2ND ORDER DIFFERENTIAL EQUATIONS

↳ basically contains $\frac{d^2y}{dx^2}$

Non Homogenous linear equations

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

gives f(x) when plugged in ODE

y_c = complementary func ↗

general solution:

$$y = c_1y_1 + c_2y_2 + y\rho$$
y_c

$y\rho$

watch for f'(x)!

What can f(x) be such that it results to f(x) when plugged into left side

Homogenous linear equations

solve as if DE eq = 0

$$\frac{d^2y}{dx^2} = f(x) \rightarrow \text{don't forget to integrate +C}$$

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$
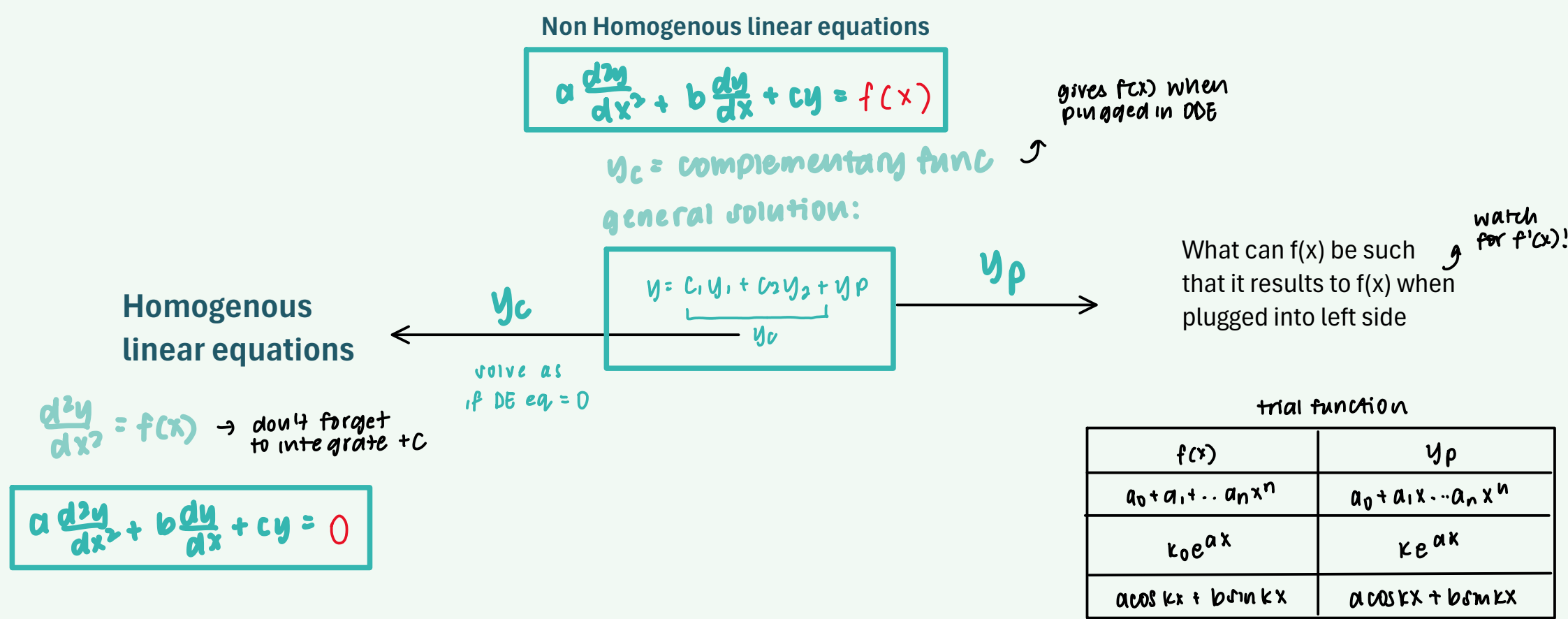
general sol: $y = Ae^{m_1x} + Be^{m_2x}$
where m_1, m_2 are the roots of $am^2 + bm + c$

characteristic equation

1) $m_1 \neq m_2$

eg: $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2$

$m^2 - m = 2$ $y = Ae^{-x} + Be^{2x}$
$m = -1$ or 2

2) $m_1 \neq m_2$ (not real)

$y = e^{ax}(A\cos\beta x + B\sin\beta x)$

proof: $e^{i\theta} = \cos\theta + i\sin\theta$

3) $m_1 = m_1$ (real)

$y = (A + Bx)e^{mx}$

trial function

f(x)	$y\rho$
$a_0 + a_1 + \cdots a_n x^n$	$a_0 + a_1 x \cdots a_n x^n$
$k_0 e^{ax}$	ke^{ax}
$a\cos kx + b\sin kx$	$a\cos kx + b\sin kx$

eg. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 16e^{-2x}$

y_c
char eq: $m^2 + 4m + 4 = 16e^{-2x}$
$(m+2)^2 = 16e^{-2x}$
$y_c = (A+Bx)e^{-2x}$

$y\rho$
kx^2e^{-2x}
$y\rho' = k(2xe^{-2x} - 2xe^{-})$
$y\rho'' = k(2 - 8x + 4x^2)e^{-2x}$
$ke^{-2x}[(2-8x+4x^2) + 4(2x-2x^2) + 4x^2] = 16e^{-2x}$
$k = 8$

$$y = (Ax+B)e^{-2x} + 8x^2e^{-2x}$$
y_c $y\rho$

modeling oscillation kinematics ↗

Real life Application

● **HOOKE'S LAW**

$$F_R = -kx = mx''$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$
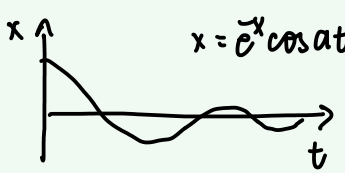
$$x = A\cos(wt - \phi)$$

● **Damping**

mmx—▨——▯ → opposing force ← damping constant

let x = displacement

$$mx'' = -kx + (-cx')$$

Fnet on mass | restoring force by string | force by dashboard ⎫ prevent the door from slamming when closed (loud)
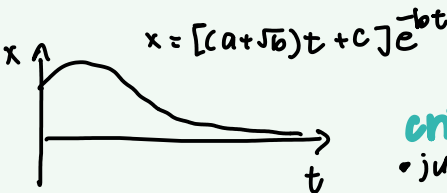
$$\frac{d^2x}{dt^2} + \frac{c}{m}\left(\frac{dx}{dt}\right) + \frac{k}{m}x = 0$$

$x = e^{-t}\cos at$

underdamping
• not periodic
• door oscillates
• trigo function

D < 0

$x = [(a+\sqrt{b})t + c]e^{-bt}$
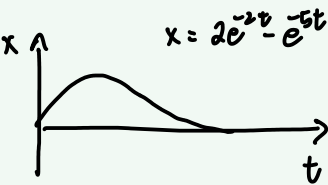
critical damping
• just enough for body to reach equilibrium
• ideal door
• always contains decreasing exponential function

D = 0

$x = 2e^{-2t} - e^{-5t}$

overdamping
• x → 0 in a gradual manner

D > 0