 + 1) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n \cdot 1)$$

$$= \sum_{i=0}^{n-1} \left(n \cdot \sum_{j=0}^{n-1} 1 \right) = \sum_{i=0}^{n-1} n^2$$

$$= n^3$$

$$\sum_{i=l}^{u} ca_i = c \sum_{i=l}^{u} a_i$$

Example: Matrix Multiplication

THE UNIVERSITY OF MELBOURNE

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n-1) - 0 + 1) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n \cdot 1)$$

$$= \sum_{i=0}^{n-1} \left(n \cdot \sum_{j=0}^{n-1} 1 \right) = \sum_{i=0}^{n-1} n^2$$

$$\sum_{i=l}^{u} ca_i = c \sum_{i=l}^{u} a_i$$

$$= n^3$$

$$\in \Theta(n^3)$$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
    if $n = 0$ **then return** 1
    else return $\mathrm{F}(n-1) \cdot n$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
    if $n = 0$ **then return** 1
    else return $\mathrm{F}(n-1) \cdot n$

$F(5)$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
    if $n = 0$ **then return** 1
    else return $\mathrm{F}(n-1) \cdot n$

$$F(5) = F(4) \cdot 5$$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
    if $n = 0$ **then return** 1
    else return $\mathrm{F}(n-1) \cdot n$

$$\begin{aligned} F(5) &= F(4) \cdot 5 \\ &= (F(3) \cdot 4) \cdot 5 \end{aligned}$$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
    if $n = 0$ **then return** 1
    else return $\mathrm{F}(n-1) \cdot n$

$$
\begin{aligned}
F(5) &= F(4) \cdot 5 \\
&= (F(3) \cdot 4) \cdot 5 \\
&= ((F(2) \cdot 3) \cdot 4) \cdot 5
\end{aligned}
$$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
  if $n = 0$ **then return** 1
  else return $\mathrm{F}(n-1) \cdot n$

$$
\begin{aligned}
F(5) &= F(4) \cdot 5 \\
&= (F(3) \cdot 4) \cdot 5 \\
&= ((F(2) \cdot 3) \cdot 4) \cdot 5 \\
&= (((F(1) \cdot 2) \cdot 3) \cdot 4) \cdot 5
\end{aligned}
$$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
  if $n = 0$ **then return** 1
  else return $\mathrm{F}(n-1) \cdot n$

$$
\begin{aligned}
F(5) &= F(4) \cdot 5 \\
&= (F(3) \cdot 4) \cdot 5 \\
&= ((F(2) \cdot 3) \cdot 4) \cdot 5 \\
&= (((F(1) \cdot 2) \cdot 3) \cdot 4) \cdot 5 \\
&= ((((F(0) \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5
\end{aligned}
$$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
  if $n = 0$ **then return** 1
  else return $\mathrm{F}(n-1) \cdot n$

$$
\begin{aligned}
F(5) &= F(4) \cdot 5 \\
&= (F(3) \cdot 4) \cdot 5 \\
&= ((F(2) \cdot 3) \cdot 4) \cdot 5 \\
&= (((F(1) \cdot 2) \cdot 3) \cdot 4) \cdot 5 \\
&= ((((F(0) \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5 \\
&= ((((1 \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5
\end{aligned}
$$

Analysing Recursive Algorithms

THE UNIVERSITY OF MELBOURNE

function $\mathrm{F}(n)$
  if $n = 0$ **then return** 1
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$$
\begin{aligned}
F(5) &= F(4) \cdot 5 \\
&= (F(3) \cdot 4) \cdot 5 \\
&= ((F(2) \cdot 3) \cdot 4) \cdot 5 \\
&= (((F(1) \cdot 2) \cdot 3) \cdot 4) \cdot 5 \\
&= ((((F(0) \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5 \\
&= ((((1 \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5 \\
&= 5!
\end{aligned}
$$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
    if $n = 0$ **then return** 1
    else return $\mathrm{F}(n-1) \cdot n$

Analysing Recursive Algorithms

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function $\mathrm{F}(n)$
    if $n = 0$ **then return** 1
    else return $\mathrm{F}(n-1) \cdot n$

Basic operation: multiplication

Analysing Recursive Algorithms

function $\mathrm{F}(n)$
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Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**)

Analysing Recursive Algorithms

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Basic operation: multiplication

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$$M(0) =$$

Analysing Recursive Algorithms

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Analysing Recursive Algorithms

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Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**)

$$M(0) = \quad 0$$
$$M(n) =$$

Analysing Recursive Algorithms

function $\mathrm{F}(n)$
  if $n = 0$ **then return** 1
  else return $\mathrm{F}(n-1) \cdot n$

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**)

$$M(0) = 0$$
$$M(n) = M(n-1) + 1$$

Analysing Recursive Algorithms

function $\mathrm{F}(n)$
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Analysing Recursive Algorithms

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Analysing Recursive Algorithms

THE UNIVERSITY OF MELBOURNE

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$$M(0) = 0$$
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Need to express M(n) in **closed form** (i.e. non-recursively)

Analysing Recursive Algorithms

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We express the cost **recursively** (as a **recurrence relation**)

$$M(0) = 0$$
$$M(n) = M(n-1) + 1$$

Need to express M(n) in **closed form** (i.e. non-recursively)

Try: **“telescoping”** aka **“backward substitution”**

Telescoping
(aka Backward Substitution)

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$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

Telescoping
(aka Backward Substitution)

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$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1) ?

Telescoping
(aka Backward Substitution)

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$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ? $M(n-1) = M((n-1)-1) + 1$

Telescoping
(aka Backward Substitution)

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$\begin{aligned} M(n-1) &= M((n-1)-1) + 1 \\ &= M(n-2) + 1 \end{aligned}$$

Telescoping
(aka Backward Substitution)

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$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$M(n) =$$

Telescoping
(aka Backward Substitution)

THE UNIVERSITY OF MELBOURNE

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$M(n) = (M(n-2) + 1) + 1$$

Telescoping
(aka Backward Substitution)

THE UNIVERSITY OF MELBOURNE

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$M(n) = (M(n-2) + 1) + 1$$
$$= M(n-2) + 2$$

Telescoping
(aka Backward Substitution)

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$M(n) = (M(n-2) + 1) + 1$$
$$= M(n-2) + 2$$
$$= (M(n-3) + 1) + 2$$

Telescoping
(aka Backward Substitution)

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$M(n) = (M(n-2) + 1) + 1$$
$$= M(n-2) + 2$$
$$= (M(n-3) + 1) + 2$$
$$= M(n-3) + 3$$

Telescoping
(aka Backward Substitution)

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$M(n) = (M(n-2) + 1) + 1$$
$$= M(n-2) + 2$$
$$= (M(n-3) + 1) + 2$$
$$= M(n-3) + 3$$
$$\cdots$$

Telescoping
(aka Backward Substitution)

THE UNIVERSITY OF MELBOURNE

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$M(n) = (M(n-2) + 1) + 1$$
$$= M(n-2) + 2$$
$$= (M(n-3) + 1) + 2$$
$$= M(n-3) + 3$$
$$\cdots$$
$$= M(n-n) + n$$

Telescoping
(aka Backward Substitution)

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

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$$= M(n-2) + 2$$
$$= (M(n-3) + 1) + 2$$
$$= M(n-3) + 3$$
$$\cdots$$
$$= M(n-n) + n$$
$$= M(0) + n$$

Telescoping
(aka Backward Substitution)

THE UNIVERSITY OF MELBOURNE

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

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$$\cdots$$
$$= M(n-n) + n$$
$$= M(0) + n$$
$$= n$$

Telescoping
(aka Backward Substitution)

THE UNIVERSITY OF MELBOURNE

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$M(n) = (M(n-2) + 1) + 1$$
$$= M(n-2) + 2$$
$$= (M(n-3) + 1) + 2$$
$$= M(n-3) + 3$$
$$\cdots$$
$$= M(n-n) + n$$
$$= M(0) + n$$
$$= n$$

Closed form:

Telescoping
(aka Backward Substitution)

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$\begin{aligned}
M(n) &= (M(n-2)+1)+1 \\
&= M(n-2)+2 \\
&= (M(n-3)+1)+2 \\
&= M(n-3)+3 \\
&\cdots \\
&= M(n-n)+n \\
&= M(0)+n \\
&= n
\end{aligned}$$

Closed form:

$$M(n) = n$$

Telescoping
(aka Backward Substitution)

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$\begin{aligned}
M(n) &= (M(n-2)+1)+1 \\
&= M(n-2)+2 \\
&= (M(n-3)+1)+2 \\
&= M(n-3)+3 \\
&\cdots \\
&= M(n-n)+n \\
&= M(0)+n \\
&= n
\end{aligned}$$

Closed form:

$$M(n) = n$$

Complexity:

Telescoping
(aka Backward Substitution)

$$M(n) = M(n-1) + 1 \qquad M(0) = 0$$

What is M(n-1) ?

$$M(n-1) = M((n-1)-1) + 1$$
$$= M(n-2) + 1$$

$$\begin{aligned}
M(n) &= (M(n-2)+1)+1 \\
&= M(n-2)+2 \\
&= (M(n-3)+1)+2 \\
&= M(n-3)+3 \\
&\cdots \\
&= M(n-n)+n \\
&= M(0)+n \\
&= n
\end{aligned}$$

Closed form:

$$M(n) = n$$

Complexity:

$$M(n) \in \Theta(n)$$

Example:
Binary Search in Sorted Array

THE UNIVERSITY OF MELBOURNE

```
function BinSearch(A[·], lo, hi, key)
    if lo > hi then return −1
    mid ← lo + (hi − lo)/2
    if A[mid] = key then return mid
    else
        if A[mid] > key then
            return BinSearch(A, lo, mid − 1, key)
        else return BinSearch(A, mid + 1, hi, key)
```

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

Example:
Binary Search in Sorted Array

THE UNIVERSITY OF MELBOURNE

```
function BinSearch(A[·], lo, hi, key)
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BinSearch(A,0,6,41)

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```

lo: 0

A:

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```

lo: 0

hi: 6

A:

4	9	13	22	41	83	96
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BinSearch(A,0,6,41)

Example:
Binary Search in Sorted Array

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        if A[mid] > key then
            return BinSearch(A, lo, mid − 1, key)
    else return BinSearch(A, mid + 1, hi, key)
```

lo: 0

hi: 6

key: 41

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

BinSearch(A,0,6,41)

Example:
Binary Search in Sorted Array

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```

lo: 0

hi: 6

key: 41

mid: 3

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

BinSearch(A,0,6,41)

Example:
Binary Search in Sorted Array

THE UNIVERSITY OF MELBOURNE

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    else return BinSearch(A, mid + 1, hi, key)
```

lo: 0

hi: 6

key: 41

mid: 3

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

BinSearch(A,0,6,41)

BinSearch(A,4,6,41)

Example:
Binary Search in Sorted Array

THE UNIVERSITY OF MELBOURNE

```
function BinSearch(A[·], lo, hi, key)
    if lo > hi then return −1
    mid ← lo + (hi − lo)/2
    if A[mid] = key then return mid
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```

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

BinSearch(A,4,6,41)

Example:
Binary Search in Sorted Array

THE UNIVERSITY OF MELBOURNE

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A:

0	1	2	3	4	5	6
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BinSearch(A,4,6,41)

Example:
Binary Search in Sorted Array

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```

lo: 4

A:

4	9	13	22	41	83	96
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BinSearch(A,4,6,41)

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Binary Search in Sorted Array

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```

lo: 4

hi: 6

key: 41

mid: 5

A:

0	1	2	3	4	5	6
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BinSearch(A,4,6,41)

Example:
Binary Search in Sorted Array

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lo: 4

hi: 6

key: 41

mid: 5

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4	9	13	22	41	83	96
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BinSearch(A,4,6,41)

BinSearch(A,4,4,41)

Example:
Binary Search in Sorted Array

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A:

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BinSearch(A,4,4,41)

Example:
Binary Search in Sorted Array

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A:

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Binary Search in Sorted Array

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        else return BinSearch(A, mid + 1, hi, key)
```

lo: 4

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

BinSearch(A,4,4,41)

Example:
Binary Search in Sorted Array

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            return BinSearch(A, lo, mid − 1, key)
    else return BinSearch(A, mid + 1, hi, key)
```

lo: 4

hi: 4

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

BinSearch(A,4,4,41)

Example:
Binary Search in Sorted Array

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```

lo: 4

hi: 4

key: 41

A:

0	1	2	3	4	5	6
4	9	13	22	41	83	96

BinSearch(A,4,4,41)

Example:
Binary Search in Sorted Array

THE UNIVERSITY OF MELBOURNE

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    else return BinSearch(A, mid + 1, hi, key)
```

lo: 4

hi: 4

key: 41

mid: 4

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

BinSearch(A,4,4,41)

Example:
Binary Search in Sorted Array

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```

lo: 4

hi: 4

key: 41

mid: 4

A:

4	9	13	22	41	83	96
0	1	2	3	4	5	6

BinSearch(A,4,4,41)

returns 4

Example:
Binary Search in Sorted Array

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```
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Telescoping

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$$C(n) \in \Theta(\log n)$$

Logarithmic Functions Have Same Rate of Growth

In *O*, *Ω*, *Θ*, expressions we can just write "log" for any logarithmic function no matter what the base is

Asymptotically, all logarithmic behaviour is the same, since

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Back to Euclid

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function EUCLID(m, n)
  while $n \neq 0$ **do**
    $r \leftarrow m \bmod n$
    $m \leftarrow n$
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  return m

function EUCLID(m, n)
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since the value of m (and n) is at least halved in every two iterations.

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Why? After two iterations, m becomes m mod n; also

$$1 < n < m \implies m \bmod n < m/2$$

Summarising Reasoning with Big-Oh

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Summarising Reasoning with Big-Oh

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(for nested loops: count number of times outer loop is executed, multiply by cost of inner loop)

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See also Cormen's Appendix A or Levitin's Appendix A.

Levitin's Appendix B is a tutorial on recurrence relations.

The Road Ahead

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- You'll get much more familiar with asymptotic analysis as we use it on algorithms we meet in this course.

- Next week we begin our study of algorithms by looking at **brute force** approaches