

# Sequence Tagging: Hidden Markov Models

COMP90042

Natural Language Processing

Lecture 6

Semester 1 2021 Week 3  
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# POS Tagging Recap

- Janet will back the bill
- Janet/**NNP** will/**MB** back/**VP** the/**DT** bill/**NN**
- Local classifier: prone to **error propagation**
- What about treating the full sequence as a “class”?
  - ▶ Output: “NNP\_MB\_VP\_DT\_NN”
- Problems:
  - ▶ Exponentially many combinations:  $|Tags|^M$ , for length  $M$
  - ▶ How to tag sequences of different lengths?

# A Better Approach

- Tagging is a sentence-level task but as humans we **decompose** it into small word-level tasks.
  - ▶ Janet/**NNP** will/**MD** back/**VP** the/**DT** bill/**NN**
- Solution:
  - ▶ Define a model that decomposes process into individual word level steps
  - ▶ But that takes into account the whole sequence when learning and predicting (no error propagation)
- This is the idea of **sequence labelling**, and more general, **structured prediction**.

# A Probabilistic Model

- Goal: obtain best tag sequence  $\mathbf{t}$  from sentence  $\mathbf{w}$

- $\hat{\mathbf{t}} = \operatorname{argmax}_{\mathbf{t}} P(\mathbf{t} \mid \mathbf{w})$

- $\hat{\mathbf{t}} = \operatorname{argmax}_{\mathbf{t}} \frac{P(\mathbf{w} \mid \mathbf{t}) P(\mathbf{t})}{P(\mathbf{w})} = \operatorname{argmax}_{\mathbf{t}} P(\mathbf{w} \mid \mathbf{t}) P(\mathbf{t})$  [Bayes]

- Let's decompose:

- $P(\mathbf{w} \mid \mathbf{t}) = \prod_{i=1}^n P(w_i \mid t_i)$  [Prob. of a word depends only on the tag]

- $P(\mathbf{t}) = \prod_{i=1}^n P(t_i \mid t_{i-1})$  [Prob. of a tag depends only on the previous tag]

- This is a Hidden Markov Model (HMM)

# Two Assumptions of HMM

$$\hat{t} = \operatorname{argmax}_t P(\mathbf{w} | t) P(t)$$

$$P(\mathbf{w} | t) = \prod_{i=1}^n P(w_i | t_i)$$

$$P(t) = \prod_{i=1}^n P(t_i | t_{i-1})$$

- Output independence
  - ▶ An observed event (word) depends only on the hidden state (tag)
- Markov assumption
  - ▶ The current state (tag) depends only on previous state

# HMMs - Training

- Parameters are the individual probabilities
  - ▶  $P(w_i | t_i) = \text{emission (O) probabilities}$
  - ▶  $P(t_i | t_{i-1}) = \text{transition (A) probabilities}$
- Training uses Maximum Likelihood Estimation (MLE)
  - ▶ This is done by simply counting word frequencies according to their tags (just like  $N$ -gram LMs!)

$$\bullet \quad P(\textit{like} | VB) = \frac{\textit{count}(VB, \textit{like})}{\textit{count}(VB)}$$

$$\bullet \quad P(NN | DT) = \frac{\textit{count}(DT, NN)}{\textit{count}(DT)}$$

# HMMs - Training

- What about the first tag?
  - Assume we have a symbol “<s>” that represents the start of your sentence

$$P(NN \mid < s >) = \frac{\text{count}( < s > , NN )}{\text{count}( < s > )}$$

- What about unseen (word, tag) and (tag, previous\_tag) combinations?
  - Smoothing techniques

# Transition Matrix

	<b>NNP</b>	<b>MD</b>	<b>VB</b>	<b>JJ</b>	<b>NN</b>	<b>RB</b>	<b>DT</b>
<b>&lt;s&gt;</b>	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
<b>NNP</b>	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
<b>MD</b>	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
<b>VB</b>	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
<b>JJ</b>	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
<b>NN</b>	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
<b>RB</b>	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
<b>DT</b>	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

**Figure 8.7** The A transition probabilities  $P(t_i|t_{i-1})$  computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus  $P(VB|MD)$  is 0.7968.



# Emission (Observation) Matrix

	Janet	will	back	the	bill
<b>NNP</b>	0.000032	0	0	0.000048	0
<b>MD</b>	0	0.308431	0	0	0
<b>VB</b>	0	0.000028	0.000672	0	0.000028
<b>JJ</b>	0	0	0.000340	0	0
<b>NN</b>	0	0.000200	0.000223	0	0.002337
<b>RB</b>	0	0	0.010446	0	0
<b>DT</b>	0	0	0	0.506099	0

**Figure 8.8** Observation likelihoods  $B$  computed from the WSJ corpus without smoothing, simplified slightly.

# HMMs – Prediction (Decoding)

$$\begin{aligned}\hat{t} &= \operatorname{argmax}_t \prod_n P(\mathbf{w} | t) P(t) \\ &= \operatorname{argmax}_t \prod_{i=1} P(w_i | t_i) P(t_i | t_{i-1})\end{aligned}$$

- Simple idea: for each word, take the tag that maximises  $P(w_i | t_i) P(t_i | t_{i-1})$ . Do it left-to-right, in *greedy* fashion.
- This is wrong! We are looking for  $\operatorname{argmax}_t$ , not individual  $\operatorname{argmax}_{t_i}$  terms.
  - ▶ This is a local classifier: error propagation
- Correct way: consider **all** possible tag combinations, evaluate them, take the max

What is the complexity of such a system that considers all possible tag combinations?

If sentence length =  $N$  and tag size =  $T$

- $NT$
- $N^T$
- $T^N$
- $N$

[PollEv.com/jeyhanlau569](https://pollev.com/jeyhanlau569)



**What is the complexity of such a system that considers all possible tag combinations? If sentence length =  $N$  and tag size =  $T$**



# The Viterbi Algorithm

- Dynamic Programming to the rescue!
  - ▶ We can still proceed sequentially, as long as we are careful.
- POS tag: “can play”
- Best tag for “can” is easy:  $\operatorname{argmax}_t P(\text{can} \mid t) P(t \mid \langle s \rangle)$
- Suppose best tag for “can” is NN. To get the tag for “play”, we can take  $\operatorname{argmax}_t P(\text{play} \mid t) P(t \mid \text{NN})$  but this is wrong.
- Instead, we keep track of **scores for each tag** for “can” and check them with the different tags of “play”.

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP					
MD					
VB					
JJ					
NN					
RB					
DT					

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	$P(\text{Janet} \text{NNP}) * P(\text{NNP} \text{<s>})$				
MD					
VB					
JJ	...				
NN	...				
RB	...				
DT	...				

$$P(\text{Janet} \mid \text{NNP}) = ?; P(\text{NNP} \mid \langle s \rangle) = ?$$

	NNP	MD	VB	JJ	NN	RB	DT
$\langle s \rangle$	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

**Figure 8.7** The  $A$  transition probabilities  $P(t_i|t_{i-1})$  computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus  $P(VB|MD)$  is 0.7968.

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

**Figure 8.8** Observation likelihoods  $B$  computed from the WSJ corpus without smoothing, simplified slightly.



# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	$0.000032 * 0.2767 = 8.8544e-06$				
MD	$P(\text{Janet} \text{MD}) * P(\text{MD} \text{<s>})$				
VB	...				
JJ	...				
NN	...				
RB	...				
DT	...				

$$P(\text{Janet} \mid \text{MD}) = ?; P(\text{MD} \mid \langle s \rangle) = ?$$

	NNP	MD	VB	JJ	NN	RB	DT
$\langle s \rangle$	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
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NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

**Figure 8.7** The  $A$  transition probabilities  $P(t_i|t_{i-1})$  computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus  $P(\text{VB}|\text{MD})$  is 0.7968.

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
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DT	0	0	0	0.506099	0

**Figure 8.8** Observation likelihoods  $B$  computed from the WSJ corpus without smoothing, simplified slightly.

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●				
MD	0				
VB	0				
JJ	0				
NN	0				
RB	0				
DT	0				

$$P(\text{Janet} \mid \text{VB/JJ/etc}) = ?$$

	NNP	MD	VB	JJ	NN	RB	DT
< <i>s</i> >	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
<b>NNP</b>	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
<b>MD</b>	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
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**Figure 8.7** The  $A$  transition probabilities  $P(t_i|t_{i-1})$  computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus  $P(\text{VB}|\text{MD})$  is 0.7968.

	Janet	will	back	the	bill
<b>NNP</b>	0.000032	0	0	0.000048	0
<b>MD</b>	0	0.308431	0	0	0
<b>VB</b>	0	0.000028	0.000672	0	0.000028
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**Figure 8.8** Observation likelihoods  $B$  computed from the WSJ corpus without smoothing, simplified slightly.

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06	$P(\text{will}   \text{NNP}) * P(\text{NNP}   t_{\text{Janet}}) * s(t_{\text{Janet}}   \text{Janet})$			
MD	0	...			
VB	0	...			
JJ	0	...			
NN	0	...			
RB	0	...			
DT	0	...			

Consider all possible tags, and take the maximum

$$\max( P(\text{will} | \text{NNP}) * P(\text{NNP} | \text{NNP}) * s(\text{NNP} | \text{Janet}), \\ P(\text{will} | \text{NNP}) * P(\text{NNP} | \text{MD}) * s(\text{MD} | \text{Janet}), \\ \dots \\ P(\text{will} | \text{NNP}) * P(\text{NNP} | \text{DT}) * s(\text{DT} | \text{Janet}) )$$
$$= P(\text{will} | \text{NNP}) * P(\text{NNP} | \text{NNP}) * 8.8544e-06$$

$$P(\text{will} \mid \text{NNP}) = ?; P(\text{NNP} \mid \text{NNP}) = ?$$

	NNP	MD	VB	JJ	NN	RB	DT
< <i>s</i> >	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
<b>NNP</b>	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
<b>MD</b>	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
<b>VB</b>	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
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<b>NN</b>	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
<b>RB</b>	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
<b>DT</b>	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

**Figure 8.7** The  $A$  transition probabilities  $P(t_i|t_{i-1})$  computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus  $P(VB|MD)$  is 0.7968.

	Janet	will	back	the	bill
<b>NNP</b>	0.000032	0	0	0.000048	0
<b>MD</b>	0	0.308431	0	0	0
<b>VB</b>	0	0.000028	0.000672	0	0.000028
<b>JJ</b>	0	0	0.000340	0	0
<b>NN</b>	0	0.000200	0.000223	0	0.002337
<b>RB</b>	0	0	0.010446	0	0
<b>DT</b>	0	0	0	0.506099	0

**Figure 8.8** Observation likelihoods  $B$  computed from the WSJ corpus without smoothing, simplified slightly.

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0			
MD	0	...			
VB	0	...			
JJ	0	...			
NN	0	...			
RB	0	...			
DT	0	...			

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0			
MD	0	$P(\text{will}   \text{MD}) * P(\text{MD}   t_{\text{Janet}}) * s(t_{\text{Janet}}   \text{Janet})$			
VB	0	...			
JJ	0	...			
NN	0	...			
RB	0	...			
DT	0	...			

$\max( P(\text{will} | \text{MD}) * P(\text{MD} | \text{NNP}) * s(\text{NNP} | \text{Janet}),$   
 $P(\text{will} | \text{MD}) * P(\text{MD} | \text{MD}) * s(\text{MD} | \text{Janet}),$   
...  
 $P(\text{will} | \text{MD}) * P(\text{MD} | \text{DT}) * s(\text{DT} | \text{Janet}) )$

$= P(\text{will} | \text{MD}) * P(\text{MD} | \text{NNP}) * 8.8544e-06$



# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0			
MD	0	3.004e-08			
VB	0	...			
JJ	0	...			
NN	0	...			
RB	0	...			
DT	0	...			

$\max( P(\text{will} \mid \text{MD}) * P(\text{MD} \mid \text{NNP}) * s(\text{NNP} \mid \text{Janet}),$   
 $P(\text{will} \mid \text{MD}) * P(\text{MD} \mid \text{MD}) * s(\text{MD} \mid \text{Janet}),$   
...  
 $P(\text{will} \mid \text{MD}) * P(\text{MD} \mid \text{DT}) * s(\text{DT} \mid \text{Janet}) )$

$= P(\text{will} \mid \text{MD}) * P(\text{MD} \mid \text{NNP}) * 8.8544\text{e-}06$

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0			
MD	0	3.004e-8			
VB	0	2.231e-13			
JJ	0	0			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

# Transition and Emission Matrix

	NNP	MD	VB	JJ	NN	RB	DT
<s>	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

**Figure 8.7** The  $A$  transition probabilities  $P(t_i|t_{i-1})$  computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus  $P(VB|MD)$  is 0.7968.

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

**Figure 8.8** Observation likelihoods  $B$  computed from the WSJ corpus without smoothing, simplified slightly.

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0			
MD	0	3.004e-8			
VB	0	2.231e-13			
JJ	0	0			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	$P(\text{back} \text{VB}) * P(\text{VB} t_{\text{will}}) * s(t_{\text{will}} \text{will})$		
JJ	0	0			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	MD: 1.6e-11 VB: 7.5e-19 NN: 9.7e-17		
JJ	0	0			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	MD: 1.6e-11 VB: 7.5e-19 NN: 9.7e-17		
JJ	0	0			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	1.6e-11		
JJ	0	0			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			



# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	1.6e-11		
JJ	0	0	5.1e-15		
NN	0	1.034e-10	5.4e-15		
RB	0	0	5.3e-11		
DT	0	0	0		

# The Viterbi Algorithm

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0	0	2.5e-17	
MD	0	3.004e-8	0	0	
VB	0	2.231e-13	1.6e-11	0	
JJ	0	0	5.1e-15	0	
NN	0	1.034e-10	5.4e-15	0	
RB	0	0	5.3e-11	0	
DT	0	0	0	1.8e-12	

Done!

	Janet	will	back	the	bill
NNP	8.8544e-06 ●	0	0	2.5e-17	0
MD	0	3.004e-8	0	0	0
VB	0	2.231e-13	1.6e-11	0	1.0e-20
JJ	0	0	5.1e-15	0	0
NN	0	1.034e-10	5.4e-15	0	2.0e-15
RB	0	0	5.3e-11	0	0
DT	0	0	0	1.8e-12	0

# Going Backwards To Get The Best Tag Sequence

	Janet	will	back	the	bill
NNP	8.8544e-06	0	0	2.5e-17	0
MD	0	3.004e-8	0	0	0
VB	0	2.231e-13	1.6e-11	0	1.0e-20
JJ	0	0	5.1e-15	0	0
NN	0	1.034e-10	5.4e-15	0	2.0e-15
RB	0	0	5.3e-11	0	0
DT	0	0	0	1.8e-12	0

# Going Backwards To Get The Best Tag Sequence

	Janet	will	back	the	bill
NNP	<b>8.8544e-06</b> ●	0	0	2.5e-17	0
MD	0	<b>3.004e-08</b>	0	0	0
VB	0	2.231e-13	<b>1.6e-11</b>	0	1.0e-20
JJ	0	0	5.1e-15	0	0
NN	0	1.034e-10	5.4e-15	0	<b>2.0e-15</b>
RB	0	0	5.3e-11	0	0
DT	0	0	0	<b>1.8e-12</b>	0

# Final Tagging Results

	Janet/ <b>NNP</b>	will/ <b>MD</b>	back/ <b>VB</b>	the/ <b>DT</b>	bill/ <b>NN</b>
NNP	8.8544e-06 ●	0	0	2.5e-17	0
MD	0	3.004e-08	0	0	0
VB	0	2.231e-13	1.6e-11	0	1.0e-20
JJ	0	0	5.1e-15	0	0
NN	0	1.034e-10	5.4e-15	0	2.0e-15
RB	0	0	5.3e-11	0	0
DT	0	0	0	1.8e-12	0

# What is the optimal POS tag sequence for: THEY FISH

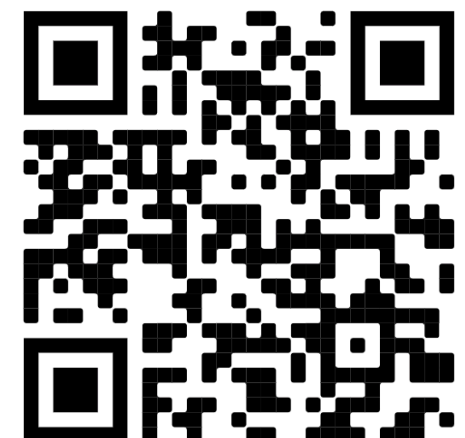
	they	fish
N	0.7	0.3
V	0.4	0.6

Word Emission Probabilities

	N	V
<S>	0.6	0.4
N	0.8	0.2
V	0.7	0.3

Transition Probabilities

[PollEv.com/jeyhanlau569](https://pollev.com/jeyhanlau569)



# THEY FISH

	they	fish
N		
V		

[PolEv.com/jeyhanlau569](https://PolEv.com/jeyhanlau569)



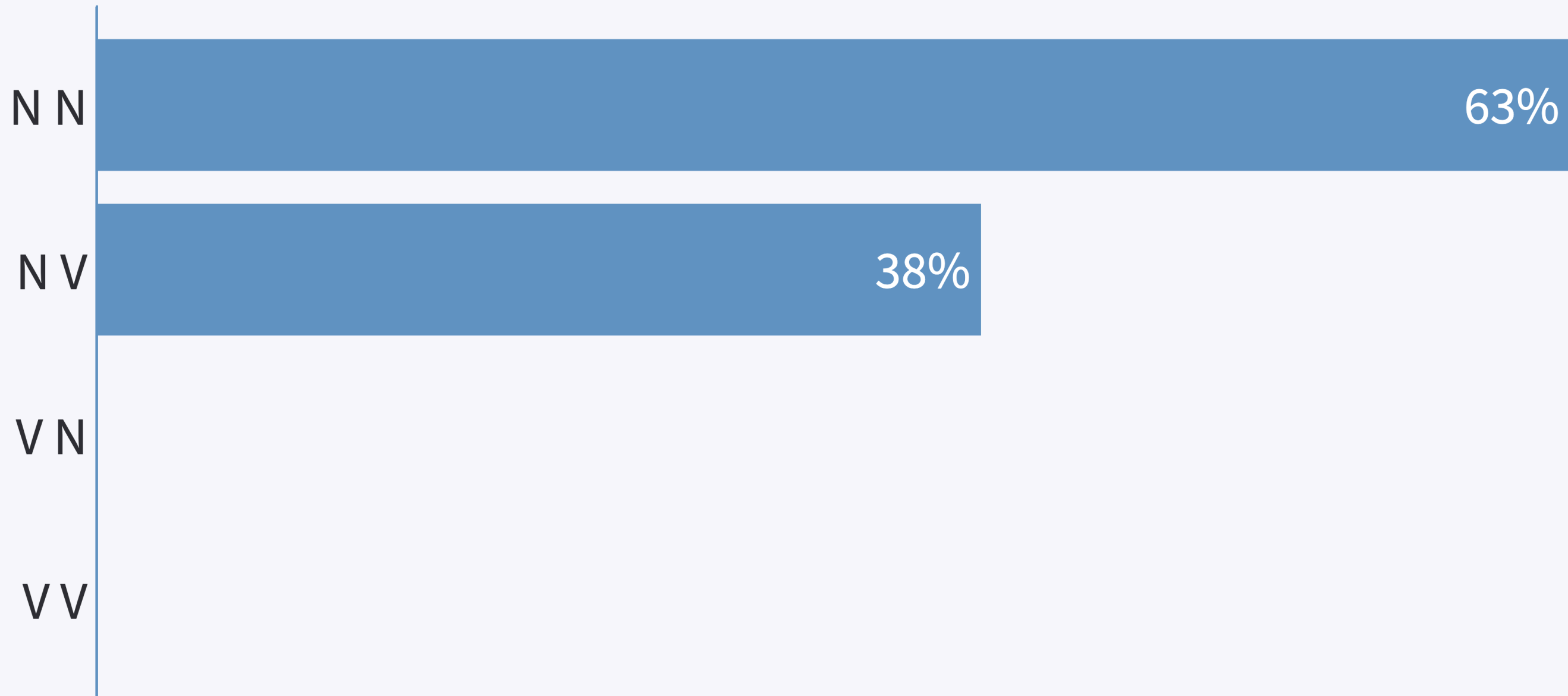


# THEY FISH

	they	<div>N for "they"</div> fish
N	<div>P(they   N) x P(N   &lt;s&gt;)</div> <div>= 0.7 x 0.6</div> <div>= 0.42</div>	<div>P(fish   N) x P(N   N) x 0.42 = 0.3 x 0.8 x 0.42 = 0.1</div> <div>P(fish   N) x P(N   V) x 0.16 = 0.3 x 0.7 x 0.16 = 0.03</div>
V	<div>P(they   V) x P(V   &lt;s&gt;)</div> <div>= 0.4 x 0.4</div> <div>= 0.16</div>	<div>P(fish   V) x P(V   N) x 0.42 = 0.6 x 0.2 x 0.42 = 0.05</div> <div>P(fish   V) x P(V   V) x 0.16 = 0.6 x 0.3 x 0.16 = 0.03</div> <div>0.05</div>

Answer: N N

## What is the optimal POS-tag sequence? THEY FISH



# The Viterbi Algorithm

- Complexity:  $O(T^2N)$ , where  $T$  is the size of the tagset and  $N$  is the length of the sequence.
  - ▶  $T * N$  matrix, each cell performs  $T$  operations.
- Why does it work?
  - ▶ Because of the **independence assumptions** that decompose the problem.
  - ▶ Without these, we cannot apply DP.

# Viterbi Pseudocode

```
alpha = np.zeros(M, T)
for t in range(T):
    alpha[1, t] = pi[t] * O[w[1], t]

for i in range(2, M):
    for t_i in range(T):
        for t_last in range(T):           # t_last means t_{i-1}
            s = alpha[i-1, t_last] * A[t_last, t_i] * O[w[i], t_i]
            if s > alpha[i, t_i]:
                alpha[i, t_i] = s
                back[i, t_i] = t_last

best = np.max(alpha[M-1, :])
return backtrace(best, back)
```

- Good practice: work with **log** probabilities to prevent underflow (multiplications become sums)
- Vectorisation (use matrix-vector operations)

# HMMs In Practice

- We saw HMM taggers based on **bigrams** (first order HMM).
  - ▶ I.e. current tag depends only the immediate previous tag
- State-of-the-art use tag **trigrams** (second order HMM).
  - ▶  $P(t) = \prod_{i=1}^n P(t_i | t_{i-1}, t_{i-2})$  Viterbi now  $O(T^3N)$
- Need to deal with sparsity: some tag trigram sequences might not be present in training data
  - ▶ Interpolation:  $P(t_i | t_{i-1}, t_{i-2}) = \lambda_3 \hat{P}(t_i | t_{i-1}, t_{i-2}) + \lambda_2 \hat{P}(t_i | t_{i-1}) + \lambda_1 \hat{P}(t_i)$
  - ▶  $\lambda_1 + \lambda_2 + \lambda_3 = 1$
- With additional features, reach 96.5% accuracy on Penn Treebank (Brants, 2000)

# Generative vs. Discriminative Taggers

- HMM is **generative**
  - ▶  $\hat{T} = \operatorname{argmax}_T P(T | W)$
  - ▶  $= \operatorname{argmax}_T P(W | T) P(T)$
  - ▶  $= \operatorname{argmax}_T \prod_i P(w_i | t_i) P(t_i | t_{i-1})$
- trained HMM can generate data (sentences)!
- allows for unsupervised HMMs: learn model without any tagged data!

# Generative vs. Discriminative Taggers

$$\begin{aligned}\hat{T} &= \operatorname{argmax}_T P(T | W) \\ &= \operatorname{argmax}_T \prod_i P(t_i | w_i, t_{i-1})\end{aligned}$$

- **Discriminative** models describe  $P(T | W)$  directly
  - ▶ supports richer feature set, generally better accuracy when trained over large supervised datasets
    - ▶  $P(t_i | w_i, t_{i-1}, x_i, y_i)$
  - ▶ E.g., Maximum Entropy Markov Model (MEMM), Conditional random field (CRF)
  - ▶ Most deep learning models of sequences are discriminative

# HMMs in NLP

- HMMs are highly effective for part-of-speech tagging
  - ▶ trigram HMM gets 96.5% accuracy
  - ▶ related models are state of the art
    - ▶ MEMMs 97%
    - ▶ CRFs 97.6%
    - ▶ Deep CRF 97.9%
  - ▶ English Penn Treebank tagging accuracy [https://aclweb.org/aclwiki/index.php?title=POS\\_Tagging\\_\(State\\_of\\_the\\_art\)](https://aclweb.org/aclwiki/index.php?title=POS_Tagging_(State_of_the_art))
- Apply out-of-the box to other sequence labelling tasks
  - ▶ named entity recognition (lecture 18), shallow parsing, ...
  - ▶ In other fields: DNA, protein sequences...



# A Final Word

- HMMs are a simple, yet effective way to perform sequence labelling.
- Can still be competitive, and fast. Natural baseline for other sequence labelling models.
- Main drawback: not very flexible in terms of feature representation, compared to MEMMs and CRFs.

# Readings

- JM3 Appendix A A.1-A.2, A.4
- See also E18 Chapter 7.3
- References:
  - ▶ Rabiner's HMM tutorial <http://tinyurl.com/2hqaf8>
  - ▶ Lafferty et al, Conditional random fields: Probabilistic models for segmenting and labeling sequence data (2001)