**DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE**

**End Semester Examination – Winter 2018**

**Course:** S.Y.B. Tech (All Branches) **Semester:** III

**Subject Name:** Engineering Mathematics-III **Subject Code:** BTBSC301

**Max Marks:**60 **Date:**30/11/2018 **Duration:** 03 Hrs

***Instructions to the Students:***

1. *Attempt **Any Five** questions of the following .All questions carry equal marks.*
2. *Use of non-programmable scientific calculators is allowed.*
3. *Figures to the right indicate full **Marks**.*

**Q. 1.** a) Show that,

$$\int_0^\infty \frac{\sin at}{t}\,dt = \frac{\pi}{2}.$$ **[4]**

b) Find the Laplace transform of **[4]**

$$\int_0^t \frac{e^{-3u}\sin 2u}{u}\,du.$$

c) Find the Laplace transform of the function

$$f(t) = \begin{cases} 2 & ,0 < t < \pi \\ 0 & ,\pi < t < 2\pi \\ \sin t & ,\quad t > 2\pi \end{cases}$$ **[4]**

**Q.2 .** a) Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$. **[4]**

b) By convolution theorem, find inverse Laplace transform of

$$\frac{s}{(s^2+1)(s^2+4)}.$$ **[4]**

c) By Laplace transform method, solve the following simultaneous equations **[4]**

$$\frac{dx}{dt} - y = e^t \;;\; \frac{dy}{dt} + x = \sin t\;;\; \textit{given that}\; x(0) = 1\,, y(0) = 0.$$

**Q. 3.** a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2\,, & |x| \le 1 \\ 0 & ,\; |x| > 1. \end{cases}$$ **[4]**

b) Find the Fourier sine transform of $e^{-|x|}$, and hence show that

$$\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx = \frac{\pi e^{-m}}{2}\,, \quad m > 0.$$ **[4]**

c) Using Parseval's Identity , prove that

$$\int_0^{\infty} \frac{t^2}{(t^2+1)^2}\, dt = \frac{\pi}{4}\ .$$ **[4]**

**Q.4.** a) Solve the partial differential equation

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$$ **[4]**

b) Use method of separation of variables to solve the equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u\ ;\ given\ that\ u(x,0) = 6e^{-3x}.$$ **[4]**

c) Find the temperature in bar of length $2\ units$ whose ends are kept at zero temperature and lateral surface insulated if initial temperature is

$$\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right).$$ **[4]**

**Q. 5.** a) If $f(z)$ is analytic function with constant modulus, show that $f(z)$ is constant . **[4]**

b) If the stream function of an electrostatic field is $\psi = 3xy^2 - x^3$ , find the potential function $\phi$, where $f(z) = \phi + i\psi$. **[4]**

c) Prove that the inversion transformation maps a circle in the z-plane into a circle in w-plane or to a straight line if the circle in the z-plane passes through the origin . **[4]**

**Q.6.** a) Evaluate $\oint_c \frac{e^z}{(z-2)} dz$ , where c is the circle $|z| = 3$ . **[4]**

b) Evaluate $\oint_c \tan z\, dz$ , where c is the circle $|z| = 2$. **[4]**

c) Evaluate , using Cauchy's integral formula : **[4]**

1) $\oint_c \frac{\cos(\pi z)}{(z^2-1)} dz$ around a rectangle with vertices $2 \pm i\ , -2 \pm i$.

2) $\oint_c \frac{\sin^2 z}{(z-\frac{\pi}{6})^3} dz$ , where C is the circle $|z| = 1$.

***** End *****