

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE**RAIGAD-402 103.****Semester Examination – May - 2019**

Branch: First year B. Tech.(ALL)

Semester: I

Subject with Code: Engg. Math-I (BTMA101)

Time: 3 hrs.

Date: 14/05/2019

Max. Marks: 60

Instructions to Students:

- (1) All questions are compulsory.
- (2) Use of non-programmable calculator is allowed.
- (3) Figures to right indicate full marks.
- (4) Illustrate your answer with neat sketches, diagram etc. whatever necessary.
- (5) If some part or parameter is noticed to be missing you may appropriately assume it and should mention it clearly.

Q.1 Attempt any three from the following: 4 X 3 = 12

- (a) Reduce the following matrix into normal form & find its rank

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

- (b) Test the consistency & solve

$$4x - 2y + 6z = 8 ;$$

$$x + y - 3z = -1 ;$$

$$15x - 3y + 9z = 21.$$

- (c) Find the Eigen values & Eigen vectors of the following matrix

$$\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

- (d) Find
- A^{-1}
- by using Cayley-Hamilton Theorem

$$\text{Where } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Q.2 Attempt any three from the following:**4 X 3 = 12**

- (a) If
- $u = \log(x^3 + y^3 + z^3 - 3xyz)$
- then prove that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}.$$

- (b) If $f(u)$ is a homogeneous function of degree n in x & y , then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = G(u) [G'(u) - 1] \quad \text{where } G(u) = n \frac{f(u)}{f'(u)}.$$

- (c) Verify Euler's theorem for $u = \frac{x^2 + y^2}{x + y}$.

- (d) If $z = f(u, v)$ where $u = lx + my$ and $v = ly - mx$, then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right).$$

Q.3 Attempt any three from the following:

4 X 3 = 12

- (a) Show that $J.J' = 1$ If $x = e^u \cos v$ & $y = e^u \sin v$.
- (b) Expand $f(x, y) = \sin(xy)$ in the powers of $(x - 1)$ & $(y - \frac{\pi}{2})$ by using Taylor's Theorem.
- (c) Discuss the maxima & minima of $xy(a - x - y)$.
- (d) If $u = \frac{a^3}{x^2} + \frac{b^3}{y^2} + \frac{c^3}{z^2}$ where $x + y + z = 1$, then find the stationary values by using Lagrange's method of multipliers.

Q.4 Attempt any three from the following:

4 X 3 = 12

- (a) Evaluate $\int_0^4 x^3 \sqrt{4x - x^2} dx$.
- (b) Trace the curve $y^2(4 - x) = x(x - 2)^2$.
- (c) Trace the curve $r = a \cos(2\theta)$.
- (d) Trace the curve $x = a(t + \sin t)$
 $y = a(1 - \cos t)$.

Q.5 Attempt any three from the following:

4 X 3 = 12

- (a) Change the order of integration & evaluate it

$$\int_0^1 \int_y^1 x^2 e^{xy} dx dy.$$

- (b) Change to polar co-ordinates & evaluate it

$$\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2) dx dy$$

- (c) Evaluate $\int_{-1}^1 \int_0^z \int_{(x-z)}^{(x+z)} (x + y + z) dx dy dz$.

- (d) Find the area outside the circle $r = a$
& inside the cardioid $r = a(1 + \cos \theta)$.

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