DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

SEMESTER EXAMINATION

Mechanical/ Electrical/ EXTC/ Chemical/ Petrochemical/ Computer/ IT/ Civil

Subject: Engineering Mathematics-I | **Sem-I**

Time: 03 Hrs. 04 MAY 2017 | **Max Marks:** 70

INSTRUCTION TO STUDENTS

Solve **SIX** questions in all in the following manner:

1. **Q. No. 1** is compulsory.
2. Solve the remaining **FIVE** questions from **Q. No. 2** to **Q. No. 7.**

Q.1 **Pick-up the correct answer of the choices given:** [2 x 5=10 marks]

a) If every minor of order r of a matrix A is zero, then the rank of A is

(i) Greater than r (ii) Equal to r (iii) Less than or equal to r (iv) none.

b) If $\vec{r} := x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$, then $\nabla \times \vec{r}$ is equal to

(i) 0 (ii) 3 (iii) 1 (iv) none.

c) If $\sum u_n$ is convergent, then $\lim\limits_{n\to\infty} u_n$ is equal to

(i) 0 (ii) less than1 (iii) $\neq 0$ (iv) none.

d) If $\vec{F} := x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then $\nabla . \vec{F}$ is equal to

(i) 3 (ii) 0 (iii) 1 (iv) none.

e) The value of $\int_0^1 \int_0^x \int_0^y x\, dx\, dy\, dz$ is equal to

(i) $\frac{1}{6}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{8}$ (iv) none.

Q.2 (a) Use Gauss-Jordan method to find A^{-1}, where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}.$$ [4 marks]

(b) Determine the rank of the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. [4 marks]

(c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$. [4 marks]

Q.3 (a) Find the nth order derivative of $y = \frac{1}{x^2+a^2}$. [4 marks]

(b) If $y = (x^2-1)^n$, prove that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$. [4 marks]

(c) Use Taylor's series to prove that

$$tan^{-1}(x+h) = tan^{-1}x + (h\sin z).\frac{\sin z}{1} - (h\sin z)^2.\frac{\sin 2z}{2} + (h\sin z)^3.\frac{\sin 3z}{3} - \dots\dots\dots$$

where $z = cot^{-1}x$. [4 marks]

Q.4 (a) If a particle describes a curve $r = 2a\cos\theta$ with constant angular speed ω, find the radial and transverse components of velocity and acceleration. [6 marks]

(b) If a particle moves in $xy-$plane with velocity $\vec{v}$, acceleration $\vec{a}$ and ρ is the radius of curvature at a point P on the path of the motion, then show that

$$\rho|\vec{v}\times\vec{a}| = v^3 \text{, where } |\vec{v}| = v.$$

If $\vec{v} = a\,\hat{\imath} + b\,\hat{\jmath}$ and $\vec{a} = c\,\hat{\imath} + d\,\hat{\jmath}$, show that $\frac{(a^2+b^2)^{3/2}}{ad-bc}$; $ad - bc \neq 0$. [6 marks]

Q.5 (a) Prove that the necessary and sufficient condition for the vector $\vec{F}(t)$ to have constant direction is $\vec{F}(t)\times\frac{d\vec{F}(t)}{dt} = 0$. [4 marks]

(b) Find the directional derivative of $\phi = xyz$ at the point $(1, 1, 1)$ in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. [4 marks]

(c) Prove that $\nabla^2 e^r = e^r + \frac{4}{r}e^r$, where $r = |\vec{r}|$. [4 marks]

Q.6 (a) By changing the order of integration, evaluate $I = \int_0^3\int_{x^2}^9 x^3 e^{y^3}\, dxdy$. [4 marks]

(b) Evaluate $\iint \frac{r\,dr\,d\theta}{\sqrt{a^2+r^2}}$ over one loop of lemniscates $r^2 = a^2\cos 2\theta$. [4 marks]

(c) Evaluate $I = \int_0^a\int_0^{z^2}\int_0^{z^2-x} xz\;\, dx\,dy\,dz$. [4 marks]

Q.7 (a) Check the convergence of the series $\sum_{n=1}^{\infty}\frac{2n^3+5}{4n^5+1}$. [4 marks]

(b) Check the convergence of the series $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2+1}$. [4 marks]

(c) Find the radius of convergence of the series: $\sum\frac{n!}{n^n}x^n$. [4 marks]
