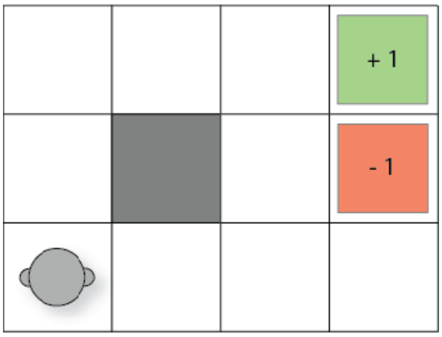
Probabilistic planning - Markov Decision Processes (MDPs)



An agent has a goal to navigate cells

- The grey square is a wall (like the edges of grid)
- The two coloured cells giving rewards: 1 and -1

Actions have **non-deterministic** outcomes (effects)!

- If the agent tries to move north, 80% of the time, this works as planned (provided the wall is not in the way)
- 10% of the time, trying to move north takes the agent east (provided the wall is not in the way)
- 10% of the time, trying to move north takes the agent west (provided the wall is not in the way);
- If wall is in the way of the cell that would have been taken, the agent stays put
- Similar for all other directions

MDPs:

- Set of states S
- Initial state I
- Probabilistic state transitions: $\sum_{s'} P_a(s'|s) = 1.0$
- Reward function r(s, a, s') in Real
- Discount factor γ (*gamma*)

Classical Planning:

- Set of states S
- Initial state I
- Transition function A $s \xrightarrow{a} s'$
- Goals G
- Costs

γ discount factor in $[0,1)$

Discounted rewards

$$G_t = r_1 + \gamma r_2 + \gamma^2 r_3 + \gamma^3 r_4 \ldots$$
$$= r_1 + \gamma(r_2 + \gamma(r_3 + \gamma(r_4 \ldots)))$$
$$= r_t + \gamma G_{t+1}$$

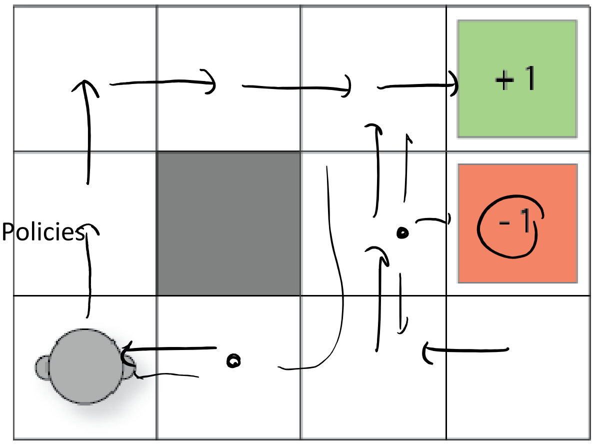
Modelling MDPs --- Probabilistic PDDL

```
(define (domain bomb-and-toilet)
    (:requirements :conditional-effects :probabilistic-effects)
    (:predicates (bomb-in-package ?pkg) (toilet-clogged)
                      (bomb-defused))

    (:action dunk-package
        :parameters (?pkg)

  :effect (and (when (bomb-in-package ?pkg)
              (bomb-defused))
           (probabilistic 0.05 (toilet-clogged))))
```

Solutions to MDPs -- Policies

Solution for MDP is a *policy*:

at(0,0) => move_up
at(0,1) => move_up
at(0,2) => move_right
at(1,0) => move_left
at(1,2) => move_right
at(2,0) => move_up
at(2,1) => move_up
at(2,2) => move_right
at(3,0) => move_left



$[\ \pi(s) \rightarrow A$ deterministic

$\pi(s,a) \in [0,1]$ stochastic

Solving MDPs

Expected return exercise:

You can steal:

A) An iPhone, which you think you have a 20% chance of selling for \$500, or an 80% chance of selling for \$250.

B) A Samsung, which you think you have a 50% chance of selling for \$500, or a 50% chance of selling for \$200.

A: 0.2*500 + 0.8*250 = 300
B: 0.5*500 + 0.5*200 = 350

Bellman equation:

$$V(s) = \max_{a \in A} \underbrace{\sum_{s' \in S} P_a(s'|s)\left[r(s,a,s') + \gamma V(s')\right]}_{\text{expected reward action } a}$$