

TRIGONOMETRY

Sine Sin [Soh] $\rightarrow \frac{\text{opposite}}{\text{hypotenuse}}$

Cosecant csc $\rightarrow \frac{\text{hypotenuse}}{\text{opposite}} \left(\frac{1}{\sin \theta}\right)$

Cosine Cos [Cah] $\rightarrow \frac{\text{adjacent}}{\text{hypotenuse}}$

Secant sec $\rightarrow \frac{\text{hypotenuse}}{\text{adjacent}} \left(\frac{1}{\cos \theta}\right)$

Tangent Tan [Toa] $\rightarrow \frac{\text{opposite}}{\text{adjacent}}$

Cotangent cot $\rightarrow \frac{\text{adjacent}}{\text{opposite}} \left(\frac{1}{\tan \theta}\right)$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$ unit circle: (x,y) = (cos, sin)

S+	STC+
T+	C+

TRIGONOMETRY LAWS

(ASA, SSA) **SIN LAW** $\rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

(SAS, SSS) **COSINE LAW** $\rightarrow c^2 = a^2 + b^2 - 2ab(\cos C)$ $[ABC] = \frac{1}{2}ab(\sin c)$

BASIC IDENTITIES

2nd Quadrant (Sine)	Complementary Angles:
$\sin(180 - \theta) = \sin \theta$ $\cos(180 - \theta) = -\cos \theta$ $\tan(180 - \theta) = -\tan \theta$	$\sin(90 - \theta) = \cos \theta$ $\cos(90 - \theta) = \sin \theta$ $\tan(90 - \theta) = \frac{1}{\tan \theta}$
3rd Quadrant (Tangent)	4th Quadrant (Cosine)
$\sin(180 + \theta) = -\sin \theta$ $\cos(180 + \theta) = -\cos \theta$ $\tan(180 + \theta) = \tan \theta$	$\sin(360 - \theta) = \sin(-\theta) = -\sin \theta$ $\cos(360 - \theta) = \cos(-\theta) = \cos \theta$ $\tan(360 - \theta) = \tan(-\theta) = -\tan \theta$

Pythagorean	Negatives	
$\sin^2 \theta + \cos^2 \theta = 1$	$\sin(-\theta) = -\sin \theta$	$\sec(180 - \theta) = -\sec \theta$
$\tan^2 \theta + 1 = \sec^2 \theta$	$\cos(-\theta) = \cos \theta$	$\csc(180 - \theta) = \csc \theta$

$\cot^2 \theta + 1 = \csc^2 \theta$	$\tan(-\theta) = -\tan \theta$	$\cot(180 - \theta) = -\cot \theta$
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SUM AND DIFFERENCES

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

DOUBLE AND HALF IDENTITIES

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \frac{g}{2} = \pm \sqrt{\frac{1 - \cos g}{2}}$$

$$\cos \frac{g}{2} = \pm \sqrt{\frac{1 + \cos g}{2}}$$

$$\tan \frac{g}{2} = \frac{\sin g}{1 + \cos g} = \frac{1 - \cos g}{\sin g}$$

SUM TO PRODUCT

$$\cos a + \cos b = 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

$$\cos a - \cos b = -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)$$

$$\sin a + \sin b = 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

$$\sin a - \sin b = 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)$$

PRODUCT AND SUM

$$\cos a \cos b = \frac{1}{2} (\cos(a+b) + \cos(a-b))$$

$$\sin a \sin b = \frac{1}{2} (\cos(a-b) - \cos(a+b))$$

$$\sin a \cos b = \frac{1}{2} (\sin(a+b) + \sin(a-b))$$

$$\cos a \sin b = \frac{1}{2} (\sin(a+b) - \sin(a-b))$$

R FORMULAE

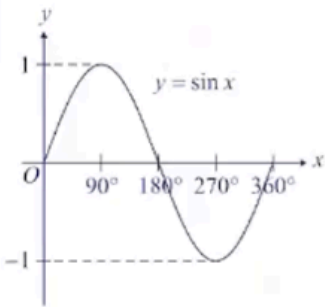
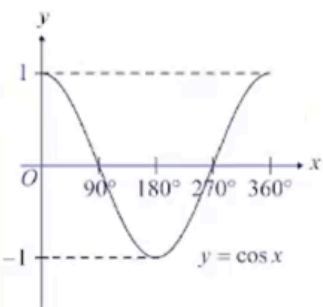
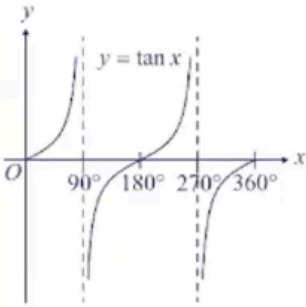
$$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$$

$$a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha)$$

$$\text{Where } a, b > 0, R = \sqrt{a^2 + b^2} \text{ and } \alpha = \tan^{-1}\left(\frac{b}{a}\right)$$

Degree	Radian	Sin θ	Cos θ	Tan θ
0°	0	0	1	0
15°	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2 - \sqrt{3}$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	-
180°	$\frac{3\pi}{2}$	0	-1	0
360°	2π	0	1	0

GRAPHS

	$y = \sin x$	$y = \cos x$	$y = \tan x$
Graph			
Range	$-1 \leq y \leq 1$	$-1 \leq y \leq 1$	All real values

Degree	Sin graph	Cos graph	Tan graph
0°	0	1	0
90°	1	0	error
180°	0	-1	0
270°	-1	0	error
360°	0	1	0

	Sin graph	Cos graph	Tan graph
Amplitude	1	1	NIL
Period	360°	360°	180°
Lines of symmetry	$x = 0^\circ$ $x = \pm 180^\circ$	$x = 90^\circ$ $x = \pm 270^\circ$	No line of symmetry

$$Y = a \sin(bx) + c$$

a - scales the **amplitude/height** of the graph

b - scales the **period** of the graph (horizontally) - $\frac{2\pi}{\text{period}}$

b > 1, the period decreases by a factor of b, if b < 1, the period increases by a factor of b

c - affects **max** and **min** - shifts the graph up (if pos) and down (if neg) by c

Amplitude - vertical distance from the centerline to the max or min point

Period - the length of the interval that a periodic function goes through before it repeats itself