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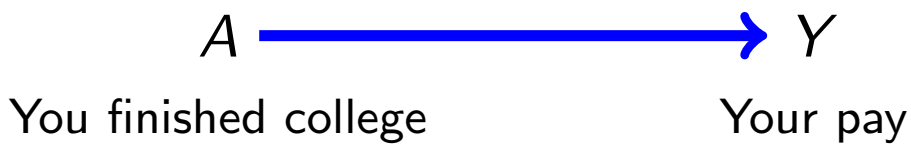

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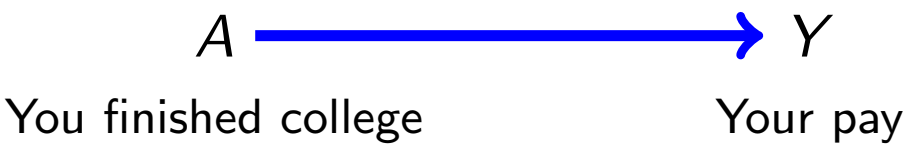

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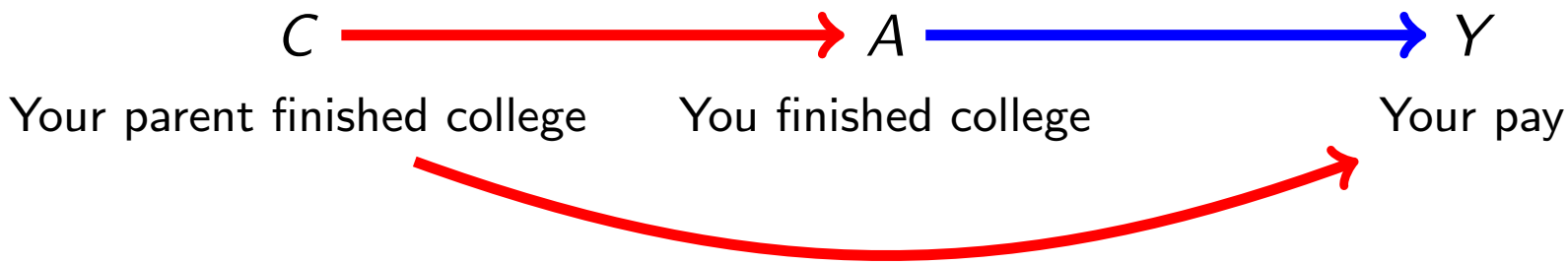

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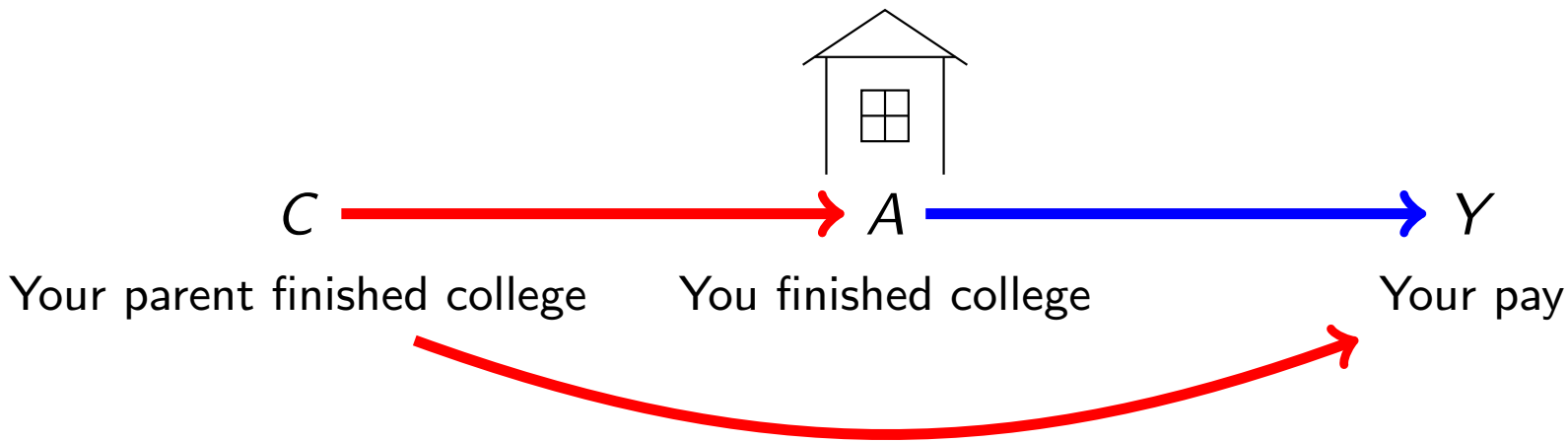

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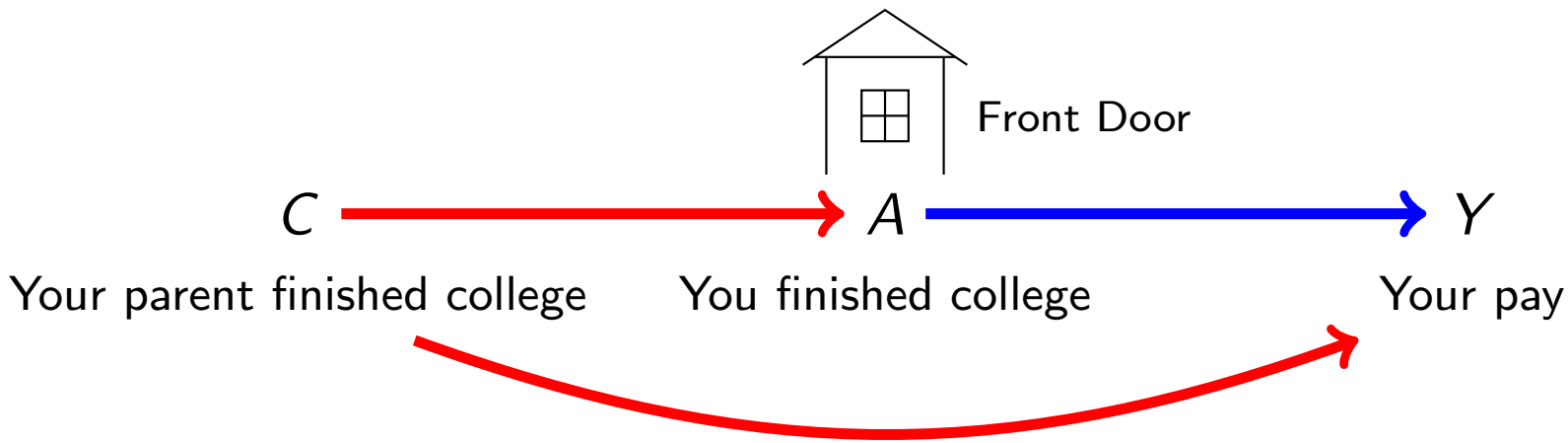

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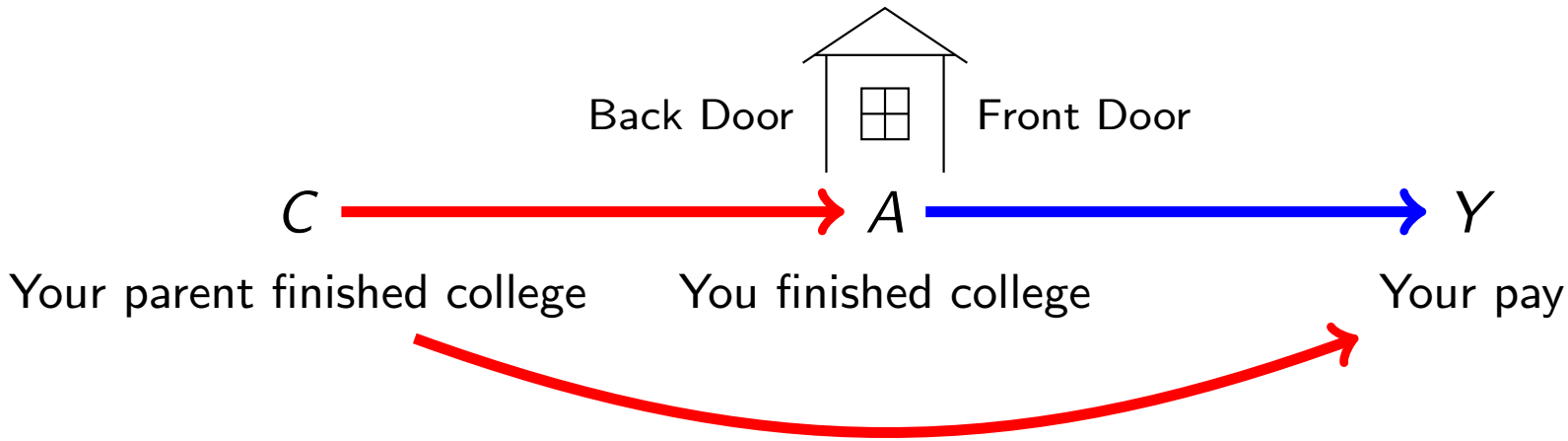

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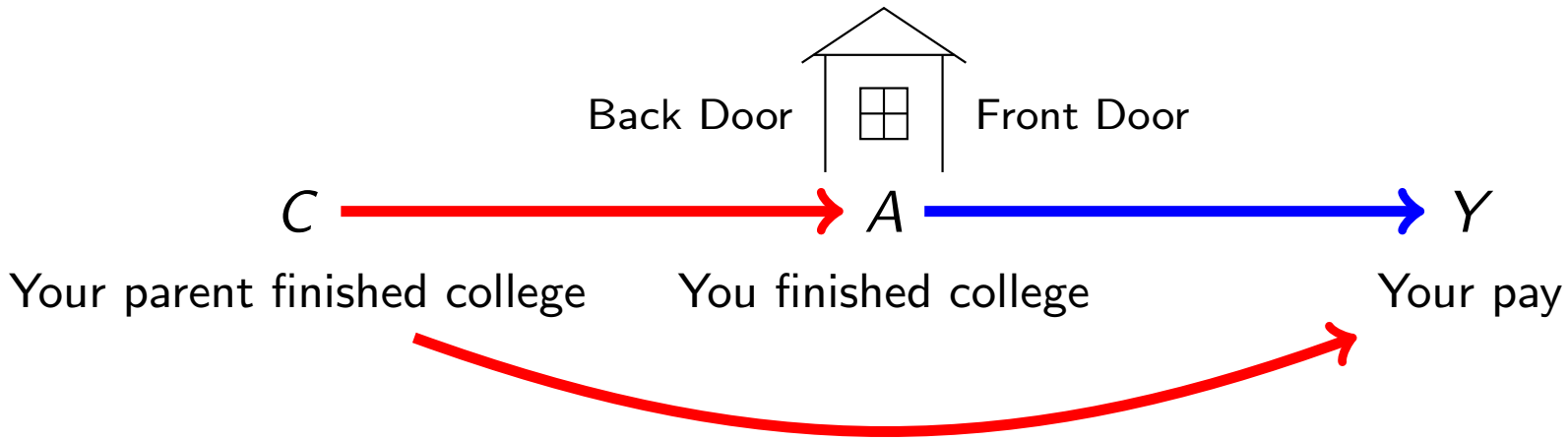

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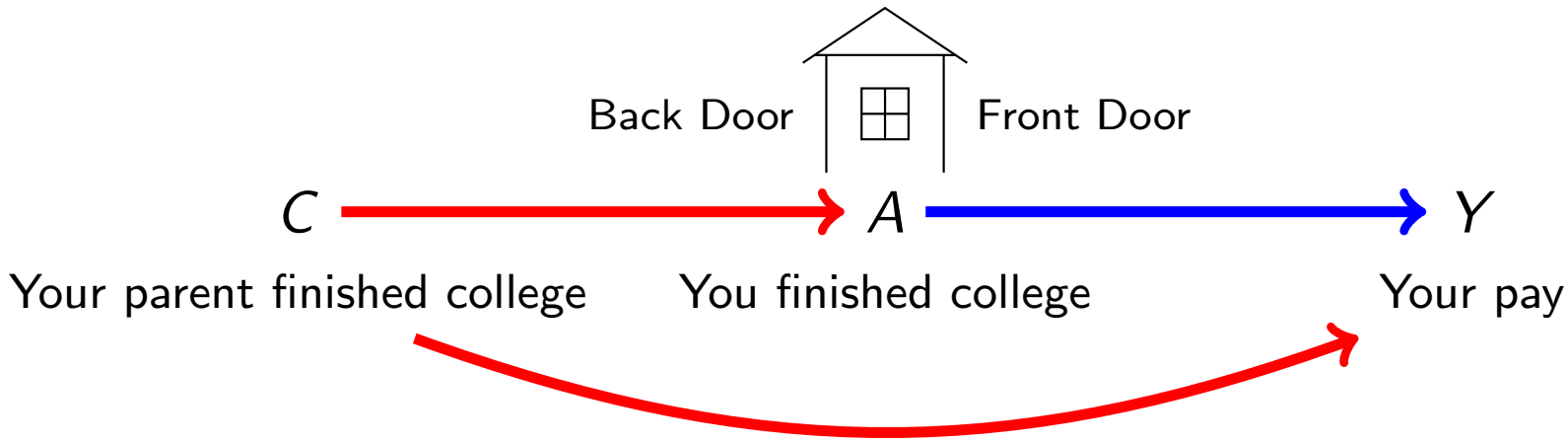

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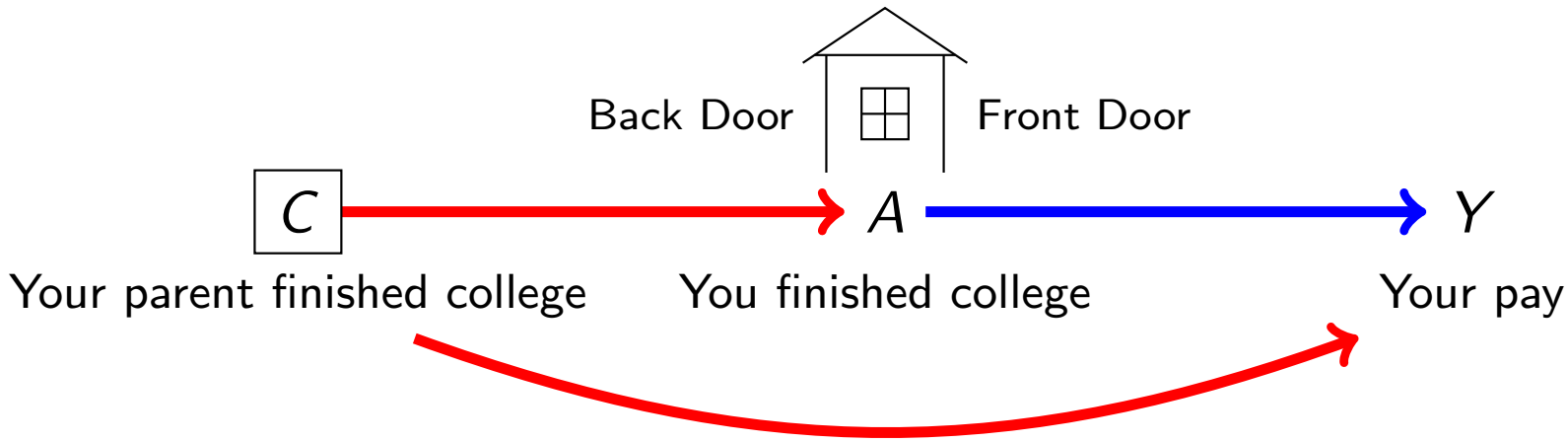

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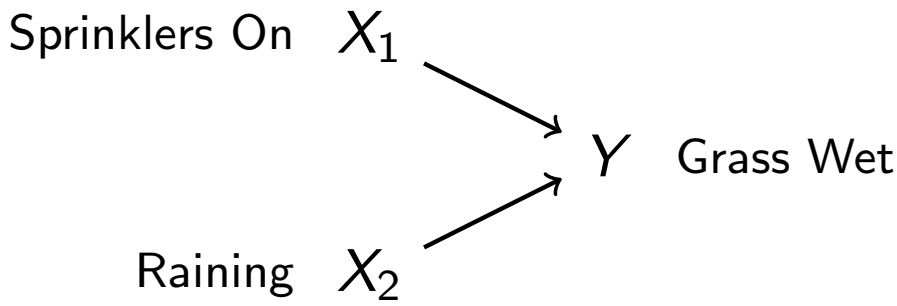

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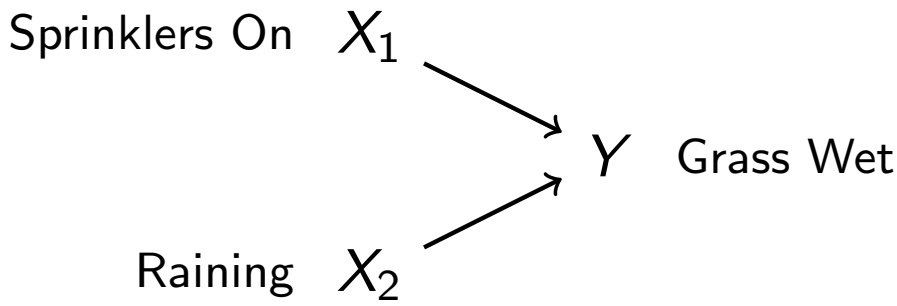

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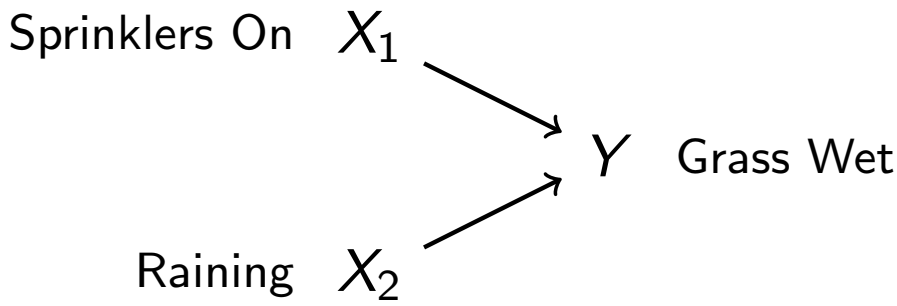

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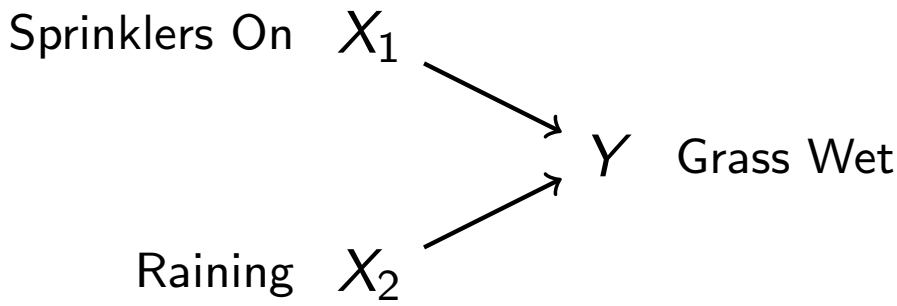

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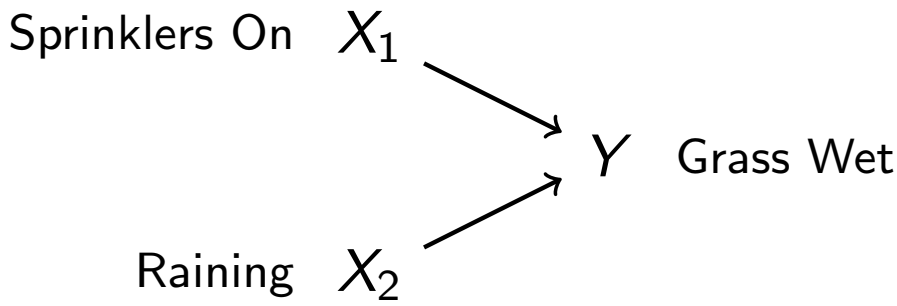

We say Y is a **collider** along the path $X_1 \to Y \leftarrow X_2$

- The collider blocks the path
- X_1 is independent of X_2
  - (Sprinklers On) is uninformative about (Raining)
- Conditioning on Y opens the path
  - If the grass is wet (conditional on $Y = 1$), then either (Sprinklers On) or (Raining)

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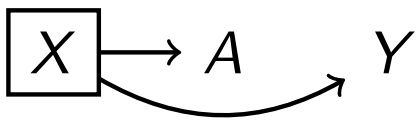
Ancestors vs. Colliders

Conditioning on an ancestor **closes** an open path



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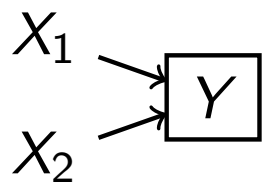

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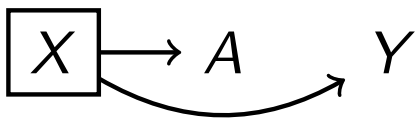

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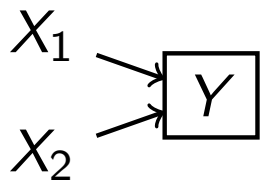

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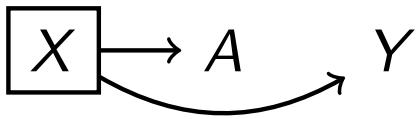

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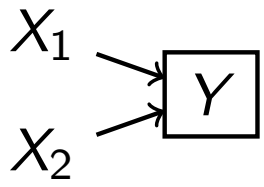

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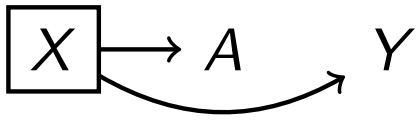

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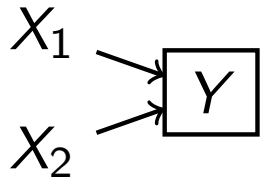

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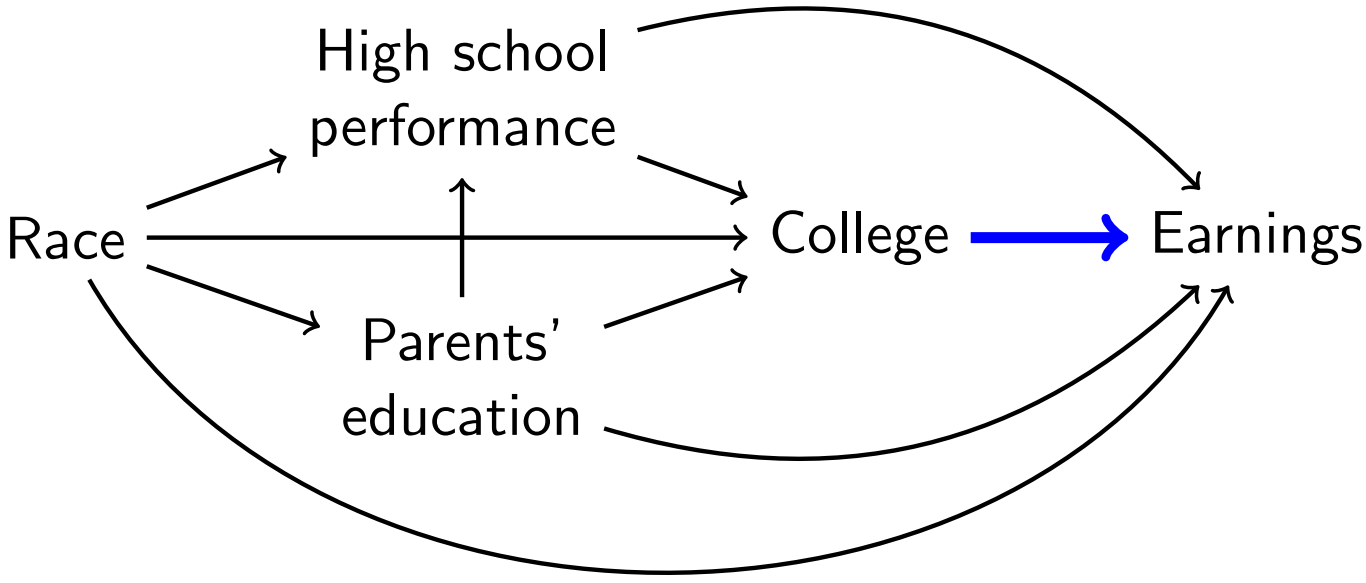
High school
performance
Race
College
Earnings
Parents'
education

How to find adjustment variables to identify causal effects

Goal:

Block all backdoor paths so treatment A and outcome Y are associated only by the causal path

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Backdoor path: Any sequence of edges $A \leftarrow \texttt{nodes} \rightarrow Y$

Blocked if it contains an adjusted variable along a fork

$$A \leftarrow \boxed{C} \rightarrow Y$$
$$A \leftarrow \boxed{C} \leftarrow \cdots \rightarrow Y$$
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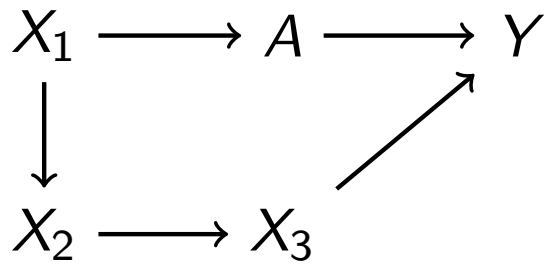
Blocked if it contains an unadjusted collider

$$A \rightarrow C \leftarrow Y$$

Exercise 1

Find adjustment sets that identify the effect of A on Y



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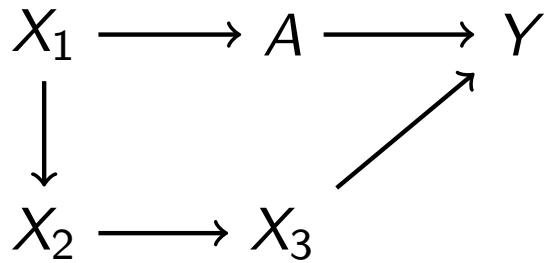

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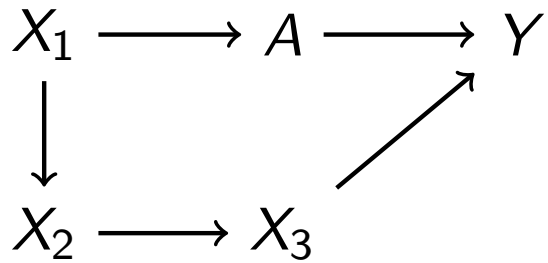

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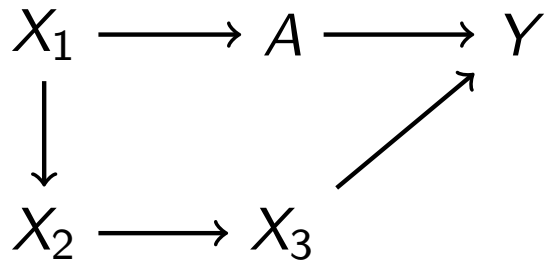

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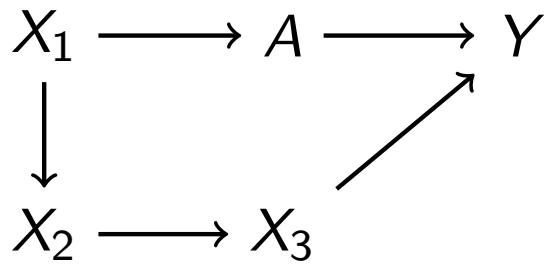

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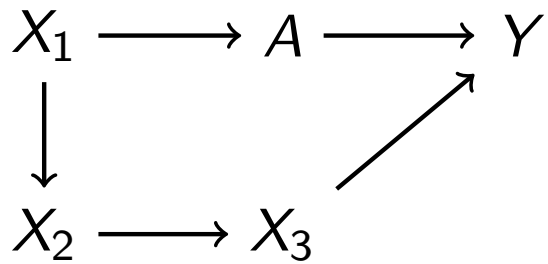

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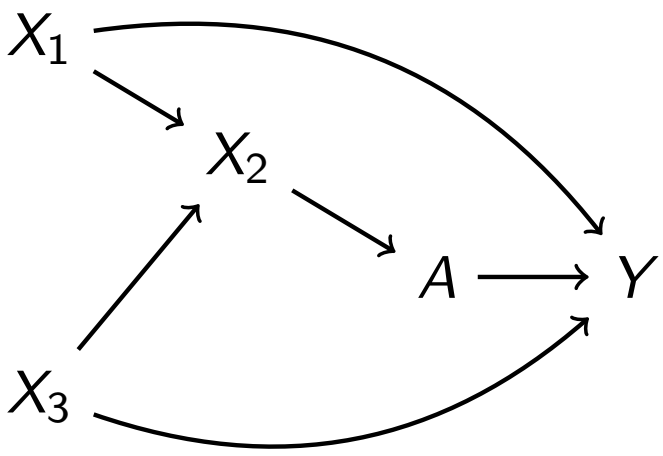
- Condition on X_3: $A \leftarrow X_1 \rightarrow X_2 \rightarrow \boxed{X_3} \rightarrow Y$
- Any combination of the above

Exercise 2

Find 3 sufficient adjustment sets to identify $A \rightarrow Y$



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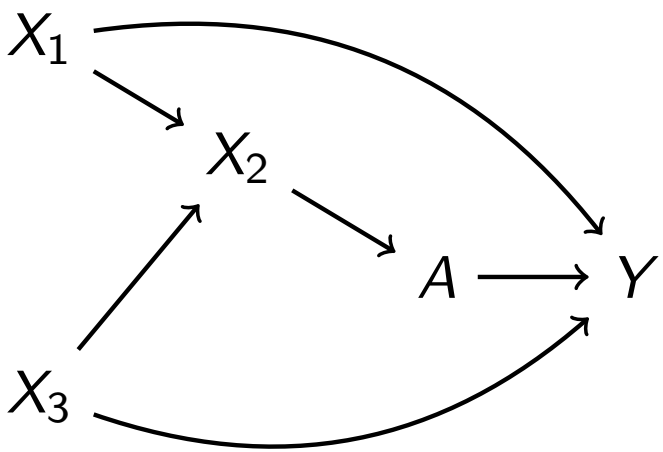
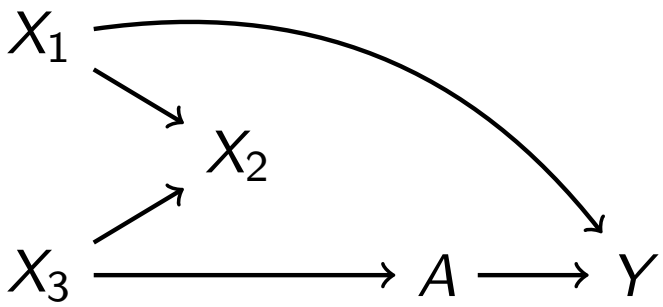


Answer: $\{X_2\}, \{X_1, X_3\}, \{X_1, X_2, X_3\}$

Exercise 3

What is the smallest adjustment set that identifies $A \to Y$?



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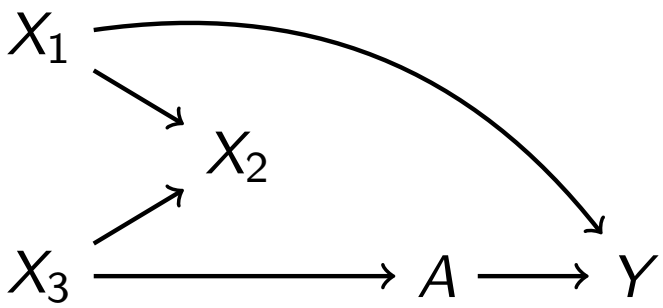

Answer: The empty set! Don't condition on anything.
The collider X_2 already blocks the path.

Learning goals for today

By the end of class, you will be able to

- Formalize causal assumptions in Directed Acyclic Graphs (DAGs)
- Use DAGs to find a sufficient adjustment set of variables within which a statistical association is causal