Differentiation: finding gradient of a curve in a more accurate manner
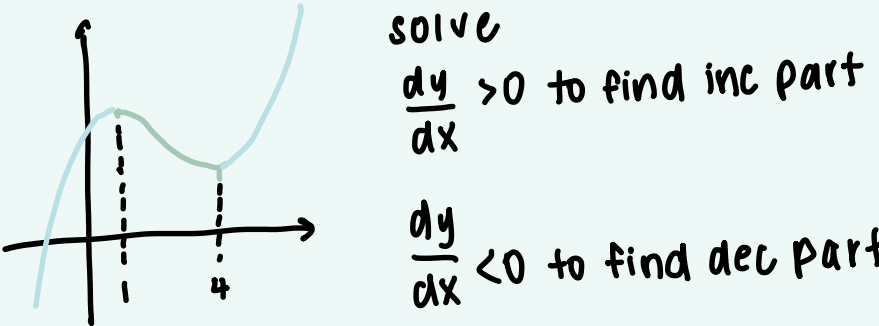
notations

$\dfrac{d}{dx}$ — differentiate with relation to x

$f'(x) = \dfrac{dy}{dx}$ — 1st derivative of y — Gives the gr. Gives the gradient of a curve for any point $x = a$

$f''(x) = \dfrac{d^2y}{dx^2}$ — 2nd derivative — Rate of change of gradient

If $\dfrac{dy}{dx} > 0$, function is increasing

If $\dfrac{dy}{dx} < 0$, function is decreasing

solve
$\dfrac{dy}{dx} > 0$ to find inc part

$\dfrac{dy}{dx} < 0$ to find dec part

Ex: $\dfrac{dy}{dx} = x^2 - 5x + 4$

func is increasing for $x<1$ or $x>4$
func is decreasing for $1<x<4$

scalar multiple rule

$\dfrac{d}{dx} ax^n = anx^{n-1}$ or $\dfrac{d}{dx} kf(x) = kf'(x)$

Ex: $\dfrac{d}{dx} 4x^2 = 4(2)x^{2-1}$
$= 8x$

$\dfrac{d}{dx} \dfrac{1}{5}x^{-1} = -\dfrac{1}{5}(2)x^{-1-1}$
$= -\dfrac{2}{5}x^{-2}$

power functions

$\dfrac{d}{dx} ax = a$ (linear)

$\dfrac{d}{dx} x^n = nx^{n-1}$

1) multiply power
2) subtract power

$\dfrac{d}{dx} a^n$ $\dfrac{dy}{dx} = \ln 3(y)$
$= 3^x (\ln 3)$

$\ln y = \ln 3^x$
$= x \ln 3$

$\dfrac{1}{y} \dfrac{dy}{dx} = \ln 3$

Chain rule

$\dfrac{d}{dx} \left[f(x)\right]^n = n\left[f(x)\right]^{n-1} f'(x)$

$\dfrac{d}{dx} a\left[f(x)\right]^n = an\left[f(x)\right]^{n-1} f'(x)$

$\dfrac{d}{dx} \left[af(x)\right]^n = n\left[af(x)\right]^{n-1} af'(x)$

Ex: $\dfrac{d}{dx} (2x^2+5)^{10} = 10(2x^2+5)^9 (4x)$
$= 40x(2x^2+5)^{11}$

$\dfrac{d}{dx} ((2x-2))^{\frac{1}{2}} = \dfrac{1}{2}(2x-2)^{\frac{1}{2}-1}(2)$
$= \dfrac{1}{2}(2x-2)^{-\frac{1}{2}}(2)$
$= (2x-2)^{-\frac{1}{2}}$

Addition / subtraction rule

$\dfrac{d}{dx}\left[f(x) \pm g(x)\right] = \dfrac{d}{dx} f(x) \pm \dfrac{d}{dx} g(x)$

differentiate $y = \dfrac{2}{x^2} + 5x + 3$

$\dfrac{dy}{dx}(2x^{-2}) + \dfrac{dy}{dx}(5x) + \dfrac{dy}{dx}(3)$

$= (-2)(2x^{-3}) + (5) + 0$

$= -4x^{-3} + 5$

Product Rule

$\dfrac{d}{dx} f(x)g(x) = f(x)g'(x) + g(x)f'(x)$

Ex: $f(x) = (x^2-2x)(x^2+x-1)$

$f'(x) = (x^2-2x)(2x+1) + (x^2+x-1)(2x-2)$
$= 2x^3 - 3x^2 - 2x + 2x^3 - 4x + 2$
$= 4x^3 - 3x^2 - 6x + 2$

Quotient Rule

$\dfrac{d\ f(x)}{dx\ g(x)} = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$\dfrac{d}{dx}\left(\dfrac{x}{4x-2}\right) = \dfrac{(4x-2)(1) - (x)(4)}{(4x-2)^2}$

$= \dfrac{4x-2-4x}{(4x-2)^2}$

$= \dfrac{-2}{(4x-2)^2}$

$= -\dfrac{1}{2(2x-1)^2}$

Tangent of a curve

1) Find the derivative
• plug in x in og eq to find y
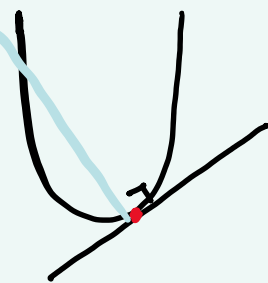• plug in x in $\dfrac{dy}{dx}$ to find slope
of eq $\perp$ to curve

🔺 gradient of tangent to a curve $f(x) = f'(a)$

gradient of normal to a curve $f(x) = \dfrac{1}{-f'(a)}$

Equation of the normal

$\perp$ to tangent

1) Find $f'(a)$
2) Find $\dfrac{-1}{f'(a)}$
3) Write $y = mx + c$ where $m = \dfrac{-1}{f'(a)}$
4) sub $x=a$ and $y=f(a)$ to find c

$\dfrac{d}{dx} \sin x = \cos x$		$\dfrac{d}{dx} \csc x = -\csc x \cot x$	
$\dfrac{d}{dx} \cos x = -\sin x$		$\dfrac{d}{dx} \sec x = \sec x \tan x$	
$\dfrac{d}{dx} \tan x = \sec^2 x$		$\dfrac{d}{dx} \cot x = -\csc^2 x$	
$\dfrac{d}{dx} e^x = e^x$		$\dfrac{d}{dx} e^{f(x)} = f'(x)e^{f(x)}$	
$\dfrac{d}{dx} \ln x = \dfrac{1}{x}$		$\dfrac{d}{dx} \ln f(x) = \dfrac{f'(x)}{f(x)}$	

$\dfrac{\cos x}{\sin^2 x}$

$\dfrac{\sin x}{\cos^2 x}$

Stationary points — used for max/min problems

When $\dfrac{dy}{dx} = 0$
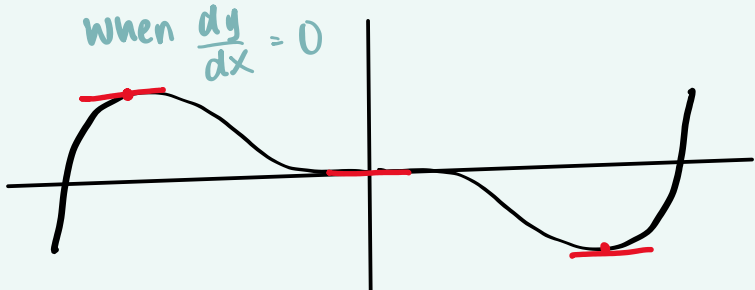
$\boxed{\dfrac{dy}{dx} = 0}$

$\dfrac{d^2y}{dx^2} > 0$ — min point

$\dfrac{d^2y}{dx^2} = 0$ — stationary point of inflection (have to check first test)

$\dfrac{d^2y}{dx^2} < 0$ — max point

1) $\dfrac{dy}{dx} = 0$
2) sub vals of x in og equation to find y

determining nature of each point
1) find $\dfrac{dx}{dy}$ for stationary points

x	$a<x$	x	$b>0$
$\dfrac{dy}{dx}$	plug in into $\frac{dy}{dx}$	0	plug in into $\frac{dy}{dx}$
tangent	/ or \	—	/ or \
shape			