	DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE **Supplementary Examination – Summer 2023** **Course: B. Tech. (Common to all Branches) Semester : III** **Subject Name & Code: Engineering Mathematics – III (BTBSC 301)** **Max Marks: 60 Date:08/08/2023 Duration: 3 Hrs.**	
	Instructions to the Students: 1. *Attempt any **FIVE** of the following questions.* 2. *All questions carry equal marks.* 3. *Use of non-programmable scientific calculators is allowed.* 4. *Assume suitable data wherever necessary and mention it clearly.*	
		Marks
Q. 1	**Solve Any Two of the following.**	**12**
A)	Find the Laplace transform of $F(t) = \frac{e^{-at}-e^{-bt}}{t}$	**6**
B)	Find the Laplace transform of $F(t) = \sin 2t \cos 3t$	**6**
C)	Find the Laplace transform of $erf\left(\sqrt{t}\right)$.	**6**
Q.2	**Solve Any Two of the following:**	**12**
A)	State and prove the convolution theorem for finding the inverse Laplace transform.	**6**
B)	Using Partial Fraction method, find the inverse Laplace transform of $\bar{f}(s) = \frac{5s+3}{(s-1)(s^2+2s+5)}$	**6**
C)	Find the inverse Laplace transform of $\bar{f}(s) = \cot^{-1}\left(\frac{s+3}{2}\right)$	**6**
Q. 3	**Solve any Two of the following:**	**12**
A)	Find the Fourier sine transform of $e^{-\|x\|}$, and hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}$, $m > 0$	**6**
B)	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \|x\| \le 1 \\ 0, & \|x\| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3}\right) \cos\frac{x}{2} dx$.	**6**
C)	Evaluate the integral $\int_0^\infty \frac{t^2}{(t^2+1)^2} dt = \frac{\pi}{4}$. .	**6**
Q.4	**Solve any Two of the following:**	**12**
A)	The partial differential equation by eliminating the arbitrary function from $z = x + y + f(xy)$	**6**
B)	The partial differential equations by eliminating the arbitrary constant $z = (x^2 + a)(y^2 + b)$	**6**
C)	Solve the following partial differential equations $(mz - ny)p + (nx + lz)q = ly - mx$ where the symbols have got their usual meanings.	**6**
Q. 5	**Solve any Two of the following:**	**12**
A)	Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is a harmonic function and hence determine the corre-	**6**

	sponding analytic function	
B)	If $\boldsymbol{f(z)}$ is an analytic function with constant modulus, show that $\boldsymbol{f(z)}$ is constant	**6**
C)	Under the transformation $\boldsymbol{W = \frac{1}{z}}$, find the image of $\boldsymbol{\lvert z - 2i \rvert = 2}$.	**6**
Q. 6	**Solve any Two of the following:**	
A)	Evaluate $\boldsymbol{\int_0^{1+i} (x^2 + iy)\, dz}$ along the path $\boldsymbol{y = x}$ and $\boldsymbol{y = x^2}$	**6**
B)	Evaluate $\oint_C \frac{e^{-z}}{z+1} \boldsymbol{dz}$ where C is the circle $\boldsymbol{\lvert z \rvert = 2}$ and $\boldsymbol{\lvert z \rvert = \frac{1}{2}}$	**6**
C)	Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} \boldsymbol{dz}$, where $\boldsymbol{C}$ the circle is $\boldsymbol{\lvert z \rvert = 2}$.	**6**
	***** End *****	