DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE
End Semester Examination – May 2019

Course: B. Tech **Sem: III**

Subject Name: Engineering Mathematics-III **Subject Code: BTBSC301**

Max Marks: 60 **Date: 28-05-2019** **Duration: 3 Hr.**

Instructions to the Students:

1. *Solve **ANY FIVE** questions out of the following.*
2. *The level question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.*
3. *Use of non-programmable scientific calculators is allowed.*
4. *Assume suitable data wherever necessary and mention it clearly.*

			(Level/CO)	Marks
Q. 1		**Attempt any three.**		**12**
	A)	Find $L\{f(t)\}$, where $f(t) = t^2\, e^{-3t} sinhat$	**Understand**	**4**
	B)	Express $f(t)$ in terms of Heaviside's unit step function and hence find its Laplace transform where $f(t) = \begin{cases} cost, & 0 < t < \pi \\ sint, & t > \pi \end{cases}$	**Understand**	**4**
	C)	Find $L\{f(t)\}$, where $f(t) = 2^t \int_0^t \frac{\sin 3u}{u}\, du$	**Understand**	**4**
	D)	By using Laplace transform evaluate $\int_0^\infty e^{-t}\left(\frac{1-cos2t}{t}\right)\, dt$	**Evaluation**	**4**
Q. 2		**Attempt the following.**		**12**
	A)	Using convolution theorem find $L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$	**Application**	**4**
	B)	Find $L^{-1}\{\bar{f}(s)\}$, where $\bar{f}(s) = cot^{-1}\left(\frac{s+3}{2}\right)$	**Application**	**4**
	C)	Using Laplace transform solve $y'' - 3y' + 2y = 12e^{-2t}; y(0) = 2,$ $y'(0) = 6$	**Application**	**4**
Q. 3		**Attempt any three.**		12
	A)	Express $f(t) = \begin{cases} 1, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral and hence deduce that $\int_0^\infty \frac{1-cos\pi\lambda}{\lambda} \sin \pi\lambda\, d\lambda = \frac{\pi}{4}$.	**Evaluation**	**4**
	B)	Using Parseval's identity for cosine transform, prove that $\int_0^\infty \frac{sinat}{t(a^2+t^2)}\, dt = \frac{\pi}{2}\left(\frac{1-e^{-a^2}}{a^2}\right)$	**Application**	**4**

	C)	Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & if\ \lvert x\rvert \le 1 \\ 0, & if\ \lvert x\rvert > 1 \end{cases}$. Hence prove that $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\ dx = -\frac{3\pi}{16}$	Understand	4
	D)	Find Fourier sine transform of $5e^{-2x} + 2e^{-5x}$	Understand	4
Q. 4		**Attempt the following.**		**12**
	A)	Form the partial differential equation by eliminating arbitrary function f from $f(x+y+z, x^2+y^2+z^2) = 0$	Synthesis	4
	B)	Solve $xz(z^2+xy)p - yz(z^2+xy)q = x^4$	Analysis	4
	C)	Find the temperature in a bar of length two units whose ends are kept at zero temperature and lateral surface insulated if the initial temperature is $sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$.	Application	4
Q. 5		**Attempt Any three.**		**12**
	A)	If the function $f(z) = (x^2+axy+by^2) + i(cx^2+dxy+y^2)$ is analytic, find the values of the constants a, b, c and d.	Understand	4
	B)	If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant.	Understand	4
	C)	Find the bilinear transformation which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$.	Understand	4
	D)	Prove that the function $u = e^x(x\cos y - y\sin y)$ satisfies the Laplace's equation. Also find the coresponding analytic function.	Synthesis	4
Q. 6		**Attempt ANY TWO of the following.**		**12**
	A)	Evaluate $\oint_C \frac{z+4}{z^2+2z+5}\ dz$, where C is the circle $\lvert z+1-i\rvert = 2$.	Evaluation	6
	B)	Find the residues of $f(z) = \frac{\sin z}{z\cos z}$ at its poles inside the circle $\lvert z\rvert = 2$.	Understand	6
	C)	Evaluate $\oint_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)}\ dz$, where C is the circle $\lvert z\rvert = 3$.	Evaluation	6

*** End ***