

COMP90054 — AI Planning for Autonomy

5. Delete Relaxation Heuristics

It's a Long Way to the Goal, But How Long Exactly?

Part I: *Acting As If the World Can Only Get Better*

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Agenda

- 1 Motivation
- 2 The Delete Relaxation
- 3 The Additive and Max Heuristics
- 4 Relaxed Plans
- 5 Conclusion

Motivation

→ Delete relaxation is a method to relax planning tasks, and thus automatically compute heuristic functions h .

→ Every h yields good performance **only in some domains!** (Search reduction vs. computational overhead)

→ We must come up with as many alternative methods as possible!

We cover the 4 different methods currently known:

- Critical path heuristics:
- Delete relaxation. Soon to be Done.
- Abstractions.
- Landmarks.

→ Delete relaxation is very wide-spread, and highly successful for satisficing planning!

We introduce the method in STRIPS.

Reminder: Relaxing the World by Ignoring Delete Lists

“What was once true remains true forever.”

Relaxed world: (after)



The Delete Relaxation

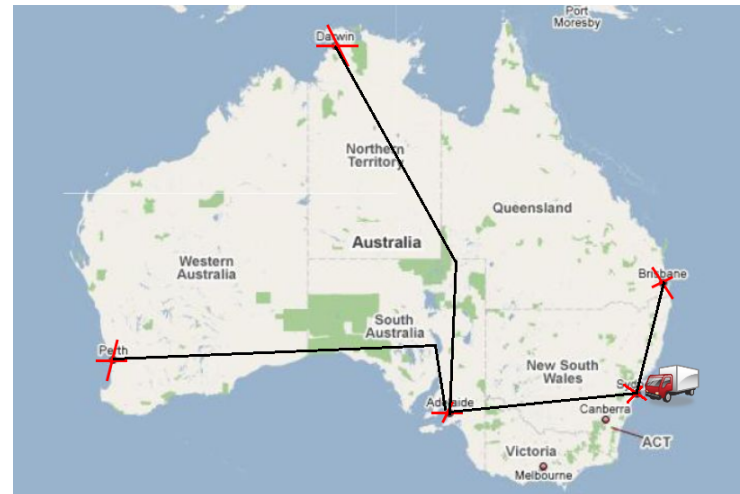
Definition (Delete Relaxation).

- (i) For a STRIPS action a , by a^+ we denote the corresponding *delete relaxed action*, or short *relaxed action*, defined by $pre_{a^+} := pre_a$, $add_{a^+} := add_a$, and $del_{a^+} := \emptyset$.
 - (ii) For a set A of STRIPS actions, by A^+ we denote the corresponding set of relaxed actions, $A^+ := \{a^+ \mid a \in A\}$; similarly, for a sequence $\vec{a} = \langle a_1, \dots, a_n \rangle$ of STRIPS actions, by \vec{a}^+ we denote the corresponding sequence of relaxed actions, $\vec{a}^+ := \langle a_1^+, \dots, a_n^+ \rangle$.
 - (iii) For a STRIPS planning task $\Pi = (F, A, c, I, G)$, by $\Pi^+ := (F, A^+, c, I, G)$ we denote the corresponding *(delete) relaxed planning task*.
- “+” super-script = delete relaxed. We’ll also use this to denote states encountered within the relaxation. (For STRIPS, s^+ is a fact set just like s .)

Definition (Relaxed Plan). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, and let s be a state. An (optimal) *relaxed plan* for s is an (optimal) plan for Π_s^+ . A relaxed plan for I is also called a relaxed plan for Π .

→ Anybody remember what Π_s is? $\Pi_s = (F, A, c, s, G)$

A Relaxed Plan for “TSP” in Australia



- 1 **Initial state:** $\{at(Sy), v(Sy)\}$.
- 2 **Apply** $drive(Sy, Br)^+$: $\{at(Br), v(Br), at(Sy), v(Sy)\}$.
- 3 **Apply** $drive(Sy, Ad)^+$: $\{at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}$.
- 4 **Apply** $drive(Ad, Pe)^+$: $\{at(Pe), v(Pe), at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}$.
- 5 **Apply** $drive(Ad, Da)^+$: $\{at(Da), v(Da), at(Pe), v(Pe), at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}$.

State Dominance

Definition (Dominance). Let $\Pi^+ = (F, A^+, c, I, G)$ be a STRIPS planning task, and let s^+, s'^+ be states. We say that s'^+ **dominates** s^+ if $s'^+ \supseteq s^+$.

→ For example, on the previous slide, who dominates who? Each state along the relaxed plan dominates the previous one, simply because the actions don't delete any facts.

Proposition (Dominance). Let $\Pi^+ = (F, A^+, c, I, G)$ be a STRIPS planning task, and let s^+, s'^+ be states where s'^+ dominates s^+ . We have:

- (i) If s^+ is a goal state, then s'^+ is a goal state as well.
- (ii) If \vec{a}^+ is applicable in s^+ , then \vec{a}^+ is applicable in s'^+ as well, and $\text{appl}(s'^+, \vec{a}^+)$ dominates $\text{appl}(s^+, \vec{a}^+)$.

Proof. (i) is trivial. (ii) by induction over the length n of \vec{a}^+ . Base case $n = 0$ is trivial. Inductive case $n \rightarrow n + 1$ follows directly from induction hypothesis and the definition of $\text{appl}(\cdot, \cdot)$.

→ It is always better to have more facts true.

The Delete Relaxation and State Dominance

Proposition. Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, let s be a state, and let $a \in A$. Then $\text{appl}(s, a^+)$ dominates both (i) s and (ii) $\text{appl}(s, a)$.

Proof. Trivial from the definitions of $\text{appl}(s, a)$ and a^+ .

⇒ Optimal relaxed plans admissibly estimate the cost of optimal plans:

Proposition (Delete Relaxation is Admissible). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, let s be a state, and let \vec{a} be a plan for Π_s . Then \vec{a}^+ is a relaxed plan for s .

Proof. Prove by induction over the length of \vec{a} that $\text{appl}(s, \vec{a}^+)$ dominates $\text{appl}(s, \vec{a})$. Base case is trivial, inductive case follows from (ii) above.

⇒ It is now clear how to find a relaxed plan:

- Applying a relaxed action can only ever make more facts true ((i) above).
- That can only be good, i.e., cannot render the task unsolvable (dominance proposition).

→ So? Keep applying relaxed actions, stop if goal is true (see next slide).

Greedy Relaxed Planning

Greedy Relaxed Planning for Π_s^+

```

 $s^+ := s; \vec{a}^+ := \langle \rangle$ 
while  $G \not\subseteq s^+$  do:
  if  $\exists a \in A$  s.t.  $pre_a \subseteq s^+$  and  $appl(s^+, a^+) \neq s^+$  then
    select one such  $a$ 
     $s^+ := appl(s^+, a^+); \vec{a}^+ := \vec{a}^+ \circ \langle a^+ \rangle$ 
  else return “ $\Pi_s^+$  is unsolvable” endif
endwhile
return  $\vec{a}^+$ 

```

Proposition. *Greedy relaxed planning is sound, complete, and terminates in time polynomial in the size of Π .*

Proof. Soundness: If \vec{a}^+ is returned then, by construction, $G \subseteq appl(s, \vec{a}^+)$. Completeness: If “ Π_s^+ is unsolvable” is returned, then no relaxed plan exists for s^+ at that point; since s^+ dominates s , by the dominance proposition this implies that no relaxed plan can exist for s . Termination: Every $a \in A$ can be selected at most once because afterwards $appl(s^+, a^+) = s^+$.

⇒ It is easy to decide whether a relaxed plan exists!

Greedy Relaxed Planning to Generate a Heuristic Function?

Using greedy relaxed planning to generate h

- In search state s during forward search, run greedy relaxed planning on Π_s^+ .
- Set $h(s)$ to the cost of \vec{a}^+ , or ∞ if “ Π_s^+ is unsolvable” is returned.

→ **Is this heuristic safe?** Yes: $h(s) = \infty$ only if no relaxed plan for s exists, which by admissibility of delete relaxation implies that no plan for s exists.

→ **Is this heuristic goal-aware?** Yes, we'll have $G \subseteq s^+$ right at the start.

→ **Is this heuristic admissible?** Would be if the relaxed plans were optimal; but they clearly aren't. So h isn't consistent either.

→ To be informed (accurately estimate h^*), a heuristic needs to approximate the *minimum effort* needed to reach the goal. Greedy relaxed planning doesn't do this because it may select arbitrary actions that aren't relevant at all.

h^+ : The Optimal Delete Relaxation Heuristic

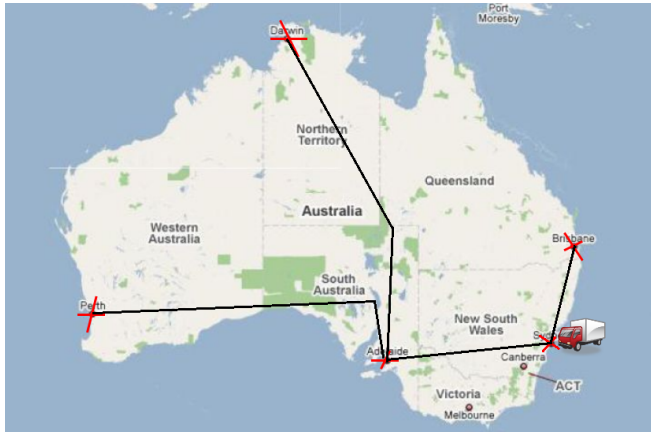
Definition (h^+). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task with state space $\Theta_\Pi = (S, A, c, T, I, G)$. The *optimal delete relaxation heuristic* h^+ for Π is the function $h^+ : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$ where $h^+(s)$ is defined as the cost of an optimal relaxed plan for s .

Corollary (h^+ is Admissible). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. Then h^+ is admissible, and thus safe and goal-aware. (By admissibility of delete relaxation.)

→ To be informed (accurately estimate h^*), a heuristic needs to approximate the *minimum effort* needed to reach the goal. h^+ naturally does so by asking for the cheapest possible relaxed plans.

[→ You might rightfully ask “But won’t optimal relaxed plans usually under-estimate h^* ?” Yes, but that’s just the effect of considering a relaxed problem, and arbitrarily adding actions useless within the relaxation does not help to address it.]

h^+ in “TSP” in Australia



■ $P: at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.

■ $A: drive(x, y)$ where x, y have a road.

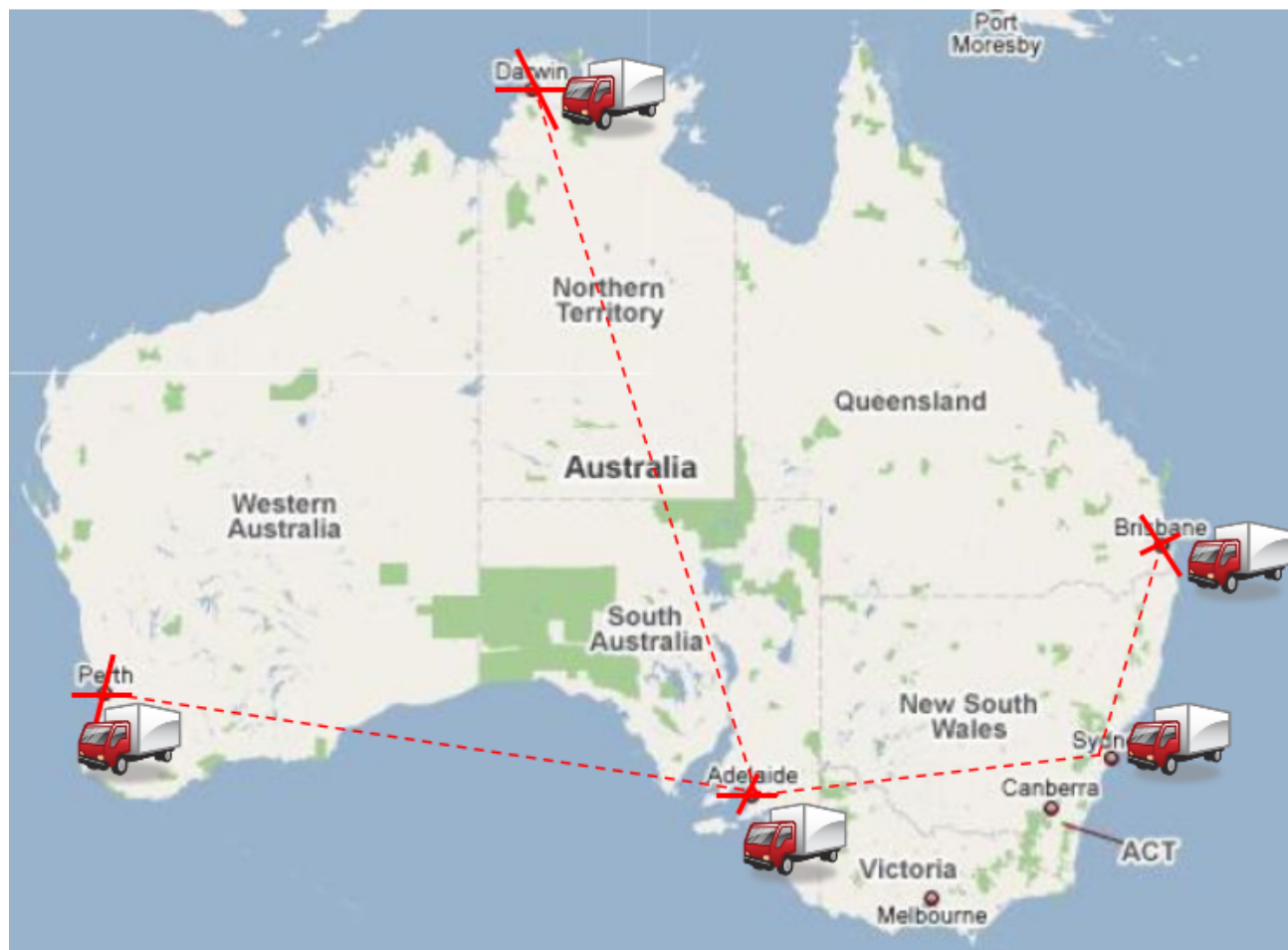
$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

■ $I: at(Sy), v(Sy)$; $G: at(Sy), v(x)$ for all x .

Planning vs. Relaxed Planning:

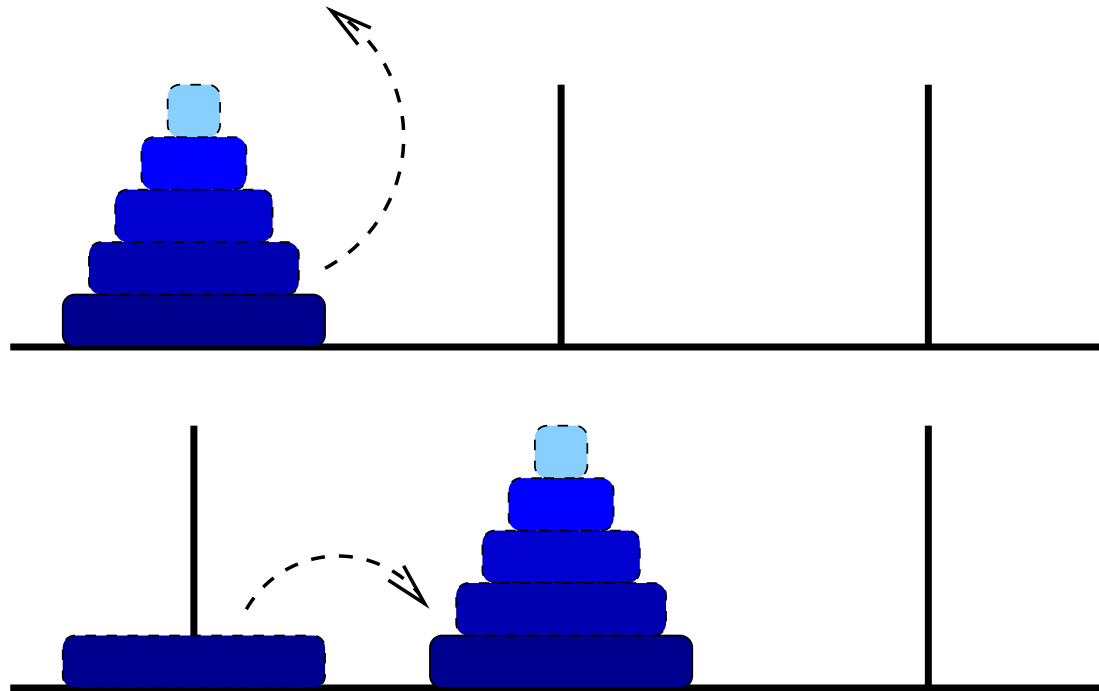
- **Optimal plan:** $\langle drive(Sy, Br), drive(Br, Sy), drive(Sy, Ad), drive(Ad, Pe), drive(Pe, Ad), drive(Ad, Da), drive(Da, Ad), drive(Ad, Sy) \rangle$.
- **Optimal relaxed plan:** $\langle drive(Sy, Br), drive(Sy, Ad), drive(Ad, Pe), drive(Ad, Da) \rangle$.
- $h^*(I) = 20$; $h^+(I) = 10$.

Reminder: h^+ in (the real) TSP



$$h^+(\text{TSP}) = \text{Minimum Spanning Tree!}$$

Reminder: h^+ in Hanoi



$$h^+(\text{Hanoi}) = n, \text{ not } 2^n$$

But How to Compute h^+ ?

Definition (Optimal Relaxed Planning). By PlanOpt^+ , we denote the problem of deciding, given a STRIPS planning task $\Pi = (F, A, c, I, G)$ and $B \in \mathbb{R}_0^+$, whether there exists a relaxed plan for Π whose cost is at most B .

→ By computing h^+ , we would solve PlanOpt^+ .

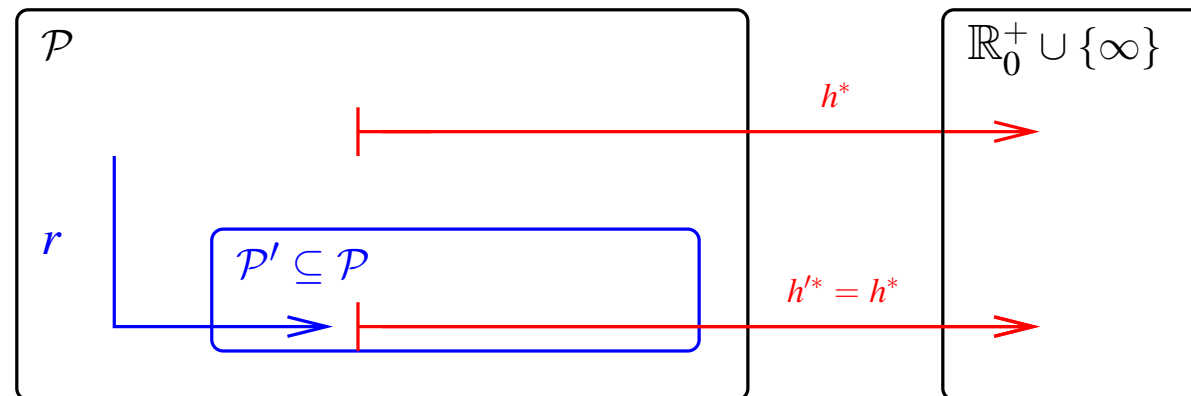
Theorem (Optimal Relaxed Planning is Hard). PlanOpt^+ is **NP**-complete.

Proof. Membership: Guess action sequences of length $|A|$ – in a relaxed plan, each action is applied at most once!

Hardness: By reduction from SAT.

- For each variable $v_i \in \{v_1, \dots, v_m\}$ in the CNF, three facts v_i , $\text{not}v_i$, and $\text{set}v_i$; for each clause $c_j \in \{c_1, \dots, c_n\}$ in the CNF, one fact $\text{sat}c_j$.
- Actions $\text{set}v_{\text{true}}_i$: $(\emptyset, \{v_i, \text{set}v_i\}, \emptyset)$ and $\text{set}v_{\text{false}}_i$: $(\emptyset, \{\text{not}v_i, \text{set}v_i\}, \emptyset)$.
- Actions $\text{make} \text{sat}c_j$: $(\{v_i\}, \{\text{sat}c_j\}, \emptyset)$ where v_i appears positively in clause c_j ; $(\{\text{not}v_i\}, \{\text{sat}c_j\}, \emptyset)$ where v_i appears negatively in clause c_j .
- Initial state \emptyset , goal $\{\text{set}v_1, \dots, \text{set}v_m, \text{sat}c_1, \dots, \text{sat}c_n\}$; $B := m + n$.

h^+ as a Relaxation Heuristic



where, for all $\Pi \in \mathcal{P}$, $h^*(r(\Pi)) \leq h^*(\Pi)$.

For $h^+ = h^* \circ r$:

- Problem \mathcal{P} : All STRIPS planning tasks.
- Simpler problem \mathcal{P}' : All STRIPS planning tasks with empty deletes.
- Perfect heuristic h'^* for \mathcal{P}' : Optimal plan cost = h^* on \mathcal{P}' .
- Transformation r : Drop the deletes.

- Is this a native relaxation? Yes.
- Is this relaxation efficiently constructible? Yes.
- Is this relaxation efficiently computable? No.

What shall we do with this relaxation?

Reminder:

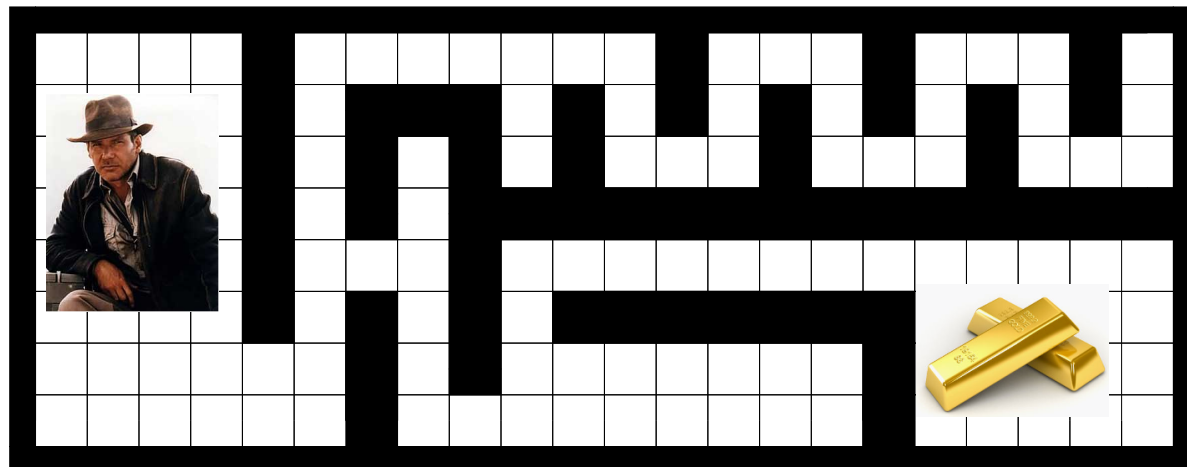
→ **Lecture 4**

What if \mathcal{R} is not efficiently computable?

- Either (a) approximate h'^* , or (b) design h'^* in a way so that it will typically be feasible, or (c) just live with it and hope for the best.
- Many known relaxations (in planning) are efficiently computable, some aren't (like h^+). The latter use (a); (b) and (c) are not used anywhere right now.

→ The delete relaxation heuristic we want is h^+ . Unfortunately, this is hard to compute so the computational overhead is very likely to be prohibitive. All implemented systems using the delete relaxation approximate h^+ in one or the other way. We now look at the the most wide-spread approaches to do so.

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Question!

In this domain, h^+ is equal to?

(A): Manhattan Distance.

(B): h^* .

(C): Horizontal distance.

(D): Vertical distance.

→ (A): No, relaxed plans can't walk through walls. (B): Yes, optimal plan = shortest path = relaxed plan (deletes do not matter because "shortest paths never walk back"). (C), (D): No, relaxed plans must move both horizontally and vertically.

The Additive and Max Heuristics

Definition (h^{add}). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. The *additive heuristic* h^{add} for Π is the function $h^{\text{add}}(s) := h^{\text{add}}(s, G)$ where $h^{\text{add}}(s, g)$ is the point-wise greatest function that satisfies $h^{\text{add}}(s, g) =$

$$\begin{cases} 0 & g \subseteq s \\ \min_{a \in A, g \in \text{add}_a} c(a) + h^{\text{add}}(s, \text{pre}_a) & |g| = 1 \\ \sum_{g' \in g} h^{\text{add}}(s, \{g'\}) & |g| > 1 \end{cases}$$

Definition (h^{max}). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. The *max heuristic* h^{max} for Π is the function $h^{\text{max}}(s) := h^{\text{max}}(s, G)$ where $h^{\text{max}}(s, g)$ is the point-wise greatest function that satisfies $h^{\text{max}}(s, g) =$

$$\begin{cases} 0 & g \subseteq s \\ \min_{a \in A, g \in \text{add}_a} c(a) + h^{\text{max}}(s, \text{pre}_a) & |g| = 1 \\ \max_{g' \in g} h^{\text{max}}(s, \{g'\}) & |g| > 1 \end{cases}$$

The Additive and Max Heuristics: Properties

Proposition (h^{\max} is Optimistic). $h^{\max} \leq h^+$, and thus $h^{\max} \leq h^*$.

Proposition (h^{add} is Pessimistic). For all STRIPS planning tasks Π , $h^{\text{add}} \geq h^+$. There exist Π and s so that $h^{\text{add}}(s) > h^*(s)$.

→ Both h^{\max} and h^{add} approximate h^+ by assuming that singleton sub-goal facts are achieved independently. h^{\max} estimates optimistically by the most costly singleton sub-goal, h^{add} estimates pessimistically by summing over all singleton sub-goals.

The Additive and Max Heuristics: Properties, ctd.

Proposition (h^{\max} and h^{add} Agree with h^+ on ∞). For all STRIPS planning tasks Π and states s in Π , $h^+(s) = \infty$ if and only if $h^{\max}(s) = \infty$ if and only if $h^{\text{add}}(s) = \infty$.

→ States for which no relaxed plan exists are easy to recognize, and that is done by both h^{\max} and h^{add} . Approximation is needed only for the cost of an optimal relaxed plan, if it exists.

Bellman-Ford for h^{\max} and h^{add}

Bellman-Ford variant computing h^{add} for state s

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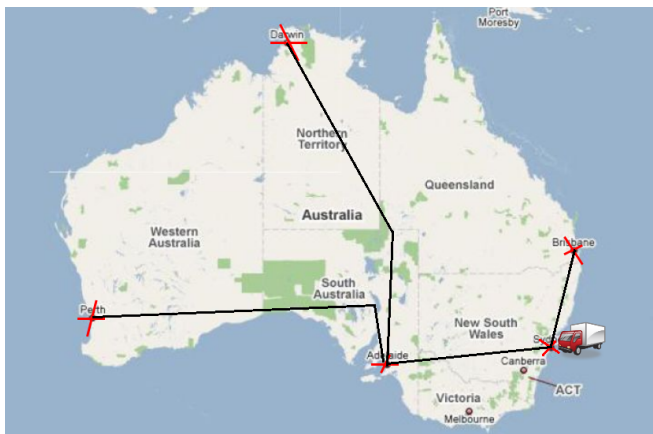
new table  $T_0^{\text{add}}(g)$ , for  $g \in F$ 
For all  $g \in F$ :  $T_0^{\text{add}}(g) := \begin{cases} 0 & g \in s \\ \infty & \text{otherwise} \end{cases}$ 
fn  $c_i(g) := \begin{cases} T_i^{\text{add}}(g) & |g| = 1 \\ \sum_{g' \in g} T_i^{\text{add}}(g') & |g| > 1 \end{cases}$ 
fn  $f_i(g) := \min[c_i(g), \min_{a \in A, g \in \text{add}_a} c(a) + c_i(\text{pre}_a)]$ 
do forever:
  new table  $T_{i+1}^{\text{add}}(g)$ , for  $g \in F$ 
  For all  $g \in F$ :  $T_{i+1}^{\text{add}}(g) := f_i(g)$ 
  if  $T_{i+1}^{\text{add}} = T_i^{\text{add}}$  then stop endif
   $i := i + 1$ 
enddo

```

→ Basically the same algorithm works for h^{\max} , just change \sum for \max

Proposition. Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. Then the series $\{T_i^{\text{add}}(g)\}_{i=0, \dots}$ converges to $h^{\text{add}}(s, g)$, for all g . (Proof omitted.)

Bellman-Ford for h^{\max} in “TSP” in Australia



■ $F: at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.

■ $A: drive(x, y)$ where x, y have a road.

$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

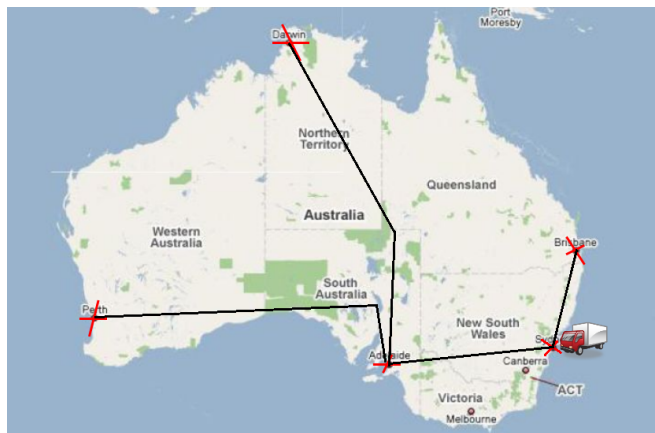
■ $I: at(Sy), v(Sy)$; $G: at(Sy), v(x)$ for all x .

Content of Tables T_i^1 :

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

$\rightarrow h^{\max}(I) = 5.5 < 20 = h^*(I).$

Bellman-Ford for h^{add} in “TSP” in Australia



■ $F: at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.

■ $A: drive(x, y)$ where x, y have a road.

$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

■ $I: at(Sy), v(Sy)$; $G: at(Sy), v(x)$ for all x .

Content of Tables T_i^{add} :

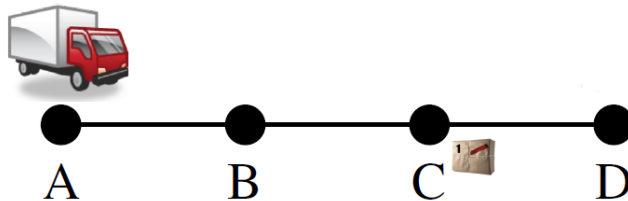
i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

→ $h^{\text{add}}(I) = 1.5 + 1 + 5 + 5.5 = 13 > 10 = h^+(I)$. But $< 20 = h^*(I)$.

→ $h^{\text{add}}(I) > h^+(I)$ because it counts the cost of $drive(Sy, Ad)$ 3 times:

As part of $h^{\text{add}}(I, \{v(Ad)\})$, $h^{\text{add}}(I, \{v(Pe)\})$, and $h^{\text{add}}(I, \{v(Da)\})$!

Bellman-Ford for h^{add} in “Logistics”



- Initial state $I: t(A), p(C)$.
- Goal $G: t(A), p(D)$.
- Actions $A: dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add} : (Table content T_i^1 , where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

→ $h^{\text{add}}(I) = 7 > h^+(I) = 5$. But $< 8 = h^*(I)$.

→ $h^{\text{add}}(I) > h^+(I)$ **because?** It counts the cost of $dr(A, B), dr(B, C)$ 2 times, for the two preconditions $p(T)$ and $t(D)$ of achieving $p(D)$.

→ **So, what if $G = \{t(D), p(D)\}$?** $h^{\text{add}}(I) = 10 > 5 = h^*(I) = h^+(I)$ because now $dr(A, B), dr(B, C), dr(C, D)$ is counted also as part of the goal $t(D)$.

The Additive and Max Heuristics: So What?

Summary of typical issues in practice with h^{add} and h^{max} :

- Both h^{add} and h^{max} can be computed reasonably quickly.
- h^{max} is **admissible**, but is typically **far too optimistic**.
- h^{add} is **not admissible**, but is typically **a lot more informed than h^{max}** .
- h^{add} is sometimes better informed than h^+ , but for the “wrong reasons”: rather than accounting for deletes, it overcounts by **ignoring positive interactions**, i.e., sub-plans shared between sub-goals.
- Such overcounting can result in **dramatic over-estimates of h^*** !!

→ On slide 28 with goal $t(D)$, if we have 100 packages at C that need to go to D , what is $h^{\text{add}}(I)$? $703 \gg 203 = h^*(I) = h^+(I)$: For every package, a count of 7 which includes $dr(A, B)$, $dr(B, C)$ for getting the package into the truck, and $dr(A, B)$, $dr(B, C)$, $dr(C, D)$ for getting the truck to D .

→ Relaxed plans (up next) are a means to reduce this kind of over-counting.

Relaxed Plans, Basic Idea

→ First compute a **best-supporter function** bs , which for every fact $p \in F$ returns an action that is deemed to be the cheapest achiever of p (within the relaxation). Then **extract a relaxed plan** from that function, by applying it to singleton sub-goals and collecting all the actions.

→ The best-supporter function can be based directly on h^{\max} or h^{add} , simply selecting an action a achieving p that minimizes the sum of $c(a)$ and the cost estimate for pre_a .

And now for the details:

Popular Best-Supporter Functions

Definition (Best-Supporters from h^{\max} and h^{add}). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, and let s be a state.

The h^{\max} supporter function $bs_s^{\max} : \{p \in F \mid 0 < h^{\max}(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs_s^{\max}(p) := \arg \min_{a \in A, p \in \text{add}_a} c(a) + h^{\max}(s, \text{pre}_a)$.

The h^{add} supporter function $bs_s^{\text{add}} : \{p \in F \mid 0 < h^{\text{add}}(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs_s^{\text{add}}(p) := \arg \min_{a \in A, p \in \text{add}_a} c(a) + h^{\text{add}}(s, \text{pre}_a)$.

Example h^{add} in "Logistics":

Heuristic Values:

	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
h^{add}	0	1	2	3	3	4	5	0	7

Yields best-supporter function:

	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
bs^{add}	—	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	—	$ul(D)$

Relaxed Plan Extraction

Relaxed Plan Extraction for state s and best-supporter function bs

```

Open := G \ s; Closed := ∅; RPlan := ∅
while Open ≠ ∅ do:
    select  $g \in \text{Open}$ 
    Open := Open \ { $g$ }; Closed := Closed ∪ { $g$ };
    RPlan := RPlan ∪ { $bs(g)$ }; Open := Open ∪ (pre $bs(g)$  \ (s ∪ Closed))
endwhile
return RPlan

```

→ Starting with the top-level goals, iteratively close open singleton sub-goals by selecting the best supporter.

This is fast! Number of iterations bounded by $|P|$, each near-constant time.

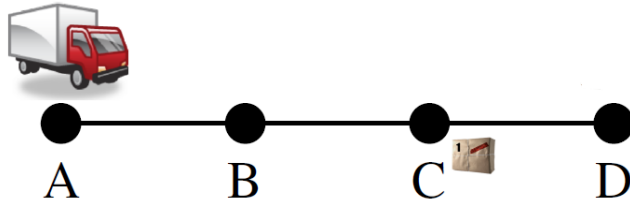
But is it correct?

→ **What if $g \notin add_{bs(g)}$?** Doesn't make sense. → Prerequisite (A).

→ **What if $bs(g)$ is undefined?** Runtime error. → Prerequisite (B).

→ **What if the support for g eventually requires g itself as a precondition?** Then this does not actually yield a relaxed plan. → Prerequisite (C).

Relaxed Plan Extraction from h^{add} in “Logistics”



- Initial state $I: t(A), p(C)$.
- Goal $G: t(A), p(D)$.
- Actions $A: dr(X, Y), lo(X), ul(X)$.

	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
bs^{add}	—	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	—	$ul(D)$

Extracting a relaxed plan:

- 1 $bs_s^{\text{add}}(p(D)) = ul(D)$; opens $t(D), p(T)$.
 - 2 $bs_s^{\text{add}}(t(D)) = dr(C, D)$; opens $t(C)$.
 - 3 $bs_s^{\text{add}}(t(C)) = dr(B, C)$; opens $t(B)$.
 - 4 $bs_s^{\text{add}}(t(B)) = dr(A, B)$; opens nothing.
 - 5 $bs_s^{\text{add}}(p(T)) = lo(C)$; opens nothing.
 - 6 **Anything more?** No, open goals empty at this point.
- $h^{\text{FF}}(I) = 5 = h^+(I) < 7 = h^{\text{add}}(I) < 8 = h^*(I)$.

→ **What if $G = \{t(D), p(D)\}$?** $h^{\text{FF}}(I) = 5 = h^+(I) = h^*(I)$ because relaxed plan extraction selects the drive actions only once. By contrast, $h^{\text{add}}(I) = 10$ overcounts these actions, cf. slide 28.

Best-Supporter Functions

→ For relaxed plan extraction to make sense, it requires a *closed well-founded* best-supporter function:

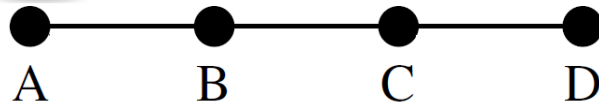
Definition (Best-Supporter Function). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, and let s be a state. A *best-supporter function* for s is a partial function $bs : (F \setminus s) \mapsto A$ such that $p \in add_a$ whenever $a = bs(p)$.

The *support graph* of bs is the directed graph with vertices $F \cup A$ and arcs $\{(p, a) \mid p \in pre_a\} \cup \{(a, p) \mid a = bs(p)\}$. We say that bs is *closed* if $bs(p)$ is defined for every $p \in (F \setminus s)$ that has a path to a goal $g \in G$ in the support graph. We say that bs is *well-founded* if the support graph is acyclic.

- “ $p \in add_a$ whenever $a = bs(p)$ ”: Prerequisite (A).
- bs is closed: Prerequisite (B).
- bs is well-founded: Prerequisite (C).

→ Intuition for (C): Relaxed plan extraction starts at the goals, and chains backwards in the support graph. If there are cycles, then this backchaining may not reach the currently true state s , and thus not yield a relaxed plan.

Support Graphs and Prerequisite (C) in “Logistics”



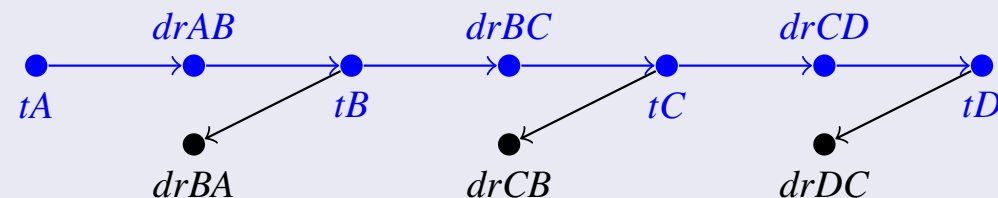
- Initial state: tA .
- Goal: tD .
- Actions: $drXY$.

How to do it (well-founded)

Best-supporter function:

p	$bs(p)$
$t(B)$	$dr(A, B)$
$t(C)$	$dr(B, C)$
$t(D)$	$dr(C, D)$

Yields support graph backchaining:

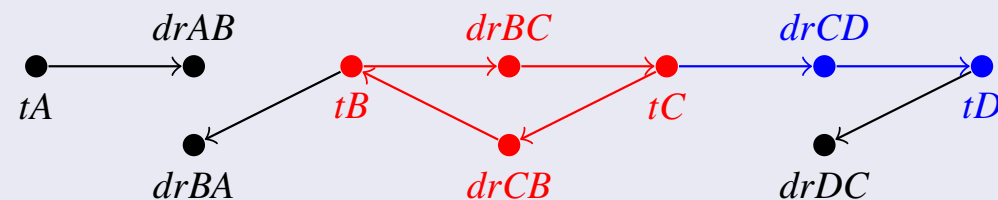


How to NOT do it (not well-founded)

Best-supporter function:

p	$bs(p)$
$t(B)$	$dr(C, B)$
$t(C)$	$dr(B, C)$
$t(D)$	$dr(C, D)$

Yields support graph backchaining:



How to obtain closed well-founded bs ?

Definition (Best-Supporters from h^{\max} and h^{add}). Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, and let s be a state.

The h^{\max} supporter function $bs_s^{\max} : \{p \in F \mid 0 < h^{\max}(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs_s^{\max}(p) := \arg \min_{a \in A, p \in \text{add}_a} c(a) + h^{\max}(s, \text{pre}_a)$.

The h^{add} supporter function $bs_s^{\text{add}} : \{p \in F \mid 0 < h^{\text{add}}(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs_s^{\text{add}}(p) := \arg \min_{a \in A, p \in \text{add}_a} c(a) + h^{\text{add}}(s, \text{pre}_a)$.

Proposition. Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task such that, for all $a \in A$, $c(a) > 0$. Let s be a state where $h^+(s) < \infty$. Then both bs_s^{\max} and bs_s^{add} are closed well-founded supporter functions for s .

Proof. Since $h^+(s) < \infty$ implies $h^{\max}(s) < \infty$, it is easy to see that bs_s^{\max} is closed (details omitted). If $a = bs_s^{\max}(p)$, then a is the action yielding $0 < h^{\max}(s, \{p\}) < \infty$ in the h^{\max} equation. Since $c(a) > 0$, we have $h^{\max}(s, \text{pre}_a) < h^{\max}(s, \{p\})$ and thus, for all $q \in \text{pre}_a$, $h^{\max}(s, \{q\}) < h^{\max}(s, \{p\})$. Transitivity, if the support graph contains a path from fact vertex r to fact vertex t , then $h^{\max}(s, \{r\}) < h^{\max}(s, \{t\})$. Thus there can't be cycles in the support graph and bs_s^{\max} is well-founded. Similar for bs_s^{add} .

The Relaxed Plan Extraction: Correctness

Proposition. Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, let s be a state, and *let bs be a closed well-founded best-supporter function for s . Then the action set $RPlan$ returned by relaxed plan extraction can be sequenced into a relaxed plan \vec{a}^+ for s .*

Proof. Order a before a' whenever the support graph contains a path from a to a' . Since the support graph is acyclic, such a sequencing $\vec{a} := \langle a_1, \dots, a_n \rangle$ exists. We have $p \in s$ for all $p \in pre_{a_1}$, because otherwise $RPlan$ would contain the action $bs(p)$, necessarily ordered before a_1 . We have $p \in s \cup add_{a_1}$ for all $p \in pre_{a_2}$, because otherwise $RPlan$ would contain the action $bs(p)$, necessarily ordered before a_2 . Iterating the argument shows that \vec{a}^+ is a relaxed plan for s .

The Relaxed Plan Heuristic

Definition (Relaxed Plan Heuristic). A heuristic function is called a *relaxed plan heuristic*, denoted h^{FF} , if, given a state s , it returns ∞ if no relaxed plan exists, and otherwise returns $\sum_{a \in RPlan} c(a)$ where *RPlan* is the action set returned by relaxed plan extraction on a closed well-founded best-supporter function for s .

→ Recall: If a relaxed plan exists, then there also exists a closed well-founded best-supporter function, see previous slide.

The Relaxed Plan Heuristic, ctd.

Proposition (h^{FF} is Pessimistic and Agrees with h^* on ∞). For all STRIPS planning tasks Π , $h^{\text{FF}} \geq h^+$; for all states s , $h^+(s) = \infty$ if and only if $h^{\text{FF}}(s) = \infty$. There exist Π and s so that $h^{\text{FF}}(s) > h^*(s)$.

Proof. $h^{\text{FF}} \geq h^+$ follows directly from the previous proposition. Agrees with h^+ on ∞ : direct from definition. Inadmissibility: Whenever bs makes sub-optimal choices. → **Exercise, perhaps**

→ Relaxed plan heuristics have the same theoretical properties as h^{add} .

So what's the point?

- Can h^{FF} over-count, i.e., count sub-plans shared between sub-goals more than once? No, due to the set union in " $RPlan := RPlan \cup \{bs(g)\}$ ".
- h^{FF} may be inadmissible, just like h^{add} , but for more subtle reasons.
- In practice, h^{FF} typically does not over-estimate h^* (or not by a large amount, anyway); cf. example on previous "Logistics" slide.

Helpful Actions

Definition (Helpful Actions) Let h^{FF} be a relaxed plan heuristic, let s be a state, and let $RPlan$ be the action set returned by relaxed plan extraction on the closed well-founded best-supporter function for s which underlies h^{FF} . Then an action a **applicable** to s is called **helpful** if it is **contained in $RPlan$** .

Remarks

- Initially introduced in FF [Hoffmann and Nebel (2011)], restricting Enforced Hill-Climbing to use **only** the helpful actions
- Expanding only helpful actions does not guarantee completeness.
- Other planners use helpful actions as preferred operators, expanding first nodes resulting from helpful actions.

Questionnaire

Question!

How does ignoring delete lists simplify FreeCell?

(A): You can move all cards immediately to their goal.

(B): Free cells remain free.

→ (A): No, we don't get any new moves in the relaxation. (B): Yes, when putting a card into a free cell, it's still free for another card.

Question!

How does ignoring delete lists simplify Sokoban?

(A): Free positions remain free.

(B): You can walk through walls.

(C): You can push 2 stones to same position.

(D): Nothing ever becomes blocked.

→ (A), (C), (D): Yes (similar to above). (B): No, we don't get any new moves.

Summary

- The **delete relaxation** simplifies STRIPS by removing all delete effects of the actions.
- The cost of **optimal relaxed plans** yields the heuristic function h^+ , which is admissible but hard to compute.
- We can approximate h^+ optimistically by h^{\max} , and pessimistically by h^{add} . h^{\max} is admissible, h^{add} is not. h^{add} is typically much more informative, but can suffer from **over-counting**.
- Either of h^{\max} or h^{add} can be used to generate a **closed well-founded best-supporter function**, from which we can **extract a relaxed plan**. The resulting **relaxed plan heuristic** h^{FF} does not do over-counting, but otherwise has the same theoretical properties as h^{add} ; it typically does not over-estimate h^* .

Example Systems

HSP [*Bonet and Geffner, AI-01*]

1. Search algorithm: Greedy best-first search.
2. Search control: h^{add} .

FF [*Hoffmann and Nebel, JAIR-01*]

1. Search algorithm: Enforced hill-climbing.
2. Search control: h^{FF} extracted from h^{max} supporter function; helpful actions pruning (basically expand only those actions contained in the relaxed plan).

LAMA [*Richter and Westphal, JAIR-10*]

1. Search algorithm: Multiple-queue greedy best-first search.
2. Search control: h^{FF} + a landmarks heuristic (→ similar to goal counting); for each, one search queue all actions, one search queue only helpful actions.

BFWS [*Lipovetzky and Geffner, AAAI-17*]

1. Search algorithm: best-first width search (→ next lecture).
2. Search control: novelty (→ next lecture) + variant of h^{FF} + goal counting.

Remarks

- The delete relaxation is aka [ignoring delete lists](#).
- HSP was competitive in the 1998 International Planning Competition (IPC'98); FF outclassed the competitors in IPC'00.
- The delete relaxation is still used at large, specially since the wins of LAMA in the satisficing planning tracks of IPC'08 and IPC'11.

Remarks, ctd.

- → More generally, the relaxation principle is very generic and potentially applicable in many different contexts, as are all relaxation principles covered in this course.
- While h^{\max} is not informative in practice, other lower-bounding approximations of h^+ are very important for optimal planning: [admissible landmarks heuristics](#) [Karpas and Domshlak, IJCAI-09]; [LM-cut heuristic](#) [Helmert and Domshlak, ICAPS-09].
- It has always been a challenge to take *some* delete effects into account. Recent work done to interpolate smoothly between h^+ and h^* : [explicitly represented fact conjunctions](#) [Keyder, Hoffmann, and Haslum ICAPS-12].

Reading

■ *Planning as Heuristic Search [Bonet and Geffner, AI-01].*

Available at:

<http://www.dtic.upf.edu/~hgeffner/html/reports/hsp-aij.ps>

Content: This is “where it all started”: the first paper¹ explicitly introducing the notion of heuristic search and automatically generated heuristic functions to planning. Introduces the additive and max heuristics h^{add} and h^{max} .

¹Well, this is the first full journal paper treating the subject; the same authors published conference papers in AAAI'97 and ECP'99, which are subsumed by the present paper.

Reading, ctd.

- *The FF Planning System: Fast Plan Generation Through Heuristic Search* [Hoffmann:nebel:jair-01]. **JAIR Best Paper Award 2005.**

Available at:

<http://fai.cs.uni-saarland.de/hoffmann/papers/jair01.pdf>

Content: The main reference for delete relaxation heuristics (cited > 1000 times). Introduces the relaxed plan heuristic, extracted from the h^{\max} supporter function.² Also introduces helpful actions pruning, and enforced hill-climbing.

²Done in a uniform-cost setting presented in terms of relaxed planning graphs instead of h^{\max} , and not identifying the more general idea of using a well-founded best-supporter function (I used the same simpler presentation in the AI'12 core course). The notion of best-supporter functions (handling non-uniform action costs) first appears in [Keyder and Geffner, ECAI-08].

Reading, ctd.

- *Semi-Relaxed Plan Heuristics* [Keyder, Hoffmann, and Haslum ICAPS-12]. **Best Paper Award at ICAPS'12.**

Available at: <http://fai.cs.uni-saarland.de/hoffmann/papers/icaps12a.pdf>

Content: Computes relaxed plan heuristics within a compiled planning task Π_{ce}^C , in which a subset C of all fact conjunctions in the task is represented explicitly as suggested by [Haslum, ICAPS-12]. C can in principle always be chosen so that $h_{\Pi_{ce}^C}^+$ is perfect (equals h^* in the original planning task), so the technique allows to interpolate between h^+ and h^* . In practice, small sets C sometimes suffice to obtain dramatically more informed relaxed plan heuristics.