An Efficient Scheduling Scheme of Swapping and Purification Operations for End-to-End Entanglement Distribution in Quantum Networks

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***Abstract*—Entanglement-based quantum networks provide unconditionally secure quantum communication by distributing end-to-end entangled pairs. To reach such goal, entanglement swapping and purification are two necessary operations, which can expand the entanglement distance and improve the fidelity of entangled pairs, respectively. During the end-to-end entanglement distribution process, the execution sequence of these two operations has a significant impact on the final performance, such as throughput and fidelity. Thus, how to schedule the sequence of swapping and purification becomes a critical problem in quantum networks. Our work focuses on such scheduling problem and we devise an efficient solution with fidelity maintenance. In our paper, we first propose a concurrent swapping method for any-length path to reduce delay. Secondly, by modeling the scheduling sequence as a permutation problem, we obtain an optimal scheduling principle called "prioritized purification (PP)". Finally, to meet the fidelity requirement of the end-to-end entangled pairs, we propose a fidelity maintenance scheme based on PP (FMSPP). The simulation results show that the proposed FMSPP can achieve higher distribution rates and less resource overhead than other scheduling solutions, and the advantages are more significant when there are more hops on the path.**

***Index Terms*—Quantum Networks; Swapping; Purification; Fidelity Maintenance; Scheduling Sequence.**

I. INTRODUCTION

Along with on-going breakthroughs of physical devices, quantum information technology has been rapidly developed in academic community. In the past decade, many experimental studies have demonstrated that long-distance quantum information sharing is achievable via quantum networks [1]. Specifically, a quantum network is an interconnection of quantum nodes that can generate, exchange, and process quantum information [1]–[3]. It fully utilizes the characteristics of quantum physics and enables numerous ground-breaking applications [2], such as clock synchronization [4], secure communications [5] and distributed quantum computation [6].

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One of the most important capabilities in quantum networks is creating entanglement between two end nodes regarded as entanglement distribution [7]. In principle, it merges short-distance entangled pairs into long-distance entangled pairs by exploiting *entanglement swapping* operations [8]–[11]. Once the end-to-end entangled pairs are distributed, a pair of quantum end nodes in quantum networks can realize information sharing through *teleportation* [12]. However, due to the imperfect quantum channel (e.g., lossy optical fiber) and operations in practice, end-to-end entanglement distribution faces several tough challenges. On the one hand, the successful probability of generating an entangled pair decays exponentially with the physical distance between the two nodes [13]. On the other hand, the quality of entangled pairs, i.e., *fidelity* [14], declines with the increase of waiting time and transmission distance, which can lead to poor performance for some specific applications such as distributed quantum computation.

To meet the fidelity requirements for specific quantum applications, a technique called *entanglement purification* can be used to improve the fidelity of target entangled pair by consuming extra entanglement resources [15]–[17]. For the end-to-end entanglement distribution, swapping expands the entanglement distance, and purification enhances the fidelity. Through multiple swapping and purification operations, it can theoretically meet the requirements of entanglement connection of any distance with desired fidelity. Unfortunately, purification and swapping are not independent of each other. For example, the reduced fidelity caused by swapping decreases the success probability of purification [18]. Due to the inseparability of the two operations, different scheduling strategies result in different end-to-end distribution rate and final fidelity. Thus, how to provide the optimal scheduling solution of swapping and purification becomes a critical but challenging problem. Although some critical problems, such as entanglement routing, have been well studied, [19]–[21], the scheduling problem of swapping and purification has not yet been explored in-depth.

In this paper, we focus on the scheduling problem of swapping and purification during the process of end-to-end entanglement distribution. In more detail, the scheduling problem can be decomposed into two sub-problems: 1) Designing a swapping sequence according to the timeline in order to minimize the distribution delay. 2) In a fixed swapping sequence, arranging the minimum number of purification operations to meet the fidelity requirements by using the least possible overhead. To solve such a scheduling problem, we propose concurrent swapping method to decide the swapping sequence, and model the scheduling of swapping sequences as a permutation problem, called prioritized purification (PP), by taking purification sequence into consideration. Finally, the fidelity maintenance scheme based on PP, named FMSPP for short, is proposed to maintain end-to-end fidelity.

The main contributions of this paper are summarized as follows:

- We propose a concurrent swapping algorithm to decide the swapping sequence, which explicitly tell any repeater which two entangled pairs should be swapped in each time slot. Compared to the simple hop-by-hop method, concurrent swapping can greatly reduce the end-to-end entanglement distribution delay.
- We formulate the scheduling problem of both swapping and purification as a permutation problem and design a state-transition model to analyze the impact of various scheduling schemes on the performance of entanglement distribution. We find out that the best scheduling scheme, prioritized purification (PP), has obvious superiority over other schemes. We then propose a PP-based scheme, called FMSPP, to realize the end-to-end entanglement distribution with arbitrary fidelity.
- We conduct extensive evaluations to verify that the FMSPP scheme has a higher distribution rate and less resource overhead than other scheduling solutions, and the advantages are even more significant as more hops.

The rest of this paper is organized as follows: Section II shows the latest related work about routing and scheduling. Section III presents the system model: preliminaries, network elements and network management. A concurrent swapping method and the proposed FMSPP scheme are presented in Section IV. In Section V, the performance evaluation of the proposed schemes is conducted. Finally, conclusions are drawn in Section VI.

II. Related Work

Many studies have been proposed to address the routing and transmission problem of entangled pairs in quantum networks. Above all, Li *et al.* [22] presented a novel design of a cluster-based structure to describe how quantum nodes are interconnected, and how the structure can improve the performance of qubit transmission and reduce the network complexity. Pant *et al.* [19] introduced a greedy algorithm for routing problems on quantum lattice networks. While this algorithm works well on networks having one entangled pair shared between each pair of adjacent nodes and one connection request being processed per time slot. On this basis, Li *et al.* [20] proposed an effective routing scheme to enable automatic responses for multiple requests of entanglement generation between source-terminal stations on a quantum lattice network with finite edge capacities. Shi *et al.* [1] proposed a comprehensive entanglement routing model which reflected the differences between quantum networks and classical networks but without considering the fidelity requirement of entangled pairs. Chakraborty *et al.* [21] proposed a resource-efficient decentralized routing protocol for the network of ring or sphere topology. Schoute *et al.* [23] presented two simple hierarchical routing schemes for a quantum network of N nodes in ring or sphere topology. For concurrent multiple requests, Cicconetti *et al.* [24] introduced a general framework of heuristic algorithms, for which three illustrative instances with the objective of keeping the application delay small while achieving a good system utilization, in terms of high entanglement rate. Li *et al.* [25] proposed purification-enabled entanglement routing designs to provide fidelity guarantee for multiple Source-Destination (S-D) pairs in quantum networks, which only focuses on the influence of fidelity on routing decisions.

When entanglement routing is studied and solved in depth, we should focus on how to use the basic link-level entanglement resources to distribute end-to-end entanglement on the given path. The current research mainly focuses on how to schedule swapping or purification. Meter *et al.* [26] outlined the end-to-end protocol architecture and introduced four different purification scheduling algorithms to raise the fidelity of the entangled pairs. However, these schemes do not provide specific entanglement distribution processes of coupling swapping and purification o achieve a specific fidelity. Victora *et al.* [27] optimized both the choice of path over the quantum network and the choice of purification schemes used between nodes. It is worth noting that this work explores how network parameters influence the performance of path-finding algorithms necessary for optimizing routing. Unfortunately, no quantitative conclusion has been provided. Chen *et al.* [28] proposed a heuristic algorithm by dividing the original Entanglement Graph (EG) into several sub-problems for the maximization of entanglement distribution rate (EDR) on a memory-limited path. However, this work does not involve the consideration of purification scheme, so the demand for fidelity cannot be guaranteed. In conclusion, both of these works ignore the coupling between swapping and purification and only solve the optimization problem unilaterally.

In summary, most existing works are focused on implementing an appropriate routing algorithm to cope with different network conditions, and do not take fidelity into account. However, fidelity maintenance in entanglement distribution becomes more complicated due to the mutual effect between swapping and purification. Actually, scheduling for swapping and purification so far has been limited to unilateral optimization objectives. Thus far, there is still no work that takes both swapping and purification into account in the design of scheduling entanglement distribution. In this paper, we thus attempt to design an optimal end-to-end scheduling mechanism of swapping and purification to improve the efficiency of entanglement distribution and meet the fidelity requirements of quantum applications.

III. System model

In this section, we first introduce some basic content about entanglement: fidelity and three important functions

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that quantum repeaters must provide: entanglement generation, entanglement swapping and entanglement purification. Then we introduce abstract network elements such as nodes, links, and paths in quantum networks, and elaborate centralized controllers and time slots.

A. Preliminaries

1) ***Fidelity****:* The fidelity is physically interpreted as the probability that the pair are mistaken for each other upon a measurement of the accuracy of transmission, and it can be defined as $F = \langle\Psi^+|\rho|\Psi^+\rangle$ [29], where ρ is the density matrix representation of the state, and $|\Psi^+\rangle$ is the state we are trying to create. For practical quantum applications such as QKD, a certain fidelity constraint, e.g., $F > 0.9$, is required to guarantee successful key exchange [30].

2) ***Entanglement Generation****:* As one of the most profound features of quantum mechanics, entanglement is a phenomenon in which particles interact in a system composed of two or more particles, regardless of physical distance[1]. The entanglement source (QS) emits locally entangled pairs toward both of the adjacent repeaters through either free space or optical fiber, as shown in Fig. 1(a). The noise makes the transmission of photons to the repeater probabilistic. In particular, the probability that a photon arrives at a node decreases exponentially with the distance that the photon travels. If we let p be the probability that both photons of a pair fired along the edge reach the end nodes, which can be modeled as $p = e^{-\alpha l}$, where l represents the length of the edge, and α is a value that characterizes the medium, typically, $\alpha = \frac{1}{22km}$ [31]. The newly generated entangled pairs are easy to decoherence, resulting in the decrease of fidelity. As the fidelity degrades to 0.5, the less entangled pair becomes useless [26].

3) ***Entanglement Swapping****:* In particular, an entanglement swapping process can be described as follows. As shown in Fig. 1(b), for arbitrary two entangled pairs on adjacent links, the repeater performs Bell-state measurement (BSM) [32] and merges two short-distance entangled pairs into a long-distance entangled pair. Swapping has two key characteristics: (a) Because of the imperfect BSM at middle node, swapping has a successful probability. If the swapping is successful, the entangled pair with longer entanglement distance can be generated. On the contrary, the two entangled pairs will be invalid. (b) After swapping, the fidelity of the newly formed entangled pair will decrease. If the imperfect quantum operation of each node is considered, the fidelity of the newly established entangled pair can be written as:

$$F_{AC} = F_{AB} \cdot F_{BC}, \tag{1}$$

where F_{AB} and F_{BC} respectively refer to the fidelity of the two short-distance pairs, and F_{AC} refers to the fidelity of the newly established long-distance pair. Furthermore, for multiple swapping operations, the resulting fidelity can be calculated by:

$$F_n = \prod_{i=1}^{n-1} F_{i,i+1}, \tag{2}$$

where n represents the number of hops along the path, and $F_{i,i+1}$ represents the fidelity of the hop i. Hence, performing entanglement swapping results in a longer-distance entangled pair with lower fidelity.

4) ***Entanglement Purification****:* Entanglement purification refers to the process of extracting high-fidelity maximum entangled states from mixed entangled states. After experiencing entanglement distribution and quantum storage, the maximum entangled state becomes a mixed entangled state. For example, when an entangled pair have suffered a bit flip error, the mixed state can be represented by a density operator,

$$\rho = F\,|\Phi^+\rangle\langle\Phi^+| + (1-F)\,|\Psi^+\rangle\langle\Psi^+|. \tag{3}$$

In practice, such as using the polarizing beam splitter (PBS) scheme [33], as shown in Fig. 1(c), purifying two entangled pairs always destroys one pair and returns its physical resources to the pool of free qubits. For two entangled pairs with fidelity F_1, F_2, the purification operation succeeds with probability $F_1F_2 + (1-F_1)(1-F_2)$, failing with probability $F_1 + F_2 - 2F_1F_2$ (then both pairs are invalid), when purification succeeds, the resulting fidelity is:

$$F' = \frac{F_1F_2}{F_1F_2 + (1-F_1)(1-F_2)}. \tag{4}$$

Several low-fidelity entangled pairs need to be purified into high-fidelity entangled pairs. Because performing one purification involves two entangled pairs, there is a combination problem of entangled pairs to be purified within the node. Meter *et al.*'s work [26] has discussed this problem. In our scheme, we use the simplest symmetrical purification. The symmetric purification always uses the entangled pairs with the same fidelity to perform purification. The recursive calculation of fidelity can be obtained as follows.

$$F_n = \frac{F_{n-1}^2}{F_{n-1}^2 + (1-F_{n-1})^2}, \tag{5}$$

where F_n refers to the resulting fidelity and F_{n-1} means the last fidelity. All of the purification operations involved in this paper default to symmetrical purification.

B. Network Elements

The topology of a quantum network can be modeled as a graph $G = (V, E)$, where V represents the set of quantum nodes (end users or repeaters) and E represents the set of quantum links connecting adjacent nodes.

1) ***Quantum node****:* According to the functions of the nodes, quantum nodes can generally be classified into end nodes and repeater nodes. An end node, located on the edge of the network, wishes to establish end-to-end entanglement connection with another end node through the network. The repeaters interconnect end nodes and other repeaters to offer a variety of quantum manipulation capabilities, such

[1]The particular type of entanglement that we deal with is called Bell states. The Bell entangled states are the maximally entangled states in the Hilbert space of two qubits. Bell entangled states (Hereinafter referred to as entangled pairs) have four states and form a complete orthonormal basis for a 4-dimensional vector space of states.

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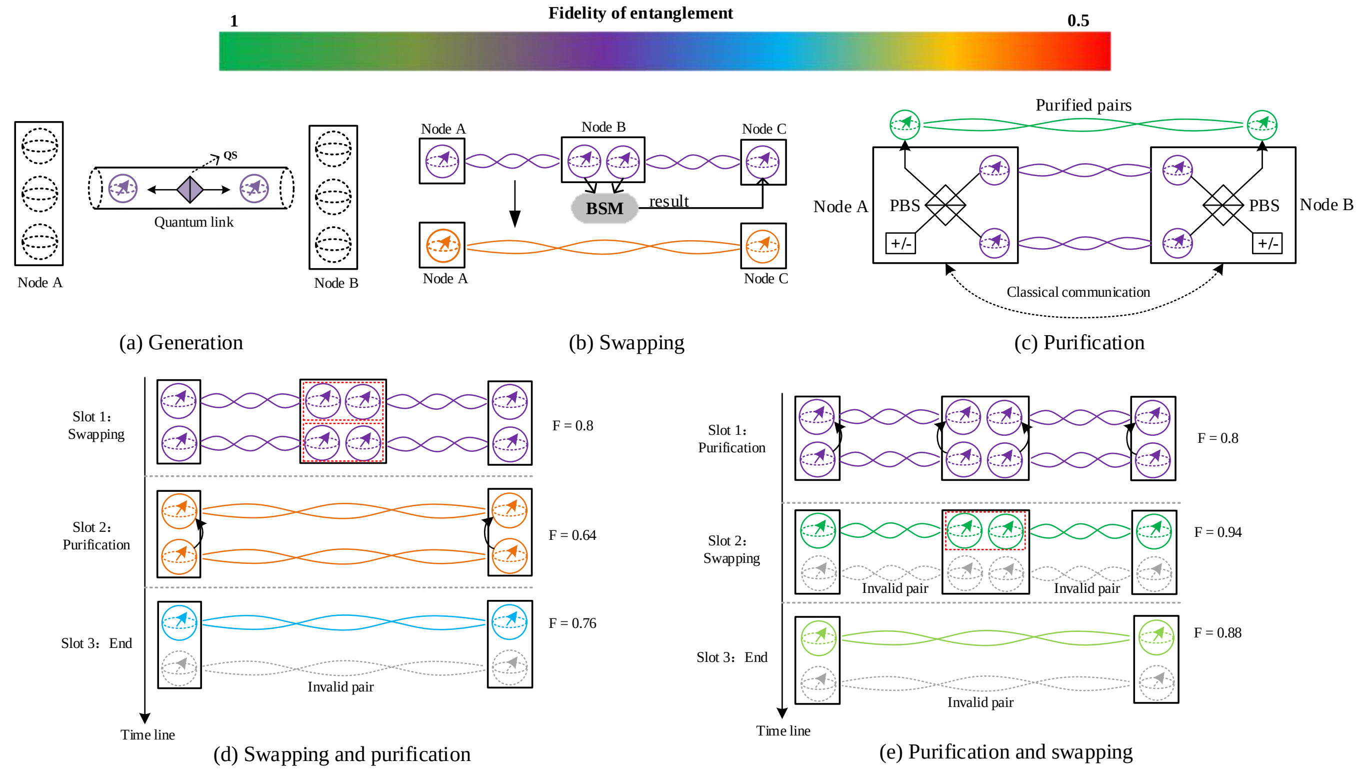

Fig. 1: Key technologies in quantum networks and examples of different scheduling sequences. (a) QS generates entangled pairs which are distributed to the internal memory of nodes through quantum links. (b) The longer-distance entangled pair can be created by swapping which also decreases the fidelity. (c) PBS purification: improving the fidelity of another pair by sacrificing one pair. (d-e) Scheduling problems on a 3-nodes path. Entangled pairs of different fidelity can be distributed by changing the sequence of swapping and purification.

as gate manipulation for swapping and purification. Each node accommodates quantum memory [2] to store constantly generated entangled pairs. The memory capacity of each node is limited. If the quantum measurement fails, the corresponding qubit will be emptied for reuse. The newly arrived entangled pairs will be discarded when memory is full.

*2) **Quantum link**:* A quantum link is abstracted as the entanglement relationship between adjacent nodes, and the entangled pairs are used as link resources. The width of the quantum link can be defined as the maximum number of entangled pairs that can exist on this link. Thus, the link width is related to memory capacity of the nodes on the link. When a quantum request is submitted, the network spontaneously (or under control) forms an entangled path, which refers to the logical concatenation of several quantum links. These links will reserve the same memory space upon request, and this capacity is called the path width.

C. Network Management

*1) **Network controller**:* There is a so-called quantum network controller which is a classical computing device connecting to quantum nodes through Internet connections. Network controller is in charge of the following: collecting the requests for an end-to-end entanglement connection, routing decisions based on network conditions, and determining where it is better for swapping or purification to be performed. In subsequent descriptions, we assume that the device exists for global control.

*2) **Time slot**:* The link entanglement generation, swapping and purification, as well as the propagation of control information and quantum operation are time-consuming. Similar to existing works, e.g., [1], [19], [24], the time slot is introduced to synchronize the behavior of each node which can be achieved by the current synchronization protocols via Internet connections. Each time slot is a device-technology-dependent constant and is set to an appropriate duration by the link layer. It is assumed that only one of the three operations for a qubit, namely, entanglement generation, swapping, and purification, can be performed within any time slot. In each time slot, the swapping or purification of each entangled pair in the repeater is determined by the scheduling algorithm. Some new entangled pairs are then generated by QS, transmitted by quantum channels and stored inside the repeater as a supplement to the resources.

D. Scheduling Problem Statement

In order to realize end-to-end entanglement distribution, multiple swapping and purification operations are necessary.

[2]Actually, it has been recently demonstrated that NV centers are capable of memory lifetimes approaching one minute [34] in nodes not yet interfaced to the network. Other platforms, such as ion traps [35] and neutral atoms [36], are with similar capabilities .

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For example, as shown in Fig. 1(d-e), a target entangled pair can be distributed by different sequences of swapping and purification operations, such as swapping and then purification or purification and then swapping. Obviously, in such a simple setting, it is better to do purification first because of the higher final fidelity. However, when the network grows more complex (with longer paths and wider paths), scheduling plays even more important role in it, and there must exist scheduling problems between swapping and purification operations in the two dimensions of time and space. In addition, swapping and purification are coupled. For example, swapping reduces the fidelity, which may further affect the fidelity gain and successful probability of purification. The optimal scheduling solution can be obtained by exhaustive search of all possible solutions, but without universality. For example, initial fidelity, path width, path length, final fidelity requirements, and swapping success probability are the parameters that are changeable, and it is thus quite likely to affect the optimal scheduling solution.

The scheduling problem is difficult to solve directly due to coupling and other factors mentioned above. Therefore, we can decompose the holistic scheduling problem into two subproblems: **swapping sequence** and **purification sequence**. The swapping sequence is solved preferentially because the number of swapping is only determined by the path length, then we can get the swapping sequence to decide where and when to do swapping in each time slot. In contrast, the round number of purification will be affected by other parameters such as initial fidelity and final fidelity requirement. Then we need to study when and where to insert several purification operations into the swapping sequence in order to get a higher distribution rate. Detailed analysis and solutions are shown in Section IV.

IV. End-to-End Entangled Pair Distribution

In this section, we focus the scheduling problem of a given path $P = \{N_1, N_2, ..., N_l\}$, and l means the path length, $N_i (i < l)$ means a node along the path. Each node along this path reserves W memory units, which is equivalent to path width W. Next, the different swapping sequences are compared first, and the concurrent swapping sequence is generalized so as to be suitable for paths of arbitrary length. Then we take purification sequence into consideration and model it as a permutation problem, given the recursive calculation between success probability and fidelity. Finally, a fidelity maintenance scheme based on prioritized purification is proposed.

A. Swapping Sequence

Similar to building a railway, we can start construction from one end, or we can construct multiple segments at the same time. Therefore, how to design a long-distance entanglement distribution sequence can be boiled down to a similar problem. Correspondingly, we propose the following two representative swapping sequences.

1) Hop-By-Hop Swapping: Hop-By-Hop swapping means that the process of swapping starts from the source node and advances hop by hop by increasing the length of a quantum link each time to expand the entanglement distance. By sequentially performing such serial operations $l-2$ times, the entanglement will be extended to both ends of the path. As shown in Fig. 2(a), it takes 4 chain swapping operations to achieve entanglement distribution across 5 nodes. Such a swapping sequence has the following characteristics: 1) there is no need for complex centralized decision-making and control. In each time slot, the node performing the swapping is treated as the current forwarding node. A distributed swapping protocol can be designed to achieve this function. 2) The number of time slots is $O(l)$. As the path length increases, the entanglement distribution delay will increase linearly. Too long waiting time is obviously not conducive to maintaining the fidelity of entanglement. When the scale of quantum entanglement networks becomes larger, this shortcoming of long delay will become even more fatal.

2) Nested Swapping: Considering that centralized control will be deployed in the quantum network, the advantages of centralized control (such as concurrent operations between nodes) can be used. In principle, in each time slot, as many swapping operations as possible are performed concurrently. Therefore, we adopt a swapping sequence similar to merging (or nested entanglement swapping scheme [26]). The core idea of this method is to complete as many swapping operations as possible concurrently without causing measurement conflicts of entangled quantum bits. As shown in Fig. 2(b), only two time slots are required. The number of time slots required by this method is $O(\log_2 l)$, which obviously shortens the time needed compared to the hop-by-hop swapping method.

The paths that can be symmetrically merged, as shown in Fig. 2(b), are limited in length. The length of such a path needs to satisfy the form of $2^i + 1 (i \geq 1)$. Such a symmetric merging sequence is easy to make. However, when the path is arbitrarily long, the urgent challenge is how the two entangled pairs stored on each repeater should perform entanglement swapping.

3) Concurrent Swapping: We observe that each repeater performs a merge, which can be distinguished by the entanglement distance (hop) (abbreviated as ED). If we label the nodes on the path in order from the source to the destination as $N_1, N_2, ..., N_l$ (e.g. from left to right). A repeater merges two entangled pairs with entanglement distance ED_1, ED_2 into the new entangled pair with entanglement distance ED, which can be abbreviated as $(ED_1, ED_2) \rightarrow ED$, and $ED = ED_1 + ED_2$. It can be found that nodes which achieve $(1,1) \rightarrow 2$ are $\{N_2, N_4, N_6, ...\}$, nodes which achieve $(2,2) \rightarrow 4$ are $\{N_3, N_7, N_{11}, ...\}$. Unfortunately, when the path does not satisfy the length of symmetric merging $(l \neq 2^i + 1)$, then there will be an asymmetric merger, such as $(2,1) \rightarrow 3$ on the 4-hops path. When $ED_1 > ED_2$, it can be found that ED_2 is the longest entanglement distance on the right side.

Remark 1. *For the nodes N_i whose sequence i satisfies*

$$i = 2^{j-1} + k * 2^j + 1, \tag{6}$$

where $k, j \in \mathbb{N}$, $i < l, ED_l = 2^{j-1}$, the swapping that merges $(ED_l, ED_r) \rightarrow ED_l + ED_r$, $ED_r = min\{ED_l, l-i\}$ should happen in N_i, in which $l-i$ means the entanglement distance

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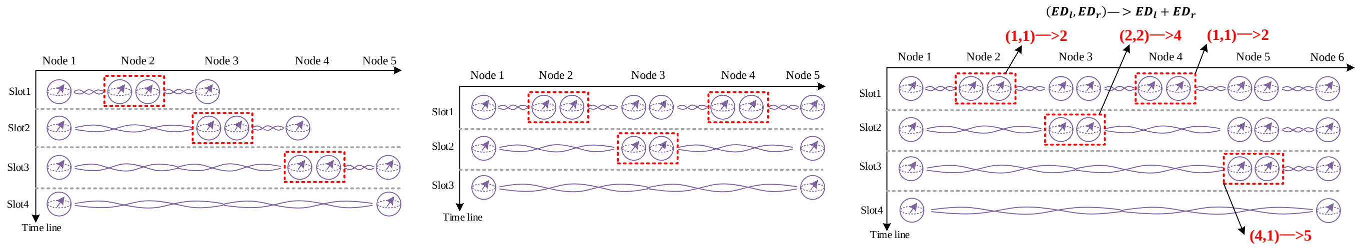

(a) Hop-by-hop swapping (b) Nested swapping (c) Concurrent swapping

Fig. 2: (a) Forward hop-by-hop swapping. (b) Nested swapping on paths with limited hop. (c) Concurrent Swapping on any hop path.

between N_i and the right end node N_l, k, j are non-negative integer parameters, which are flexible and variable to construct the right ED.

Based on Remark1, the entanglement merge at what distance that the node should perform can be calculated based on its location. In each time slot, each node searches for a pair of entangled qubits in its own internal memory to meet the distance requirement, and then the swapping can be achieved by BSM. The details of the decision mechanism of the swapping sequence are given in **Algorithm 1**.

Algorithm 1: Concurrent swapping

Input: The node set on the path $\mathbb{P} = \{N_1, N_2, ..., N_l\}$, l is the path length.

Output: All matching results $M = \{(qubit_l, qubit_r)\}$ inside the node.

Select a node N_i from $\mathbb{P}$ except for end nodes N_1, N_l;
The left qubit set of N_i is $\mathbb{L}$, C_l is the number of $\mathbb{L}$;
The right qubit set of N_i is $\mathbb{R}$, C_r is the number of $\mathbb{R}$;
Find the $ED_l = 2^{j-1}$, let $2^{j-1} + k * 2^j + 1 == i$, $k, j \in \mathbb{N}$;
$ED_r = min\{ED_l, l - i\}$;
`/* Traverse all entangled qubits. */`
for $qubit_l$ *in* $\mathbb{L}$ **do**
  for $qubit_r$ *in* $\mathbb{L}$ **do**
    if $ED(qubit_l) == ED_l$ *and* $ED(qubit_r) == ED_r$ **then**
      Match $(qubit_l, qubit_r)$ and do BSM;
      $(ED_l, ED_r) \rightarrow ED_l + ED_r$;
      `/* the successful swapping */`
    end
  end
end
return All matching results M inside the node;

B. Purification Sequence

In Section IV-A, the swapping sequence has been solved by a similar merging method. Next, purification also needs to be considered due to end-to-end fidelity requirements. Specifically, in the determined swapping sequence, where should the purification operations be inserted? It is very similar to the permutation problem in discrete mathematics.

Considering the following static scenario: there is a given path P, in which path width is $W = 2^i$, path length is $L = 2^j + 1$, and all qubits are occupied with initial entangled pairs. Such path parameters are for the convenience of subsequent analysis. Now these initial entangled resources will achieve the ultimate target through a series of swapping and purification operations: a purified end-to-end entangled pair. For the convenience of description, we use a path state tuple $[W, L]$ to describe the width and length of the path after each time slot. Thus, if we want to get a pair of end-to-end entangled pairs with as high fidelity as possible, the whole entanglement distribution process can be described as $[2^i, 2^j + 1] \rightarrow [1, 2]$. Next, we focus on the state transition in depth.

Remark 2. *After performing one symmetrical purification (all operations are successful), we have $[2^i, 2^j + 1] \rightarrow [2^{i-1}, 2^j + 1]$, and after performing one concurrent swapping (all operations are successful), we have $[2^i, 2^j + 1] \rightarrow [2^i, 2^{j-1} + 1]$. Therefore, $i + j$ operations are required to achieve $[2^i, 2^j + 1] \rightarrow [1, 2]$.*

Scheduling sequence is a permutation problem with constraints. i swapping operations and j purification operations are required to achieve the final single end-to-end entangled pair. The constraints are that there is no difference between swapping operations and no difference between purification operations. Therefore, there are A^i_{i+j} kinds of execution sequence:

$$A^i_{i+j} = A^j_{i+j} = \frac{(i+j)!}{i! * j!}. \tag{7}$$

Next, for a path with width $W = 4$ and length $L = 5$, we illustrate the different scheduling sequences to realize $[4, 5] \rightarrow [1, 2]$, as shown in Fig 3(a). The upward arrow represents purification, and the downward arrow represents swapping. For example, there will be two extreme examples, namely implementing purification operations to enough higher fidelity and then implementing swapping operations (the process formed by the green arrow). On the contrary, completing all the swapping operations and then implementing purification operations (the process formed by the red arrow). The process of other arrow combinations means that swapping and purification alternate with each other. So the problem turns out to be which execution sequence setting is more conducive to distributing a higher fidelity end-to-end entangled pair?

Considering an arbitrary sub-branch in Fig 3(a), e.g. as the one shown in Fig 3(b). A sub-branch represents starting from the current state $[2^i, 2^j + 1]$, either performing a round

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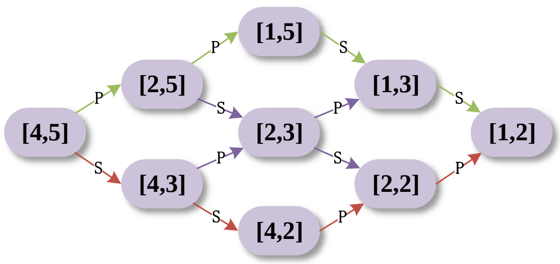
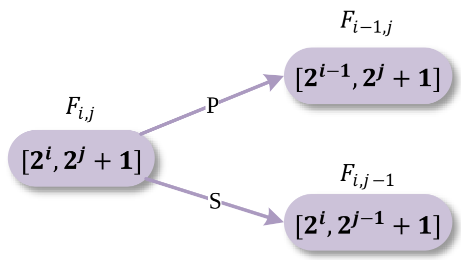


(a) States transition in different scheduling sequences.



(b) The sub-branch.

Fig. 3: Multiple solutions to the permutation problem of swapping and purification and recursive solution on sub-branches.

of purification or performing a round of swapping, and then reaching two sub-states $[2^{i-1}, 2^j + 1]$ or $[2^i, 2^{j-1} + 1]$, respectively. Among them, the conversion is successful with probability P_p or P_s and will cause the fidelity $F_{i,j}$ to change to $F_{i-1,j}$ or $F_{i,j-1}$. The recursive calculations of these parameters are as follows:

$$\begin{aligned} P_p &= \{[(F_{i,j})^2 + (1 - F_{i,j})^2]^{2^{i-1}}\}^{2^j} \\ &= [(F_{i,j})^2 + (1 - F_{i,j})^2]^{2^{i+j-1}}, \end{aligned} \tag{8}$$

$$P_s = [(P_{swap})^j]^{2^i} = (P_{swap})^{j*2^i}, \tag{9}$$

$$F_{i-1,j} = \frac{(F_{i,j})^2}{(F_{i,j})^2 + (1 - F_{i,j})^2}, \tag{10}$$

$$F_{i,j-1} = (F_{i,j})^2. \tag{11}$$

Obviously, from the recursive calculation formulas, it can be found that swapping affects the fidelity, and the fidelity in turn affects the effect of purification, which reflects the coupling between the two operations. Based on the recursive calculation, we can calculate the fidelity F_f and success probability P_f of the final single end-to-end entangled pair. The success probability P of the distributing process can be calculated as:

$$P_f = \prod_{k=0}^{i+j} P_k, \ 0 < P_k < 1. \tag{12}$$

Then we consider calculating the expected value $E(F)$ of the final fidelity as:

$$E(F_f) = P_f * F_f + (1 - P_f) * 0 = P_f * F_f. \tag{13}$$

Therefore, $E(F_f)$ will become very small, so $\lg E(F_f)$ will be considered as follow:

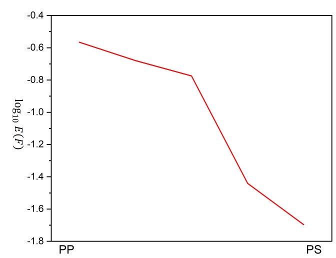
$$\lg E(F_f) = \lg F_f + \sum_{k=0}^{i+j} \lg P_k, \ and \ \lg E(F_f) < 0. \tag{14}$$

The $E(F)$ is small and $\lg E(F_f)$ is even smaller due to the monotonicity of the logarithmic function.

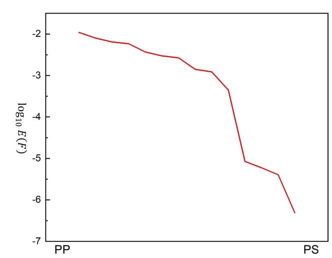
We design 4 scenarios for numerical simulation, namely $[4,5] \to [1,2]$, $[8,9] \to [1,2]$, $[16,9] \to [1,2]$, $[32,17] \to [1,2]$, and there are 6, 20, 35, and 126 achievable state-transition choices, respectively. Since the number of scheduling sequences will be different in four scenarios, for unified description and comparison, the scheduling sequence is classified according to the priority of purification. Here, we use a binary bit string to represent the scheduling sequence of purification and swapping in $(i+j)$ operations. An example can be recorded as,

$$S = 101...100, \tag{15}$$

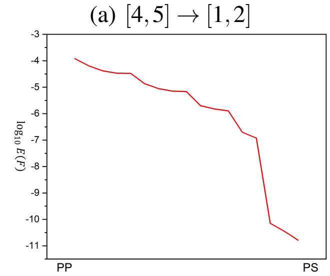
where "1" represents purification with j times, "0" represents swapping with i times. Thus, each binary string represents one possible scheduling sequence. And the larger the value of this string, the more preferentially the purification operation is performed. The results are shown in Fig 4.



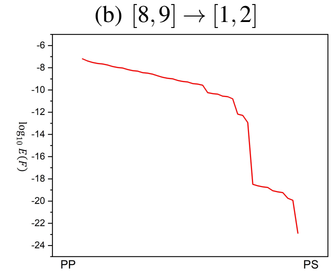
(a) $[4,5] \to [1,2]$



(b) $[8,9] \to [1,2]$



(c) $[16,9] \to [1,2]$



(d) $[32,17] \to [1,2]$

Fig. 4: The $\lg E(F)$ of different scheduling sequences.

The four numerical simulation results of Fig. 4 (a)-(d) show that in the variable direction, from left to right, the priority of purification is getting lower. For example, the leftmost side of the abscissa indicates that the purification has the highest priority (e.g. 1100, and prioritized purification is abbreviated as PP). All purification operations are performed first before swapping, and the rightmost is the opposite, and the swapping has the higher priority (e.g. 0011, prioritized swapping is abbreviated as PS). As the priority of purification decreases, it can be seen that $lgE(F)$ of the target entangled pair will gradually decrease. It should be noted here that not all scheduling orders can successfully distribute target

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entangled pairs. For example, a sequence like "000111" may cause the entanglement property to disappear ($F < 0.5$) due to too many consecutive swapping operations. Therefore, by modeling scheduling sequence as a permutation problem, it can be concluded that **prioritized purification (PP)** is more conducive to obtaining higher-fidelity end-to-end entangled pairs with limited resources.

C. Fidelity Maintenance Scheme

End-to-end entanglement distribution will be completed on a given path, and specific fidelity requirements F_f need to be guaranteed. Then, when the result obtained by the routing algorithm is determined, the biggest factor that affects the distribution efficiency is the scheduling sequence mentioned in the previous section. Combining the **prioritized purification** mentioned above, we propose the following end-to-end fidelity maintenance scheme **FMSPP**.

FMSPP means fidelity maintenance based prioritized purification. This method means that we first purify the initial entangled pairs repeatedly to such a limitation so that fidelity is sufficient to resist the fidelity attenuation caused by the swapping. Assumed the minimum fidelity after all purification is F^*, then

$$(F^*)^{l-1} \geq F_f, \ and \ F^* \geq (F_f)^{\frac{1}{l-1}}, \tag{16}$$

where l means the path length. Since F^* is available, the initial link-level entanglement fidelity needs to be continuously purified to F^*. Purification will not be carried out in the subsequent process, because the fidelity is high enough to offset the negative impact of swapping. FMSPP is shown in **Algorithm 2**.

Algorithm 2: Fidelity maintenance scheme based on prioritized purification (FMSPP)

Input: The fidelity set $\mathbb{F} = \{F_1, F_2, ..., F_{l-1}\}$ of each link along the path;
Output: End-to-end entangled pairs satisfying fidelity requirements.

```
/* path length is l.                          */
Define the minimum fidelity after purification F*;
(F*)^{l-1} ≥ F_R;
F* ≥ (F_R)^{1/(l-1)};
/* final required fidelity is F_f.            */
for F_i in 𝔽 do
    while F_i < F* do
        F_i = F_i^2 / (F_i^2 + (1 - F_i)^2);
        /* symmetrical purification.          */
    end
end
Do concurrent swapping;
return Final entangled pairs;
```

V. Performance Evaluation

A. Simulation Setup

We first introduce the simulation setup. 1) Path length (i.e., node number): considering the scale of the quantum network, we set the four path length $6, 10, 15, 20$. 2) Memory capacity C: capacity of the end nodes are 300 units, and the memory capacity of the repeater nodes are twice of the end node, which is 600 units. 3) The generation rate of initial entangled pairs R and initial fidelity F_0: in each time slot, each link can generate 10 initial entangled pairs. The initial fidelity $F_0 = 0.8$. 4) Swapping successful probability $Q = 0.6$ or 0.9. 5) Final fidelity requirement $F_f = 0.8$. 6) Time slot: in our simulation, for convenience, it is assumed that each operation (entanglement generation, swapping, and purification) can be performed at most once for a memory unit. Specifically, in each time slot, the controller will first perform entanglement generation, then check whether entanglement purification is required, and finally perform entanglement swapping for entangled pairs that meet the swapping conditions(e.g., above fidelity threshold). Our simulation runs for 500 time slots, counts relevant parameters after each time slot, and then repeats 100 times to obtain the average value.

Then, we define three important parameters EDR, ECR, and ave_op to compare the performance of different scheduling schemes. If the total running time is $\mathbb{T} = nT$ (n = 500), where T means the time span of a time slot and n represents the number of time slots in simulation, then the detailed explanation is as follows:

EDR: Entanglement distribution rate EDR represents the total number of end-to-end entangled pairs in $\mathbb{T}$.

$$EDR = \sum_{i=1}^{n} num(ep), \tag{17}$$

where $num(ep)$ means the number of end-to-end entangled pairs in each slot.

ECR: Entanglement consumption ratio ECR to reflect the number of initial entangled pairs that need to be consumed to distribute an available end-to-end entangled pair on average.

$$ECR = \frac{\sum_{i=1}^{n} \sum_{j=1}^{l} O(lp)}{\sum_{i=1}^{n} num(ep)}, \tag{18}$$

where $O(lp)$ reflects the number of initial entangled pairs generated on each link, and $num(ep)$ means the number of end-to-end entangled pairs in each slot. The higher ECR means the lower distribution efficiency.

ave_op: Since purification and swapping consume a lot of entanglement resources, the number of these operations can reflect the resource overhead. We define the average number of purification and swapping times needed to distribute an end-to-end entangled pair as O_{ave}, which reflects the resource overhead of scheduling schemes. It can be expressed as

$$ave\_op = \frac{\sum_{i=1}^{n} O(S) + \sum_{i=1}^{n} O(P)}{\sum_{i=1}^{n} num(ep)}, \tag{19}$$

where $O(S)$ refers to the number of times of swapping operations, $O(P)$ refers to the number of times of purification operations.

B. Comparison Schemes

Since the FMSPP scheme is designed based on the PP, the influence of other scheduling sequences on the distribution performance should be compared. There have been quite a few feasible scheduling solutions, and it is difficult for us to traverse all of them to compare. According to the characteristics of scheduling, other scheduling schemes can be divided into the following two categories:

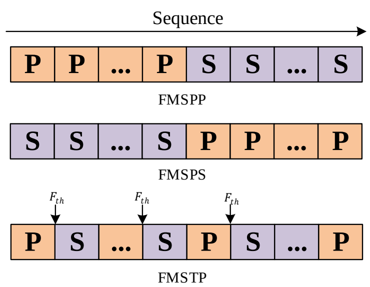

Fig. 5: Three methods to maintain fidelity are illustrated. Orange squares represent the purification, purple squares represent the swapping, and left to right are considered the entanglement establishment process.

1) Fidelity Maintenance Scheme Based on Prioritized Swapping (FMSPS): As opposed to FMSPP, swapping has the highest priority, which means that the entanglement distance is extended first, and the required purification operations are done finally. It is important to note that this scheme is not fully applicable in all situations, because purification gain of the entangled pairs may be very low after all the swapping operations are completed due to too low fidelity. Indeed too many consecutive swapping operations may make the entangled pairs lose their entanglement properties, resulting in no recovery of any number of purification operations. Therefore, this method is not applicable when the hop is more, i.e., in a large quantum network.

2) Fidelity Maintenance Scheme Based on Threshold Purification (FMSTP): Between FMSPP and FMSPS, swapping and purification are interspersed and complementary to each other. There are many permutation results, and it is impossible to traverse all scheduling sequences in the simulation. Therefore, we introduce a fidelity threshold F_{th}. If the current fidelity of an entangled pair fidelity is found to be lower than F_{th}, then multiple rounds of purification are performed until the fidelity is higher than F_{th}, and then the swapping operations can be performed. This process is repeated until an end-to-end entangled pair is formed. Certainly, if the final fidelity is found to be lower than the requirement F_f, the purification should be continued until the fidelity is satisfied. Different thresholds represent various scheduling sequences. For example, the higher threshold means that the purification priority is higher, and on the contrary, the lower threshold corresponds to a higher swapping priority. A simple example of the three schemes is shown in Fig. 5.

C. Discussion

We first compare the performance of various solutions in the FMSTP scheme to obtain the optimal purification threshold. Then, for the three schemes of FMSPP, FMSPS, and FMSTP (with the optimal threshold), we compare the performance in terms of *EDR*, *ECR*, and *ave_op*, and give analysis and conclusions.

1) Optimal Fidelity Threshold of FMSTP: The FMSTP scheme can be mapped to various scheduling solutions, and it is difficult for us to exhaustively search them to find the optimal threshold. Also, the optimal solution changes with the initial setting of the path. Thus, it is impossible to find a fixed optimal threshold. Therefore, we set four representative path settings to find their optimal thresholds through multiple simulations, which are shown in Table. I. Specifically, we set the variable of fidelity threshold changing in the interval between 0.85 and 0.99, and divide it into 15 fidelity thresholds in 0.1 step. For the four different path settings, we keep other parameters consistent, such as memory capacity, distribution rate, successful swapping probability, and so on. In each row, the threshold corresponding to the highest EDR value (with red highlight) represents the best threshold for the FMSTP scheme on the current path. For example, when we consider the node number is 6, when the Q is 0.6, the optimal threshold is 0.92. when the Q is 0.9, the optimal threshold is 0.89. We select the FMSTP scheme based on the optimal threshold (abbreviated as FMSTP_opt) and compare it with our proposed FMSPP scheme for simulation.

As expected, the FMSPS is not suitable for long paths with the fast decay of *EDR*, which is not comparable with the other two methods. We separately list the *EDR* of FMSPS scheme in Table. II when the node number varies from 4 to 10 nodes, respectively. The simulation results can confirm that the distribution performance of FMSPS scheme is extremely poor, especially dealing with long distribution paths. Therefore, in subsequent simulations, we ignored the discussion of FMSPS. Our focus is on comparing the performance differences between FMSPP and FMSTP_opt.

2) Analysis about EDR: The comparison of *EDR* results between FMSPP and FMSTP_opt is shown in Fig. 6. First of all, it can be clearly observed that the *EDR* gradually decreases with the increase of the node number on the path, because more nodes lead to more entanglement swapping, and the success probability of end-to-end entanglement distribution will be lower. Secondly, more entanglement swapping makes the decrease of fidelity more obvious. It is necessary to perform more rounds of purification at the repeaters to improve the fidelity, which will greatly reduce the *EDR*. Comparing the two schemes, we can see that even the FMSTP_opt based on the optimal threshold, its *EDR* is still lower than that of FMSPP, especially when the path becomes longer, the disadvantage will become more prominent.

How to explain this phenomenon? First, due to the imperfection of entanglement swapping, purification is performed earlier, and there is a higher comprehensive income. On the contrary, the purification performed last requires entangled pairs that can be obtained after multiple rounds of successful swapping as input, which is obviously more difficult. Then, as the discontinuity of purification gain (i.e. inevitably exceeding the desired threshold), multiple intermittent purification operations are more likely to cause fidelity overflow in the FMSTP_opt scheme. When the path length is longer, the “fidelity overflow” of FMSTP_opt scheme

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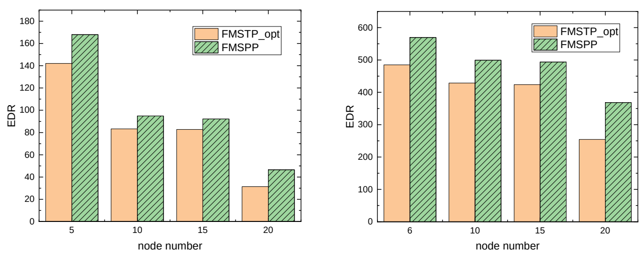
TABLE I: The optimal threshold of FMSTP scheme.

node number = 6															
threshold	0.85	0.86	0.87	0.88	0.89	0.9	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
Q = 0.6	119.33	118.01	117.87	118.01	140.11	141.27	141.75	**142.03**	141.02	140.92	92.07	89.48	95.88	93.3	64.01
Q = 0.9	402.02	402.18	400.98	405.17	**484.8**	484.45	483.36	481.43	481.87	481.19	313.51	304.74	326.46	316.78	218.85
node number = 10															
threshold	0.85	0.86	0.87	0.88	0.89	0.9	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
Q = 0.6	68.99	68.12	26.14	26.99	82.6	82.49	**83.27**	82.37	82.67	35.94	50.32	52.72	55.99	54.76	37.64
Q = 0.9	354.46	353.34	135.75	135.51	425.73	**428.55**	428.41	427.21	425.26	187.69	262.11	268.44	287.51	282.16	194.2
node number = 15															
threshold	0.85	0.86	0.87	0.88	0.89	0.9	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
Q = 0.6	25.09	24.36	23.94	23.14	81.03	81.66	80.4	**82.8**	79.47	36.62	49.31	51.16	53.68	53.07	35.31
Q = 0.9	128.6	127.23	127.07	126.62	423.96	422.32	**423.96**	423.48	422.93	187.43	261.44	265.65	284.81	277.29	190.05
node number = 20															
threshold	0.85	0.86	0.87	0.88	0.89	0.9	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
Q = 0.6	12.02	12.51	14.69	15.08	18.59	18.48	18.57	18.96	18.61	20.73	28.82	30.22	**31.41**	30.22	20.86
Q = 0.9	98.13	96.76	117.37	118.58	148.5	148.46	148.3	148.16	146.98	165.43	230.43	238.09	**254.41**	248.92	170.08

TABLE II: *EDR* of FMSPS scheme.

node number	4	5	6	7	8	9	10
Q = 0.6	28.6	15	1.7	1.2	0.7	0.5	0.1
Q = 0.9	59.2	30.4	5.3	4.3	2.9	2.3	0.2



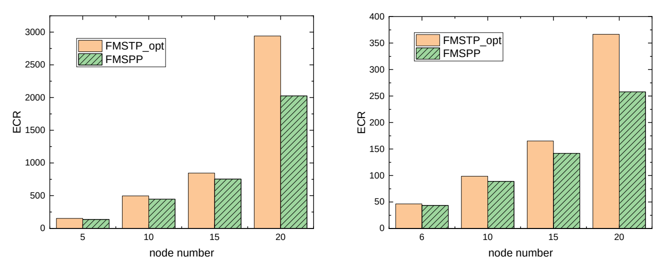
(a) $Q = 0.6$ (b) $Q = 0.9$

Fig. 6: Entanglement distribution rate *EDR* on different length paths. (a) Entanglement swapping probability $Q = 0.6$. (b) Entanglement swapping probability $Q = 0.9$.

is more fatal for *EDR*. Although this may result in higher final fidelity but with declining in *EDR*. In contrast, since all purification operations are performed centrally on the initial link, the fidelity of FMSPP schemes can be considered "just enough" without serious fidelity overflow. Therefore, the *EDR* of the FMSPP scheme is always better than the *EDR* of the FMSTP_opt for long paths.

3) Analysis about ECR and ave_op: The comparison of *ECR* results and *ave_op* results between FMSPP and FMSTP_opt are shown in Fig. 7 and Fig. 8. We can clearly observe that as the number of nodes increases, both *ECR* and *ave_op* are rising, because more swapping and purification cause more failed operations, and it is more difficult to successfully establish an end-to-end available entangled pair. When the path is short (6 nodes), the difference between the two schemes is unimportant. There are fewer operations at this time, and the difference caused by different scheduling sequences will not be obvious. When the path is long, the efficiency of the two schemes will be quite different. FMSPP puts the enhancement of fidelity at the beginning as much as possible. In this way, the process of improving fidelity does not involve entanglement swapping that may fail, so the gain will be greater and it is easier to form end-to-end available entangled pairs. Moreover, when the path becomes longer, the more entanglement swapping failures, the advantage of preferential purification becomes more obvious, and the performance gap between these two schemes is more prominent.

Finally, by comparing two subfigures in Fig. 6, Fig. 7 and Fig. 8, respectively, the improvement of entanglement swapping probability is often huge for the improvement of distribution performance, as the entanglement swapping between nodes on the path is independent, and the probability of end-to-end distribution of available entangled pairs is equal to the product of entanglement swapping probability at each node. Therefore, the gain obtained from swapping probability Q is not linear and almost satisfies the exponential relationship.



(a) $Q = 0.6$ (b) $Q = 0.9$

Fig. 7: Entanglement consumption ratio *ECR* on different length paths. (a) Entanglement swapping probability $Q = 0.6$. (b) Entanglement swapping probability $Q = 0.9$.

VI. Conclusion

In this paper, we decomposed the complicated scheduling problem of swapping and purification into two sub-problems: swapping sequence and purification sequence. For the swapping sequence, the concurrent swapping method was proposed to decide the swapping location in each time slot with less entanglement distribution delay. Then for the purification sequence, we got the "prioritized purification" (PP) principle by modeling it as a permutation problem. Based on PP, we proposed the FMSPP to maintain the end-to-end fidelity of any demand. Finally, we developed simulations, and

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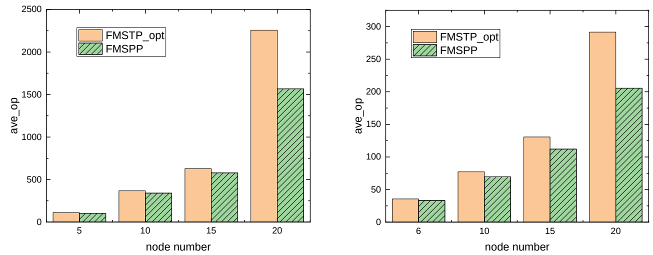

(a) $Q = 0.6$ (b) $Q = 0.9$

Fig. 8: Average number of operations *ave_op* on different length paths. (a) Entanglement swapping probability $Q = 0.6$. (b) Entanglement swapping probability $Q = 0.9$.

results shown that the FMSPP scheme has a higher *EDR* and less resource overhead, and the advantages are more significant as there are more hops on the selected path.

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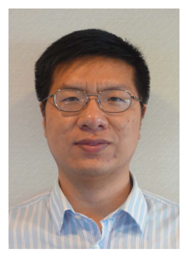

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