DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End – Semester Examination (Supplementary): May 2019

Branch: B. Tech (Common to all) **Semester:** I

Subject with code: Engineering Mathematics – I (MATH 101) **Marks:** 60

Date: 28.05.2019 **Duration:** 03 Hrs.

INSTRUCTION: Attempt any **FIVE** of the following questions. All questions carry equal marks.

Q.1

(a) Solve the equations:

$$x + 2y + 4z + w = 0 \; ; 2x + 4y + 8z + 2w = 0 \; ; \; 3x + 6y + 2z = 0$$ [6 Marks]

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [6 Marks]

Q.2

(a) Find the nth order derivative of $y = cos^4 x$. [6 Marks]

(b) Using Taylor's theorem, express the polynomial $f(x) = 2x^3 + 7x^2 + x - 6$

in the powers of $(x - 1)$. [6 Marks]

Q.3 Solve any TWO:

(a) Evaluate $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ if $z = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$. [6 Marks]

(b) If z is a homogeneous function of degree n in $x \, and \, y$, prove that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$ [6 Marks]

(c) If $u = f(y - z, \; z - x, \; x - y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. [6 Marks]

Q.4

(a) Expand $f(x, y) = e^{xy}$ at $(1, 1)$ by using Taylor's theorem. [4 *Marks*]

(b) Find the percentage error in the measurement of the area of an ellipse when an error of **1.5 %** is made in measuring its major and minor axes. [4 *Marks*]

(c) Find the maximum value of $x^m y^n z^p$, when $x + y + z = c$. [4 *Marks*]

Q.5

(a) Evaluate the integral $I = \int_1^a \int_1^b \frac{dy\,dx}{xy}$. [6 *Marks*]

(b) Change the order of integration and evaluate $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y} \, dx\, dy$. [6 *Marks*]

(c) Evaluate the integral $I = \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz\, dx\, dy$. [6 *Marks*]

Q.6

(a) State D' Alembert's ratio test, and hence check the convergence of the series:

$$\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2} \right).$$ [6 *Marks*]

(b) State Cauchy's root test, and hence check the convergence of the series:

$$\sum \frac{[(2n+1)\, x\,]^n}{n^{n+1}} \quad (x > 0)$$ [6 *Marks*]

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