Entropic Dynamics of Monte Carlo

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June 12, 2020

Abstract

We develop a Monte Carlo method using Entropic Dynamics for solving dynamical systems with some probability distribution $\rho(x)$.

1 Introduction

2 Entropic Dynamics

3 Entropic Dynamics for Monte Carlo

We desire a method for updating a probability distribution $\rho(x,t)$ defined over some sample space $\mathbf{X} \subseteq \mathbb{R}^n$ where the distribution necessarily satisfies a continuity equation,

$$\partial_t \rho(x,t) = -\partial_k (v^k \rho), \qquad k = \{1,\ldots,n\}, \tag{3.1}$$

where $\rho v^k \stackrel{\text{def}}{=} j^k$ is the probability current. If we can integrate (3.1), then we have solved the problem. For most applications, this will be done by some kind of approximation. For example, one may throw a sample distribution $\mathcal{X} \subset \mathbf{X}$ according to j^k and ρ and numerical integrate using Monte Carlo integration. Whenever the space $\mathbf{X}$ has a large dimension, this method can be computationally expensive.

Instead of integrating (3.1), we will take a different approach and attempt to approximate the expression,

$$\rho(x',t') = \int dx\, P(x',t'|x,t)\rho(x,t), \tag{3.2}$$

where $P(x',t'|x,t)$ is some specified *transition probability*. This equation is the dual to (3.1) and is often referred to as a *Chapman-Kolmogorov* equation.

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The goal is to determine the set of transition probabilities $P(x'_i|x_i)$ by maximizing the relative entropy,

$$\mathcal{S}[P,Q] = -\sum_{x'_i} P(x'_i|x_i) \log \frac{P(x'_i|x_i)}{Q(x'_i|x_i)}, \tag{3.3}$$

subject to the constraints,

$$\sum_{x_i'} P(x_i'|x_i) = 1 \qquad \text{and} \qquad \sum_{x_i'} P(x_i'|x_i) f_n(x_i', x_i) = \kappa_n(x_i) \tag{3.4}$$

where $\{f_\ell\}_{\ell=1}^n$ constitute a set of n functions on the joint space $\mathbf{X}' \times \mathbf{X}$.

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We construct the transition probability matrix $\mathbf{P}$ as,

$$\mathbf{P} = \begin{bmatrix} P(x_1'|x_1) & P(x_1'|x_2) & \cdots & P(x_1'|x_n) \\ P(x_2'|x_1) & P(x_2'|x_2) & \dots & P(x_2'|x_n) \\ \vdots & \vdots & \ddots & \vdots \\ P(x_n'|x_1) & P(x_n'|x_2) & \cdots & P(x_n'|x_n) \end{bmatrix}. \tag{3.5}$$

Then, the update can be written,

$$\mathbf{p}' = \mathbf{P}\mathbf{p}, \tag{3.6}$$

or

$$p_i' = P^j{}_i p_j. \tag{3.7}$$

4 Transition probabilities