

Constructing Schedules for Sports Leagues with Divisional and Round-robin Tournaments

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Abstract

We analyze a sports league that wishes to augment its traditional double round-robin tournament into a longer season. The method for doing so, chosen by the top Swedish handball league Elitserien, is to form two divisions that hold an additional single round-robin tournament to start the season. This format introduces new constraints since pairs of teams in the same division meet three times during the season, while others only meet twice. Though motivated by the concerns of a specific league, the requirements addressed are general enough to be useful for other leagues. We enumerate the number of minimum break home-away pattern sets that satisfy the league's requirements, not all of which are schedulable. We propose a sequence of increasingly restrictive necessary conditions that remove most of the unschedulable home-away pattern sets from consideration. We lastly discuss the final steps of assigning teams to a schedulable home-away pattern set; such an approach was used to construct the 2013-14 Elitserien schedule.

1 Introduction

The prevailing league format in team sports is the double round robin tournament (DRRT), where each team hosts every other team once. Sports leagues wishing to increase exposure and revenue may look to add games to their double round-robin tournament schedule. Expanding the league by adding teams (thereby increasing games in the season schedule) is a possibility, but dilution of the talent pool can adversely affect the quality of play within the league, e.g., Kahn (2007). It is also possible to extend the season by having each pair of teams play three times in a triple round-robin tournament (TRRT), as shown by Rasmussen (2008) for the top Danish soccer league. This 50% increase in the number of games played may be too large for some leagues. In this paper, we address a third option for increasing a league's season length: forming two division which hold concurrent single round-robin tournaments (SRRTs) in addition to the DRRT.

Elitserien, the top Swedish handball league, has taken such an approach to increase its season schedule length. The 14 team owners considered its traditional 26-game DRRT to be too short, while a possible 39-game TRRT was deemed too long. The schedule was therefore augmented by dividing teams into two seven-team divisions (based on geographic proximity), each of which plays an intra-division SRRT before the traditional DRRT. The league's schedule is therefore separated into three parts: Part I (periods 1-7) consisting of two concurrent, intra-division SRRTs; Part II (periods 8-20) consisting of an RRT between all teams, and Part III (periods 21-33) which is the complement (away games become home games and vice-versa) of Part II. This schedule results in teams in the same division meeting three times, once in each part; teams in different divisions only meet in Parts II and III. This asymmetry introduces new concerns about the schedule. In addition to previously studied constraints (e.g., minimizing consecutive home or away matches, called breaks; ensuring equal number of home and away games; etc.) the league desires that any consecutive meetings between pairs of teams alternated venues. We call this constraint the *Alternating Venue Requirement* (AVR). For two teams from separate divisions, this is readily ensured since Part III is the complement of Part II. For teams in the same division, no obvious solution exists.

Many sports leagues have formed divisions of teams who meet each other more often than other teams. To the best of our knowledge, no other league has a format where a divisional RRT precedes a DRRT to form a complete season. Nevertheless, there exists some pertinent research relating to groups within an RRT. Though the groups considered in the literature are often *strength groups* (teams of similar ability), results from such papers can possibly be applied

to groups constructed based on geographic proximity. For example, Briskorn (2009), Briskorn and Knust (2010) analyze schedules where no team plays against a team from the same group in consecutive matches (*group-changing*) and schedules where no team plays a team from the same group within a stretch of p games (*group-balanced*). The combinatorial properties of these strength groups are summarized in (Briskorn, 2008b, Chapter 4). Multiple strength groups are considered in the paper by van't Hof, Post, and Briskorn (2010) where they construct a minimum break SRRT, where some teams never play at home during the same period (complementary schedules), and teams from different strength groups are sufficiently spread throughout the tournament. Ensuring desirable home-away patterns when playing teams from different groups has yet to be addressed.

In this paper we present a methodology for scheduling general tournaments where each division holds an RRT before a DRRT. We first show that it is always possible to construct a tournament satisfying the Alternating Venue Requirement (if we relax other requirements). Then, we construct home-away patterns (HAPs) for each team such that the league can always be scheduled (if we relax the Alternating Venue Requirement). A particular effort is devoted to constructing and characterizing HAP sets where a maximal number of pairs of teams never play at home during the same period. We further present some necessary conditions for a HAP set to be scheduled in a manner satisfying the Alternating Venue Requirement. These conditions reduce the number of possible HAP sets to a reasonable number, which, after a quick search, can locate a schedule satisfying the league's requirements. We use this approach to construct as a tournament template for Elitserien, the top Swedish handball league. This template outlines a full-season schedule with generic team numbers rather than specific teams (e.g., Team 1 plays Team 2 in period 4). To form a schedule each year, we show how teams can be assigned numbers to further increase the schedule's attractiveness. While we only address one league, we consider the league requirements to be reasonable; we hope this makes the problem and its solution relevant to any other league looking to augment to a DRRT in a similar fashion.

We also believe this research is relevant for leagues containing divisions that do not necessarily follow a DRRT format. For example, the two stages of the Chilean football league Durán, Guajardo, and Wolf-Yadlin (2012) or the National Basketball Association's current schedule format contains an asymmetric structure where teams in a division may play teams in another division a different number of times. Though the NBA does not ensure consecutive meetings between teams satisfy the AVR (in 2013-14, Chicago plays home-home-away-away in four meetings with Charlotte, and home-away-away in three Washington meetings), the methods within this paper are general enough to be applied to most league formats.

The outline of the paper follows: In Section 2, we define the constraints of the Elitserien schedule and justify the use of HAP sets to construct a solution. We construct tournament templates satisfying the AVR in Section 3.1 (possibly not satisfying other requirements) and construct schedulable HAP sets in Section 3.2 (possibly not satisfying the Alternating Venue Requirement). In Section 3.3 we enumerate the number of HAP sets which satisfy a list of general requirements and then outline necessary conditions in Section 3.4 for such a HAP set to satisfy the AVR. We then search over the remaining HAP sets to construct a tournament template satisfying every league requirement in Section 3.5. Lastly, in Section 4 we outline the league's concerns when assigning teams to the template. Section 5 concludes the paper.

2 Problem Statement

The requirements on the Elitserien schedule can be broadly classified into two categories: the first addresses schedule structure and fairness in terms of breaks, periods without games (called byes), and the sequence of home and away games; the second concerns stadium and referee availabilities, the desire to support various match-ups (such as rivalries), and wishes from the media. Historically, Elitserien has determined their schedule by first proposing a tournament template which addresses the fairness constraints. This tournament template has numbers in place of actual teams in the schedule. Every year, the league collects information about unavailabilities and particular wishes from the clubs and assigns teams to numbers in the tournament template to form the season schedule.

We find the intermediate process of constructing a tournament template to be a useful step. In a relatively straightforward fashion, one can convey the strengths and weaknesses of a schedule in a single table without involving any actual team names. This generality allows individuals to articulate what they desire in a schedule more easily than when team names (and the memories/biases that such names invoke) are involved. The template also provides clarity when analyzing properties that can be difficult to optimize over. For example, we find that complementary schedules

can be more easily recognized when analyzing a template with numbers than when inspecting a schedule with team names.

The assignment of actual teams to the numbers of the tournament template is rather straightforward and will be dealt with in Section 4. The majority of this paper addresses the more interesting and challenging problem of constructing and characterizing tournament templates which satisfy the Elitserien requirements. We state this problem in detail by declaring the Elitserien schedule requirements in Table 1.

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1. Each 7-team division must hold a SRRT to start the season.
 2. This must be followed by two SRRTs between the entire league, the second SRRT being a complement of the first.
 3. There must be a minimum number of breaks in the schedule.
 4. Each team has one bye during the season to occur during the divisional RRT.
 5. At no point during the season can the number of home and away games played by a team differ by more than 1.
 6. Any pair of teams must have consecutive meetings occur at different venues. (AVR)
 7. Each division must have 3 pairs of complementary schedules.
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Table 1: Elitserien schedule requirements.

To be clear, Elitserien considers any consecutive home or away matches to be breaks. Therefore, a team which plays away-by-away or home-by-home constitutes a break, as does a team ending the SRRT with the same type of game (home or away) and starting the DRRT with the same type of game.

Before constructing tournament templates, we first provide a lower bound on the minimum number of breaks required to schedule a league such as Elitserien.

Proposition 2.1 *In an n -team league (n even) with a schedule consisting of two concurrent divisional RRTs followed by two consecutive full-league RRTs, if only one bye is allowed and it must occur during the divisional RRT, any schedule must have at least $2n - 4$ breaks.*

Proof: Since we are looking to minimize breaks in the schedule two results apply. First, de Werra (1981) proves an n -team RRT (n even) must have at least $n - 2$ breaks and also constructs schedules meeting this lower bound. Second, Fronček and Mészka (2005) constructs unique RRT with one bye and no breaks (and also shows such RRTs are unique). Therefore the divisional RRTs can be scheduled without any breaks, since we allow a bye in Part I, and the two full-league RRTs must each have $n - 2$ breaks, resulting in a total of $2n - 4$ breaks. ■

We will show that it is possible to construct schedules achieving this lower bound. That is, one can transition from one tournament to another without introducing breaks. Since Fronček and Mészka (2005) prove the HAP set for the divisional tournament is unique, and Ribeiro and Urrutia (2007) shows that HAP sets must break in odd periods if they ensure that the cumulative number of home and away games played never differs by more than 1 at any point in the season, we use the home-away patterns from these two papers to construct our HAP set. The divisional RRT home-away patterns in Figure 1(a) will be combined with two copies of a full-season RRT home-away pattern in Figure 1(b) without introducing additional breaks. These combinations will then construct the full-season HAP set.

3 Constructing Tournament Templates

We first show that we can construct a HAP set which can be scheduled as a tournament satisfying every constraint in Table 1, except for possibly the Alternating Venue Requirement. We then show that we can construct a tournament satisfying the AVR, but possibly not the other constraints. We lastly demonstrate some necessary conditions which remove many HAP sets as unschedulable.

BAHAHAH	AHAHAHAHAHAHA
HBAHAHA	AHAHAHAHAHAHH
AHBAHAH	AHAHAHAHAHHAH
HAHBAHA	AHAHAHAHHAHAH
AHAHBAH	AHAHAHHHAHAHAH
HAHAHBA	AHAHAHHHAHAHAH
AHAHAHB	AHAHHHAHAHAHAH
or	AHHHAHAHAHAHAH
BHAHAHA	HAAHAHAHAHAHAH
ABHAHAH	HAHAHAHAHAHAHA
HABHAHA	HAHAHAHAHAHAHA
AHABHAH	HAHAHAHAHAHAHA
HAHABHA	HAHAHAHAHAHAHA
AHAHABH	HAHAHAHAHAHAHA
HAHAHAB	HAHAHAHAHAHAHA

(a) Two unique HAP sets for a 7-team, zero-break RRT. (b) Unique HAP set satisfying the Elitserien requirements for a 14-team, 12-break RRT.

Figure 1: The HAP sets which will define the season schedule.

3.1 Alternating Venue Schedulability

Proposition 3.1 *It is always possible to construct a schedule which satisfies the AVR (but might not satisfy other requirements).*

Proof: A tournament which only satisfies the Alternating Venue Requirement can be constructed in the following manner. Take the unique 1-bye tournament on $\frac{n}{2}$ teams for one division and its complement for the other division. This will be Part I of the full schedule. Let the first $\frac{n}{2}$ periods of Part II be the complement of Part I, except if a team is scheduled for a bye. In that case, pair it with the team from the other division also scheduled for a bye (assigning home arbitrarily). Complete the remaining $\frac{n}{2} - 1$ matches of Part II by cycling through the remaining teams in the other division in the following manner, assigning home arbitrarily. Number the teams in each division $\{1, \dots, \frac{n}{2}\}$. Since each team has played one interdivisional game against a unique team, ensure both have the same number. Proceed by assigning team i to play teams $\{(i+1) \bmod \frac{n}{2}, (i+2) \bmod \frac{n}{2}, \dots, (i+\frac{n}{2}-1) \bmod \frac{n}{2}\}$ from the other division in the order listed. Lastly, complete the schedule by letting Part III be identical to Part II except switch the home and away teams for each game. ■

An example of one such tournament for $n = 6$ is in Table 2. In Table 2, a positive (negative) j entry (i, p) denotes team i plays at home (away) against team j in period p .

0	3	-2	4	-3	2	-5	6	-4	3	-2	5	-6
-3	0	1	3	5	-1	6	-4	-3	-5	1	-6	4
2	-1	0	-2	1	-6	4	-5	2	-1	6	-4	5
0	-6	5	-1	6	-5	-3	2	1	-6	5	3	-2
6	0	-4	-6	-2	4	1	3	6	2	-4	-1	-3
-5	4	0	5	-4	3	-2	-1	-5	4	-3	2	1

Table 2: A tournament template satisfying the Alternating Venue Requirement

3.2 Home-Away Pattern Set Construction

For each team, we can take one row of Figure 1(a), append one row of Figure 1(b), and then append the reflected, complementary of the same row from Figure 1(b) to yield a HAP for a given team. An example of a possible HAP set for the Elitserien is given in Table 3.

B H A H A H A	H A H A A H A H A H A H A	H A H A H A H A H H A H A
H A B H A H A	H A H A H A H A A H A H A	H A H A H H A H A H A H A
H A H A B H A	H A A H A H A H A H A H A	H A H A H A H A H A H H A
H A H A H A B	H A H A H A H A H A A H A	H A H H A H A H A H A H A
A B H A H A H	A H A H A H A H H A H A H	A H A H A A H A H A H A H
A H A B H A H	A H H A H A H A H A H A H	A H A H A H A H A H A A H
A H A H A B H	A H A H A H A H A H H A H	A H A A H A H A H A H A H
B A H A H A H	A H A H A H A H A H A H H A A	A H A H A H A H A H A H
A H B A H A H	A H A H H A H A H A H A H	A H A H A H A H A A H A H
A H A H B A H	A H A H A H A H A H A H A	H A H A H A H A H A H A H
A H A H A H B	A H A H A H H A H A H A H	A H A H A H A A H A H A H
H B A H A H A	H A H A H A A H A H A H A	H A H A H A H H A H A H A
H A H B A H A	H A H A H A H A H A H A A H H	A H A H A H A H A H A H
H A H A H B A	H A H A H A H A H A H A H	A H A H A H A H A H A H

Table 3: One possible HAP set with a minimal number of breaks

This schedule has many attractive properties. Taking the unique no-break, 7-team tournament HAP set and its complement ensures that 7 teams play at home and 7 teams play away in period 8 without introducing a break. If we did not take the complement, we would have 8 teams needing to play at home without introducing a break in period 8, an impossibility. Since we are reflecting and taking the complement of Part II to schedule Part III, and breaks only occur during odd periods (to ensure the number of home and away games never differ by more than 1 at any point in the season), there are no breaks in period 9. This implies no breaks to end the season; in other words, each team plays at home one of the last two periods of the season.

Even though the HAP sets generated in this fashion have some appealing features, and they can be scheduled without regard for the Alternating Venue Requirement using results from Fronček and Meszka (2005) to schedule Part I and results from de Werra (1981) to schedule Parts II and III, one cannot guarantee they can be scheduled so that the AVR is satisfied. As an example, the HAP set in Table 3 can not be scheduled so the AVR is satisfied since it is impossible for team 13 to host team 14 in Part I or Part II of the schedule.

At first glance, the reflecting and taking the complement of Part II to form Part III forces teams to play the same team in periods 20 as they do in period 21 (at the opposite venue). This could be undesirable, depending on the league, but it is a non-issue for the Elitserien. Period II ends before Christmas, allowing for a month-long break for Champions League competitions before Period III starts at the beginning of February.

3.3 Maximally Complementary Home-Away Pattern Sets

Any HAP set where each team's HAP is created with one row of Figure 1(a), one row from Figure 1(b), and the reflected complement of that same row of Figure 1(b) can be scheduled satisfying the constraints from the Elitserien, except for (possibly) the Alternating Venue Requirement. Since there are $14!$ (over 87 billion) possible combinations of rows from Figure 1, a search over all of them to find a HAP set satisfying the AVR would be prohibitively expensive. That is, one could form an integer program to assign games to the HAP set and see if a feasible solution can be found which satisfies the AVR. Of course, calling an integer program for 87 billion HAP sets is computationally impractical and, as is later shown, a tiny fraction of the $n!$ combinations *a priori* by ensuring the maximal number of complementary HAPs are within each division.

When $\frac{n}{2}$ is even (resp. odd), this means we desire $\frac{n}{4}$ (resp. $\frac{n-2}{4}$) pairs of complementary teams in each division. The following two propositions enumerate the number of HAP sets which satisfy this requirement.

Proposition 3.2 *For an n -team tournament, $\frac{n}{2}$ even, with a divisional RRT before a full-league DRRT, there are $\frac{n}{2}!$ unique HAP sets satisfying the requirements in Table 1, except for possibly the AVR, with $\frac{n}{4}$ pairs of complementary schedules within each division.*

Proof: When $\frac{n}{2}$ is even, and each division must have $\frac{n}{4}$ pairs of complementary schedules in Parts I, II, and III, the analysis is straightforward. Each Part I HAP is complementary with only one other HAP within its division and each Part II HAP has only one complement. Therefore, once any of the Part II HAPs is appended to any Part I HAP, the complementary pair is uniquely determined. There are $\frac{n}{2}!$ ways to assign each complementary pair from Part II to any pair from Part I. Therefore, there are $\frac{n}{2}!$ possible HAP sets. ■

Proposition 3.3 *For an n -team tournament, $\frac{n}{2}$ odd, with a divisional RRT before full-league DRRT, if ${}_nP_r = \frac{n!}{(n-r)!}$, then there are*

$$\frac{n}{2}! \left(\frac{n+2}{4} \right)^2 \quad (1)$$

unique HAP sets satisfying the requirements in Table 1, except possibly for the AVR, with $\frac{n-2}{4}$ pairs of complementary schedules within each division.

Proof: It is always possible to order the divisional HAP sets so each HAP is complementary with the team above and below it. (See Figure 1(a) as an example for $n = 14$.) The first and last HAPs are complementary, but both end with the same type of game (home in the top HAP set, away in the bottom HAP set of Figure 1(a)). They therefore can not be made complementary throughout the season without introducing additional breaks into the schedule. For example, if we construct a schedule where the first and last teams in the bottom HAP set of Figure 1(a) are complementary, we must introduce a break in period 8.

If a division will have $\frac{n-2}{4}$ pairs of complementary teams, the team without a complementary counterpart inside the division must be numbered odd since the first and last divisional HAPs cannot be completed to complementary HAPs for the entire season. (In a 7-team division for example, if 3 is not complementary with anyone, then 1 must be complementary with 2, 4 with 5, and 6 with 7.)

If we only consider which teams will not be complementary, there are $\frac{n+2}{4}$ possible teams in each division, or $\left(\frac{n+2}{4}\right)^2$ possibilities. After the non-complementary teams in each division are decided, there are $\frac{n-2}{2}$ total pairs of patterns in Part I that need to be assigned one of $\frac{n}{2}$ complementary Part II HAPs. Considering the non-complementary teams from each division as a pair themselves, we have $\frac{n}{2}!$ possible ways of assigning Part II patterns to Part I patterns, once the non-complementary teams are decided. ■

For the Elitserien with $n = 14$, Proposition 3.3 leaves 80640 HAP sets which satisfy the league's schedule requirements, except for possibly the AVR. Next, we will demonstrate that also accounting for the AVR allows us to rule out even more HAP sets.

3.4 Necessary Conditions for AVR Schedulability

Several researchers have attempted to address when a schedule can be assigned to a HAP set. A necessary condition for general round-robin tournaments, proposed in Miyashiro, Iwasaki, and Matsui (2003), is that there must be "enough opportunities" for any subset of teams to play each other. Explicitly, if $c_A(T', p)$ (resp. $c_H(T', p)$) counts the number of away (resp. home) games in the HAPs for teams in the subset T' in period p , then

$$\sum_{p \in P} \min(c_A(T', p), c_H(T', p)) \geq \binom{|T'|}{2} \quad \forall T' \subseteq T. \quad (2)$$

Notice that the left side of (2) counts the number of possible matches that could possibly be played between teams in T' , and the right side of (2) is the number of necessary matches between teams in T' . Using condition (2), we see

Team 1	AHAHA
Team 2	AAHAH
Team 3	AHHAH
Team 4	HAHAH
Team 5	HHAHA
Team 6	HAAHA

Table 4: An unschedulable HAP set for a 6-team RRT

that the 6-team HAP set in Table 4 can not be scheduled. (Let $T' = \{1, 5, 6\}$). It should be noted that all HAP sets generated using the patterns in Figure 1 are schedulable and therefore satisfy (2).

In the following, we propose some possible generalizations to the necessary condition (2) to account for the Alternating Venue Requirement. For example, since all pairs of teams in the same division must play home-away or away-home in Parts I and II, an obvious modification of (2) is:

$$\sum_{p \in \{\text{Part I}\} \cup \{\text{Part II}\}} \min(c_A(T', p), c_H(T', p)) \geq 2 \binom{|T'|}{2} \quad \forall T' \subseteq \{\text{Division I}\} \text{ or } T' \subseteq \{\text{Division II}\}.$$

That is, any subset of teams in the same division must be able to meet the required number of times. Our HAP sets satisfy this condition by construction, so we do not need to check it. Another necessary condition is that there are “enough” home and away games to satisfy the AVR requirement. For an arbitrary HAP set S , define

$$S(t, p) = \begin{cases} H & \text{if team } t \text{ plays home in period } p, \\ A & \text{if team } t \text{ plays away in period } p, \\ B & \text{if team } t \text{ has a bye in period } p. \end{cases}$$

Then for a HAP set S to be schedulable, for any two teams t_1 and t_2 in the same division, there must exist two periods, p_1 in Part I and p_2 in Part II, such that

$$\begin{aligned} S(t_1, p_1) = H \quad \text{and} \quad S(t_2, p_1) = A, \\ S(t_1, p_2) = A \quad \text{and} \quad S(t_2, p_2) = H. \end{aligned} \tag{3}$$

As we have already noted, the HAP set in Table 3 does not satisfy this requirement for $t_1 = 13$, $t_2 = 14$. We can therefore determine it is unschedulable with respect to the AVR.

Since the divisional tournament (who plays whom in each period) is unique, as shown by Fronček and Meszka (2005), we can improve the pairwise bound by specifying which type of games are required in the Part I RRT. For the HAP set in Table 3, team 8 must host team 10 in period 3 in the unique Part I tournament. (To see this, notice that 8 must play 9 in period 2 and 9 must play 10 in period 4.) However, since team 10 can never host team 8 in Part II, the HAP set in Table 3 is unschedulable with respect to the AVR. This simple condition allows to rule out 30720 of the 80640 HAP sets (roughly 39%) generated by accounting for the other tournament template requirements. Table 5 demonstrates the efficiency of the condition for other league sizes.

n	HAP sets from (1)	Sets removed by simple condition (3)	% removed
6	24	8 (of 20 unschedulable)	40%
10	1080	396 (of 998 unschedulable)	$\approx 40\%$
14	80640	30720 (of 79024 unschedulable)	$\approx 39\%$

Table 5: Measure of the efficiency of a simple necessary condition.

In addition to this simple condition, we can check if i or j is already “committed” to play another team in every period when they could possibly meet. For example, if for a given HAP set, i can only play j in periods p_1 or p_2 , but i must play k_1 in p_1 and j must play k_2 in p_2 , this test will determine such a HAP set is unschedulable. This is only

slightly more expensive computationally to check than condition (3), but it catches many “deeper” contradictions; this condition allows us to remove 46944 of the 80640 HAP sets (59%).

Rather than dealing with conditions on the HAP set, we can instead build an $n \times n$ array, where each entry (i, j) is a vector of periods when it is possible for teams i and j to meet. We can then use various logical arguments to update this array. This reduces the problem of determining if a HAP set can be scheduled so it satisfies the AVR into the problem of completing a Latin square. (A Latin square is a $n \times n$ matrix with entries containing one of n different elements. Each element must occur exactly once in each row and column (Graham, Grötschel, and Lovász, 1995).) We can then check the logical conditions in Table 6 iteratively to determine if a HAP set is unschedulable. Checking all of the conditions in Table 6 for every entry (i, j) can result in three possibilities. First, if an entry (i, j)

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1. If (i, j) has only one entry, remove that value (if possible) from any vector (i, k) , $k \neq j$ and any vector (k, j) , $k \neq i$.
 2. If (i, j) has more than one entry, see if any value is unique in a row or column. Replace (i, j) by that value.
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Table 6: Possible Latin square logical tests.

of the Latin square becomes an empty vector, then there is no feasible period for team i to meet team j and the HAP set can be ruled unschedulable. Second, if every entry of the Latin square becomes a 1-dimensional vector, then the corresponding HAP set is schedulable and we have discovered unique tournament template. Lastly, if the Latin square remains unchanged after checking every entry (i, j) , the HAP set cannot be ruled unschedulable. The efficacy of using the conditions in Table 6 are shown in Table 7.

n	HAP sets from (1)	Sets removed by Latin square test	% removed
6	24	20 (of 20 unschedulable)	100%
10	1080	949 (of 998 unschedulable)	$\approx 95\%$
14	80640	68541 (of 79024 unschedulable)	$\approx 87\%$

Table 7: Measure of the efficiency of the Latin square condition.

This Latin square approach has many advantages, namely that we can often determine when HAP set is unschedulable. Even if the logic does not decide a HAP set to be infeasible, further constraints on the schedule are nearly always discovered (e.g., when attempting to see if a HAP set can be scheduled to satisfy the AVR, it is determined that team i must play team j in period p). This information can then be used to greatly reduce the search space for an integer program attempting to schedule a HAP set. Sadly, the above logic is insufficient for determining if a Latin square has a completion; in fact, the problem of completing a general Latin square is known to be NP-complete (Colbourn, 1984).

It is possible to extend the Latin square logic even further, for example, if a Latin square L can not be reduced further using the conditions in Table 6 and there exists an entry (i, j) in the Latin square containing two periods p_1 and p_2 . One could make two new Latin squares, L_1 and L_2 , that are identical to L except entry (i, j) is p_1 in L_1 and p_2 in L_2 . Each of L_1 and L_2 can then be analyzed using the conditions in Table 6. If both L_1 and L_2 eventually have an empty vector as an entry, then the HAP set that formed L can be ruled infeasible. If only L_1 has an empty vector as an entry, then we replace L with L_2 and continue working. If neither L_1 nor L_2 can be decided, the schedulability of the HAP set which generated L can not be determined. The efficacy of this a method is shown in Table 8.

It is not surprising that this extended Latin square approach is both powerful and time consuming. In Table 8, we see that such a condition is able to remove almost all unschedulable HAP sets from the search space for the 14-team case, but the time required to check all 80640 HAP sets is approximately 1 day in MATLAB. Nevertheless, the basic Latin square method still has the ability to quickly remove many HAP sets in much less time. The basic method requires less than 30 minutes in MATLAB for the 14-team case, but still detects a large portion of unschedulable HAP sets, as seen in Table 7.

n	HAP sets from (1)	Sets removed by extended Latin square test	% removed
6	24	20 (of 20 unschedulable)	100%
10	1080	998 (of 998 unschedulable)	100%
14	80640	75995 (of 79024 unschedulable)	$\approx 96\%$

Table 8: Measure of the efficiency of the extended Latin square condition.

3.5 From HAP set to Tournament Template

Once a large number of HAP sets have been ruled out, one must assign games to a HAP set to form a tournament template. We used a simple extension of the integer programming scheduling formulation in Briskorn (2008a) that also ensures that the AVR requirement is met. We implemented this model in a constraint programming environment and used the solution collector to retrieve multiple feasible tournament templates for each home-away pattern. These templates were then ranked using a number of criteria, including carry-over-effect. A representative tournament template is shown in Table 9 and a simple model for constructing a template from a HAP set is presented in Appendix A.

This template is an improvement over previous Elitserien schedules in two key respects. First, the previous template used identical HAP sets for both division, which necessitated an additional break when transition from divisional to DRRT play. Second, the base HAP set was not feasible with respect to the Alternating Venue Requirement. Therefore, there were a few pairs of teams from each division which played at the same venue in Parts I and II.

0	-2	3	-4	5	-6	7	-8	9	-5	6	-7	10	-11	12	-13	4	-14	2	-3	3	-2	14	-4	13	-12	11	-10	7	-6	5	-9	8
-7	1	0	-3	4	-5	6	-9	7	-12	10	-6	5	-4	11	-14	13	3	-1	8	-8	1	-3	13	14	-11	4	-5	6	-10	12	-7	9
-6	7	-1	2	0	-4	5	-10	8	-7	9	-5	11	-13	6	-12	14	-2	4	1	-1	-4	2	-14	12	-6	13	-11	5	-9	7	-8	10
-5	6	-7	1	-2	3	0	-11	10	-6	5	-13	12	2	-14	8	-1	9	-3	7	-7	3	-9	1	-8	14	-2	-12	13	-5	6	-10	11
4	0	-6	7	-1	2	-3	13	-14	1	-4	3	-2	8	-7	9	-10	-11	6	-12	12	-6	11	10	-9	7	-8	2	-3	4	-1	14	-13
3	-4	5	0	-7	1	-2	12	-13	4	-1	2	-8	14	-3	7	-9	10	-5	-11	11	5	-10	9	-7	3	-14	8	-2	1	-4	13	-12
2	-3	4	-5	6	0	-1	14	-2	3	-8	1	-9	-10	5	-6	11	-12	13	-4	4	-13	12	-11	6	-5	10	9	-1	8	-3	2	-14

0	9	-10	11	-12	13	-14	1	-3	-9	7	-11	6	-5	10	-4	12	-13	14	-2	2	-14	13	-12	4	-10	5	-6	11	-7	9	3	-1
14	-8	0	10	-11	12	-13	2	-1	8	-3	-10	7	-12	13	-5	6	-4	11	-14	14	-11	4	-6	5	-13	12	-7	10	3	-8	1	-2
13	-14	8	-9	0	11	-12	3	-4	14	-2	9	-1	7	-8	-11	5	-6	12	-13	13	-12	6	-5	11	8	-7	1	-9	2	-14	4	-3
12	-13	14	-8	9	-10	0	4	-12	13	-14	8	-3	1	-2	10	-7	5	-9	6	-6	9	-5	7	-10	2	-1	3	-8	14	-13	12	-4
-11	0	13	-14	8	-9	10	-6	11	2	-13	14	-4	9	-1	3	-8	7	-10	5	-5	10	-7	8	-3	1	-9	4	-14	13	-2	-11	6
-10	11	-12	0	14	-8	9	-5	6	-11	12	4	-14	3	-9	1	-2	8	-7	10	-10	7	-8	2	-1	9	-3	14	-4	-12	11	-6	5
-9	10	-11	12	-13	0	8	-7	5	-10	11	-12	13	-6	4	2	-3	1	-8	9	-9	8	-1	3	-2	-4	6	-13	12	-11	10	-5	7

Table 9: A possible tournament template for Elitserien.

4 Assigning Teams to Numbers

The final step of the scheduling process is to assign actual teams to the numbers in the template. This assignment allows one to account for a number of factors in a priority order agreed by the Swedish Handball Association and representatives from the Elitserien clubs. Such an assignment of teams to the template in Table 9 was used to construct the 2013-14 Elitserien season schedule.

After assigning teams which share the same home venue to complementary home-away patterns, the next most important consideration when assigning teams to numbers is stadium and referee availability. Luckily for Elitserien, such constraints are rarely “hard” since each period of play where teams are scheduled to meet spans multiple days.

For example, if two teams are scheduled to meet in period 7, this means that the teams can play on one of several possible game dates (e.g., Friday, Saturday, or Sunday). On the rare occasion that a venue is occupied every day in a period or no team of referees will be available during a period, it is possible to ensure that the relevant teams are not assigned to certain numbers in the template. More often than not, teams have “soft” preferences (collected by the league before the season) where they would prefer not to play at home.

Next in priority, the league wishes to support any program or initiative from the clubs that raises the visibility of handball. This includes the preference of a club to play home in a certain period to inaugurate a new stadium or in connection with an important local event (e.g., a large youth tournament in the area). This also encompasses the desire to organize derby games. In addition to standard derbies, the Elitserien often accommodates events such as the “Battle of Scania,” where the four Elitserien teams from the region of Scania play each other on the same day.

The last priority concerns placing certain “desired games” throughout the schedule. On the one hand, there is a wish to spread interesting games over the season, to make sure that there are good games to air on TV each week. On the other hand, there is also a wish to schedule many games among the teams that are likely to fight for playoff positions to occur late in the season, so that the league remains undecided for as long as possible. By a similar argument, games between teams that are likely to fight against relegation should also be scheduled late in the season.

Explicitly addressing all of these concerns can be somewhat cumbersome. For the interested reader, we formally declare the model which we use to assign teams to numbers in Appendix B.

5 Conclusion

In this paper, we analyzed the situation where a league augments a traditional DRRT schedule by forming two divisions of teams, each of which hold an SRRT to start the season. This asymmetry (pairs of teams play three times if they are in the same division, twice otherwise) makes constructing feasible schedules an interesting problem. We highlighted the concerns of Elitserien, which we consider to be general enough to apply to many other leagues, and enumerated the number of HAP sets which satisfy these concerns. We next constructed necessary conditions for a HAP set to satisfy the Alternating Venue Requirement; this allowed us to remove many unschedulable HAP sets from the search space. After constructing a schedule template, we finally highlighted the principles used to assign teams to the template to form the 2013-14 Elitserien schedule.

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A From HAP set to Template

To attempt assign numbers to a given HAP set, we can find a feasible point satisfying the following constraints. Let N be the set of numbers to be assigned to the HAP set, let N_1 (resp. N_2) be the first (resp. second) half of elements in N , and let P be the set of periods within the HAP set divided into Parts I, II, and, III (represented by P_1 , P_2 , and P_3).

- **Variables:**

1. A variable to decide who hosts whom in a given period.

$$x_{ijp} = \begin{cases} 1 : & \text{if team } i \text{ hosts team } j \text{ in period } p, \\ 0 : & \text{otherwise,} \end{cases} \quad \forall i, j, \in N, p \in P.$$

- **Parameters:**

1. A parameter denoting whether the HAP set calls for a home, away, or bye game.

$$A_{ip} = \begin{cases} 1 & \text{if team } i \text{ plays at home in period } p, \\ 0 & \text{if team } i \text{ has a bye in period } p, \\ -1 & \text{if team } i \text{ plays away in period } p \end{cases} \quad \forall i \in N, p \in P$$

• **Constraints:**

1. Ensure that no team plays during a bye period.

$$\sum_j x_{ijp} + \sum_j x_{jip} = 0 \quad \forall i \in N, p \in P \text{ s.t. } A_{np} = 0.$$

2. Ensure that no team is home during an away game or away during a home game.

$$\begin{aligned} \sum_j x_{ijp} &= 0 & \forall i \in N, p \in P \text{ s.t. } A_{np} = -1. \\ \sum_j x_{ijp} &= 1 & \forall i \in N, p \in P \text{ s.t. } A_{np} = 1. \end{aligned}$$

3. Ensure at most one game per round.

$$\sum_j x_{ijp} + \sum_j x_{jip} \leq 1 \quad \forall i \in N, p \in P.$$

4. Ensure numbers in N_1 meet in Part I (and similarly for numbers in N_2).

$$\begin{aligned} \sum_{p \in P_1} (x_{ijp} + x_{jip}) &= 1 & \forall i, j \in N_1 \text{ s.t. } i \neq j. \\ \sum_{p \in P_1} (x_{ijp} + x_{jip}) &= 1 & \forall i, j \in N_2 \text{ s.t. } i \neq j. \end{aligned}$$

5. Ensure that each number hosts each other number in Parts II or III.

$$\sum_{p \in P_2 \cup P_3} x_{ijp} = 1 \quad \forall i, j \in N, i \neq j$$

6. Ensure numbers in N_1 (resp. N_2) meet Home-Away or Away-Home in periods P_1 and P_2 .

$$\begin{aligned} \sum_{p \in P_1 \cup P_2} x_{ijp} &= 1 & \forall i, j \in N_1, i \neq j \\ \sum_{p \in P_1 \cup P_2} x_{ijp} &= 1 & \forall i, j \in N_2, i \neq j \end{aligned}$$

7. Ensure a Part III mirrors Part II.

$$x_{ij8} = x_{ji33}, \quad x_{ij9} = x_{ji32}, \quad \dots, \quad x_{ij20} = x_{ji21} \quad \forall i, j \in N$$

B From Template to Schedule

To assign teams to numbers in the template, we can form an IP model of small enough size to be easily solved for any size league (at most, a few hundred binary variables). Let T be the set of teams (indexed by t), let P be the set of periods (indexed by p), and let N be the set of numbers in the template (indexed by n). Let T_1 (T_2) be the first (second) division of teams and let N_1 (N_2) be the first (second) group of numbers, each containing 3 pairs of complementary teams. Lastly, introduce sets D_p and M_p for all periods p where entires in D_p are pairs of teams that are desired to meet in period p and M_p contains pairs of numbers that meet in period p .

- **Variables:**

1. A variable to determine which number each team is assigned to.

$$x_{tn} = \begin{cases} 1 : & \text{if team } t \text{ is assigned to number } n, \\ 0 : & \text{otherwise,} \end{cases} \quad \forall t \in T, n \in N.$$

2. A variable to determine which group of numbers each division is assigned to.

$$\Delta = \begin{cases} 1 : & \text{if division } T_1 \text{ is assigned to the group of numbers } N_1, \\ 0 : & \text{otherwise.} \end{cases}$$

3. A variable indicating if a desired game occurs during a given period.

$$\delta_{(t_1, t_2), p, (n_1, n_2)} = \begin{cases} 1 : & \text{if teams } (t_1, t_2) \text{ assigned to } (n_1, n_2) \\ & \text{play a desired match in period } p, \\ 0 : & \text{otherwise,} \end{cases} \quad \begin{matrix} \forall (t_1, t_2) \in D_p, \\ \forall p \in P, (n_1, n_2) \in M_p. \end{matrix}$$

- **Parameters:**

1. A parameter denoting hard venue unavailabilities.

$$A_{tp} = \begin{cases} 0 : & \text{if venue } t \text{ is unavailable throughout period } p, \\ 1 : & \text{otherwise,} \end{cases} \quad \forall t \in T, p \in P.$$

2. A parameter denoting soft venue unavailabilities.

$$S_{tp} = \begin{cases} 1 : & \text{if venue } t \text{ would prefer not to host during period } p, \\ 0 : & \text{otherwise,} \end{cases} \quad \forall t \in T, p \in P.$$

3. A numerical value for HAP set entries.

$$H_{np} = \begin{cases} 1 : & \text{if number } n \text{ plays at home during period } p, \\ 0 : & \text{if number } n \text{ has a bye during period } p, \\ -1 : & \text{if number } n \text{ plays away during period } p, \end{cases} \quad \forall n \in N, p \in P.$$

4. A parameter to count the number of soft violations.

$$V_{tn} = \sum_p (H_{np} S_{tp})^+ \quad \forall t \in T, \forall n \in N.$$

5. Define $\alpha \in (0, 1)$ as the trade-off between minimizing soft conflicts and maximizing desired games. In practice, the IP can be solved for a variety of values of α to provide a few schedules. The league can then be made aware of these options to decide the relative merit of such choices. For example, is 10 desired games worth 1 additional soft conflict.

- **Constraints:**

1. Ensure each number is assigned a team.

$$\sum_t x_{tn} = 1 \quad \forall n \in N.$$

2. Ensure each team is assigned a number.

$$\sum_n x_{tn} = 1 \quad \forall t \in T.$$

3. Ensure teams in T_1 are in the same subgroup of numbers.

$$\sum_{t \in T_1} \sum_{n \in N} x_{tn} = \Delta |N_1|.$$

4. Ensure hard venue unavailabilities are not violated.

$$\sum_n x_{tn} H_{np} \leq A_{tp} \quad \forall p \in P, \forall t \in T.$$

5. Ensure $\delta_{(t_1, t_2), p, (n_1, n_2)}$ can only be 1 if a desired matchup occurs between teams t_1 and t_2 in period p .

$$x_{t_1 n_1} + x_{t_2 n_2} + x_{t_1 n_2} + x_{t_2 n_1} \geq 2\delta_{(t_1, t_2), p, (n_1, n_2)} \quad \forall (t_1, t_2) \in D_p, p \in P, (n_1, n_2) \in M_p.$$

• **Objective Function:**

$$\underset{x}{\text{minimize}} (1 - \alpha) \sum_t \sum_n x_{tn} V_{tn} - \alpha \sum_p \sum_{(t_1, t_2) \in D_p} \sum_{(n_1, n_2) \in M_p} \delta_{(t_1, t_2), p, (n_1, n_2)}.$$

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