DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

SEMESTER EXAMINATION: May 2015

Subject: Engineering Mathematics-II (NEW COURSE) Sem-II

Time: 03 Hrs. Max Marks: 70

Instruction to Students

Attempt **SIX** questions in all in the following manner:

1. **Q.1** is compulsory.
2. Attempt the remaining **FIVE** questions from **Q.2** to **Q.7**.

Q.1 Pick up the correct answer of the choices given: [2 x 5=10 marks]

a) Which, out of the following, is an analytic function?

(i) $f(z) = \sin z$ (ii) $f(z) = \bar{z}$ (iii) $f(z) = Im(z)$ (iv) none.

b) If $f(x) = x^2$ in $(-\pi, \pi)$, then the Fourier series contains

(i) only sine terms (ii) only cosine terms (iii) both sine and cosine terms (iv) none.

c) The order and degree of the differential equation $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

(i) 2,2 (ii) 1,2 (iii) 1,1 (iv) none.

d) P.I. of $y'' - 3y' + 2y = 12$ is

(i) 6 (ii) 3 (iii) 0 (iv) none.

e) If $u = \log\left(\frac{x^2}{y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(i) u (ii) $2u$ (iii) 1 (iv) none.

Q.2 (a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is analytic function of $z = x + iy$, find $f(z)$ in terms of z. [4 marks]

(b) Determine whether the function $f(z) = \frac{x}{x^2+y^2} + i\left(\frac{y}{x^2+y^2}\right)$ is an analytic function. [4 marks]

(c) Evaluate $\int_0^{1+i}(x - y + ix^2)dz$ along the straight line from $z = 0$ to $z = 1 + i$. [4 marks]

Q.3 (a) Find the Fourier series to represent the function $f(x)$ given by

$$f(x) = \begin{cases} x & , 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}.$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots = \frac{\pi^2}{8}$. [6 marks]

(b) Obtain the half-range Fourier cosine series for $f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l - x), & l/2 \le x \le l \end{cases}$.

Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots = \frac{\pi^2}{8}$. [6 marks]

P.T.O.

Q.4 (a) Solve $xdx + sin^2\left(\frac{y}{x}\right)(ydx - xdy) = 0$. [4 marks]

(b) Solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$. [4 marks]

(c) When a switch is closed in a circuit containing a battery E, a resistance R and an inductance L, the current i builds up at the rate given by $L\frac{di}{dt} + Ri = E$. Find i as a function of t. How long will it be, before the current has reached one-half its maximum value if $E = 6\ volts, R = 100\ ohms$ and $L = 0.1\ henry$? [4 marks]

Q.5 (a) Solve: $\frac{d^2y}{dx^2} + 4y = x \sin x$. [4 marks]

(b) Solve: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$. [4 marks]

(c) By variation of parameters method, solve the equation $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$. [4 marks]

Q.6 (a) If $x^x y^y z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x\, \partial y} = -(x \log ex)^{-1}$. [4 marks]

(b) If $f(x, y, z)$ is a homogeneous function of degree n in x, y and z, prove that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf.$$ [4 marks]

(c) Transform the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar co-ordinates. [4 marks]

Q.7 (a) Expand $f(x, y) = e^x \sin y$ in the powers of x and y as far as the terms of third degree by using Maclaurin's theorem. [4 marks]

(b) Find the possible error in percents in computing the resistance r from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ if r_1 and r_2 are in error by 2 %. [4 marks]

(c) Find the points on the surface $z^2 = xy + 1$ nearest to the origin. [4 marks]

P.T.O.