İstanbul Bilgi University

Department of Computer Engineering

SPRING, 2022
Campus: Santral

CMPE 407
MACHINE LEARNING

HW on PCA

(Duration: minutes)

Name :

Student ID :

Make sure that you explain in detail all your steps - thoughts. You may get extra points for an appropriate observation, you may lose some marks due to an obscure solution.

1. {40 points} A norm is a mathematical concept associated to the length of a vector. Let $\mathbb{R}^N$ be a vector space with elements x; the norm of x, $||x||$, is a real valued function which satisfies some mathematical requirements. Let us define:

- L_2 norm= $||x||_2 = (\sum_{i=1}^{N} |x_i|^2)^{1/2}$
- A 'normalized' vector is a vector with unit norm.

Knowing that it is always possible to normalize a vector, except vector 0:

(a) {20 points} Calculate the L_2 norm of v=(3, -1, 2)'

(b) {20 points} Normalize vector v.

2. {60 points} An eigenvector of a square matrix A is a non-zero vector v that, when the matrix is multiplied by v, yields a constant multiple of v, the multiplier being commonly denoted by λ. That is: $A \cdot v = \lambda \cdot v$ The number λ is called the eigenvalue of A corresponding to v.

(a) {20 points} Decide which, if any, of the following vectors are eigenvectors of A

(b) {20 points} Give the corresponding eigenvalue

(c) {20 points} Normalize the eigenvector (by imposing unit norm)

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

$$\mathbf{v1} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \mathbf{v2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \mathbf{v3} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \mathbf{v4} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$