DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE
End – Semester Examination (Supplementary): November 2018

Branch: B. Tech (Common to all) **Semester:** I
Subject with code: Engineering Mathematics – I (MATH 101)
Date: 26/11/2018 **Marks:** 60 **Duration:** 03 Hrs.

INSTRUCTION: Attempt any **FIVE** of the following questions. All questions carry equal marks.

Q.1 (a)Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ by reducing it to normal form

$[6\ Marks]$

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. $[6\ Marks]$

Q.2 (a) If $y = e^{a\sin^{-1}x}$, prove that$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$.

$[6\ Marks]$

(b) Using Taylor's theorem, express the polynomial

$f(x) = 2x^3 + 7x^2 + x - 6$in powers of $(x - 1)$. $[6\ Marks]$

Q.3 **Solve any TWO:**

(a) If$v = \log(x^2 + y^2 + z^2)$, prove that$(x^2 + y^2 + z^2)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = 2$.

$[6\ Marks]$

(b) If z is a homogeneous function of degree $n$in $, y$, prove that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$ $[6\ Marks]$

(c) If $z = f(x, y)$ where $x = e^u + e^{-v}$& $y = e^{-u} - e^v$, then show that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$ $[6\ Marks]$

Q.4 (a) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$ and $w = \frac{xy}{z}$, show that $\frac{\partial(u\ ,\ v,\ w)}{\partial(x,\ y,\ z)} = 4$. $[4\ Marks]$

(b) The focal length of a mirror is found from the formula $\frac{2}{f} = \frac{1}{v} - \frac{1}{u}$. Find the percentage error in f if u & v are both in error by 2% each. $[4\ Marks]$

(c) Find the maximum value of $x^m y^n z^p$, when $x + y + z = c$. $[4\ Marks]$

Q.5 (a) Evaluate the integral $I = \int_0^1 \int_0^x e^{x+y}\ dy\, dx.$

$[6\ Marks]$

(b) Change to polar co-ordinates to evaluate $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$. $[6\ Marks]$

(c) Evaluate the integral $I = \int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx\, dy$. $[6\ Marks]$

Q.6 (a) State D' Alembert's ratio test, and hence check the convergence of the series:

$$\sum_{n=1}^{\infty} \frac{n}{(n^n)^2} .$$ $[6\ Marks]$

(b) State Cauchy's root test, and hence check the convergence of the series:

$$\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}} .$$ $[6\ Marks]$

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