DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

SEMESTER EXAMINATION: MAY : 2015

Mechanical/ Electrical/ EXTC/ Chemical/ Petrochemical/ Computer/ IT/ Civil

Subject: Engineering Mathematics-I (NEW COURSE) **Sem-I**

Time: 03 Hrs. **Max Marks:** 70

Instruction to Students

Solve **SIX** questions in all in the following manner:

1. **Q. No. 1** is compulsory.
2. Solve the remaining **FIVE** questions from **Q. No. 2** to **Q. No. 7**.

Q.1 Pick-up the correct answer of the choices given: [2 x 5=10 marks]

a) If every minor of order r of a matrix A is zero, then the rank of A is

(i) Greater than r (ii) Equal to r (iii) Less than r (iv) none.

b) A unit tangent vector to the surface $x = t, y = t^2, z = t^3$ at $t = 1$ is

(i) $\frac{1}{\sqrt{14}}(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ (ii) $\frac{1}{\sqrt{14}}(\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$ (iii) $\hat{\imath} + \hat{\jmath} + \hat{k}$ (iv) none.

c) If $\vec{F}$ is such that $\nabla \times \vec{F} = 0$, then $\vec{F}$ is called

(i) Solenoidal (ii) Cylindrical (iii) Irrotational (iv) none.

d) If $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = K$, then $\sum u_n$ diverges for

(i) $K > 1$ (ii) $K < 1$ (iii) $K = 1$ (iv) none.

e) The value of $\int_0^{\frac{\pi}{2}} \int_0^1 r \sin\theta \; dr d\theta$ is equal to

(i) $\frac{1}{2}$ (ii) $\frac{\pi}{2}$ (iii) 0 (iv) none.

Q.2 (a) Use Gauss-Jordan method to find A^{-1}, where

$$A = \begin{bmatrix} 8 & 4 & -3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$ [4 marks]

(b) Find the rank of the matrix A by reducing it to normal form, where

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}.$$ [4 marks]

(c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [4 marks]

P.T.O.

Q.3 (a) If $y = x\log(1+x)$, prove that $y_n = \frac{(-1)^{n-2}(n-2)!(x+n)}{(x+1)^n}$. [4 marks]

(b) If $y = \frac{sinh^{-1}x}{\sqrt{1+x^2}}$, show that $(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2 y_n = 0$. [4 marks]

(c) Using Taylor's theorem, express the polynomial $f(x) = 2x^3 + 7x^2 + x - 6$ in the powers of $(x-1)$. [4 marks]

Q.4 (a) A point moves in a plane so that its tangential and normal components of acceleration are equal and the angular velocity of the tangent is constant and equal to ω. Show that the path of the particle is an equiangular spiral $\omega s = Ae^{\omega t} + B$, where A and B are constants. [6 marks]

(b) If a particle moves along the curve $y = a\log\sec\left(\frac{x}{a}\right)$ in such a way that the tangent to the curve rotates uniformly, prove that the resultant acceleration of the particle varies as square of the radius of curvature. [6 marks]

Q.5 If $\vec{r}$ is the position vector such that $r = |\vec{r}|$ and $\vec{a}$, $\vec{b}$ are constant vectors, then prove that

(a) $\nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^n}\right) = \left[\frac{2-n}{r^n}\right]\vec{a} + nr^{-(n+2)}(\vec{a} \cdot \vec{r})\vec{r}$

(b) Curl $\left[(\vec{r} \times \vec{a}) \times \vec{b}\right] = \vec{b} \times \vec{a}$

(c) $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$. [4 marks]

Q.6 (a) Test the convergence of the series: $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^3}$. [4 marks]

(b) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$. [4 marks]

(c) Test the convergence of the series: $1 - 2x + 3x^2 - 4x^3 + \dots\dots\dots\dots \infty$ $\left(x < \frac{1}{2}\right)$. [4 marks]

Q.7 (a) Evaluate $I = \int_0^{\infty}\int_0^{x} x\, e^{-\frac{x^2}{y}} dxdy$ by changing the order of integration. [4 marks]

(b) Change to polar co-ordinates to evaluate $I = \int_0^{a}\int_0^{\sqrt{a^2-y^2}} e^{-x^2-y^2}\, dxdy$. [4 marks]

(c) Evaluate $I = \int_0^1\int_{y^2}^1\int_0^{1-x} x \; dz\, dx\, dy$. [4 marks]

P.T.O.