

Lecture 16: Ensemble Learning

COMP90049

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Introduction

- We have discussed individual classification algorithms and considered each of them in isolation
- We have discussed ways of comparing the performance of individual classifiers over a given dataset/task, which allows us to choose the "optimal" classifier for a dataset
- When evaluating, we only get one shot at classifying a given test instance and are stuck with the bias inherent in a given algorithm

- **Ensemble learning (aka. Classifier combination):** constructs a set of base classifiers from a given set of training data and aggregates the outputs into a single meta-classifier
- **Intuition 1:** the combination of lots of weak classifiers can be at least as good as one strong classifier
- **Intuition 2:** the combination of a selection of strong classifiers is (usually) at least as good as the best of the base classifiers

- When does ensemble learning work?
 - the base classifiers should not make the same mistakes
 - the base classifiers are reasonably accurate

	t_1	t_2	t_3
C_1	✓	✓	x
C_2	x	✓	✓
C_3	✓	x	✓
C^*	✓	✓	✓

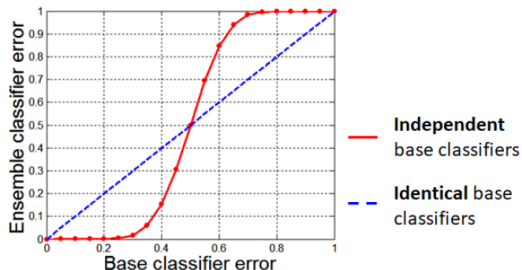
	t_1	t_2	t_3
C_1	✓	✓	x
C_2	✓	✓	x
C_3	✓	✓	x
C^*	✓	✓	x

	t_1	t_2	t_3
C_1	✓	x	x
C_2	x	✓	x
C_3	x	x	✓
C^*	x	x	x

- Assume we have a set of 25 binary base classifiers, each with an error rate of $\epsilon = 0.35$. If the base classifiers are independent and we perform classifier combination by voting, the error rate of the combined classifier is:

$$\sum_{i=13}^{25} \binom{25}{i} \epsilon^i (1 - \epsilon)^{25-i} \approx 0.06$$

- When does ensemble learning work?



- The simplest means of classification over multiple base classifiers is simple **voting**:
 - for a nominal class set, run multiple base classifiers over the test data and select the class predicted by the most base classifiers (e.g. k-NN)
 - for a continuous class set, average over the numeric predictions of our base classifiers

- **Instance manipulation:** generate multiple training datasets through sampling, and train a base classifier over each dataset
- **Feature manipulation:** generate multiple training datasets through different feature subsets, and train a base classifier over each dataset
- **Class label manipulation:** generate multiple training datasets by manipulating the class labels in a reversible manner
- **Algorithm manipulation:** semi-randomly tweak internal parameters within a given algorithm to generate multiple base classifiers over a given dataset

Stacking

- **Intuition:** smooth errors over a range of algorithms with different biases
- **Simple Voting:** generate multiple training datasets through different feature subsets, and train a base classifier over each dataset
 - presupposes the classifiers have equal performance
- **Meta Classification:** train a classifier over the outputs of the base classifiers
 - train using nested cross validation to reduce bias

- Given training dataset (X, y) :
 - Train Neural Network
 - Train Naive Bayes
 - Train Decision Tree
- Discard (or keep) X , add new attributes for each instance:
 - predictions (labels) of the classifiers above
 - other data as available (NB scores etc.)
- Train meta-classifier (usually Logistic Regression)

- Mathematically simple but computationally expensive method
- Able to combine heterogeneous classifiers with varying performance
- Generally, stacking results in as good or better results than the best of the base classifiers
- Widely seen in applied research; less interest within theoretical circles (esp. statistical learning)

Bagging

- **Intuition:** the more data, the better the performance (lower the variance), so how can we get ever more data out of a fixed training dataset?
- **Method:** construct novel datasets through a combination of random sampling and replacement
 - Randomly sample the original dataset N times, with replacement (bootstrap)
 - Thus, we get a new dataset of the same size, where any individual instance is absent with probability $(1 - \frac{1}{N})^N$
 - construct k random datasets for k base classifiers, and arrive at prediction via voting

- Original dataset:

1	2	3	4	5	6	7	8	9	10
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- Bootstrap Samples

7	2	6	7	5	4	8	8	1	10
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1	3	8	10	3	5	8	10	1	9
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2	9	4	2	7	9	3	10	1	10
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- The same (weak) classification algorithm is used throughout
- As bagging is aimed towards minimising variance through sampling, the algorithm should be unstable (=high-variance) ... e.g.?

- Simple method based on sampling and voting
- Possibility to parallelise computation of individual base classifiers
- Highly effective over noisy datasets (outliers may vanish)
- Performance is generally significantly better than the base classifiers and only occasionally substantially worse

Bagging - Random Forest

A Random Tree is a Decision Tree where:

- At each node, only some of the possible attributes are considered
- For example, a fixed proportion of all of the attributes, except the ones used earlier in the tree
- Attempts to control for unhelpful attributes in the feature set
- Much faster to build than a deterministic Decision Tree, but increases model variance

A Random Forest is:

- An ensemble of Random Trees (many trees = forest)
- Each tree is built using a different Bagged training dataset
- As with Bagging the combined classification is via voting
- The idea behind them is to minimise overall model variance, without introducing (combined) model bias

Hyperparameters:

- number of trees B (can be tuned, e.g. based on out-of-bag error rate)
- feature sub-sample size (e.g. $(\log |F| + 1)$)

Interpretation:

- logic behind predictions on individual instances can be tediously followed through the various trees
- logic behind overall model: ???



Practical Properties of Random Forests:

- Generally a very strong performer
- Embarrassingly parallelisable
- Surprisingly efficient
- Robust to overfitting
- Interpretability sacrificed

Boosting

- Intuition: tune base classifiers to focus on the hard to classify instances
- Approach: iteratively change the distribution and weights of training instances to reflect the performance of the classifier on the previous iteration
 - start with each training instance having a probability of $\frac{1}{N}$ being included in the sample
 - over T iterations, train a classifier and update the weight of each instance according to whether it is correctly classified
 - combine the base classifiers via weighted voting

Boosting II

- Original dataset:

1	2	3	4	5	6	7	8	9	10
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- Boosting samples:

<i>Iteration 1:</i>	7	2	6	7	5	4	8	8	1	10
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<i>Iteration 2:</i>	1	3	8	4	3	5	4	10	1	4
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<i>Iteration 3:</i>	4	9	4	2	4	4	3	10	1	4
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- Base classifiers: C_1, C_2, \dots, C_T
- Training instances $(x_j, y_j) | j = 1, 2, \dots, N$
- Initial instance weights $w_j^{(0)} = \frac{1}{N} | j = 1, 2, \dots, N$
- Construct classifier C_i in iteration i :

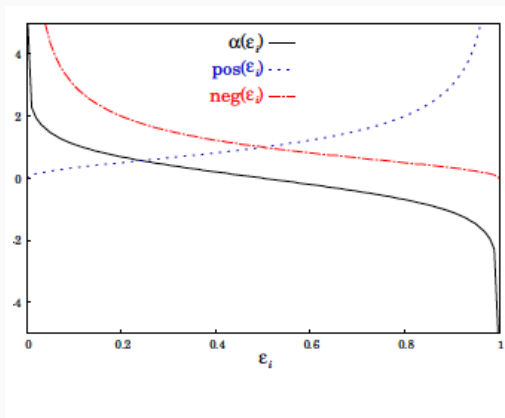
Error rate for C_i :

$$\epsilon_i = \sum_{j=1}^N w_j^{(i)} \delta(C_i(x_j) \neq y_j)$$



- Importance of C_i (i.e. the weight associated with the classifiers votes):

$$\alpha_i = \frac{1}{2} \log_e \frac{1 - \epsilon_i}{\epsilon_i}$$



- Weights for instance j ($i > 0$):

$$w_j^{(i+1)} = \frac{w_j^{(i)}}{Z_i} \times \begin{cases} e^{-\alpha_i} & \text{if } C_i(x_j) = y_j \\ e^{\alpha_i} & \text{if } C_i(x_j) \neq y_j \end{cases}$$

- Continue iterating for $i = 1, 2, \dots, T$, but reinitialise the instance weights whenever $\epsilon_i > 0.5$
- Classification:

$$C^*(x) = \underset{y}{\operatorname{argmax}} \sum_{j=1}^T \alpha_j \delta(C_j(x) = y)$$

- Base classification algorithm: decision stumps (OneR) or decision trees



- Mathematically complicated but computationally cheap method based on iterative sampling and weighted voting
- More computationally expensive than bagging
- The method has guaranteed performance in the form of error bounds over the training data
- Interesting effect with convergence of the error rate over the training vs. test data
- In practical applications, boosting has the tendency to overfit

Bagging vs. Boosting

Bagging

- Parallel sampling
- Simple voting
- Single classification algorithm
- Minimise variance
- Not prone to overfitting

Boosting

- Iterative sampling
- Weighted voting
- Single classification algorithm
- Minimise (instance) bias
- Prone to overfitting



Summary

- What is classifier combination?
- What is bagging and what is the basic thinking behind it?
- What is boosting and what is the basic thinking behind it?
- What is stacking and what is the basic thinking behind it?
- How do bagging and boosting compare?

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