DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

SEMESTER EXAMINATION: MAY 2017

Mechanical/Electrical/ExTC/Chemical/Petrochemical/Computer/IT/Civil

Subject: Engineering Mathematics-II (OLD) **Sem-II**

Time: 03 Hrs. **Max Marks:** 70

Instruction to Students

Solve **SIX** questions in all in the following manner:

1. **Q.1** is compulsory.
2. Solve any **FIVE** questions from amongst **Q.2** to **Q.7**.

Q.1 **Pick up the correct option of the choices given.** [2 x 5=10 marks]

a) If $2x + x^2 + \alpha y^2$ is to be harmonic, then the value of α is

(i) 1 (ii) -1 (iii) 2 (iv) none.

b) The period of the function $cos3x$ is

(i) 2π (ii) $\frac{2\pi}{3}$ (iii) π (iv) none.

c) The order of the differential equation $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = c\frac{d^2y}{dx^2}$ is

(i) 2 (ii) 1 (iii) 0 (iv) none.

d) Complementary Function (C.F.) of $y'' - 3y' + 2y = 12$ is

(i) $c_1e^x + c_2e^{2x}$ (ii) $c_1e^x + c_2e^{-2x}$ (iii) $c_1e^{-x} + c_2e^{2x}$ (iv) none.

e) If $u = x^y$, then $\frac{\partial u}{\partial y}$ is equal to

(i) $x^y \log x$ (ii) 0 (iii) yx^{y-1} (iv) none.

Q.2 (a) Find the analytic function whose imaginary part is $e^x(xsiny + ycosy)$. [4 marks]

(b) Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is a harmonic function and hence determine the corresponding analytic function. [4 marks]

(c) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is the circle $|z| = 2$. [4 marks]

Q.3 (a) Find the Fourier series expansion for $f(x) = x^2$ in the interval $(0,2\pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots \infty$. [6 marks]

(b) Find the Fourier series to represent the function $f(x)$ given by

$$f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}.$$

Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots\dots\dots = \frac{\pi^2}{8}$. [6 marks]

Q.4 (a) Solve $(x - y - 2)dx + (x - 2y - 3)dy = 0$. [4 marks]

(b) Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$. [4 marks]

(c) Determine the minimum velocity with which if a particle is projected vertically upwards, the particle will cross earth's gravitational attraction and will never return on the surface of the earth. Assume that the particle is acted upon by the gravitational attraction of earth only. [4 marks]

Q.5 (a) Solve $(D^2 + 16)y = x \sin 3x$. [4 marks]

(b) Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$. [4 marks]

(c) Solve the equation $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ by the method of variation of parameters. [4 marks]

Q.6 (a) If $x^x y^y z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \, \partial y} = -(x \log ex)^{-1}$. [4 marks]

(b) If $u = sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -7 \tan u$. [4 marks]

(c) If $u = f(r,s,t)$ and $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. [4 marks]

Q.7 (a) Expand $f(x,y) = e^{xy} \cos y$ at $(1, 1)$ by using Taylor's theorem for two variables. [4 marks]

(b) If the sides and angles of a plane triangle vary in such a way that its circum-radius remains constant, prove that $\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0$ where da, db, dc are small increments in the sides a, b, c respectively. [4 marks]

(c) Find the points on the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minimum value, and hence find this minimum value of f. [4 marks]